Introduction

- Fluid is a matter in a state which can flow. Liquids, gases, molten metal and tar are examples of fluids.

- Fluid mechanics is studied in two parts:

  (i) Fluid statics - Study of the forces and pressures acting on stationary fluid. Pascal's law and Archimedes' principle and surface tension are discussed in fluid statics.

  (ii) Fluid dynamics - Study of motion of fluid and properties related to it as a result of forces acting on fluid. Bernoulli's theorem and its applications and viscosity of fluid are discussed here. Fluid dynamics is studied in two sections: Hydrodynamics and Aerodynamics.

10.1 Pressure

Pressure is the force acting on a surface per unit area in a direction perpendicular to it. It is a scalar quantity and its SI unit is N/m^2 named pascal (Pa) in honour of the French scientist Blasé Pascal. Its dimensional formula is $M^1 L^{-1} T^{-2}$. Thus,

$$\text{Pressure, } P\ (Pa) = \frac{\text{Force, } F\ (N)}{\text{Area, } A\ (m^2)}.$$

A bigger unit of pressure is 'bar'. 1 bar = 10^5 Pa.

1 atmosphere pressure (atm) = 1.013×10^5 Pa or N/m^2 = 760 mm (76 cm) of Hg column.

Density:

Density is the ratio of mass to volume of an object. It is a scalar quantity and its S I unit is kg/m^3.

Liquids are almost incompressible. Hence, the density of a liquid remains almost constant at a given temperature for small change in the value of pressure. Gases are compressible. Hence, the volume of gas decreases and density increases with increase of pressure.

Relative density / Specific density / Specific gravity:

"Relative density also known as specific density or specific gravity of a given substance is the ratio of its density to the density of water at 277 K (i.e., 4° C)."

It is a dimensionless quantity and hence does not have a unit. Also,

$$\text{Relative (specific) density of an object} = \frac{\text{Mass of an object}}{\text{Mass of the same volume of water at 277 K}}$$

10.2 Pascal's Law

"A change in pressure applied to an enclosed (incompressible) fluid is transmitted undiminished to every point of the fluid and the walls of the containing vessel." This statement is known as Pascal's law.

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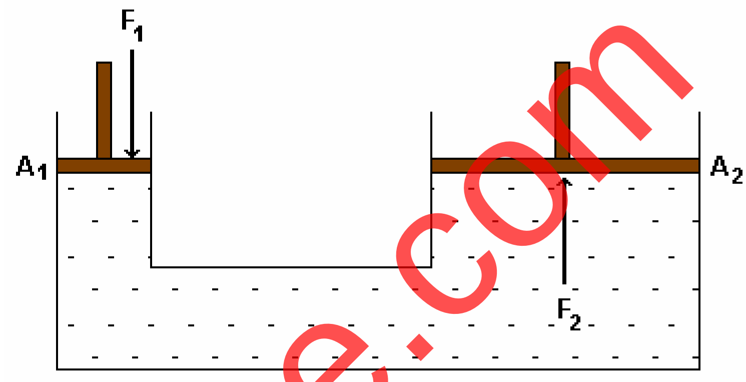
Pascal's law is also given as **"If the effect of gravitation is neglected, the pressure at every point in an incompressible liquid, in equilibrium, is the same."**

Applications of Pascal's Law:

The figure shows the principle of a hydraulic lift used to raise heavy loads. This device has two vertical cylinders of different diameters connected by a horizontal tube. A liquid is filled in this vessel. Air-tight pistons having cross-sectional areas A_1 and A_2 ($A_1 < A_2$) are fitted touching the liquid surface in both the cylinders. According to Pascal's law, in equilibrium, the pressure on liquid in both the arms is the same. Hence,



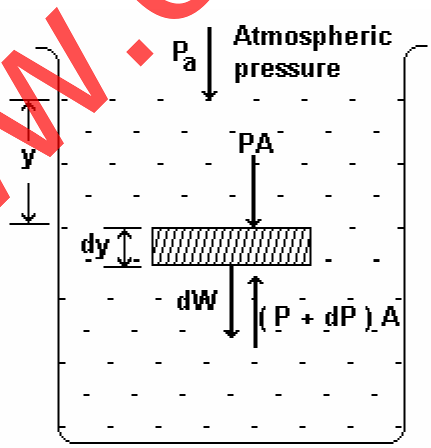
Principle of a hydraulic lift

$$\frac{F_1}{A_1} = P_1 = P_2 = \frac{F_2}{A_2} \Rightarrow F_2 = F_1\left(\frac{A_2}{A_1}\right)$$

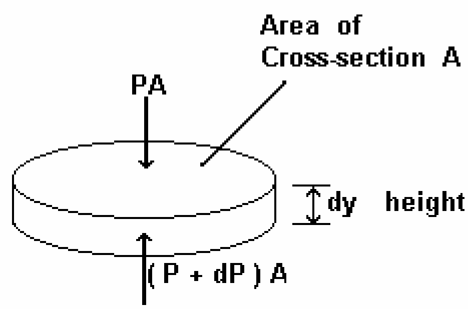
Thus, a large force, F_2, is generated using a small force, F_1, as the magnitude of F_2 is $\left(\frac{A_2}{A_1}\right)$ times the magnitude of F_1. Using Pascal's law, devices like hydraulic lift, hydraulic jack, hydraulic brake and hydraulic press are developed.

Pressure due to a fluid column:

For liquid of density ρ in a static equilibrium in a container, pressure at all points at the same depth (or in other words, at the same horizontal layer) is the same.



Pressure due to liquid column



Cylindrical volume of element

Consider an imaginary cylindrical volume element of height dy and cross-sectional area A at the depth y from the surface of liquid as shown in the figure.

