

**9 - INTEGRAL CALCULUS**  
( Answers at the end of all questions )

(1) If  $I_1 = \int_0^1 2^{x^2} dx$ ,  $I_2 = \int_0^1 2^{x^3} dx$ ,  $I_3 = \int_1^2 2^{x^2} dx$ ,  $I_4 = \int_1^2 2^{x^3} dx$ , then

- (a)  $I_2 > I_1$    (b)  $I_1 > I_2$    (c)  $I_3 = I_4$    (d)  $I_3 > I_4$  [ AIEEE 2005 ]

(2) The area enclosed between the curve  $y = \log_e(x + e)$  and the coordinate axes is

- (a) 1   (b) 2   (c) 3   (d) 4 [ AIEEE 2005 ]

(3) The parabolas  $y^2 = 4x$  and  $x^2 = 4y$  divide the square region bounded by the lines  $x = 4$ ,  $y = 4$  and the coordinate axes. If  $S_1, S_2, S_3$  are respectively the area of these parts numbered from top to bottom, then  $S_1 : S_2 : S_3$  is

- (a) 1 : 2 : 1   (b) 1 : 3 : 1   (c) 2 : 1 : 2   (d) 1 : 1 : 1 [ AIEEE 2005 ]

(4)  $\int \left\{ \frac{(\log x - 1)}{1 + (\log x)^2} \right\}^2 dx$  is equal to

- (a)  $\frac{\log x}{(\log x)^2 + 1} + c$    (b)  $\frac{x}{x^2 + 1} + c$   
 (c)  $\frac{xe^x}{1 + x^2} + c$    (d)  $\frac{x}{(\log x)^2 + 1} + c$  [ AIEEE 2005 ]

(5) Let  $f(x)$  be a non-negative continuous function such that the area bounded by the curve  $y = f(x)$ , X-axis and the ordinates  $x = \frac{\pi}{4}$  and  $x = \beta > \frac{\pi}{4}$  is

$(\beta \sin \beta + \frac{\pi}{4} \cos \beta + \sqrt{2} \beta)$ . Then  $f(\frac{\pi}{2})$  is

- (a)  $\frac{\pi}{4} + \sqrt{2} - 1$    (b)  $\frac{\pi}{4} - \sqrt{2} + 1$   
 (c)  $1 - \frac{\pi}{4} - \sqrt{2}$    (d)  $1 - \frac{\pi}{4} + \sqrt{2}$  [ AIEEE 2005 ]

(6) The value of  $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1 + a^x} dx$ ,  $a > 0$  is

- (a)  $a\pi$    (b)  $\frac{\pi}{2}$    (c)  $\frac{\pi}{a}$    (d)  $2\pi$  [ AIEEE 2005 ]

(7)  $\lim_{n \rightarrow \infty} \sum_{r=1}^n e^{\frac{r}{n}}$  is

- (a)  $e$       (b)  $e - 1$       (c)  $1 - e$       (d)  $e + 1$

[ AIEEE 2004 ]

(8) If  $\int \frac{\sin x}{\sin(x - \alpha)} dx = Ax + B \log \sin(x - \alpha) + C$ , then the value of (A, B) is

- (a)  $(\sin \alpha, \cos \alpha)$       (b)  $(\cos \alpha, \sin \alpha)$   
(c)  $(-\sin \alpha, \cos \alpha)$       (d)  $(-\cos \alpha, \sin \alpha)$

[ AIEEE 2004 ]

(9)  $\int \frac{dx}{\cos x - \sin x}$  is equal to

- (a)  $\frac{1}{\sqrt{2}} \log \left| \tan \left( \frac{x}{2} - \frac{\pi}{8} \right) \right| + C$       (b)  $\frac{1}{\sqrt{2}} \log \left| \cot \left( \frac{x}{2} \right) \right| + C$   
(c)  $\frac{1}{\sqrt{2}} \log \left| \tan \left( \frac{x}{2} - \frac{3\pi}{8} \right) \right| + C$       (d)  $\frac{1}{\sqrt{2}} \log \left| \tan \left( \frac{x}{2} + \frac{3\pi}{8} \right) \right| + C$

[ AIEEE 2004 ]

(10) The value of  $\int_{-1}^3 |1 - x^2| dx$  is

- (a)  $\frac{28}{3}$       (b)  $\frac{14}{3}$       (c)  $\frac{7}{3}$       (d)  $\frac{1}{3}$

[ AIEEE 2004 ]

