01 - SETS, RELATIONS AND FUNCTIONS

(Answers at the end of all questions)

(1) Let R = { (3, 3) (6, 6) ((9, 9) (12, 12), (6, 12) (3, 9) (3, 12), (3, 6) } be a relation on the set A = { 3, 6, 9, 12 }. The relation is

(a) reflexive and transitive (b) reflexive only
(c) an equivalence relation (d) reflexive and symmetric only [AIEEE 2005]

(2) Let f : (- 1, 1) $\rightarrow$ B be a function defined by f (x) = $\tan^{-1}\frac{2x}{1-x^2}$, then f is both one-one and onto when B is the interval

(a) $\left(0, \frac{\pi}{2}\right)$ (b) $\left[0, \frac{\pi}{2}\right]$ (c) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (d) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ [AIEEE 2005]

(3) If a real valued function f (x) satisfies the functional equation f (x - y) = f (x) f (y) - f (a - x) f (a + y), where 'a' is a given constant and f (0) = 1, then f (2a - x) is equal to

(a) - f (x) (b) f (x) (c) f (a) + f (a - x) (d) f (- x) [AIEEE 2005]

(4) Let R = { (1, 3), (4, 2), (2, 4), (2, 3), (3, 1) } be a relation on the set A = { 1, 2, 3, 4 }. The relation R is

(a) a function (b) transitive (c) not symmetric (d) reflexive [AIEEE 2004]

(5) The range of the function f (x) = $^{7-x}P_{x-3}$ is

(a) { 1, 2, 3 } (b) { 1, 2, 3, 4, 5, 6 }
(c) { 1, 2, 3, 4 } (d) { 1, 2, 3, 4, 5 } [AIEEE 2004]

(6) If f : R $\rightarrow$ S, defined by f(x) = sin x - $\sqrt{3}$ cos x + 1 is onto, then the interval of S is

(a) [0, 3] (b) [- 1, 1] (c) [0, 1] (d) [- 1, 3] [AIEEE 2004]

(7) The graph of the function f (x) is symmetrical about the line x =2, then

(a) f (x + 2) = f (x - 2) (b) f (2 + x) = f (2 - x)
(c) f (x) = f (- x) (d) f (x) = - f (- x) [AIEEE 2004]

(8) The domain of the function $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is

(a) [2, 3] (b) [2, 3) (c) [1, 2] (d) [1, 2) [AIEEE 2004]

(9) If $f : \{1, 2, 3, \ldots.\} \to \{0, \pm 1, \pm 2, \ldots..\}$ is defined by

$$f(x) = \begin{cases} \frac{x}{2}, & \text{if } x \text{ is even} \\ -\frac{(x-1)}{2}, & \text{if } x \text{ is odd} \end{cases}$$

then value of $f^{-1}(-100)$ is

(a) 100 (b) 199 (c) 200 (d) 201 [AIEEE 2003]

(10) Domain of definition of the function $f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$ is

(a) (1, 2) (b) (- 1, 0) $\cup$ (1, 2)
(c) (1, 2) $\cup$ (2, ∞) (d) (- 1, 0) $\cup$ (1, 2) $\cup$ (2, ∞) [AIEEE 2003]

(11) The function $f(x) = \log(x + \sqrt{x^2 + 1})$ is a / an

(a) even function (b) odd function
(c) periodic function (d) none of these [AIEEE 2003]

(12) The function $f : R \to R$ defined by $f(x) = \sin x$ is

(a) into (b) onto (c) one-one (d) many-one [AIEEE 2002]

(13) The range of the function $f(x) = \frac{2+x}{2-x}$, $x \neq 2$ is

(a) R (b) R - { - 1 } (c) R - { 1 } (d) R - { 2 } [AIEEE 2002]

(14) If $f(x) = \begin{cases} x, & x \in Q \\ 0, & x \notin Q \end{cases}$ and $g(x) = \begin{cases} 0, & x \in Q \\ x, & x \notin Q \end{cases}$, then $(f - g)$ is