The weight of liquid in this volume element is $dW = \rho g A \, dy$

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If P and $P + dP$ are the pressures on the upper and lower faces of the element, then PA and $(P + dP)A$ are the forces acting on them respectively. In equilibrium,

$$PA + dW = (P + dP)A$$

$$\therefore \quad PA + \rho g A\,dy = PA + A\,dP$$

$$\therefore \quad \frac{dP}{dy} = \rho g$$

This equation is valid for any fluid (liquid or gas). It shows that the pressure increases with increase in the depth. Here, ρg is the weight density, i.e., weight per unit volume of the fluid. Its value for water is $9800\ N/m^3$. Pressure P at the depth $y = h$ can be obtained by integration as under.

$$\int_{P_a}^{P} dP = \int_{0}^{h} \rho g\,dy$$

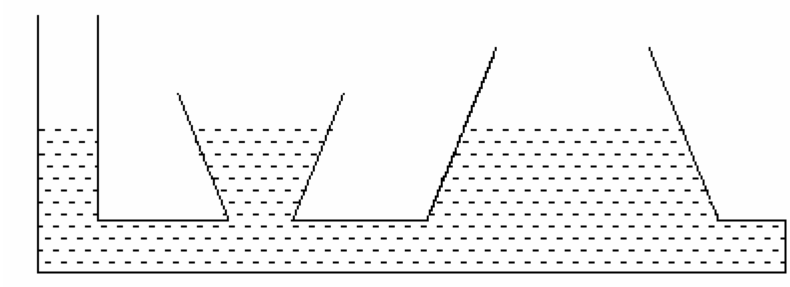
As ρ is independent of pressure and constant for liquid, the above integration gives

$$P - P_a = \rho g h \qquad \therefore \quad P = P_a + \rho g h$$

This equation is valid only for incompressible fluid, i.e., liquid and gives the pressure at depth h in a liquid of density ρ.

Here, $P\ (= P_a + \rho g h)$ is the absolute pressure whereas $P - P_a\ (= \rho g h)$ is the gauge pressure also known as the hydrostatic pressure.

The pressure at any point in a liquid does not depend on the shape or cross-sectional area of its container. This is known as hydrostatic paradox.



When liquid is filled in the containers of different shapes and sizes, joined at the bottom as shown in the figure, the height of liquid columns in all the containers is found to be the same.

10.3 Buoyancy and Archimedes' principal

<u>" When a body is partially or completely immersed in a liquid, the buoyant force acting on it is equal to the weight of the displaced liquid and it acts in the upward direction at the centre of gravity of the displaced liquid."</u>

This statement was given by Archimedes and is known as Archimedes' principle.

Buoyant force = weight of the displaced liquid (or any fluid, i.e., liquid or gas)
= decrease in the weight of the immersed body

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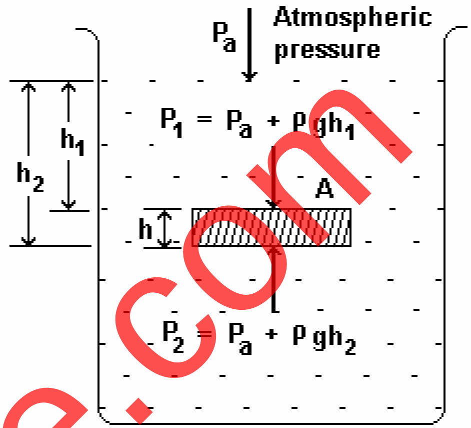
Proof of Archimedes' Principle:

Suppose a solid of height h, uniform cross-sectional area A and volume V is immersed in a liquid of density ρ. Net force on the vertical surface of the solid is zero.



Pressure due to liquid column

The pressure, P_1, on top of the solid in the downward direction and, P_2, on the bottom of the solid in the upward direction are respectively given by

$$P_1 = P_a + \rho g h_1 \quad \text{and} \quad P_2 = P_a + \rho g h_2$$

$$\therefore \; P_2 - P_1 = \rho g (h_2 - h_1) = \rho g h$$

$$(\because \; h_2 - h_1 = h)$$

Hence, the resultant (buoyant) force acting in the upward direction is

$$F_b = \rho g h \cdot A = \rho g V \qquad (\because \; h \cdot A = V)$$

$$= \text{weight of the displaced liquid}$$

Although the above result has been proved for a symmetrical solid of uniform height and cross-sectional area, it is valid for a solid of any shape.

With the help of Archimedes' principle, specific density of solid or liquid, volume of a body of irregular shape, the constituents of the mixture and its proportion in an alloy may be known.

Law of floatation:

When a body is partially or completely immersed in a liquid, two forces act on it.
(i) Weight of the body, $W = \rho_s g V_s \downarrow$ and (ii) Buoyant force, $F_b = \rho_f g V_f \uparrow$

Here, ρ_s and V_s are the density and volume of the solid body, while ρ_f and V_f are the density and volume of the displaced liquid respectively. When the body is fully immersed in the liquid, $V_s = V_f$ (provided the solid body is not hollow or has no cavities).

If $W > F_b$, the body sinks. If $W = F_b$, the body remains in equilibrium at any depth. Submarine works on this principle.

If $W < F_b$, the body floats in liquid and if its partially immersed volume is V_p, then the buoyant force $F_b = \rho_f g V_p$ balances the weight W of the body.

$$\therefore \; W = F_b \quad \text{which implies} \quad \rho_s g V_s = \rho_f g V_p. \quad \therefore \; \frac{\rho_s}{\rho_f} = \frac{V_p}{V_s}.$$

Thus, " <u>When the weight of a body is equal to the weight of the liquid displaced by a part of the body immersed in it, the body floats on the surface of the liquid.</u>"

This statement is known as the law of floatation which is used in designing a steamer.

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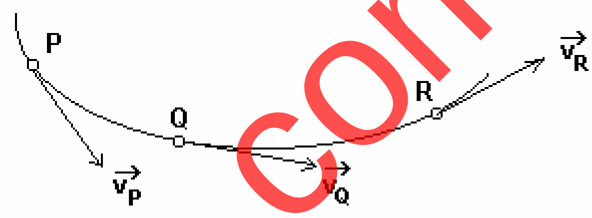
Fluid Dynamics:

10.4 Characteristics of fluid flow

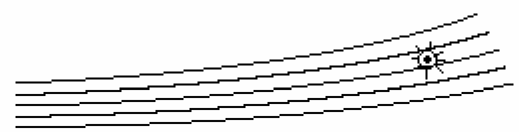
(1) Steady flow: In a steady flow of fluid, the velocity of the fluid at each point remains constant with time. Every particle of the fluid passing through a given point will have the same velocity. Let the particles of fluid, P, Q and R have velocities $\vec{v}_P$, $\vec{v}_Q$ and $\vec{v}_R$ respectively which may all be different. But these velocities do not change with time and all particles of the fluid in its flow passing through these points will have these velocities at all times. Such a condition is achieved only at low speeds, e.g., a gently flowing stream.



(2) Unsteady flow: In an unsteady flow of fluid, the velocity of the fluid at a given point keeps on changing with time as in motion of water during ebb and tide.

(3) Turbulent flow: In turbulent flow, the velocity of fluid changes erratically from point to point and from time to time as in waterfalls, breaking of the sea waves.

(4) Irrotational flow: In irrotational flow, no element of fluid has net angular velocity. A small paddle wheel placed in such a flow will move without rotating.