(11) The value of  $I = \int_0^{\frac{\pi}{2}} \frac{(\sin x + \cos x)^2}{\sqrt{1 + \sin 2x}} dx$  is

- (a) 0      (b) 1      (c) 2      (d) 3

[ AIEEE 2004 ]

(12) If  $\int_0^{\pi} x f(\sin x) dx = A \int_0^{\frac{\pi}{2}} f(\sin x) dx$ , then A is equal to

- (a) 0      (b)  $\pi$       (c)  $\frac{\pi}{4}$       (d)  $2\pi$

[ AIEEE 2004 ]

**9 - INTEGRAL CALCULUS**  
( Answers at the end of all questions )

(13) If  $f(x) = \frac{e^x}{1 + e^x}$ ,  $I_1 = \int_{f(-a)}^{f(a)} x g\{x(1-x)\} dx$  and  $I_2 = \int_{f(-a)}^{f(a)} g\{x(1-x)\} dx$ ,

then the value of  $\frac{I_2}{I_1}$  is

- (a) 2      (b) -3      (c) -1      (d) 1

[ AIEEE 2004 ]

(14) The area of the region bounded by the curves  $y = |x - 2|$ ,  $x = 1$ ,  $x = 3$  and X-axis is

- (a) 1      (b) 2      (c) 3      (d) 4

[ AIEEE 2004 ]

(15) The value of  $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} \sec^2 t dt}{x \sin x}$  is

- (a) 3      (b) 2      (c) 1      (d) 0

[ AIEEE 2003 ]

(16) The value of the integral  $I = \int_0^1 x(1-x)^n dx$  is

- (a)  $\frac{1}{n+1}$       (b)  $\frac{1}{n+2}$       (c)  $\frac{1}{n+1} - \frac{1}{n+2}$       (d)  $\frac{1}{n+1} + \frac{1}{n+2}$

[ AIEEE 2003 ]

(17) If  $f(y) = e^{-y}$ ,  $g(y) = y$ ,  $y > 0$  and  $F(t) = \int_0^t f(t-y)g(y)dy$ , then

- (a)  $F(t) = te^t$       (b)  $F(t) = te^{-t}$   
(c)  $F(t) = e^t - (1+t)$       (d)  $F(t) = 1 - e^{-t}(1+t)$

[ AIEEE 2003 ]

(18) If  $f(a+b-x) = f(x)$ , then the value of  $\int_a^b x f(x) dx$  is

- (a)  $\frac{b-a}{2} \int_a^b f(x) dx$       (b)  $\frac{a+b}{2} \int_a^b f(x) dx$   
(c)  $\frac{a+b}{2} \int_a^b f(b-x) dx$       (d)  $\int_a^b f(a+b-x) dx$

[ AIEEE 2003 ]

**9 - INTEGRAL CALCULUS**  
(Answers at the end of all questions)

(19) Let  $\frac{d}{dx} F(x) = \frac{e^{\sin x}}{x}$ ,  $x > 0$ . If  $\int_1^4 \frac{3}{x} e^{\sin x^3} dx = F(K) - F(1)$ , then one of the possible values of K is

- (a) 15      (b) 16      (c) 63      (d) 64

[ AIEEE 2003 ]

(20) Let  $f(x)$  be a function satisfying  $f'(x) = f(x)$  with  $f(0) = 1$  and  $g(x)$  be a function that satisfies  $f(x) + g(x) = x^2$ . The value of the integral  $\int_0^1 f(x)g(x) dx$  is

- (a)  $e - \frac{e^2}{2} - \frac{5}{2}$       (b)  $e + \frac{e^2}{2} - \frac{3}{2}$   
(c)  $e - \frac{e^2}{2} - \frac{3}{2}$       (d)  $e + \frac{e^2}{2} + \frac{5}{2}$

[ AIEEE 2003 ]

(21) If  $\int x \sin x dx = -x \cos x + \alpha$ , then the value of  $\alpha$  is

- (a)  $\sin x + c$       (b)  $\cos x + c$   
(c)  $x \cos x + c$       (d)  $\cos x - \sin x + c$

[ AIEEE 2002 ]

(22) The value of  $\int \frac{1 - \cos 2x}{\cos 2x + 1} dx$  is

- (a)  $\tan x - x + c$       (b)  $x + \tan x + c$   
(c)  $x - \tan x + c$       (d)  $-x - \cot x + c$

[ AIEEE 2002 ]

(23) The value of  $\int_0^{\frac{\pi}{2}} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$  is

