Eton College King's Scholarship Examination 2010

MATHEMATICS B (One and a half hours)

Answer as many questions as you can. Each of the ten questions carries ten marks. Show all your working. Calculators are not allowed.

1. (a) Evaluate $\dfrac{2\frac{1}{2}-2\frac{1}{4}\div 1\frac{1}{3}}{2\frac{1}{7}\times 3\frac{1}{2}}$ exactly.

   (b) If x horses can eat through y bags of feed in z days, how many days would u bags of feed last with v similar horses eating at the same rate?

   (c) An orchard has r trees each with s leaves. If half of the trees lose half of their leaves, and a third of the trees lose a third of their leaves, how many leaves will be left on the trees?

2. Given that $12345\times 6789 = 83810205$:

   (a) Explain briefly why $12.345\times 6789 = 83810.205$.

   Hence evaluate the following exactly:

   (b) 1.2345×67.89

   (c) $\dfrac{838.10205}{0.012345}$

   (d) $\dfrac{0.0012345\times 67.89}{83.810205}$

   (e) $\dfrac{1.2345^2}{838.10205}\times\dfrac{6.789}{0.012345}$.

3. (a) Expand and simplify $(a+b)(a-b)$.

   (b) Hence factorise the following:

   (i) a^2-b^2 (ii) $a^2b^2-c^2d^2$

   (c) (i) Combine the simultaneous equations $\begin{aligned}2x+y&=31\\(2x)^2-y^2&=31\end{aligned}$ to show that $2x-y=1$.

   (ii) Hence solve the simultaneous equations $2x+y=31$, $(2x)^2-y^2=31$ for x and y.

   (d) Solve the simultaneous equations $3x+4y=44$, $9x^2-16y^2=176$.

4. (a) Solve the equation $\frac{x}{3} - \frac{1}{2}(5x+1) = \frac{2}{3} - x$.

(b) In ten years' time Tom will be twice as old as Sally. Eight years ago, Tom was 5 times older than Sally. Let Tom's current age, in years, be x and Sally's current age, in years, be y.

(i) Write down and simplify two different equations connecting their ages.

(ii) Solve these equations simultaneously to find Tom and Sally's current ages.

5. In diagram 1, four points A, B, C, D lie on the circle with centre O. T is a point on the line passing through D and C. The radii OA, OB, OC and OD are drawn and four angles are marked.

(a) Prove that $p + q + x + y = 180^\circ$.

(b) (i) Find angle BCT in terms of the angles marked.

(ii) Deduce that angle BAD = angle BCT.

(c) Use your answer to part (b) to show that in diagram 2, XY is parallel to UV.

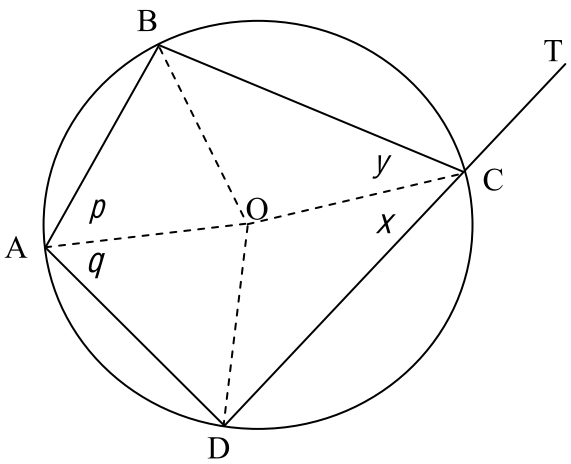


Diagram 1

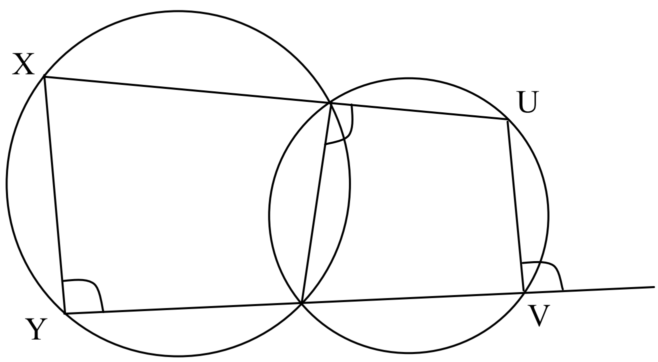
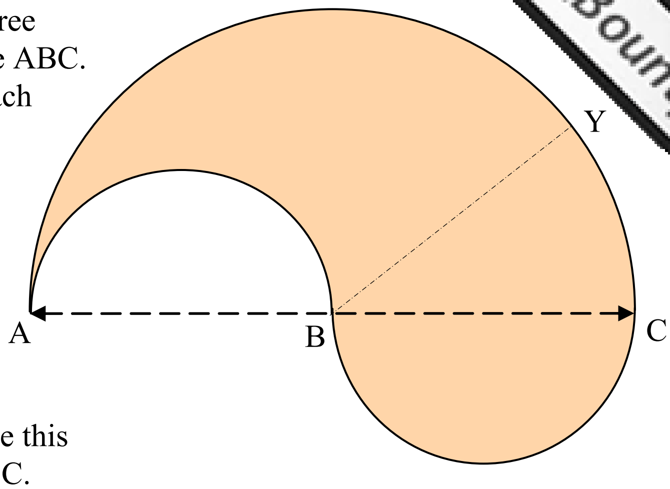


Diagram 2

6. The shaded shape shown is formed from three semi-circles whose diameters lie on the line ABC. The larger semi-circle has radius $2R$ and each of the smaller semi-circles has radius R.



