08 - DIFFERENTIAL CALCULUS

(Answers at the end of all questions)

(1) $\lim_{n \to \infty} \left[\frac{1}{n^2} \sec^2 \frac{1}{n^2} + \frac{2}{n^2} \sec^2 \frac{4}{n^2} + \ldots\ldots\ldots + \frac{1}{n} \sec^2 1 \right]$ is

(a) $\frac{1}{2} \sec 1$ (b) $\frac{1}{2} \operatorname{cosec} 1$ (c) $\tan 1$ (d) $\frac{1}{2} \tan 1$ [AIEEE 2005]

(2) The normal to the curve $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$ at any point 'θ' is such that

(a) it passes through the origin

(b) it makes angle $\frac{\pi}{2} + \theta$ with the X-axis

(c) it passes through $\left(a\frac{\pi}{2}, -a \right)$

(d) it is at a constant distance from the origin

[AIEEE 2005]

(3) A function is matched below against an interval where it is supposed to be increasing. Which of the following pairs is incorrectly matched ?

	Interval	Function		Interval	Function
(a)	$(-\infty, \infty)$	$x^3 - 3x^2 + 3x + 3$	(b)	$[2, \infty)$	$2x^3 - 3x^2 + 12x + 6$
(c)	$\left(-\infty, \frac{1}{3}\right)$	$3x^3 - 2x^2 + 1$	(d)	$(-\infty, -4)$	$x^3 - 6x^2 + 6$

[AIEEE 2005]

(4) Let α and β be the distinct roots of the equation $ax^2 + bx + c = 0$. Then $\lim_{x \to \alpha} \frac{1 - \cos(ax^2 + bx + c)}{(x - \alpha)^2}$ is equal to

(a) $\frac{a^2}{2}(\alpha - \beta)^2$ (b) 0 (c) $-\frac{a^2}{2}(\alpha - \beta)^2$ (d) $\frac{1}{2}(\alpha - \beta)^2$

[AIEEE 2005]

(5) Suppose $f(x)$ is differentiable at $x = 1$ and $\lim_{h \to 0} \frac{1}{h} f(1 + h) = 5$, then $f'(1)$ equals

(a) 3 (b) 4 (c) 5 (d) 6 [AIEEE 2005]

(Answers at the end of all questions)

(6) Let f be differentiable for al x. If $f(1) = -2$ and $f'(x) \geq 2$ for $x \in [1, 6]$, then

(a) $f(6) \geq 8$ (b) $f(6) < 8$ (c) $f(6) < 5$ (d) $f(6) = 5$ [AIEEE 2005]

(7) If f is a real valued differentiable function satisfying $|f(x) - f(y)| \leq (x - y)^2$, $x, y \in R$ and $f(0) = 0$, then $f(1)$ equals

(a) -1 (b) 0 (c) 2 (d) 1 [AIEEE 2005]

(8) A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of 50 cm^3/min. When the thickness of ice is 5 cm, then the rate at which thickness of ice decreases in cm/min is

(a) $\frac{1}{36\pi}$ (b) $\frac{1}{18\pi}$ (c) $\frac{1}{54\pi}$ (d) $\frac{5}{6\pi}$ [AIEEE 2005]

(9) Let $f : R \to R$ be a differentiable function having $f(2) = 6$, $f'(2) = \frac{1}{48}$. Then

$\lim_{x \to 2} \int_6^{f(x)} \frac{4t^3}{x - 2} dt$ equals

(a) 24 (b) 36 (c) 12 (d) 18 [AIEEE 2005]

(10) If the equation $a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x = 0$, $a_1 \neq 0$, $n \geq 2$ has a positive root $x = \alpha$, then the equation $n a_n x^{n-1} + (n - 1) a_{n-1} x^{n-2} + ... + a_1 = 0$ has a positive root which is

(a) greater than α (b) smaller than α
(c) greater than or equal to α (d) equal to α [AIEEE 2005]

(11) If $\lim_{x \to \infty} \left(1 + \frac{a}{x} + \frac{b}{x^2} \right)^{2x} = e^2$, then the values of a and b are

(a) $a \in R$, $b \in R$ (b) $a = 1$, $b \in R$
(c) $a \in R$, $b = 2$ (d) $a = 1$, $b = 2$ [AIEEE 2004]