(5) Rotational flow: In rotational flow, an element of fluid at each point has net angular velocity about that point. A paddle wheel kept in such a flow has turbulent motion while rotating. Rotational flow includes vortex motion such as whirlpools, the air thrown out of exhaust fans, etc.

(6) Incompressible flow: In incompressible flow, the density of fluid remains constant with time everywhere. Generally, liquids and sometimes even a highly compressed gas flow incompressibly. Flow of air relative to the wings of an aeroplane flying below sonic velocity is incompressible.

(7) Compressible flow: In compressible flow, density of fluid changes with position and time.

(8) Non-viscous flow: In non-viscous flow, fluid with small co-efficient of viscosity flows readily. Normally, the flow of water is non-viscous.

(9) Viscous flow: In viscous flow, fluid having large co-efficient of viscosity cannot flow readily. Castor oil, tar have viscous flow.

10.5 Streamlines, Tube of flow and Equation of continuity

The path of motion of a fluid particle is called a line of flow. In a steady flow, velocity of each particle arriving at a point on this path remains constant with time. Hence, every particle reaching this point moves in the same direction with the same speed. However, when this particle moving on the flow line reaches a different point, its velocity may be different.

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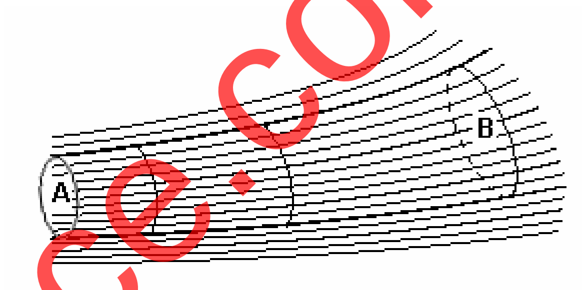
But this different velocity also remains constant with respect to time. The path so formed is called a streamline and such a flow is called a streamline flow. In unsteady flow, flow lines can be defined, but they are not streamlines as the velocity at a point on the flow line may not remain constant with time.

Streamlines do not intersect each other, because if they do then two tangents can be drawn at the point of intersection and the particle may move in the direction of any tangent which is not possible.

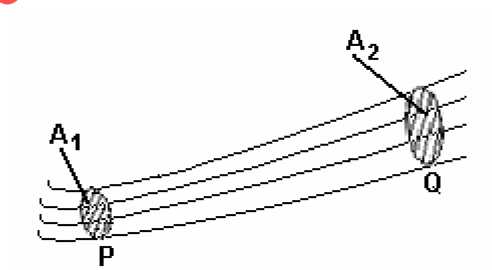
Tube of flow:



The tubular region made up of a bundle of streamlines passing through the boundary of any surface is called a tube of flow.

The tube of flow is surrounded by a wall made of streamlines. As the streamlines do not intersect, a particle of fluid cannot cross this wall. Hence the tube behaves somewhat like a pipe of the same shape.

Equation of continuity:



In a tube of flow shown in the figure, the velocity of a particle can be different at different points, but is parallel to the tube wall.

In a non-viscous flow, all particles in a given cross-section have the same velocity. Let the velocity of the fluid at cross-section P, of area A_1, and at cross-section Q, of area A_2, be v_1 and v_2 respectively. Let ρ_1 and ρ_2 represent density of the fluid at P and Q respectively. Then, as the fluid can not pass through the wall and can neither be created or destroyed, the mass flow rate (also called mass flux) at P and Q will be equal and is given by

$$\frac{dm}{dt} = \rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

This equation is known as the law of conservation of mass in fluid dynamics.

For liquids, which are almost incompressible, $\rho_1 = \rho_2$.

$\therefore$ $A_1 v_1 = A_2 v_2$ ... ... (1) which implies $Av = \text{constant}$ ... ... (2) or, $v \propto \frac{1}{A}$

Equations (1) and (2) are known as the equations of continuity in liquid flow. The product of area of cross-section, A and the velocity of the fluid, v at this cross section, i.e., Av, is known as the volume flow rate or the volume flux.

Thus, velocity of liquid is larger in narrower cross-section and vice versa. In the narrower cross-section of the tube, the streamlines are closer thus increasing the liquid velocity. Thus, widely spaced streamlines indicate regions of low speed and closely spaced streamlines indicate regions of high speed.

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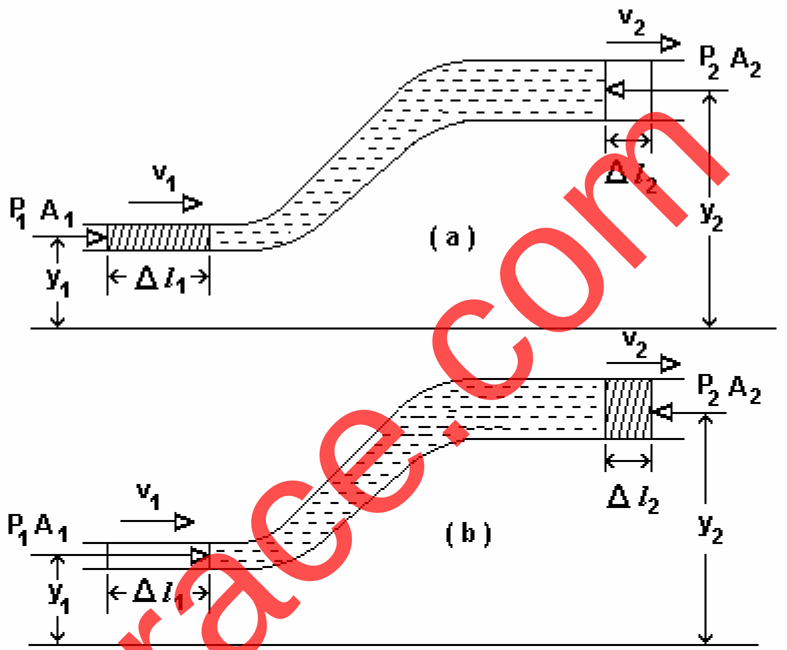
10.6 Bernoulli's equation

Bernoulli's equation is derived in fluid dynamics with the help of work-energy theorem.

The figure shows a streamline flow of a non-viscous liquid which is steady and irrotational through a hypothetical pipe or a flow tube.

One end of the pipe is horizontal at a height y_1 above some reference level and has uniform cross-section A_1 upto some length.

The pipe gradually widens and rises and becomes horizontal at the other end which is at a height y_2 from the reference level and has uniform cross-section A_2.



