13 - VECTOR ALGEBRA

(Answers at the end of all questions)

(1) If C is the midpoint of AB and P is any point outside AB, then

(a) $\vec{PA} + \vec{PB} = 2\,\vec{PC}$ (b) $\vec{PA} + \vec{PB} = \vec{PC}$

(c) $\vec{PA} + \vec{PB} + 2\,\vec{PC} = \vec{0}$ (d) $\vec{PA} + \vec{PB} + \vec{PC} = \vec{0}$ [AIEEE 2005]

(2) For any vector $\vec{a}$, the value of $(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{k})^2$ is equal to

(a) $3\,\vec{a}^{\,2}$ (b) $\vec{a}^{\,2}$ (c) $2\,\vec{a}^{\,2}$ (d) $4\,\vec{a}^{\,2}$ [AIEEE 2005]

(3) Let a, b and c be distinct non-negative numbers. If the vectors $a\hat{i} + a\hat{j} + \hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ lie in a plane, then c is

(a) the Geometric Mean of a and b (b) the Arithmetic Mean of a and b
(c) equal to zero (d) the Harmonic Mean of a and b
[AIEEE 2005]

(4) If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-coplanar vectors and λ is a real number, then
$\lambda(\vec{a} + \vec{b}) \cdot [\lambda^2 \vec{b} \times \lambda \vec{c}] = \vec{a} \cdot [(\vec{b} + \vec{c}) \times \vec{b}]$ for

(a) exactly one value of λ (b) no value of λ
(c) exactly three values of λ (d) exactly two values of λ [AIEEE 2005]

(5) Let $\vec{a} = \hat{i} - \hat{k}$, $\vec{b} = x\hat{i} + \hat{j} + (1 - x)\hat{k}$ and $\vec{c} = y\hat{i} + x\hat{j} + (1 + x - y)\hat{k}$.
Then $[\vec{a}\,\vec{b}\,\vec{c}]$ depends on

(a) only y (b) only x (c) both x and y (d) neither x nor y [AIEEE 2005]

(6) Let $\bar{a}$, $\bar{b}$, $\bar{c}$, be three non-zero vectors such that no two of these are collinear. If the vector $\bar{a} + 2\bar{b}$ is collinear with $\bar{c}$, and $\bar{b} + 3\bar{c}$ is collinear with $\bar{a}$, then $\bar{a} + 2\bar{b} + 6\bar{c}$, for some non-zero scalar λ equals

(a) $\lambda\bar{a}$ (b) $\lambda\bar{b}$ (c) $\lambda\bar{c}$ (d) 0 [AIEEE 2004]

(Answers at the end of all questions)

(7) A particle is acted upon by constant forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$ which displace it from a point $\hat{i} + 2\hat{j} + 3\hat{k}$ to the point $5\hat{i} + 4\hat{j} + \hat{k}$. The work done in standard units by the forces is given by

(a) 40 (b) 30 (c) 25 (d) 15 [AIEEE 2004]

(8) If $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar vectors and λ is a real number, then the vectors $\bar{a} + 2\bar{b} + 3\bar{c}$, $\lambda\bar{b} + 4\bar{c}$ and $(2\lambda - 1)\bar{c}$ are non-coplanar for

(a) all values of λ (b) all except one value of λ
(c) all except two values of λ (d) no value of λ [AIEEE 2004]

(9) Let $\bar{u}, \bar{v}, \bar{w}$ be such that $|\bar{u}| = 1$, $|\bar{v}| = 2$ and $|\bar{w}| = 3$. If the projection of $\bar{v}$ along $\bar{u}$ is equal to that of $\bar{w}$ along $\bar{u}$ and $\bar{v}, \bar{w}$ are perpendicular to each other, then $|\bar{u} - \bar{v} + \bar{w}|$ equals

(a) 2 (b) $\sqrt{7}$ (c) $\sqrt{14}$ (d) 14 [AIEEE 2004]

(10) Let $\bar{a}, \bar{b}, \bar{c}$ be non-zero vectors such that $(\bar{a} \times \bar{b}) \times \bar{c} = \frac{1}{3}|\bar{b}||\bar{c}|\,\bar{a}$. If θ is the acute angle between the vectors $\bar{b}$ and $\bar{c}$, then $\sin\theta$ equals