(12) Let $f(x) = \dfrac{1 - \tan x}{4x - \pi}$, $x \neq \dfrac{\pi}{4}$, $x \in \left[0, \dfrac{\pi}{2}\right]$. If $f(x)$ is continuous in $\left[0, \dfrac{\pi}{2}\right]$, then $f\left(\dfrac{\pi}{4}\right)$ is

(a) 1 (b) $\dfrac{1}{2}$ (c) $-\dfrac{1}{2}$ (d) -1 [AIEEE 2004]

(13) If $x = e^{y + e^{y + \ldots\ldots \infty}}$, $x > 0$, then $\dfrac{dy}{dx}$ is

(a) $\dfrac{x}{1 + x}$ (b) $\dfrac{1}{x}$ (c) $\dfrac{1 - x}{x}$ (d) $\dfrac{1 + x}{x}$ [AIEEE 2004]

(14) A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is

(a) (2, 4) (b) (2, - 4) (c) $\left(-\dfrac{9}{8}, \dfrac{9}{2}\right)$ (d) $\left(\dfrac{9}{8}, \dfrac{9}{2}\right)$ [AIEEE 2004]

(15) A function $y = f(x)$ has a second order derivative $f''(x) = 6(x - 1)$. If its graph passes through the point (2, 1) and at that point the tangent to the graph is $y = 3x - 5$, then the function is

(a) $(x - 1)^2$ (b) $(x - 1)^3$ (c) $(x + 1)^3$ (d) $(x + 1)^2$ [AIEEE 2004]

(16) The normal to the curve $x = a(1 + \cos\theta)$, $y = a \sin\theta$ at 'θ' always passes through the fixed point

(a) (a, 0) (b) (0, a) (c) (0, 0) (d) (a, a) [AIEEE 2004]

(17) If $2a + 3b + 6c = 0$, then at least one root of the equation $ax^2 + bx + c = 0$ lies in the interval

(a) (0, 1) (b) (1, 2) (c) (2, 3) (d) (1, 3) [AIEEE 2004]

(18) Let $f(x)$ be a polynomial function of second degree. If $f(1) = f(-1)$ and a, b, c are in A. P., then $f'(a)$, $f'(b)$ and $f'(c)$ are in

(a) A. P. (b) G. P. (c) H. P. (d) A. G. P. [AIEEE 2003]

(19) $$\lim_{x \to \frac{\pi}{2}} \frac{\left[1 - \tan\left(\frac{x}{2}\right)\right][1 - \sin x]}{\left[1 + \tan\left(\frac{x}{2}\right)\right][\pi - 2x]^3} =$$

(a) 0 (b) ∞ (c) $\frac{1}{32}$ (d) $\frac{1}{8}$ [AIEEE 2003]

(20) If $\lim_{x \to 0} \frac{\log(3 + x) - \log(3 - x)}{x} = k$, then the value f k is

(a) 0 (b) $-\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $-\frac{2}{3}$ [AIEEE 2003]

(21) If $f(x) = \frac{\log(1 + ax) - \log(1 - bx)}{x}$ is continuous at $x = 0$, then the value of $f(0)$ is

(a) ab (b) a + b (c) a - b (d) log a - log b [AIEEE 2003]

(22) If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$, then the value of $\frac{dy}{dx}$ is

(a) 0 (b) 1 (c) x (d) y [AIEEE 2003]

(23) The value of $\lim_{n \to \infty} \frac{1 + 2^4 + 3^4 + \ldots + n^4}{n^5}$ is

(a) zero (b) $\frac{1}{4}$ (c) $\frac{1}{5}$ (d) $\frac{1}{30}$ [AIEEE 2003]

(24) If $f : R \to R$ satisfies $f(x + y) = f(x) + f(y)$, for all $x, y \in R$ and $f(1) = 7$, then the value of $\sum_{r=1}^{n} f'(r)$ is

(a) $\frac{7n}{2}$ (b) $7n(n + 1)$ (c) $\frac{7(n + 1)}{2}$ (d) $\frac{7n(n + 1)}{2}$ [AIEEE 2003]

(25) The real number x when added to its inverse gives the minimum value of the sum at x equal to

(a) 2 (b) - 2 (c) 1 (d) - 1 [AIEEE 2003]

(26) If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where $a > 0$, attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a equals

(a) 3 (b) 1 (c) 2 (d) 4 [AIEEE 2003]