Derivation of Bernoulli's equation

Now consider the portion of fluid shown by shaded area as the system. Suppose the system of fluid gets displaced from the position shown in figure (a) to that in figure (b) in a small time interval.

(1) The work done on the system, W_1, due to the pressure P_1 and hence by the force, $P_1 A_1$, during displacement Δl_1 at the left end is

$$W_1 = \text{force} \times \text{displacement} = P_1 A_1 \Delta l_1$$

(2) The work done on the system, W_2, due to the pressure P_2 and hence by the force, $P_2 A_2$, during displacement Δl_2 at the left end is

W_2 = force × displacement = $- P_2 A_2 \Delta l_2$ [negative sign is because of force, $P_2 A_2$, and displacement, Δl_2, being in opposite directions.]

(3) The work done on the system, W_3, by the gravitational force, mg, is

W_3 = force × displacement = $- mg (y_2 - y_1)$ [negative sign is due to force, mg, and displacement, $(y_2 - y_1)$, being in opposite directions.]

$\therefore$ the total work done on the system,

$$W = W_1 + W_2 + W_3 = \text{change in K. E. of the system} \quad \text{[By work-energy theorem]}$$

$$= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

$$\therefore \quad P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 - mg (y_2 - y_1) = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 \quad \dots \dots \dots \quad (1)$$

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As the liquid is incompressible, the mass flow rate at both the ends must be the same.

$$\therefore \quad \rho A_1 \Delta l_1 = \rho A_2 \Delta l_2 = m = \rho V$$

$$\therefore \quad A_1 \Delta l_1 = A_2 \Delta l_2 = \frac{m}{\rho} = V \quad \dots \quad \dots \quad \dots \quad (2)$$

From equations (1) and (2),

$$P_1 \left(\frac{m}{\rho}\right) - P_2 \left(\frac{m}{\rho}\right) - mg(y_2 - y_1) = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

$$\therefore \quad P_1 - P_2 - \rho g (y_2 - y_1) = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$$

$$\therefore \quad P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$\therefore \quad P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant} \quad (\text{for every point on a streamline})$$

This equation is known as Bernoulli's equation applicable only to steady, irrotational, incompressible and non-viscous streamline flow.

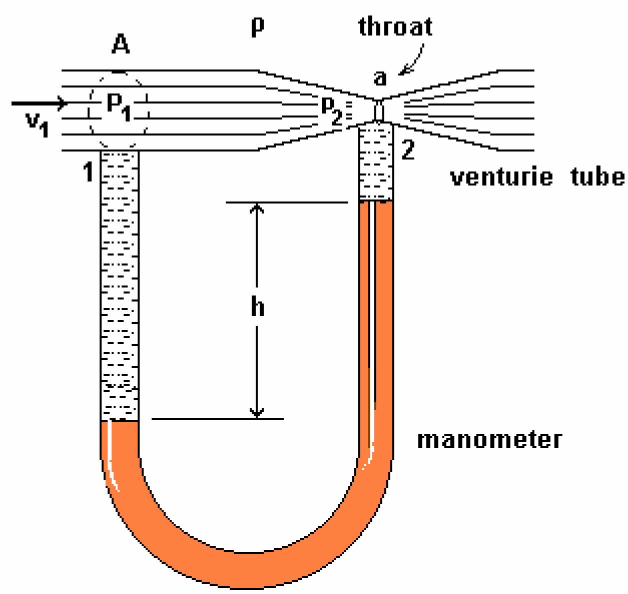
Dividing the above equation by ρg,

$$\frac{P}{\rho g} + \frac{v^2}{2g} + y = \text{constant}$$

Each term in the above equation has the dimension of length and hence every term is known as 'head'. The first term is called 'pressure head', the second 'velocity head' and the third is known as the 'elevation head'.

Applications of Bernoulli's equation

(1) Venturie meter (principle)



The figure shows the venturie meter used to measure the flow rate of liquid in a pipe. One end of a manometer is connected to the broad end of the venturie meter and the other end to the throat.

The cross-sectional area of the broad end and velocity and pressure of liquid there are A, v_1 and P_1 respectively. The cross-sectional area of the throat and velocity and pressure of liquid there are a, v_2 and P_2 respectively. The densities of the liquid flowing through the venturie meter and of the manometer liquid (mercury) are ρ and ρ' respectively.

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Using Bernoulli's theorem at points '1' and '2' which are at the same elevation,

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$\therefore \quad P_1 - P_2 = \frac{1}{2}\rho \left(v_2^2 - v_1^2 \right)$$

$$= (\rho' - \rho)\, gh \qquad \text{(from the manometer reading)}$$

$$\therefore \quad (\rho' - \rho)\, gh = \frac{1}{2}\rho \left(v_2^2 - v_1^2 \right)$$

$$= \frac{1}{2}\rho \left[\frac{A^2}{a^2} \cdot v_1^2 - v_1^2 \right] \qquad \text{(using the continuity equation, } Av_1 = av_2 \text{)}$$

$$= \frac{1}{2}\rho v_1^2 \left[\frac{A^2}{a^2} - 1 \right]$$

$$\therefore \quad v_1 = a \sqrt{\frac{2(\rho' - \rho)\, gh}{\rho\,(A^2 - a^2)}}$$

Using this value of v_1, the volume flow rate, $v_1 A$, can be calculated.

The velocity of liquid is low at the broad end and high at the throat of the venturie tube.

In an automobile carburetor, air flows through a venturie tube reducing its pressure at the throat where the fuel is sucked in and mixed with proper amount of air for satisfactory combustion. In a spray pump, the piston pushes the air which comes out of a hole with high velocity. This reduces the pressure near the hole and sucks the liquid to be sprayed through a capillary.

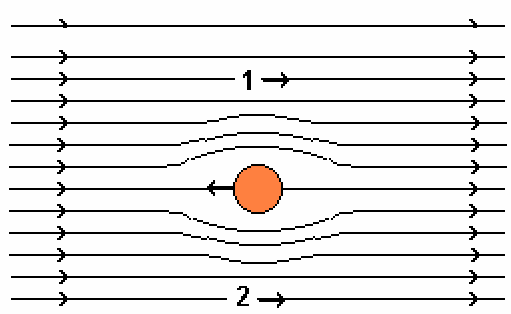
(2) The change in pressure with depth:

The expression for hydrostatic pressure in a stationary liquid can be obtained as a special case of Bernoulli's equation taking $v_1 = v_2 = 0$. Taking point 1 at a depth of h from the surface of liquid and point 2 on the surface where pressure is atmospheric pressure, P_a, and noting that $h = y_2 - y_1$, we get $P_1 = P_a + \rho gh$.