(a) $\frac{1}{3}$ (b) $\frac{\sqrt{2}}{3}$ (c) $\frac{2}{3}$ (d) $\frac{2\sqrt{2}}{3}$ [AIEEE 2004]

(11) If $\begin{vmatrix} a & a^2 & 1 + a^2 \\ b & b^2 & 1 + b^2 \\ c & c^2 & 1 + c^2 \end{vmatrix} = 0$ and vectors $(1, a, a^2)$, $(1, b, b^2)$ and $(1, c, c^2)$ are non-coplanar, then the product abc equals

(a) 2 (b) - 1 (c) 1 (d) 0 [AIEEE 2003]

(12) $\vec{a}, \vec{b}, \vec{c}$ are three vectors, such that $\vec{a} + \vec{b} + \vec{c} = 0$, $|\vec{a}| = 1$, $|\vec{b}| = 2$, $|\vec{c}| = 3$, then $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$ is equal to

(a) 0 (b) - 7 (c) 7 (d) 1 [AIEEE 2003]

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(Answers at the end of all questions)

(13) A particle acted on by constant forces $4\vec{i} + \vec{j} - 3\vec{k}$ and $3\vec{i} + \vec{j} - \vec{k}$ is displaced from the point $5\vec{i} + 4\vec{j} + \vec{k}$. The total work done by the forces is

(a) 20 units (b) 30 units (c) 40 units (d) 50 units [AIEEE 2003]

(14) If $\vec{u}$, $\vec{v}$ and $\vec{w}$ are three non-coplanar vectors, then $(\vec{u} + \vec{v} - \vec{w}).(\vec{u} - \vec{v}) \times (\vec{v} - \vec{w})$ equals

(a) 0 (b) $\vec{u}\,.\,\vec{v} \times \vec{w}$ (c) $\vec{u}\,.\,\vec{w} \times \vec{v}$ (d) $3\,\vec{u}\,.\,\vec{v} \times \vec{w}$ [AIEEE 2003]

(15) The vectors $\vec{AB} = 3\vec{i} + 4\vec{k}$ and $\vec{AC} = 5\vec{i} - 2\vec{j} + 4\vec{k}$ are the sides of a triangle ABC. The length of a median through A is

(a) $\sqrt{18}$ (b) $\sqrt{72}$ (c) $\sqrt{33}$ (d) $\sqrt{288}$ [AIEEE 2003]

(16) Consider points A, B, C and D with position vectors $7\vec{i} - 4\vec{j} + 7\vec{k}$, $\vec{i} - 6\vec{j} + 10\vec{k}$, $-\vec{i} - 3\vec{j} + 4\vec{k}$ and $5\vec{i} - \vec{j} + 5\vec{k}$ respectively. Then ABCD is a

(a) square (b) rhombus (c) rectangle (d) parallelogram [AIEEE 2003]

(17) Let $\vec{u} = \vec{i} + \vec{j}$, $\vec{v} = \vec{i} - \vec{j}$ and $\vec{w} = \vec{i} + 2\vec{j} + 3\vec{k}$. If $\hat{n}$ is a unit vector such that $\vec{u} \cdot \hat{n} = 0$ and $\vec{v} \cdot \hat{n} = 0$, then $\vec{w} \cdot \hat{n}$

(a) 0 (b) 1 (c) 2 (d) 3 [AIEEE 2003]

(18) The angle between any two diagonals of a cube is

(a) 45° (b) 60° (c) 90° (d) $\tan^{-1} 2\sqrt{2}$ [AIEEE 2003]

(19) If vector $\vec{a} = \vec{i} + \vec{j} - \vec{k}$; $\vec{b} = \vec{i} - \vec{j} + \vec{k}$ and $\vec{c} = \vec{i} - \vec{j} - \vec{k}$, then the value of $\vec{a} \times (\vec{b} \times \vec{c})$ is

(a) $\vec{i} - \vec{j} + \vec{k}$ (b) $2\vec{i} - 2\vec{j}$

(c) $3\vec{i} - \vec{j} + \vec{k}$ (d) $2\vec{i} + 2\vec{j} - \vec{k}$ [AIEEE 2002]

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(Answers at the end of all questions)