- (a)  $\pi ab$       (b)  $\pi^2 ab$       (c)  $\pi/ab$       (d)  $\pi/2ab$

[ AIEEE 2002 ]

(24) The value of  $\int e^{3 \log x} (x^4 + 1)^{-1} dx$  is

- (a)  $\log(x^4 + 1) + c$       (b)  $\frac{1}{4} \log(x^4 + 1) + c$   
(c)  $3 \log(x^4 + 1) + c$       (d)  $-\log(x^4 + 1) + c$

[ AIEEE 2002 ]

(25) The value of  $\int \frac{\log x}{x^2} dx$  is

- (a)  $\log(x + 1) + c$       (b)  $-\frac{1}{x} \log(x + 1) + c$   
(c)  $\log(x - 1) + c$       (d)  $\frac{1}{2} \log(x + 1) + c$

[ AIEEE 2002 ]

(26) The value of  $\int_0^{\sin^2 x} \sin^{-1}(\sqrt{t}) dt + \int_0^{\cos^2 x} \cos^{-1}(\sqrt{t}) dt$  is

- (a)  $\frac{\pi}{2}$       (b) 1      (c)  $\frac{\pi}{4}$       (d)  $\pi$

[ AIEEE 2002 ]

(27) If the area bounded by the X-axis, the curve  $y = f(x)$  and the lines  $x = 1$ ,  $x = b$  is equal to  $\sqrt{b^2 + 1} - \sqrt{2}$  for all  $b > 1$ , then  $f(x)$  is

- (a)  $\sqrt{x - 1}$       (b)  $\sqrt{x + 1}$       (c)  $\sqrt{x^2 + 1}$       (d)  $\frac{x}{\sqrt{1 + x^2}}$

[ AIEEE 2002 ]

(28)  $\int_{-2}^0 [x^3 + 3x^2 + 3x + 3 + (x + 1) \cos(x + 1)] dx =$

- (a) 4      (b) 0      (c) -1      (d) 1

[ IIT 2005 ]

(29) Find the area between the curves  $y = (x - 1)^2$ ,  $y = (x + 1)^2$  and  $y = \frac{1}{4}$

- (a)  $\frac{1}{3}$       (b)  $\frac{2}{3}$       (c)  $\frac{4}{3}$       (d)  $\frac{1}{6}$

[ IIT 2005 ]

(30) If  $\int_{\sin x}^1 t^2 f(t) dt = 1 - \sin x$ ,  $x \in [0, \pi/2]$ , then  $f(1/\sqrt{3})$  is

- (a) 3      (b) 1/3      (c) 1      (d)  $\sqrt{3}$

[ IIT 2005 ]

(31) If  $\int_0^{t^2} x f(x) dx = \frac{2}{5} t^5$  for  $t > 0$ , then  $f\left(\frac{4}{25}\right)$  is

- (a)  $-\frac{2}{5}$       (b) 0      (c)  $\frac{2}{5}$       (d) 1

[ IIT 2004 ]

(32)  $\int_0^1 \sqrt{\frac{1-x}{1+x}} dx$  is equal to

- (a)  $\frac{\pi}{2} + 1$       (b)  $\frac{\pi}{2} - 1$       (c) 1      (d)  $\pi$

[ IIT 2004 ]

(33) If the area bounded by the curves  $x = ay^2$  and  $y = ax^2$  is 1, then  $a$  is equal to

- (a)  $\frac{1}{\sqrt{3}}$       (b)  $\frac{1}{3}$       (c)  $\frac{1}{2}$       (d) 3

[ IIT 2004 ]

(34) If  $f(x) = \int_{x^2}^{x^2+1} e^{-t^2} dt$ , then the interval in which  $f(x)$  is increasing is

- (a)  $(0, \infty)$       (b)  $(-\infty, 0)$       (c)  $[-2, 2]$       (d) nowhere

[ IIT 2003 ]

(35) If  $I(m, n) = \int_0^1 t^m (1+t)^n dt$ ,  $m, n \in \mathbb{R}$ , then  $I(m, n)$  is

- (a)  $\frac{n}{1+m} I[(m+1), (n-1)]$       (b)  $\frac{2^n}{1+m} - \frac{m}{1+n} I[(m+1), (n-1)]$   
(c)  $\frac{2^n}{1+m} - \frac{m}{1+m} I[(m+1), (n-1)]$       (d)  $\frac{m}{n+1} I[(m+1), (n-1)]$

[ IIT 2003 ]

(36) Area bounded by the curves  $y = \sqrt{x}$ ,  $x = 2y + 3$  in the first quadrant and X-axis is