(27) If $f(x) = x^n$, then the value of $f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} + \frac{f'''(1)}{3!} + \ldots + \frac{(-1)^n f^n(1)}{n!}$ is

(a) 2^n (b) 2^{n-1} (c) 1 (d) 0 [AIEEE 2003]

(28) If $x = t^2 + t + 1$ and $y = \sin\left(\frac{\pi}{2}t\right) + \cos\left(\frac{\pi}{2}t\right)$, then at $t = 1$, the value of $\frac{dy}{dx}$ is

(a) $\frac{\pi}{2}$ (b) $-\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) $-\frac{\pi}{4}$ [AIEEE 2002]

(29) If $x = 3\cos\theta - 2\cos^3\theta$ and $y = 3\sin\theta - 2\sin^3\theta$, then the value of $\frac{dy}{dx}$ is

(a) $\sin\theta$ (b) $\cos\theta$ (c) $\tan\theta$ (d) $\cot\theta$ [AIEEE 2002]

(30) Let $f(a) = g(a) = k$ and their nth derivatives $f^n(a)$, $g^n(a)$ exist and are not equal for some n. Further if $\lim\limits_{x \to a} \dfrac{f(a)g(x) - f(a) - g(a)f(x) + g(a)}{g(x) - f(x)} = 4$, then the value of k is

(a) 4 (b) 2 (c) 1 (d) 0 [AIEEE 2002]

(31) The value of $\lim\limits_{x \to 0} \dfrac{(1 - \cos 2x)\sin 5x}{x^2 \sin 3x}$ is

(a) $\dfrac{10}{3}$ (b) $\dfrac{3}{10}$ (c) $\dfrac{6}{5}$ (d) $\dfrac{5}{6}$ [AIEEE 2002]

(32) The value of $\lim\limits_{\alpha \to \beta} \left[\dfrac{\sin^2 \alpha - \sin^2 \beta}{\alpha^2 - \beta^2} \right]$ is

(a) 0 (b) 1 (c) $\dfrac{\sin \beta}{\beta}$ (d) $\dfrac{\sin 2\beta}{2\beta}$ [AIEEE 2002]

(33) The value of $\lim\limits_{x \to 0} \dfrac{\sqrt{1 - \cos 2x}}{x}$ is

(a) 0 (b) 1 (c) $\sqrt{2}$ (d) does not exist [AIEEE 2002]

(34) If $f(x) = 2x^3 - 3x^2 - 12x + 5$ on $[-2, 4]$, then relative maximum occurs at x =

(a) -2 (b) -1 (c) 2 (d) 4 [AIEEE 2002]

(35) If $f(x) = \begin{cases} x\, e^{-\left(\frac{1}{|x|} + \frac{1}{x}\right)}, & x \neq 0, \\ 0, & x = 0 \end{cases}$ then $f(x)$ is

(a) discontinuous everywhere
(b) continuous as well as differentiable for all x
(c) neither differentiable nor continuous at x = 0
(d) continuous at all x but not differentiable at x = 0 [AIEEE 2002]

(36) If y is a twice differentiable function and $x \cos y + y \cos x = \pi$, then $y''(0) =$

(a) π (b) $-\pi$ (c) 0 (d) 1 [IIT 2005]

(37) $f(x) = \left|\, |x| - 1 \,\right|$ is not differentiable at $x =$

(a) 0, ± 1 (b) ± 1 (c) 0 (d) 1 [IIT 2005]

(38) If f is a differentiable function such that $f : R \to R$, $f\left(\frac{1}{n}\right) = 0 \ \forall \ n \in I$, $n \geq 1$, then

(a) $f(x) = 0 \ \forall \ x \in [0, 1]$ (b) $f(0) = 0$, but $f'(0)$ may or may not be 0
(c) $f(0) = 0 = f'(0)$ (d) $|f(x)| \leq 1 \ \forall \ x \in [0, 1]$ [IIT 2005]

(39) f is a twice differentiable polynomial function of x such that $f(1) = 1$, $f(2) = 4$ and $f(3) = 9$, then

(a) $f''(x) = 2, \ \forall \ x \in R$ (b) $f''(x) = f'(x) = 5, \ x \in [1, 3]$
(c) $f''(x) = 2$ for only $x \in [1, 3]$ (d) $f''(x) = 3, \ x \in (1, 3)$ [IIT 2005]

[Note: This question should have been better put as ‘polynomial function of degree two rather than twice differentiable function’.]