(20) If $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$; $\vec{b} = -\vec{i} + 2\vec{j} + \vec{k}$ and $\vec{c} = \vec{i} - 2\vec{j} + 2\vec{k}$, then a unit vector parallel to $\vec{a} + \vec{b} + \vec{c}$ is

(a) $\dfrac{\vec{i} + \vec{j} + \vec{k}}{\sqrt{3}}$ (b) $\dfrac{\vec{i} - 2\vec{j} + \vec{k}}{\sqrt{6}}$

(c) $\dfrac{\vec{i} - \vec{j} + \vec{k}}{\sqrt{3}}$ (d) $\dfrac{2\vec{i} + \vec{j} + \vec{k}}{\sqrt{6}}$ [AIEEE 2002]

(21) If $\vec{a} = 2\vec{i} + \vec{j} + 2\vec{k}$ and $\vec{b} = 5\vec{i} - 3\vec{j} + \vec{k}$, the orthogonal projection of $\vec{a}$ on $\vec{b}$ is

(a) $5\vec{i} - 3\vec{j} + \vec{k}$ (b) $9(5\vec{i} - 3\vec{j} + \vec{k})$

(c) $\dfrac{5\vec{i} - 3\vec{j} + \vec{k}}{35}$ (d) $\dfrac{9(5\vec{i} - 3\vec{j} + \vec{k})}{35}$ [AIEEE 2002]

(22) If the angle between two vectors $\vec{i} + \vec{k}$ and $\vec{i} - \vec{j} + a\vec{k}$ is $\dfrac{\pi}{3}$, then the value of a is

(a) 2 (b) 4 (c) - 2 (d) 0 [AIEEE 2002]

(23) If $\vec{a} = \vec{i} + 2\vec{j} - 3\vec{k}$ and $\vec{b} = 3\vec{i} - \vec{j} + 2\vec{k}$, then angle between $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$ is

(a) 0° (b) 30° (c) 60° (d) 90° [AIEEE 2002]

(24) The value of sine of the angle between the vectors $\vec{i} - 2\vec{j} + 3\vec{k}$ and $2\vec{i} + \vec{j} + \vec{k}$ is

(a) $\dfrac{5}{21}$ (b) $\dfrac{5}{\sqrt{7}}$ (c) $\dfrac{5}{\sqrt{14}}$ (d) $\dfrac{5}{2\sqrt{7}}$ [AIEEE 2002]

(25) If vectors $a\vec{i} + \vec{j} + \vec{k}$, $\vec{i} + b\vec{j} + \vec{k}$ and $\vec{i} + \vec{j} + c\vec{k}$ are coplanar, then

(a) a + b + c = 0 (b) abc = - 1

(c) a + b + c = abc + 2 (d) ab + bc + ca = 0 [AIEEE 2002]

(26) If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero, noncoplanar vectors and $\vec{b}_1 = \vec{b} - \dfrac{\vec{b} \cdot \vec{a}}{|\vec{a}|^2}\vec{a}$,

$\vec{b}_2 = \vec{b} + \dfrac{\vec{b} \cdot \vec{a}}{|\vec{a}|^2}\vec{a}$, $\quad \vec{c}_1 = \vec{c} - \dfrac{\vec{c} \cdot \vec{a}}{|\vec{a}|^2}\vec{a} - \dfrac{\vec{c} \cdot \vec{b}}{|\vec{b}|^2}\vec{b}$,

$\vec{c}_2 = \vec{c} - \dfrac{\vec{c} \cdot \vec{a}}{|\vec{a}|^2}\vec{a} - \dfrac{\vec{c} \cdot \vec{b}_1}{|\vec{b}_1|^2}\vec{b}_1$, and $\vec{c}_3 = \vec{c} - \dfrac{\vec{c} \cdot \vec{a}}{|\vec{a}|^2}\vec{a} - \dfrac{\vec{c} \cdot \vec{b}_2}{|\vec{b}_2|^2}\vec{b}_2$,

then the set of orthogonal vectors is

(a) $(\vec{a}, \vec{b}_1, \vec{c}_1)$ (b) $(\vec{a}, \vec{b}_1, \vec{c}_2)$

(c) $(\vec{a}, \vec{b}_1, \vec{c}_3)$ (d) $(\vec{a}, \vec{b}_2, \vec{c}_2)$ [IIT 2005]