(a) one-one, onto (b) neither one-one nor onto
(c) one-one but not onto (d) onto but not one-one [IIT 2005]

(15) If $f(x) = \sin x + \cos x$ and $g(x) = x^2 - 1$, then $g[f(x)]$ will be invertible for the domain

(a) $[0, \pi]$ (b) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ (c) $\left[0, \frac{\pi}{2}\right]$ (d) $\left[-\frac{\pi}{2}, 0\right[$ [IIT 2004]

(16) The range of the function $f(x) = \frac{x^2 + x + 2}{x^2 + x + 1}$, $x \in (-\infty, \infty)$ is

(a) $[1, \infty)$ (b) $(1, \frac{11}{7})$ (c) $(1, \frac{7}{3}]$ (d) $[1, \frac{7}{5}]$ [IIT 2003]

(17) $f : [0, \infty) \to [0, \infty)$, $f(x) = \frac{x}{1 + x}$ is

(a) one-one and onto (b) one-one but not onto
(c) onto but not one-one (d) neither one-one nor onto [IIT 2003]

(18) If $f(x) = (x + 1)^2$ for $x \geq -1$ and $g(x)$ is the function whose graph is reflection of the graph of $f(x)$ with respect to the line $y = x$, then $g(x)$ equals

(a) $-\sqrt{x} - 1$, $x \geq 0$ (b) $\frac{1}{(x + 1)^2}$, $x > -1$

(c) $\sqrt{x + 1}$, $x \geq -1$ (d) $\sqrt{x} - 1$, $x \geq 0$ [IIT 2002]

(19) If function $f : R \to R$ is defined as $f(x) = 2x + \sin x$ for $x \in R$, then f is

(a) one-one and onto (b) one-one but not onto
(c) onto but not one-one (d) neither one-one nor onto [IIT 2002]

(20) If $g(x) = 1 + x - [x]$ and $f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$, then for all x, $f[g(x)] =$

(a) x (b) 1 (c) $f(x)$ (d) $g(x)$ [IIT 2001]

(21) If $f : [1, \infty) \to [2, \infty)$ is given by $f(x) = x + \frac{1}{x}$, then $f^{-1}(x)$ equals

(a) $\frac{x + \sqrt{x^2 - 4}}{2}$ (b) $\frac{x}{1 + x^2}$ (c) $\frac{x - \sqrt{x^2 - 4}}{2}$ (d) $1 + \sqrt{x^2 - 4}$ [IIT 2001]

(22) The domain of definition of $f(x) = \frac{\log_2 (x + 3)}{x^2 + 3x + 2}$ is

(a) $R - \{-1, -2\}$ (b) $(-2, \infty)$
(c) $R - \{-1, -2, -3\}$ (d) $(-3, \infty) - \{-1, -2\}$ [IIT 2001]

(23) If $E = \{1, 2, 3, 4\}$ and $F = \{1, 2\}$, then the number of onto functions from E to F is

(a) 14 (b) 16 (c) 12 (d) 8 [IIT 2001]

(24) If $f(x) = \frac{\alpha x}{x + 1}$, $x \neq -1$, then for which value of α is $f[f(x)] = x$?

(a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) 1 (d) -1 [IIT 2001]

(25) Let $f : R \to R$ be any function. Define $g : R \to R$ by $g(x) = |f(x)|$ for all x. Then g is

(a) onto if f is onto (b) one-one if f is one-one
(c) continuous if f is continuous (d) differentiable if f is differentiable [IIT 2000]

(26) The domain of definition of the function $y(x)$ as given by the equation $2^x + 2^y = 2$ is

(a) $0 < x \leq 1$ (b) $0 \leq x \leq 1$ (c) $-\infty < x \leq 0$ (d) $-\infty < x < 1$ [IIT 2000]

(27) If the function $f : [1, \infty) \to [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$, then $f^{-1}(x)$ is

(a) $\left(\frac{1}{2}\right)^{x(x-1)}$ (b) $\frac{1}{2}(1 + \sqrt{1 + 4\log_2 x})$
(c) $\frac{1}{2}(1 + \sqrt{1 - 4\log_2 x})$ (d) not defined [IIT 1999]