(40) S is a set of polynomial of degree less than or equal to 2, $f(0) = 0$, $f(1) = 1$, $f'(x) > 0, \ \forall \ x \in [0, 1]$, then set S =

(a) $ax + (1 - a)x^2, \ a \in R$ (b) $ax + (1 - a)x^2, \ 0 < a < 2$
(c) $ax + (1 - a)x^2, \ 0 < a < \infty$ (d) ϕ [IIT 2005]

(41) Let y be a function of x, such that $\log(x + y) = 2xy$, then $y'(0)$ is

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{3}{2}$ [IIT 2004]

(42) Let $f(x) = x^{\alpha} \log x$ for $x >$ and $f(0) = 0$ follows Rolle’s theorem for $x \in [0, 1]$, then α is

(a) -2 (b) -1 (c) 0 (d) $\frac{1}{2}$ [IIT 2004]

08 - DIFFERENTIAL CALCULUS

(Answers at the end of all questions)

(43) If $f(x)$ is strictly increasing and differentiable, then $\lim\limits_{x \to 0} \dfrac{f(x^2) - f(x)}{f(x) - f(0)}$ is

(a) 1 (b) - 1 (c) 0 (d) 2 [IIT 2004]

(44) Let $f(x) = x^3 + bx^2 + cx + d$, $0 < b^2 < c$, then $f(x)$

(a) is strictly increasing (b) has local maxima
(c) has local minima (d) is a bounded curve [IIT 2004]

(45) If $f(x)$ is a differentiable function, $f'(1) = 1$, $f'(2) = 6$, where $f'(c)$ means the derivative of the function at $x = c$, then

$$\lim_{h \to 0} \frac{f(2 + 2h + h^2) - f(2)}{f(1 + h - h^2) - f(1)}$$

(a) does not exist (b) - 3 (c) 3 (d) $\dfrac{3}{2}$ [IIT 2003]

(46) If $\lim\limits_{x \to 0} \dfrac{\sin nx[(a - n)nx - \tan x]}{x^2} = 0$, where n is a non-zero positive integer, then a is equal to

(a) $\dfrac{n+1}{n}$ (b) n^2 (c) $\dfrac{1}{n}$ (d) $n + \dfrac{1}{n}$ [IIT 2003]

(47) Which function does not obey Mean Value Theorem in [0, 1] ?

(a) $f(x) = \begin{cases} \dfrac{1}{2} - x, & x < \dfrac{1}{2} \\ \left(\dfrac{1}{2} - x\right)^2, & x \geq \dfrac{1}{2} \end{cases}$ (b) $f(x) = \begin{cases} \dfrac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

(c) $f(x) = x\,|x|$ (d) $f(x) = |x|$ [IIT 2003]

(48) The domain of the derivative of the function $f(x) = \begin{cases} \tan^{-1} x, & \text{if } |x| \leq 1 \\ \dfrac{1}{2}(|x| - 1), & \text{if } |x| > 1 \end{cases}$ is

(a) R - { 0 } (b) R - { 1 } (c) R - { - 1 } (d) R - { - 1, 1 } [IIT 2002]

(49) The integer n for which $\lim_{x \to 0} \frac{(\cos x - 1)(\cos x - e^{x})}{x^{n}}$ is a finite non-zero number is

(a) 1 (b) 2 (c) 3 (d) 4 [IIT 2002]

(50) If f : R → R be such that f (1) = 3 and f ' (1) = 6, then $\lim_{x \to 0} \left(\frac{f(1+x)}{f(1)} \right)^{\frac{1}{x}}$ equals

(a) 1 (b) $e^{\frac{1}{2}}$ (c) e^{2} (d) e^{3} [IIT 2002]

(51) The point (s) on the curve $y^3 + 3x^2 = 12y$ where the tangent is vertical, is / (are)

(a) $\left(\pm \frac{4}{\sqrt{3}}, -2 \right)$ (b) $\left(\pm \sqrt{\frac{11}{3}}, 0 \right)$ (c) (0, 0) (d) $\left(\pm \frac{4}{\sqrt{3}}, 2 \right)$ [IIT 2002]

(52) Let f : R → R be a function defined by f (x) = { x, x^3 }. The set of all points where f (x) is not differentiable is