(27) If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{a} \cdot \vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$, then $\vec{b}$ is equal to

(a) $2\hat{i}$ (b) $\hat{i} - \hat{j} + \hat{k}$ (c) $\hat{i}$ (d) $2\hat{j} - \hat{k}$ [IIT 2004]

(28) A unit vector is orthogonal to $5\hat{i} + 2\hat{j} + 6\hat{k}$ and is coplanar to $2\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} - \hat{j} + \hat{k}$, then the vector is

(a) $\dfrac{3\hat{j} - \hat{k}}{\sqrt{10}}$ (b) $\dfrac{2\hat{i} + 5\hat{j}}{\sqrt{29}}$ (c) $\dfrac{6\hat{i} - 5\hat{k}}{\sqrt{61}}$ (d) $\dfrac{2\hat{i} + 2\hat{j} - \hat{k}}{3}$ [IIT 2004]

(29) The value of a so that the volume of parallelopiped formed by vectors $\hat{i} + a\hat{j} + \hat{k}$, $\hat{j} + a\hat{k}$ and $a\hat{i} + \hat{k}$ becomes minimum is

(a) $\sqrt{3}$ (b) 2 (c) $\dfrac{1}{\sqrt{3}}$ (d) 3 [IIT 2003]

(30) If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + 2\vec{b}$ and $5\vec{a} - 4\vec{b}$ are perpendicular to each other, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) 45° (b) 60° (c) $\cos^{-1}\dfrac{1}{3}$ (d) $\cos^{-1}\dfrac{2}{7}$ [IIT 2002]

(31) If $\vec{V} = 2\vec{i} + \vec{j} - \vec{k}$, $\vec{W} = \vec{i} + 3\vec{k}$ and $\vec{U}$ is a unit vector, then the maximum value of the scalar triple product $[\vec{U}\ \vec{V}\ \vec{W}]$ is

(a) -1 (b) $\sqrt{10} + \sqrt{6}$ (c) $\sqrt{59}$ (d) $\sqrt{60}$ [IIT 2002]

(32) If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors, then $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2$ does not exceed

(a) 4 (b) 9 (c) 8 (d) 6 [IIT 2001]

(33) If $\vec{a} = \hat{i} - \hat{k}$, $\vec{b} = x\hat{i} + \hat{j} + (1 - x)\hat{k}$ and $\vec{c} = y\hat{i} + x\hat{j} + (1 + x - y)\hat{k}$, then $[\vec{a}\ \vec{b}\ \vec{c}]$ depends on

(a) only x (b) only y (c) neither x nor y (d) both x and y [IIT 2001]

(34) Let the vectors a, b, c and d be such that $(a \times b) \times (c \times d) = 0$. Let P_1 and P_2 be planes determined by the pairs of vectors a, b and c, d respectively, then the angle between P_1 and P_2 is

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$ [IIT 2000]

(35) If a, b and c are unit coplanar vectors, then the scalar triple product $[2a - b,\ 2b - c,\ 2c - a] =$

(a) 0 (b) 1 (c) $-\sqrt{3}$ (d) $\sqrt{3}$ [IIT 2000]

(36) If the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ form the sides $\overline{BC}$, $\overline{CA}$ and $\overline{AB}$ of a triangle ABC, then

(a) $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = 0$ (b) $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = 0$

(c) $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a}$ (d) $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} \times \vec{a} = 0$ [IIT 2000]

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(Answers at the end of all questions)

(37) Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a} \cdot \vec{c} = |\vec{c}|$, $|\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $(\vec{a} \times \vec{b})$ and $\vec{c}$ is 30°, then $|(\vec{a} \times \vec{b}) \times \vec{c}| =$

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 2 (d) 3 [IIT 1999]

(38) Let $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and a unit vector $\vec{c}$ be coplanar. If $\vec{c}$ is perpendicular to $\vec{a}$, then $\vec{c} =$

(a) $\frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$ (b) $\frac{1}{\sqrt{3}}(-\hat{i} - \hat{j} - \hat{k})$