- (a)  $2\sqrt{3}$       (b) 18      (c) 9      (d)  $\frac{34}{3}$

[ IIT 2003 ]

(37) The area bounded by the curves  $y = |x| - 1$  and  $y = -|x| + 1$  is

- (a) 1      (b) 2      (c)  $2\sqrt{2}$       (d) 4

[ IIT 2002 ]

(38) If  $f(x) = \int_1^x \sqrt{2-t^2} dt$ , then the real roots of the equation  $x^2 - f'(x) = 0$  are

- (a)  $\pm 1$       (b)  $\pm \frac{1}{\sqrt{2}}$       (c)  $\pm \frac{1}{2}$       (d) 0 and 1

[ IIT 2002 ]

(39) Let  $T > 0$  be a fixed real number. Suppose  $f$  is a continuous function such that for all  $x \in \mathbb{R}$ ,  $f(x + T) = f(x)$ . If  $I = \int_0^T f(x) dx$ , then the value of  $\int_3^{3+3T} f(2x) dx$  is

(a)  $\frac{3}{2}I$       (b)  $I$       (c)  $3I$       (d)  $6I$       [ IIT 2002 ]

(40) The integral equals  $\int_{-\frac{1}{2}}^{\frac{1}{2}} \left( [x] + \ln\left(\frac{1+x}{1-x}\right) \right) dx$  equals

(a)  $-\frac{1}{2}$       (b)  $0$       (c)  $1$       (d)  $2 \ln \frac{1}{2}$       [ IIT 2002 ]

(41) If  $f: (0, \infty) \rightarrow \mathbb{R}$ ,  $F(x) = \int_0^x f(t) dt$  and  $F(x^2) = x^2(1+x)$ , then  $f(4)$  equals

(a)  $\frac{5}{4}$       (b)  $7$       (c)  $4$       (d)  $2$       [ IIT 2001 ]

(42) The value of  $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$ ,  $a > 0$ , is

(a)  $\pi$       (b)  $a\pi$       (c)  $\frac{\pi}{2}$       (d)  $2\pi$       [ IIT 2001 ]

(43) If  $f(x) = \begin{cases} e^{\cos x} \sin x & \text{for } |x| \leq 2, \\ 2 & \text{otherwise,} \end{cases}$  then  $\int_{-2}^3 f(x) dx =$

(a)  $0$       (b)  $1$       (c)  $2$       (d)  $3$       [ IIT 2000 ]

(44) The value of the integral  $\int_{e^{-1}}^{e^2} \left| \frac{\log_e x}{x} \right| dx$  is

(a)  $\frac{3}{2}$       (b)  $\frac{5}{2}$       (c)  $3$       (d)  $5$       [ IIT 2000 ]

(45) If  $f(x) = \int e^x (x-1)(x-2) dx$ , then  $f$  decreases in the interval

- (a)  $(-\infty, -2)$       (b)  $(-2, -1)$       (c)  $(1, 2)$       (d)  $(2, +\infty)$       [ IIT 2000 ]

(46) Let  $g(x) = \int_0^x f(t) dt$ , where  $f$  is such that  $\frac{1}{2} \leq f(t) \leq 1$  for  $t \in [0, 1]$  and  $0 \leq f(t) \leq \frac{1}{2}$  for  $t \in [1, 2]$ . Then  $g(2)$  satisfies the inequality

- (a)  $-\frac{3}{2} \leq g(2) \leq \frac{1}{2}$       (b)  $0 \leq g(2) \leq 2$   
(c)  $\frac{3}{2} \leq g(2) \leq \frac{5}{2}$       (d)  $2 < g(2) < 4$       [ IIT 2000 ]

(47) If for a real number  $y$ ,  $[y]$  is the greatest integer less than or equal to  $y$ , then the

value of the integral  $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} [2 \sin x] dx$  is

- (a)  $-\pi$       (b)  $0$       (c)  $-\frac{\pi}{2}$       (d)  $\frac{\pi}{2}$       [ IIT 1999 ]

(48)  $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1 + \cos x} =$

- (a)  $2$       (b)  $-2$       (c)  $\frac{1}{2}$       (d)  $-\frac{1}{2}$       [ IIT 1999 ]

(49) For which of the following values of  $m$ , is the area of the region bounded by the curve  $y = x - x^2$  and the line  $y = mx$  equals  $\frac{9}{2}$ ?