(3) Dynamic lift and Swing bowling:

A body in liquid experiences buoyant force which is known as static lift. When the body is in motion with respect to a fluid, it also experiences another force known as dynamic lift.

The figure shows a ball moving horizontally in air without spinning. The velocity of air at points 1 and 2 is the same. So, by Bernoulli's equation, the pressures at 1 and 2 would also be the same. Hence the dynamic lift on the ball is zero.



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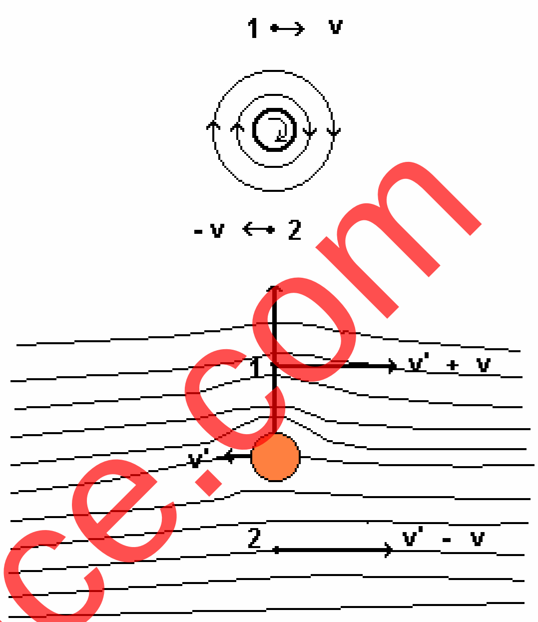
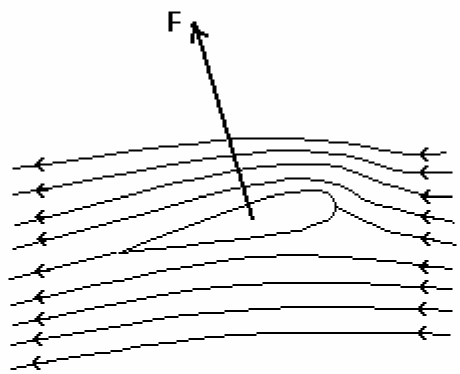
Now consider a ball which is moving horizontally with velocity v’ and also spinning about the horizontal axis passing through its centre and perpendicular to the plane of the figure with its peripheral velocity v. When the surface of the ball is rough, it drags some air along with it. This results in increase in velocity of streamline flow of air at point 1 to v’ + v and decrease at point 2 to v’ - v. Hence pressure of air at point 1 is less than the pressure at point 2 which gives dynamic lift to the ball in the upwards direction.



Again, if the ball was spun about a vertical axis lying in the plane of the figure making it rotate in the horizontal plane, it may deviate towards the off or leg stump. This is the reason for swing of the ball in fast bowling.

(4) Aerofoils:

The figure shows an aerofoil which is a solid shaped to provide an upward vertical force when it moves horizontally through air and hence it can float in air.

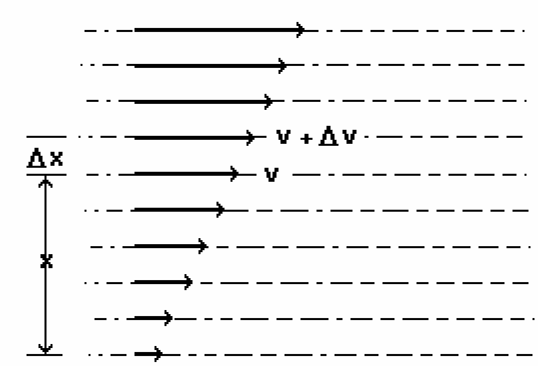


The aeroplane wings are shaped like an aerofoil. Air has streamline flow about the wings when the angle between the wing and the direction of motion, called an angle of attack, is small. The crowded streamlines above the wings results in high velocity and low pressure, while the sparse streamlines below the wings indicate low velocity and high pressure. This difference in pressure results in dynamic upward lift which helps the aeroplane in motion to float in air.

10.7 Viscosity

An appropriate external force is required to maintain the flow of fluids. A property of fluid responsible for this is called viscosity of the fluid.

Consider the steady flow of liquid on some horizontal stationary surface as shown in the figure. The layer of the liquid in contact with the surface remain stuck to it due to adhesive force and has zero velocity. The velocity of layer gradually increases on moving upwards from the surface and is the largest at the top. In a steady flow, different layers slide over each other without getting mixed which is called laminar flow.



Laminar flow

In a laminar flow, the relative velocity between the consecutive layers of fluid results in tangential force at the surfaces of the layers known as viscous force and the property of the fluid causing it is known as viscosity. To maintain the flow, some minimum external force has to be applied to balance the viscous force.

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Velocity gradient: In a laminar flow, the difference in velocity between two layers of liquid per unit perpendicular distance, in the direction perpendicular to the direction of flow, is called velocity gradient.

As shown in the figure (previous page), the velocity of the layers at distances x and $x + \Delta x$ from the surface in the direction perpendicular to the layers as well as perpendicular to the flow, are v and $v + \Delta v$ respectively.

$\therefore$ average velocity gradient of the layers lying within distance $\Delta x = \dfrac{\Delta v}{\Delta x}$

and for a given layer, velocity gradient $= \lim\limits_{\Delta x \to 0} \dfrac{\Delta v}{\Delta x} = \dfrac{dv}{dx}$

The unit of velocity gradient is s^{-1}.

Co-efficient of viscosity:

The viscous force between two adjacent layers of a laminar flow at a given temperature,

$F \propto$ area (A) between the two adjacent layers and $F \propto \dfrac{dv}{dx}$

$\therefore \quad F \propto A \dfrac{dv}{dx}$ or, $F = \eta A \dfrac{dv}{dx}$,

where η is a constant known as the co-efficient of viscosity of the fluid. Its magnitude depends on the type of the fluid and its temperature.

It's C. G. S. unit is dyne-s-cm^{-2} and is called poise and MKS unit is N-s-m^{-2}. Its dimensional formula is $M^1 L^{-1} T^{-1}$.

Taking A = 1 unit and $\dfrac{dv}{dx} = 1$ unit in the above equation, $\eta = F$.

Thus, the co-efficient of viscosity can also be defined as the viscous force acting per unit surface area of contact and per unit velocity gradient between two adjacent layers in a laminar flow of a fluid.