(c) $\frac{1}{\sqrt{5}}(\hat{i} - 2\hat{j})$ (d) $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} - \hat{k})$ [IIT 1999]

(39) Let $\vec{a}$ and $\vec{b}$ be two non-collinear unit vectors. If $\vec{u} = \vec{a} - (\vec{a} \cdot \vec{b})\vec{b}$ and $\vec{v} = \vec{a} \times \vec{b}$, then $\vec{v}$ is

(a) $|\vec{u}|$ (b) $|\vec{u}| + |\vec{u} \cdot \vec{a}|$

(c) $|\vec{u}| + |\vec{u} \cdot \vec{b}|$ (d) $|\vec{u}| + \vec{u} \cdot (\vec{a} + \vec{b})$ [IIT 1999]

(40) If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are linearly dependent vectors and $|\vec{c}| = \sqrt{3}$, then α and β respectively are

(a) 1, - 1 (b) 1, ± 1 (c) - 1, ± 1 (d) ± 1, 1 [IIT 1998]

(41) For three vectors $\vec{u}$, $\vec{v}$ $\vec{w}$ which of the following expressions is not equal to any of the remaining three ?

(a) $\vec{u} \cdot (\vec{v} \times \vec{w})$ (b) $(\vec{v} \times \vec{w}) \cdot \vec{u}$ (c) $\vec{v} \cdot (\vec{u} \times \vec{w})$ (d) $(\vec{u} \times \vec{v}) \cdot \vec{w}$ [IIT 1998]

(42) Which of the following expressions are meaningful questions ?

(a) $\vec{u} \cdot (\vec{v} \times \vec{w})$ (b) $(\vec{u} \cdot \vec{v}) \cdot \vec{w}$ (c) $(\vec{u} \cdot \vec{v})\vec{w}$ (d) $\vec{u} \times (\vec{v} \cdot \vec{w})$ [IIT 1998]

(Answers at the end of all questions)

(43) Let p, q, r be three mutually perpendicular vectors of the same magnitude. If a vector x satisfies the equation
$p \times [(x - q) \times p] + q \times [(x - r) \times q] + r \times [(x - p) \times r] = 0$, then x is given by

(a) $\frac{1}{2}(p + q - 2r)$ (b) $\frac{1}{2}(p + q + r)$

(c) $\frac{1}{3}(p + q + r)$ (d) $\frac{1}{3}(2p + q - r)$ [IIT 1997]

(44) Let $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{k} - \hat{i}$. If $\hat{d}$ is a unit vector such that $\vec{a} \cdot \vec{d} = 0 = [\vec{b}, \vec{c}, \vec{d}]$, then $\hat{d}$ equals

(a) $\pm \frac{\hat{i} + \hat{j} - 2\hat{k}}{\sqrt{6}}$ (b) $\pm \frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$ (c) $\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$ (d) $\pm \hat{k}$ [IIT 1995]

(45) Let $\vec{u}$, $\vec{v}$ and $\vec{w}$ be vectors such that $\vec{u} + \vec{v} + \vec{w} = 0$. If $|\vec{u}| = 3$, $|\vec{v}| = 4$ and $|\vec{w}| = 5$, then the value of $\vec{u} . \vec{v} + \vec{v} . \vec{w} + \vec{w} . \vec{u}$ is

(a) 47 (b) - 25 (c) 0 (d) 25 [IIT 1995]

(46) If $\vec{A}$, $\vec{B}$ and $\vec{C}$ are three non-coplanar vectors, then
$(\vec{A} + \vec{B} + \vec{C}) \cdot (\vec{A} + \vec{B}) \times (\vec{A} + \vec{C})$ equals

(a) 0 (b) $[\vec{A}, \vec{B}, \vec{C}]$ (c) $2[\vec{A}, \vec{B}, \vec{C}]$ (d) $-[\vec{A}, \vec{B}, \vec{C}]$ [IIT 1995]

(47) If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-coplanar unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b} \times \vec{c}}{\sqrt{2}}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) π [IIT 1995]

(48) Let a, b, c be distinct non-negative numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ lie in a plane, then c is

(a) AM of a and b (b) GM of a and b (c) HM of a and b (d) 0 [IIT 1993]