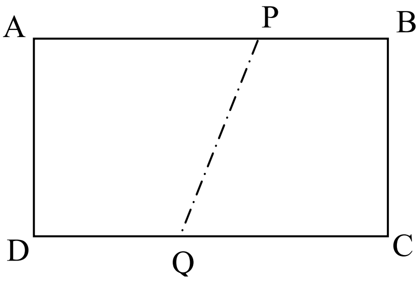
(a) Calculate the exact area of the shape.

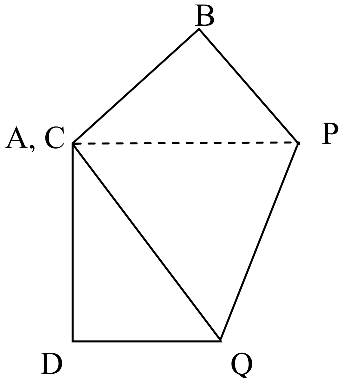
(b) Calculate the exact perimeter of the shape.

(c) A straight line BY is drawn through B so that the area either side of BY is divided into two parts of equal value. Find the exact value of the acute angle this dividing line makes with the radius BC.

7. (a) A right-angled triangle has a hypotenuse of 125cm and a short side of 35cm. Calculate the exact length of the final side.

(b) A rectangular piece of paper 15cm by 20cm is given a single fold so that one pair of opposite corners coincide. The piece of paper is drawn in the unfolded and folded positions, with the crease PQ marked. Corner C is folded on to corner A.





(i) Let x= DQ. By considering the triangle ADQ(C), in the folded position, form and solve an equation to find x.

(ii) Explain why length BP = length DQ.

(iii) Similarly, by constructing a right-angled triangle with PQ as the hypotenuse, on the unfolded paper, form equation for the crease length PQ.

(iv) Show that PQ = $\frac{75}{4}$.

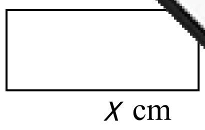
8. (a) Find the largest perfect square that is a factor of 2×3^4.

(b) A ballroom dance floor measuring 1200cm wide by 2880cm long is to be fitted exactly with rectangular wooden planks measuring n by n^2, where n is a whole number of centimetres.

(i) Write the dimensions 1200cm and 2880 cm in prime factorised form.

(ii) By factorising 1200 as $w\times n$ and 2880 as $l\times n^2$, find the least number of planks that can be used to cover the floor exactly.

9. (a) A rectangle with dimensions x cm by y cm is shown.

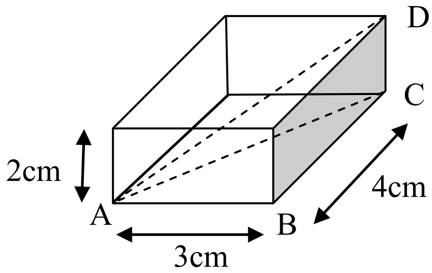


(i) Find the perimeter and diagonal in terms of x and y.

(ii) If the perimeter is 20 and diagonal is 8, show that
$x+y=10$ and $x^2+y^2=64$.

(iii) Using the expansion $(x+y)^2=x^2+y^2+2xy$, find the exact area of the rectangle.

(b) A solid cuboid has dimensions 2cm by 3cm by 4cm as shown. Four vertices, A, B, C and D have been labelled.



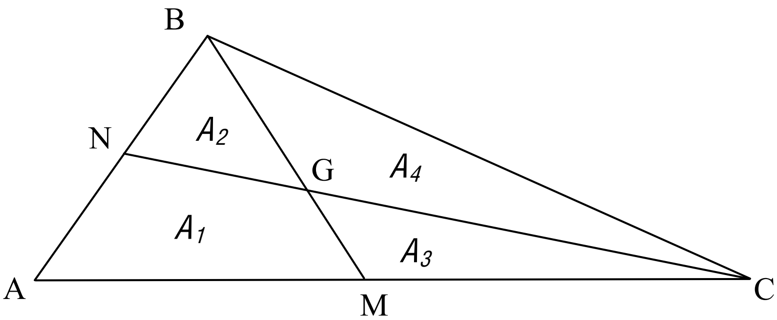
(i) Calculate the exact length of the diagonal AC of the rectangular base.

(ii) Given that angle $ACD=90^\circ$, show that the diagonal AD = $\sqrt{29}$ cm.

(c) Expand and simplify $(a+b+c)^2$.

(d) A cuboid has dimensions a cm by b cm by c cm. If its diagonal is 16cm and the perimeter is 96cm, calculate the exact surface area of the cuboid.

10.



In triangle ABC, N is at the midpoint of side AB and M is the midpoint of side AC and G is at the point where BM and CN cross.

(a) Explain why triangles ABM & CBM have equal areas, and triangles CAN & CBN have equal areas.

(b) (i) Using part (a) and considering the areas A_1 to A_4 marked, form two equations for A_1 to A_4.

(ii) Hence prove that triangles BGN & CGM have equal areas; that is $A_2=A_3$.

You are also given that the area of quadrilateral ANGM is the same as that of triangle BCG; that is $A_1=A_4$.

(c) By including line AG and first proving the result for quadrilateral ANGM, prove that triangle BCG has an area which is $\frac{1}{3}$ of the area of triangle ABC.

(d) What is the value of the fraction (area of triangle ANM)/(area of triangle BCG)?

(End of paper)