(a) { - 1, 1 } (b) { - 1, 0 } (c) { 0, 1 } (d) { - 1, 0, 1 } [IIT 2001]

(53) The left hand derivative of f (x) = [x] sin (πx) at x = k, where k is an integer, is

(a) $(-1)^{k}(k-1)\pi$ (b) $(-1)^{k-1}(k-1)\pi$
(c) $(-1)^{k} k\pi$ (d) $(-1)^{k-1} k\pi$ [IIT 2001]

(54) The left hand derivative of f (x) = [x] sin (πx) at x = k, where k is an integer, is

(a) $(-1)^{k}(k-1)\pi$ (b) $(-1)^{k-1}(k-1)\pi$
(c) $(-1)^{k} k\pi$ (d) $(-1)^{k-1} k\pi$ [IIT 2001]

(55) $\lim_{x \to 0} \frac{\sin(\pi \cos^2 x)}{x^2}$ equals

(a) - π (b) π (c) π / 2 (d) 1 [IIT 2001]

(56) If $f(x) = x e^{x(1-x)}$, then $f(x)$ is

(a) increasing on $\left[-\frac{1}{2}, 1\right]$ (b) decreasing on R

(c) increasing on R (d) decreasing on $\left[-\frac{1}{2}, 1\right]$ [IIT 2001]

(57) Which of the following functions is differentiable at $x = 0$?

(a) $\cos(|x|) + |x|$ (b) $\cos(|x|) - |x|$
(c) $\sin(|x|) + |x|$ (d) $\sin(|x|) - |x|$ [IIT 2001]

(58) If $x^2 + y^2 = 1$, then

(a) $yy'' - 2(y')^2 + 1 = 0$ (b) $yy'' + (y')^2 + 1 = 0$
(c) $yy'' + (y')^2 - 1 = 0$ (d) $yy'' + 2(y')^2 + 1 = 0$ [IIT 2000]

(59) For $x \in R$, $\lim_{x \to \infty} \left(\frac{x-3}{x+2}\right)^x =$

(a) e (b) e^{-1} (c) e^{-5} (d) e^5 [IIT 2000]

(60) Consider the following statements in S and R:

S: Both $\sin x$ and $\cos x$ are decreasing functions in the interval $\left(\frac{\pi}{2}, \pi\right)$

R: If a differentiable function decreases in an interval (a, b), then its derivative also decreases in (a, b).

Which of the following is true ?

(a) Both S and R are wrong.
(b) Both S and R are correct, but R is not the correct explanation of S.
(c) S is correct and R is correct explanation of S.
(d) S is correct and R is wrong. [IIT 2000]

(61) If the normal to the curve $y = f(x)$ at the point (3, 4) makes an angle $\frac{3\pi}{4}$ with the positive X-axis, then $f'(3) =$

(a) -1 (b) $-3/4$ (c) $4/3$ (d) 1 [IIT 2000]

(62) If $f(x) = \begin{cases} |x| & \text{for } 0 < |x| \le 2 \\ 1 & \text{for } x = 0 \end{cases}$, then at $x = 0$, f has

(a) a local maximum (b) no local maximum
(c) a local minimum (d) no extremum [IIT 2000]

(63) For all $x \in (0, 1)$, which of the following is true ?

(a) $e^x < 1 + x$ (b) $\log_e(1 + x) < x$
(c) $\sin x > x$ (d) $\log_e x > x$ [IIT 2000]

(64) The function $f(x) = \sin^4 x + \cos^4 x$ increases if

(a) $0 < x < \frac{\pi}{8}$ (b) $\frac{\pi}{4} < x < \frac{3\pi}{8}$
(c) $\frac{3\pi}{8} < x < \frac{5\pi}{8}$ (d) $\frac{5\pi}{8} < x < \frac{3\pi}{4}$ [IIT 1999]

(65) The function $f(x) = [x]^2 - [x^2]$ where $[y]$ is the greatest integer less than or equal to y, is discontinuous at

(a) all integers (b) all integers except 0 and 1
(c) all integers except 0 (d) all integers except 1 [IIT 1999]

(66) The function $f(x) = (x^2 - 1)|x^2 - 3x + 2| + \cos(|x|)$ is NOT differentiable at

(a) - 1 (b) 0 (c) 1 (d) 2 [IIT 1999]

(67) $\lim\limits_{x \to 0} \dfrac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2} =$

