05 - PERMUTATIONS AND COMBINATIONS

(Answers at the end of all questions)

(1) If the letters of the word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word 'SACHIN' appears at serial number

(a) 601 (b) 600 (c) 603 (d) 602 [AIEEE 2005]

(2) The value of $^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$ is

(a) $^{55}C_4$ (b) $^{55}C_3$ (c) $^{56}C_3$ (d) $^{56}C_4$ [AIEEE 2005]

(3) How many ways are here to arrange the letters in the word GARDEN with the vowels in alphabetical order ?

(a) 120 (b) 240 (c) 360 (d) 480 [AIEEE 2004]

(4) The number of ways of distributing 8 identical balls in 3 distinct boxes so that none of the boxes is empty is

(a) 5 (b) 21 (c) 3^8 (d) $^8C_3$ [AIEEE 2004]

(5) A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is

(a) 140 (b) 196 (c) 280 (d) 346 [AIEEE 2003]

(6) The number of ways in which 6 men and 5 women can dine at a round table, if no two women are to sit together, is given by

(a) 30 (b) $5! \times 5!$ (c) $5! \times 4!$ (d) $7! \times 5!$ [AIEEE 2003]

(7) If ^nC_r denotes the number of combinations of n things taken r at a time, then the value of expression $^nC_{r+1} + {}^nC_{r-1} + 2\,{}^nC_r$ is

(a) $^{n+2}C_r$ (b) $^{n+2}C_{r+1}$ (c) $^{n+1}C_r$ (d) $^{n+1}C_{r+1}$ [AIEEE 2003]

(8) If repetition of the digits is allowed, then the number of even natural numbers having three digits is

(a) 250 (b) 350 (c) 450 (d) 550 [AIEEE 2002]

(9) If $^{n+1}C_3 = 2\,^{n}C_2$, then the value of n is

(a) 3 (b) 4 (c) 5 (d) 6 [AIEEE 2002]

(10) If $^{n}C_{r-1} = 36$, $^{n}C_r = 84$ and $^{n}C_{r+1} = 126$, then n and r are respectively

(a) 9, 6 (b) 9, 3 (c) 6, 3 (d) 6, 2 [AIEEE 2002]

(11) If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \dots + C_n x^n$, then the value of $\frac{C_1}{C_0} + \frac{2C_2}{C_1} + \frac{3C_3}{C_2} + \dots + \frac{nC_n}{C_{n-1}}$ is

(a) $\frac{n}{2}$ (b) $n(n+1)$ (c) $\frac{n(n+1)}{12}$ (d) $\frac{n(n+1)}{2}$ [AIEEE 2002]

(12) A rectangle is constructed of lengths (2m - 1) and (2n - 1) units where m, n $\in$ I and small rectangles are inscribed in it by drawing parallel lines. Find the maximum number of rectangles that can be inscribed in it having odd unit length.

(a) $m^2 - n^2$ (b) $mn(m+1)(n+1)$
(c) 4^{m+n-2} (d) $m^2 n^2$ [IIT 2005]

(13) If $^{n-1}C_r = (k^2 - 3)\,^{n}C_{r+1}$, then k lies between

(a) $(-\infty, -2)$ (b) $(2, \infty)$ (c) $[-\sqrt{3}, \sqrt{3}]$ (d) $]\sqrt{3}, 2]$ [IIT 2004]

(14) The number of arrangements of the letters of the word BANANA in which the two N's do not appear adjacently is

(a) 40 (b) 60 (c) 80 (d) 100 [IIT 2002]

(Answers at the end of all questions)

(15) Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon on n sides. If $T_{n+1} - T_n = 21$, then n equals

(a) 5 (b) 7 (c) 6 (d) 4 [IIT 2001]

(16) For $2 \le r \le n$, $\binom{n}{r} + 2\binom{n}{r-1} + \binom{n}{r-2} =$

(a) $\binom{n+1}{r-1}$ (b) $2\binom{n+1}{r+1}$ (c) $2\binom{n+2}{r}$ (d) $\binom{n+2}{r}$ [IIT 2000]

(17) How many different nine digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even positions.

(a) 16 (b) 36 (c) 60 (d) 180

(18) If $a_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$, then $\sum_{r=0}^{n} \frac{r}{{}^nC_r}$ equals

(a) $(n - 1)a_n$ (b) $n a_n$ (c) $\frac{1}{2} n a_n$ (d) none of these [IIT 1998]

(19) An n - digit number is a positive number with exactly n digits. Nine hundred distinct n - digit numbers are to be formed using only the three digits 2, 5 and 7. The smallest value of n for which this is possible is

(a) 6 (b) 7 (c) 8 (d) 9 [IIT 1998]

(20) Number of divisors of the form $4n + 2$ $(n \ge 0)$ of the integer 240 is

(a) 4 (b) 8 (c) 10 (d) 3 [IIT 1998]

(21) A five digit number divisible by 3 is to be formed using the numerals 0, 1, 2, 3, 4 and 5 without repetition. The total number of ways in which this can be done is