Note that <u>the co-efficient of viscosity of liquids decrease with increase in temperature, while that of gases increase with the increase in temperature.</u> At normal temperature, blood has higher co-efficient of viscosity than that of water and the relative co-efficient of viscosity of blood, $\eta_{blood} / \eta_{water}$ in the temperature range of 0° C to 37° C remains almost constant.

10.8 Stokes' Law

The resistive force (viscous force) on a small, smooth, spherical, solid body of radius r, moving with velocity v through a viscous medium, of large dimensions, having co-efficient of viscosity η is given by

$F_v = 6 \pi \eta r v$.

This equation is called Stokes' Law which can be verified using dimensional analysis.

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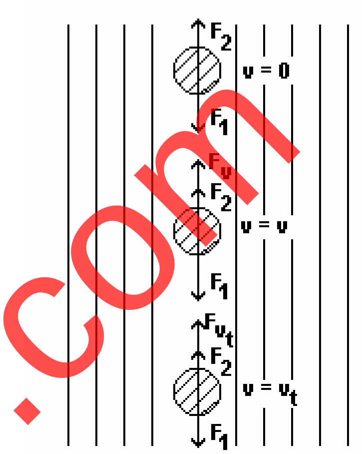
Free fall of a sphere in a fluid and its terminal velocity:

Suppose a small, smooth, solid sphere of radius r of material having density ρ falls freely in a fluid of density ρ_o ($< \rho$) and co-efficient of viscosity η as shown in the figure.



The figure shows the forces acting on the sphere at three different instants. The forces acting on the sphere at an instant when its velocity is v are:

(1) its weight, $F_1 = mg = \frac{4}{3}\pi r^3 \rho g$ (downwards),

(2) the buoyant force by the fluid,

$$F_2 = m_o g = \frac{4}{3}\pi r^3 \rho_o g \quad \text{(upwards)},$$

where m_o = mass of fluid displaced, and

(3) the resistive force opposing the motion as given by Stokes' law,

$$F_v = 6\pi\eta r v \qquad \text{(upwards)}$$

$\therefore$ the resultant force acting on the sphere is

$$F = F_1 - F_2 - F_v = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \rho_o g - 6\pi\eta r v \quad \dots \quad \dots \quad \dots \quad (1)$$

This is the equation of motion of the sphere in the viscous medium.

$v = 0$ at $t = 0$ and hence $F_v = 0$.

$$\therefore \quad F = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \rho_o g = \frac{4}{3}\pi r^3 g (\rho - \rho_o) \quad \dots \quad \dots \quad \dots \quad (2)$$

If the acceleration of the sphere is a_0 at $t = 0$,

$$F = m a_0 = \frac{4}{3}\pi r^3 \rho a_o \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (3)$$

Comparing equations (2) and (3),

$$\frac{4}{3}\pi r^3 \rho a_o = \frac{4}{3}\pi r^3 g (\rho - \rho_o) \qquad \therefore \quad a_o = \frac{(\rho - \rho_o)}{\rho} \cdot g \quad \dots \quad \dots \quad (4)$$

Initially, before the sphere starts its motion in the fluid, its velocity is zero and there is no resistive force acting on it. It accelerates downwards and its velocity increases. The upward resistive force on the sphere also increases with the velocity. This reduces the net downward force on the sphere. As a result, the velocity of the sphere increases while its acceleration decreases. At a definite high velocity, the resultant force on the sphere becomes zero and thereafter the sphere continues to move with the uniform velocity, v_t, known as the terminal velocity. At $v = v_t$, the net downward force becomes zero.

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$$\therefore \quad F = F_1 - F_2 - F_v = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \rho_o g - 6\pi\eta r v = 0$$

$$\therefore \quad v_t = \frac{2}{9}\frac{r^2 g}{\eta}\left(\rho - \rho_o\right)$$

This equation is used to determine the coefficient of viscosity of the fluid experimentally.

A bubble of air in a liquid rises up due to lower density of air as compared to water and reaches an upward terminal velocity.

10.9 Reynolds number

The type of fluid flow in a given pipe depends on

(i) the coefficient of viscosity (η) of the fluid,

(ii) the density (ρ) of the fluid,

(iii) average velocity (v) of the fluid and

(iv) the diameter (D) of the fluid

and is given by the magnitude of the dimensionless number called Reynolds number, N_R formed by the combination of these four physical quantities as under.

Reynolds number, $N_R = \dfrac{\rho v D}{\eta}$

For $N_R < 2000$, the flow is streamline,

$N_R > 3000$, the flow is turbulent and for

$2000 < N_R < 3000$, the flow is unstable and its type keeps on changing.

If $\eta = 0$ (that is fluid is non-viscous), N_R tends to infinity. Hence the flow of non-viscous fluids can never be streamline, for all values of $v > 0$.

10.10 Surface tension

- Cohesive force - inter-molecular attractive force between molecules of the same matter.
- Adhesive force - attractive force between molecules of different matters.
- Range of inter-molecular force (r_o) - the maximum distance upto which the attractive force between two molecules is significant.
- Sphere of molecular action - an imaginary sphere of radius r_o with molecule at the centre.

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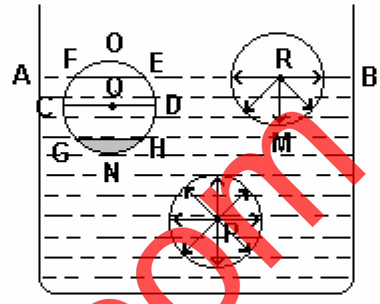
Consider molecules P, Q and R of a liquid along with their spheres of molecular action as shown in the figure.



Spheres of inter-molecular action

The sphere of action of molecule P is completely immersed in the liquid. As P is acted upon by equal forces of attraction from all sides, the resultant force on it is zero keeping it in equilibrium. All such molecules at a depth more than r_o will be in similar situation.

Molecule Q is at a depth less than r_o. Part FOEF of its sphere of action is outside the liquid and contains the molecules of air and liquid vapour. As the density of molecules in this part is less than that in GNHG part which contains only liquid molecules, the resultant force due to these two parts is in the downward direction. Molecules in CDHG and CDEF parts being equal, the resultant force on Q due to molecules in these regions is zero. Thus, there is a net downward force on molecule Q.

A layer of thickness r_o below the free surface AB of the liquid is called the <u>surface of liquid.</u> As we move upwards in this layer, the downward resultant force keeps on increasing and becomes maximum on the molecules like R on the free surface AB. Thus the molecules in this surface layer will have tendency to go inside the body of the liquid. As some of the molecules go down, density below the surface of liquid increases and it decreases on moving upwards in the surface. This results in decreased inter-molecular distance below the surface and more within the surface. As a result, the molecules lying in it experience force of tension parallel to the surface known as surface tension.