(a) 2 (b) - 2 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$ [IIT 1999]

(68) The function $f(x) = \int_{-1}^{x} t(e^t - 1)(t - 1)(t - 2)^3(t - 3)^5 \, dt$ has a local minimum at x =

(a) 0 (b) 1 (c) 2 (d) 3 [IIT 1999]

(Answers at the end of all questions)

(69) $\lim\limits_{x \to 1} \dfrac{\sqrt{1 - \cos 2(x - 1)}}{x - 1}$

(a) exists and is equal to $\sqrt{2}$ (b) exists and is equal to $-\sqrt{2}$
(c) does not exist because $x - 1 \to 0$
(d) does not exist because left hand limit ≠ right hand limit [IIT 1998]

(70) If $\int_0^x f(t)\,dt = x + \int_1^x t\,f(t)\,dt$, then the value of $f(1)$ is

(a) $\frac{1}{2}$ (b) 0 (c) 1 (d) $-\frac{1}{2}$ [IIT 1998]

(71) Let $h(x) = \min[x, x^2]$, for every real number x, then

(a) h is continuous for all x (b) h is differentiable for all x
(c) $h'(x) = 1$ for all $x > 1$ (d) h is not differentiable at two values of x [IIT 1998]

(72) If $h(x) = f(x) - [f(x)]^2$ for every real number x, then

(a) h is increasing whenever f is increasing
(b) h is increasing whenever f is decreasing
(c) h is decreasing whenever f is decreasing
(d) nothing can be said in general [IIT 1998]

(73) If $f(x) = \dfrac{x}{\sin x}$ and $g(x) = \dfrac{x}{\tan x}$, where $0 < x \le 1$, then in this interval

(a) both f (x) and g (x) are increasing functions
(b) both f (x) and g (x) are decreasing functions
(c) f (x) is an increasing function
(d) g (x) is an increasing function [IIT 1997]

(74) $\lim\limits_{n \to p} \dfrac{1}{n} \sum\limits_{1}^{2n} \dfrac{r}{\sqrt{n^2 + r^2}}$ equals

(a) $1 + \sqrt{5}$ (b) $-1 + \sqrt{5}$ (c) $-1 + \sqrt{2}$ (d) $1 + \sqrt{2}$ [IIT 1997]

(75) If $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$, (p is a constant), then $\frac{d^3}{dx^3}[f(x)]$ at x = 0 is

(a) p (b) $p + p^2$ (c) $p + p^3$ (d) independent of p [IIT 1997]

(76) The function $f(x) = [x] \cos \left[\frac{2x-1}{2}\right]\pi$, where [.] denotes the greatest integer function, is discontinuous at

(a) all x (b) all integer points
(c) no x (d) x which is not an integer [IIT 1995]

(77) If f (x) is defined and continuous for all x > 0 and satisfy $f\left(\frac{x}{y}\right) = f(x) - f(y)$ for all x, y and f (e) = 1, then

(a) f (x) is bounded (b) $\left(\frac{1}{x}\right) \to 0$ as $x \to 0$

(c) $x f(x) \to 1$ as $x \to 0$ (d) f (x) = log x [IIT 1995]

(78) On the interval [0, 1], the function $x^{25}(1 - x)^{75}$ attains maximum value at the point

(a) 0 (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$ [IIT 1995]

(79) The function $f(x) = |px - q| + r|x|$, $x \in (-\infty, \infty)$ where p > 0, q > 0, r > 0, assumes its minimum value only at one point if

(a) $p \neq q$ (b) $r \neq q$ (c) $r \neq p$ (d) p =q = r [IIT 1995]

(80) The function $f(x) = \frac{\ln(\pi + x)}{\ln(e + x)}$ is

(a) increasing on $[0, \infty)$ (b) decreasing on $[0, \infty)$
(c) increasing on $\left[0, \frac{\pi}{e}\right]$ and decreasing on $\left[\frac{\pi}{e}, \infty\right)$

(d) decreasing on $\left[0, \frac{\pi}{e}\right]$ and increasing on $\left[\frac{\pi}{e}, \infty\right)$ [IIT 1995]

(81) The function $f(x) = \max\{(1-x), (1+x), 2\}$, $x \in (-\infty, \infty)$, is

(a) continuous at all points (b) differentiable at all points
(c) differentiable at all points except at $x = 1$ and $x = -1$
(d) continuous at all points except at $x = 1$ and $x = -1$ [IIT 1995]