- (a)  $-4$       (b)  $-2$       (c)  $2$       (d)  $4$       [ IIT 1999 ]

(50) If  $f(x) = x - [x]$ , for every real number  $x$ , where  $[x]$  is the integral part of  $x$ , then

$\int_{-1}^1 f(x) dx$  is

- (a)  $1$       (b)  $2$       (c)  $0$       (d)  $\frac{1}{2}$       [ IIT 1998 ]



(51) If  $g(x) = \int_0^x \cos^4 t \, dt$ , then  $g(x + \pi)$  equals

- (a)  $g(x) + g(\pi)$    (b)  $g(x) - g(\pi)$    (c)  $g(x)g(\pi)$    (d)  $\frac{g(x)}{g(\pi)}$    [ IIT 1997 ]

(52) Let  $f$  be a positive function. If  $I_1 = \int_{1-k}^k x f[x(1-x)] dx$  and  $I_2 = \int_{1-k}^k f[x(1-x)] dx$ , where  $2k - 1 > 0$ , then  $\frac{I_1}{I_2}$  is

- (a) 2   (b)  $k$    (c)  $\frac{1}{2}$    (d) 1   [ IIT 1997 ]

(53) The slope of the tangent to a curve  $y = f(x)$  at  $[x, f(x)]$  is  $2x + 1$ . If the curve passes through the point  $(1, 2)$ , then the area of the region bounded by the curve, the X-axis and the line  $x = 1$  is

- (a)  $\frac{5}{6}$    (b)  $\frac{6}{5}$    (c)  $\frac{1}{6}$    (d) 6   [ IIT 1995 ]

(54) The value of  $\int_{\pi}^{2\pi} [2 \sin x] dx$  where  $[.]$  represents the greatest integer function, is

- (a)  $-\frac{5\pi}{3}$    (b)  $-\pi$    (c)  $\frac{5\pi}{3}$    (d)  $-2\pi$    [ IIT 1995 ]

(55) The value of  $\int_0^{\frac{\pi}{2}} \frac{dx}{1 + \tan^3 x}$  is

- (a) 0   (b) 1   (c)  $\frac{\pi}{2}$    (d)  $\frac{\pi}{4}$    [ IIT 1993 ]

(56) If  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a differentiable function and  $f(1) = 4$ , then the value of

$$\lim_{x \rightarrow 1} \int_4^{f(x)} \frac{2t}{x-1} dt \text{ is}$$

- (a)  $8f'(1)$    (b)  $4f'(1)$    (c)  $2f'(1)$    (d)  $f'(1)$    [ IIT 1990 ]

(57) If  $f: \mathbb{R} \rightarrow \mathbb{R}$  and  $g: \mathbb{R} \rightarrow \mathbb{R}$  are continuous functions, then the value of the integral

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [f(x) + f(-x)][g(x) + g(-x)] dx \text{ is}$$

- (a)  $\pi$       (b) 1      (c) -1      (d) 0

[ IIT 1990 ]

(58) For any integer  $n$ , the integral  $\int_0^{\pi} e^{\cos^2 x} \cos^3 (2n + 1)x dx$  has the value

- (a)  $\pi$       (b) 1      (c) 0      (d) none of these

[ IIT 1985 ]

(59) The value of the integral  $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx$  is

- (a)  $\frac{\pi}{4}$       (b)  $\frac{\pi}{2}$       (c)  $\pi$       (d) none of these

[ IIT 1983 ]

(60) If the area bounded by the curves  $y = f(x)$ , the X-axis and the ordinates  $x = 1$  and  $x = b$  is  $(b - 1) \sin(3b + 4)$ , then  $f(x)$  is

- (a)  $(x - 1) \cos(3x + 4)$       (b)  $\sin(3x + 4) + 3(x - 1) \cos(3x + 4)$   
(c)  $\sin(3x + 4)$       (d) none of these

[ IIT 1982 ]

(61) The value of the definite integral  $\int_0^1 (1 + e^{-x^2}) dx$  is

- (a) -1      (b) 2      (c)  $1 + e^{-1}$       (d) none of these

[ IIT 1981 ]

**9 - INTEGRAL CALCULUS**  
( Answers at the end of all questions )

**Answers**

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
b	a	d	d	d	b	b	b	a,d	a	2	b	a	a	c	c	c	b	d	b
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
a	c	d	b	b	c	d	a	a	a	c	b	a	b	c	b	b	a	c	a
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
c	c	c	b	c	b	c	a	b,d	a	a	c	a	a	d	a	d	c	a	b
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
d																			

www.examrace.com