<u>"The force exerted by the molecules lying on one side of an imaginary line of unit length, on the molecules lying on the other side of the line, which is perpendicular to the line and parallel to the surface is defined as the surface tension (T) of the liquid."</u>

The S I unit of surface tension is Nm^{-1} and its dimensional formula is $M^1 L^0 T^{-2}$.

The force of surface tension is parallel to the surface, while the resultant cohesive force on the molecules is perpendicular to the surface and towards inside the liquid.

There are molecules on either side of an imaginary line in the middle of the surface exerting forces which are equal in magnitude and opposite in direction. Hence, the effect of surface tension is not felt. At the edge of the surface, there are molecules only on one side and hence the surface tension manifests here parallel to the surface and perpendicular to the surface towards the middle of the surface.

Surface tension in context of potential energy:

Work has to be done to bring a molecule from a point below the surface to a point on top of the surface against the downward force acting on it. Hence, when such a molecule reaches the surface it acquires potential energy. Now a system has a tendency to remain in the state of minimum potential energy. Thus molecules in the surface of a liquid has a tendency to reduce its potential energy. So the surface of the liquid has a tendency to contract and minimize its area.

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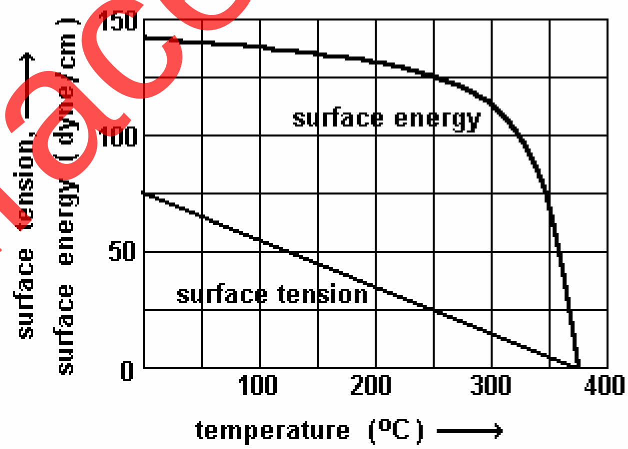
The molecules reaching the surface do not occupy place between the molecules already present in the surface, but generate a new surface. This means that the surface gets expanded. The whole surface of a liquid can be considered to have been generated this way. Thus, the molecules in the surface of a liquid possess potential energy equal to the work done on them in bringing them to the surface.

Hence surface tension can also be defined as "the potential energy stored in the surface of the liquid per unit area." By this definition, its unit is Jm^{-2} which is the same as Nm^{-1}.

Surface tension of a liquid depends on the type of the liquid and its temperature. It decreases with temperature and becomes zero at a critical temperature. It also depends upon the type of the medium which the liquid is in contact with.

Surface - energy:

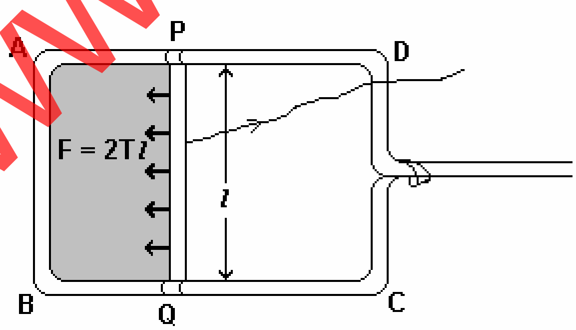
Surface tension, T, is the amount of energy to be given to the surface to increase its area by unity at constant temperature. But the temperature of the surface decreases as it expands. Hence, heat has to be supplied to the surface from outside to maintain its temperature.



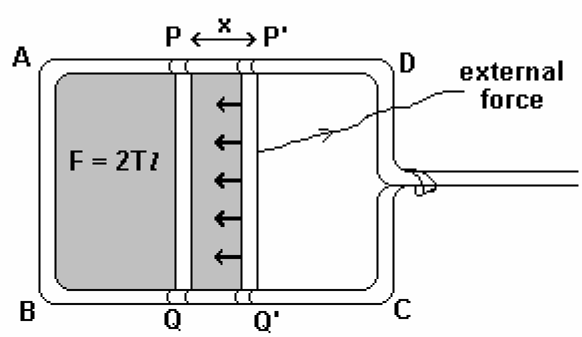
$\therefore$ Total surface energy per unit area = Potential energy (due to surface tension) + Heat energy.

Thus, at any temperature the value of surface energy is more than surface tension. Both the surface tension and surface energy decrease with temperature and become zero at the critical temperature. The figure shows the graphs of surface tension and surface energy versus temperature for water both of which become zero at the critical temperature of water which is 374° C.

Now, consider a rectangular frame ABCD on which a wire PQ can be slided over the sides AD and BC by a light string tied to PQ as shown in the figure.



A thin film of liquid formed on a rectangular frame



Expansion of the film

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A thin film ABQP is formed by dipping the frame in soap solution and holding the wire PQ in position with a thread. If the string is released, PQ slides inwards and the film contracts. This shows that the surface tension manifests itself on the edge of the surface of the liquid perpendicular to the edge and parallel to the surface.

Now displace the wire by x by pulling the string with a force slightly more than the force of surface tension. If T is the surface tension of the solution and l is the length of the wire PQ, then the force acting on the wire is

$F = 2Tl$ (The multiple '2' is because the film has two surfaces.)

$\therefore$ work W = Force $\times$ displacement $= 2Tlx = T(2lx) = T(\Delta A)$

$\therefore$ work $W = T$ for $\Delta A = 1$ unit

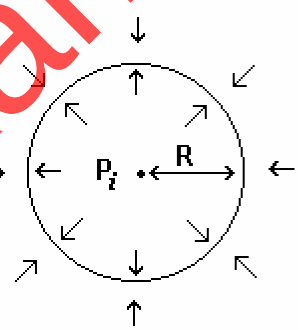
Thus, surface tension can also be defined as the work done to increase the area of the surface by 1 unit.

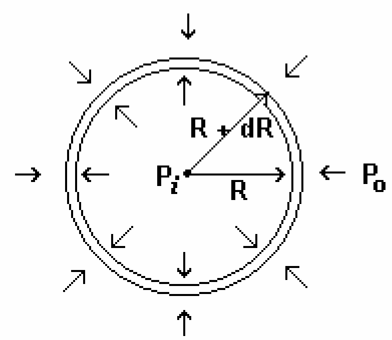
Experiments show that the surface tension does not change on expanding the surface. This indicates that the molecules coming to the surface do not occupy place between the already present molecules but create a new surface of their own.