(a) 216 (b) 600 (c) 240 (d) 3125 [IIT 1989]

(Answers at the end of all questions)

(22) If C_r stands for ${}^nC_r$, then the sum of the series

$$\frac{2\left(\frac{n}{2}\right)!\left(\frac{n}{2}\right)!}{n!}\left[C_0^2 - 2C_1^2 + 3C_2^2 - \dots + (-1)^n (n+1) C_n^2\right],$$

where n is an even positive integer, is equal to

(a) 0 (b) $(-1)^{n/2}(n+1)$ (c) $(-1)^n(n+2)$ (d) $(-1)^n n$
(e) none of these [IIT 1986]

(23) Eight chairs are numbered 1 to 8. Two women and three men wish to occupy one chair each. First the women choose the chairs from amongst the chairs marked 1 to 4, and then the men select the chairs from amongst the remaining. The number of possible arrangements is

(a) ${}^6C_3 \times {}^4C_2$ (b) ${}^4P_2 \times {}^4P_3$ (c) ${}^4C_2 \times {}^4P_3$ (d) none of these [IIT 1982]

(24) Ten different letters of an alphabet are given. Words with five letters are formed from these given letters. Then, the number of words which have at least one letter repeated is

(a) 69,760 (b) 30,240 (c) 99,748 (d) none of these [IIT 1980]

(25) The value of the expression ${}^{47}C_4 + \sum_{j=1}^{5} {}^{52-j}C_3$ is equal to

(a) ${}^{47}C_5$ (b) ${}^{52}C_5$ (c) ${}^{52}C_4$ (d) none of these [IIT 1980]

(26) ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, then r is

(a) 1 (b) 2 (c) 3 (d) none of these [IIT 1979]

(27) There are 27 points in a plane. 5, 10 and 15 points are collinear on distinct lines. By joining these points, how many distinct lines can be formed ?

(a) 194 (b) 170 (c) 435 (d) none of these

(28) In the above Q. 27, how many distinct triangles can be formed whose vertices are the given 27 points.

(a) ${}^{27}C_3$ (b) 2300 (c) 2320 (d) 2340

(29) The number of ways of putting 10 different things in 2 boxes such that there are not less than 2 things in any of the two boxes is

(a) 1024 (b) 1023 (c) 1013 (d) 1002

(30) The product of r consecutive positive integers divided by r ! is

(a) a proper fraction (b) a positive integer (c) r (d) none of these

(31) If ${}^{n}C_{r-1} = 36$, ${}^{n}C_{r} = 84$ and ${}^{n}C_{r+1} = 36$, then r =

(a) 1 (b) 2 (c) 3 (d) 4

(32) ${}^{30}C_{10} + {}^{30}C_{11} + {}^{31}C_{12} + {}^{32}C_{13} - {}^{33}C_{13} =$

(a) 0 (b) ${}^{32}C_{13}$ (c) ${}^{33}C_{14}$ (d) ${}^{32}C_{14}$

(33) A polygon has 54 diagonals. The total number of distinct triangles that can be formed using its vertices is

(a) 220 (b) 165 (c) 286 (d) 216

(34) A set of 5 parallel lines with distances 1, 2, 3, 4 between consecutive lines intersects another set of 5 parallel lines oblique to the first set with distances 1.5, 2.5, 3.5, 4.5 between consecutive lines. The number of rhombuses formed is equal to

(a) 1 (b) 2 (c) 3 (d) 4

(35) Four dice are rolled. The number of possible outcomes in which at least two dice show 6 is

(a) 216 (b) 900 (c) 150 (d) 171

(36) Six points in a plane are joined in all possible ways by indefinite straight lines. No two of them are coincident or parallel and no three pass through the same point (with the exception of the original six points). The number of distinct points of intersection is equal to

(a) 105 (b) 45 (c) 51 (d) none of these

(37) 6 men and 4 women are to be seated in a row so that no two women sit together. The number of ways they can be seated is

(a) 604800 (b) 17280 (c) 120960 (d) 518400

(38) A test consists of 10 multiple choice questions each having four alternative answers of which exactly two are correct. A student has to mark two answers and his answer is considered correct only if both the selected answers are correct. The number of ways of getting exactly 8 correct answers by a student answering all the questions is

(a) 1125 (b) 405 (c) 180 (d) none of these

(39) 10 boys and 10 girls sit alternately in a row and then alternately along a circle. The ratio of number of ways of sitting in a row to the number of ways of sitting along a circle is

(a) 5 (b) 10 (c) 15 (d) 20

(40) The sum of all 4 digits that can be formed by using the digits 2, 4, 6, 8 allowing repetition of digits is p and without allowing repetition of digits is q. The ratio of p to q is

(a) $\frac{32}{3}$ (b) $\frac{16}{3}$ (c) $\frac{64}{3}$ (d) 16

Answers

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
a	d	c	b	b	b	b	c	c	b	d	d	d	a	b	c	c	c	b	a

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
c	e	d	c	c	c	a	d	d	b	c	a	a	c	d	c	a	a	d	a