(49) Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - 2\hat{k}$ be three vectors. A vector in the plane of $\vec{b}$ and $\vec{c}$ whose projection on $\vec{a}$ is of magnitude $\sqrt{\frac{2}{3}}$ is

(a) $2\hat{i} + \hat{j} - 3\hat{k}$ (b) $2\hat{i} + 3\hat{j} + 3\hat{k}$

(c) $-2\hat{i} - \hat{j} + 5\hat{k}$ (d) $2\hat{i} + \hat{j} + 5\hat{k}$ [IIT 1993]

(50) If $\vec{a}, \vec{b}, \vec{c}$ be three non-coplanar vectors and $\vec{p}, \vec{q}, \vec{r}$ are vectors defined by the relations $\vec{p} = \frac{\vec{b} \times \vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}$; $\vec{q} = \frac{\vec{c} \times \vec{a}}{[\vec{a}\ \vec{b}\ \vec{c}]}$; $\vec{r} = \frac{\vec{a} \times \vec{b}}{[\vec{a}\ \vec{b}\ \vec{c}]}$,

then the value of the expression

$(\vec{a} + \vec{b}).\vec{p} + (\vec{b} + \vec{c}).\vec{q} + (\vec{c} + \vec{a}).\vec{r}$ is equal to

(a) 0 (b) 1 (c) 2 (d) 3 [IIT 1988]

(51) The number of vectors of unit length perpendicular to vectors $\vec{a} = (1, 1, 0)$ and $\vec{b} = (0, 1, 1)$ is

(a) one (b) two (c) three (d) infinite (e) none of these [IIT 1987]

(52) Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$ is equal to

(a) 0 (b) 1 (c) $\frac{3}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)(c_1^2 + c_2^2 + c_3^2)$

(d) $\frac{1}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)$ (e) none of these [IIT 1986]

(Answers at the end of all questions)

(53) A vector $\vec{a}$ has components 2p and 1 with respect to a rectangular Cartesian system. This system is rotated through a certain angle about the origin in the counterclockwise sense. If, with respect to the new system, $\vec{a}$ has components p + 1 and 1, then

(a) p = 0 (b) p = 1 or p = $-\frac{1}{3}$ (c) p = - 1 or p = $\frac{1}{3}$
(d) p = 1 or p = - 1 (e) none of these [IIT 1986]

(54) The volume of the parallelepiped whose sides are given by $\vec{OA}$ = 2i - 3j, $\vec{OB}$ = i + j - k, $\vec{OC}$ = 3i - k, is

(a) $\frac{4}{13}$ (b) 4 (c) $\frac{2}{7}$ (d) none of these [IIT 1983]

(55) The points with position vectors 60i + 3j, 40i - 8j, ai - 52j are collinear if

(a) a = - 40 (b) a = 40 (c) a = 20 (d) none of these [IIT 1983]

(56) For non-zero vectors $\vec{a}$, $\vec{b}$, $\vec{c}$, $|(\vec{a} \times \vec{b}) \cdot \vec{c}| = |\vec{a}|\,|\vec{b}|\,|\vec{c}|$ holds if and only if

(a) $\vec{a} \cdot \vec{b} = 0$, $\vec{b} \cdot \vec{c} = 0$ (b) $\vec{c} \cdot \vec{a} = 0$, $\vec{a} \cdot \vec{b} = 0$
(c) $\vec{b} \cdot \vec{c} = 0$, $\vec{c} \cdot \vec{a} = 0$ (d) $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$ [IIT 1982]

(57) The scalar $\vec{A} \cdot (\vec{B} + \vec{C}) \times (\vec{A} + \vec{B} + \vec{C})$ equals

(a) 0 (b) $[\vec{A}\ \vec{B}\ \vec{C}] + [\vec{B}\ \vec{C}\ \vec{A}]$ (c) $[\vec{A}\ \vec{B}\ \vec{C}]$ (d) none of these [IIT 1981]

13 - VECTOR ALGEBRA

(Answers at the end of all questions)

Answers

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
a	c	a	b	d	d	a	c	c	d	b	b	c	c	c	b	d	d	b	a

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
d	d	d	d	c	b	c	a	c	b	c	b	c	a	a	b	b	a	b,c	d

41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
c	a,c	b	a	b	d	a	b	c	d	b	d	b	b	a	d	a			