10.11 Drops and bubbles

Free surface of liquids has tendency to minimize its area due to surface tension. Since spherical surface has minimum area for a given volume, drops and bubbles of liquids are always spherical.

Consider a bubble of radius R as shown in the figure. The pressures inside and outside are P_i and P_o ($P_i > P_o$) respectively. The pressure on the concave surface is always more than that on the convex surface. Let the surface tension of the liquid forming the wall of the bubble be T.





Suppose, on blowing the bubble, its radius increases from R to R + dR and its surface area increases from S to S + dS. The work done in this process can be calculated in two ways.

(1) Work, W = force × displacement = pressure difference × area × displacement

$$= (P_i - P_o)\, 4\pi R^2 \cdot dR \quad \dots \quad (1)$$

(2) The surface area of the bubble of radius R is, $S = 4\pi R^2$

$\therefore$ the increase in the surface area is, $dS = 8\pi R\, dR$

But the bubble in air has two free surfaces

$\therefore$ total increase in its area $= 2 \times 8\pi R\, dR = 16\pi R\, dR$

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$\therefore$ work, W = surface tension $\times$ total increase in area = $16\pi T R\, dR$ ... (2)

Comparing equations (1) and (2), $(P_i - P_o)\, 4\pi R^2 \cdot dR = 16\pi T R\, dR$

$$\therefore \quad P_i - P_o = \frac{4T}{R}$$

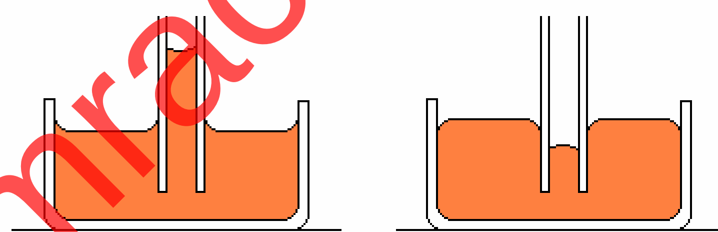
For a bubble formed in a liquid or a liquid drop which has only one free surface,

$$P_i - P_o = \frac{2T}{R}$$

10.12 Capillarity

"The phenomenon of rise or fall of a liquid in a capillary, held vertical in a liquid, due to its property of surface tension is called capillarity."

When a glass capillary of small bore is held vertical in water, water rises in the capillary and when held in mercury, mercury falls in the capillary as shown in the figure. Also, note that water wets the glass while mercury does not. The meniscus of water in the capillary is concave while the meniscus of mercury is convex.



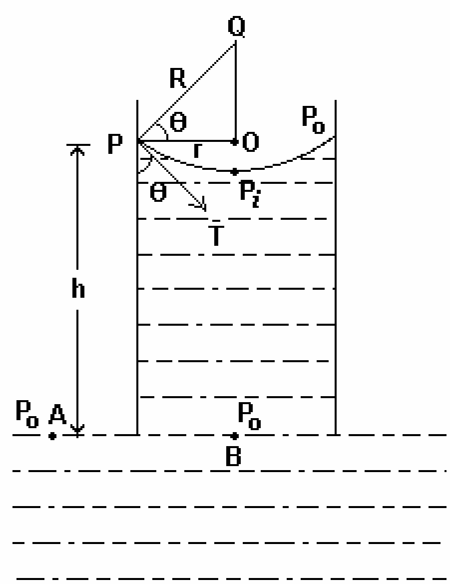
(a) in water (b) in mercury

Phenomenon of capillarity

Suppose liquid rises to height h in a capillary of radius r held vertical in the liquid as shown in the figure. The radius of concave meniscus of liquid in the capillary is R.

The tangent drawn at a point P, where the surface of meniscus is in contact with wall of the capillary, makes an angle θ with the wall. θ is known as the angle of contact of the liquid with the matter of the capillary.

Liquids with $\theta < 90°$ rise while those with $\theta > 90°$ fall in the capillary.



Column of liquid in a capillary

In the figure, $\angle OPQ = \theta$ in right-angled Δ OPQ.

$$\therefore \quad \cos\theta = \frac{OP}{OQ} = \frac{\text{Radius of the capillary } (r)}{\text{Radius of the meniscus } (R)}$$

$$\therefore \quad R = \frac{r}{\cos\theta} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

The pressure on the concave surface of the meniscus $P_o > P_i$, the pressure on the convex surface.

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$$\therefore \quad P_o - P_i = \frac{2T}{R} \quad \text{(because, the liquid has one free surface.)} \quad \ldots \quad \ldots \quad (2)$$

Also, for equilibrium, the pressure at point B is the same as at point A which is P_o as both are at the same horizontal level.

$$\therefore \quad P_o - P_i = h \rho g \quad \ldots \quad (3)$$

where ρ = density of the liquid and g = acceleration due to gravity.

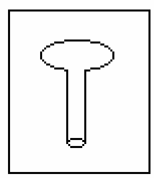
Comparing equations (2) and (3), $\frac{2T}{R} = h \rho g$

$$\therefore \quad T = \frac{R h \rho g}{2} = \frac{r h \rho g}{2 \cos \theta} \qquad \text{[putting the value of R from equation (1)]}$$

For mercury and glass, $\theta > 90°$. Hence, $\cos \theta$ is negative. Therefore, mercury falls in a glass capillary and its meniscus is convex. Also, $P_i > P_o$. Thus in equation (2), $P_i - P_o = \frac{2T}{R}$ should be taken. As $P_i - P_o = h \rho g$, the final result remains the same.

10.13 Detergent and surface tension

Stains of grease or oil on the clothes are not removed by water alone as water does not wet the grease or oil. On adding detergent, the broad end of the molecule of detergent as shown in the figure is attracted to the water molecule and the other end is attracted towards the molecule of oil or grease. This forms an interface between water and oil or grease. Thus, on adding detergent to water, the surface tension of the solution is less than that of water. Hence, it wets grease or oil and removes the dirt.



Some useful formulae

(1) Velocity, v, of a liquid having co-efficient of viscosity, η, in laminar flow through a tube of radius r and length l across which there is a pressure difference p is given by

$$v = \frac{p}{4 \eta l} (r^2 - x^2)$$

(2) Volumetric flow rate, V, of liquid in the above case is given by

$$V = \frac{\pi p r^4}{8 \eta l}$$

This equation is called Poiseiulle's Law.

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