(28) In a college of 300 students, every student reads 5 newspapers and every newspaper is read by 60 students. The number of newspapers is

(a) at least 30 (b) at most 20 (c) exactly 25 (d) none of these [IIT 1998]

(29) If $f(x) = \frac{x^2 - 1}{x^2 + 1}$, for every real number x, then the minimum value of f

(a) does not exist as f is unbounded (b) is equal to 1
(c) is not attained even though f is bounded (d) is equal to -1 [IIT 1998]

(30) If $f(x) = 3x - 5$, then $f^{-1}(x)$

(a) is given by $\frac{1}{3x - 5}$ (b) is given by $\frac{x+5}{3}$
(c) does not exist because f is not one-one
(d) does not exist because f is not onto [IIT 1998]

(31) If $g[f(x)] = |\sin x|$ and $f[g(x)] = (\sin \sqrt{x})^2$, then

(a) $f(x) = \sin^2 x$, $g(x) = \sqrt{x}$ (b) $f(x) = \sin x$, $g(x) = |x|$
(c) $f(x) = x^2$, $g(x) = \sin \sqrt{x}$ (d) f and g cannot be determined [IIT 1998]

(32) If $f(x) = (x + 1)^2 - 1$, $(x \geq -1)$, then the set $S = \{x : f(x) = f^{-1}(x)\}$ is

(a) $\left\{0, -1, \frac{-3 + i\sqrt{3}}{2}, \frac{-3 - i\sqrt{3}}{2}\right\}$ (b) $\{0, 1, -1\}$
(c) $\{0, -1\}$ (d) empty [IIT 1995]

(33) The number $\log_2 7$ is

(a) an integer (b) a rational number
(c) an irrational number (d) a prime number [IIT 1990]

(34) If S is the set of all real x such that $\frac{2x - 1}{2x^3 + 3x^2 + x}$ is positive, then S contains

(a) $\left(-\infty, -\frac{3}{2}\right)$ (b) $\left(-\frac{3}{2}, -\frac{1}{4}\right)$ (c) $\left(-\frac{1}{4}, -\frac{1}{2}\right)$
(d) $\left(\frac{1}{2}, 3\right)$ (e) none of these [IIT 1986]

(35) If $y = f(x) = \frac{x + 2}{x - 1}$, then

(a) $x = f(y)$ (b) $f(1) = 3$ (c) y increases with x for $x < 1$
(d) f is a rational function of x [IIT 1984]

(36) Let $f(x) = |x - 1|$. Then

(a) $f(x^2) = [f(x)]^2$ (b) $f(x + y) = f(x) + f(y)$
(c) $f(|x|) = |f(x)|$ (d) None of these [IIT 1983]

(37) The domain of definition of the function $y = \frac{1}{\log_{10}(1 - x)} + \sqrt{x + 2}$ is

(a) (- 3, - 2) excluding - 2.5 (b) [0, 1] excluding 0.5
(c) [- 2, 1] excluding 0 (d) None of these [IIT 1983]

(38) Which of the following functions is periodic ?

(a) $f(x) = x - [x]$ where $[x]$ denotes the largest integer less than or equal to the real number x
(b) $f(x) = \sin \frac{1}{x}$ for $x \neq 0$, $f(0) = 0$
(c) $f(x) = x \cos x$ (d) None of these [IIT 1983]

(39) If X and Y are two sets, then $X \cap (X \cup Y)^c$ equals

(a) X (b) Y (c) ϕ (d) none of these [IIT 1979]

(40) Let R be the set of real numbers. If $f : R \to R$ is a function defined by $f(x) = x^2$, then f is

(a) injective but not subjective (b) subjective but not injective
(c) bijective (d) none of these [IIT 1979]

Answers

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
a	d	a	c	a	d	b	b	d	b	a	d	b	a	b	c	b	d	a	b

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
a	d	a	d	c	d	b	c	d	b	a	c	c	a,d	a,d	d	c	a	c	d