(82) Let [.] denote the greatest integer function and $f(x) = [\tan^2 x]$. Then,

(a) $\lim\limits_{x \to 0} f(x)$ does not exist (b) $f(x)$ is continuous at $x = 0$
(c) $f(x)$ is not differentiable at $x = 0$ (d) $f'(0) = 1$ [IIT 1993]

(83) If $f(x) = \begin{cases} 3x^2 + 12x - 1, & -1 \le x \le 2 \\ 37 - x, & 2 < x \le 3 \end{cases}$, then

(a) $f(x)$ is increasing on $[-1, 2]$ (b) $f(x)$ is continuous on $[-1, 3]$
(c) $f(x)$ is maximum at $x = 2$ (d) $f'(2)$ does not exist [IIT 1993]

(84) The value of $\lim\limits_{x \to 0} \dfrac{\sqrt{\frac{1}{2}(1 - \cos 2x)}}{x}$ is

(a) 1 (b) -1 (c) 0 (d) none of these [IIT 1991]

(85) The following functions are continuous on $(0, \pi)$.

(a) $\tan x$ (b) $\int_0^{\pi} t \sin \frac{1}{t}\, dt$

(c) $\begin{cases} 1, & 0 < x \le \frac{3\pi}{4} \\ 2 \sin \frac{2x}{9}, & \frac{3\pi}{4} < x \le \pi \end{cases}$ (d) $\begin{cases} x \sin x, & 0 < x \le \frac{\pi}{2} \\ \frac{\pi}{2} \sin(\pi + x), & \frac{\pi}{2} < x < \pi \end{cases}$ [IIT 1991]

(86) If $f(x) = \dfrac{x}{2} - 1$, then, on the interval $[0, \pi]$, $\tan[f(x)]$ and

(a) $\dfrac{1}{f(x)}$ are both continuous (b) $\dfrac{1}{f(x)}$ are both discontinuous
(c) $f^{-1}(x)$ are both continuous (d) $f^{-1}(x)$ are both discontinuous [IIT 1989]

(87) If $y^2 = P(x)$, a polynomial of degree 3, then $2\dfrac{d}{dx}\left(y^3 \dfrac{d^2y}{dx^2}\right)$ equals

(a) $P'''(x) + P'(x)$ (b) $P''(x)P'''(x)$
(c) $P(x)P'''(x)$ (d) a constant [IIT 1988]

(88) The function $f(x) = \begin{cases} x - 3 & x \geq 1 \\ \dfrac{x^2}{4} - \dfrac{3x}{2} + \dfrac{13}{4} & x < 1 \end{cases}$ is

(a) continuous at $x = 1$ (b) differentiable at $x = 1$
(c) continuous at $x = 3$ (d) differentiable at $x = 3$ [IIT 1988]

(89) The set of all points where the function $f(x) = \dfrac{x}{1 + |x|}$ is differentiable is

(a) $(-\infty, \infty)$ (b) $(0, \infty)$ (c) $(-\infty, 0) \cup (0, \infty)$
(d) $(0, \infty)$ (e) none of these [IIT 1987]

(90) Let f and g be increasing and decreasing functions respectively from $(0, \infty)$ to $(0, \infty)$. Let $h(x) = f[g(x)]$. If $h(0) = 0$, $h(x) - h(1)$ is

(a) always zero (b) always negative (c) always positive
(d) strictly increasing (e) none of these [IIT 1987]

(91) Let $P(x) = a_0 + a_1x^2 + a_2x^4 + \ldots + a_nx^{2n}$ be a polynomial in a real variable x with $0 < a_0 < a_1 < a_2 < \ldots < a_n$. The function $P(x)$ has

(a) neither a maximum nor a minimum (b) only one maximum
(c) only one minimum (d) only one maximum and only one minimum
(e) none of these [IIT 1986]

(92) The function $f(x) = 1 + |\sin x|$ is

(a) continuous nowhere (b) continuous everywhere (c) differentiable
(d) not differentiable at $x = 0$ (e) not differentiable at infinite number of points
[IIT 1986]

(93) Let $[x]$ denote the greatest integer less than or equal to x. If $f(x) = [x \sin \pi x]$, then $f(x)$ is
(a) continuous at $x = 0$ (b) continuous in $(-1, 0)$ (c) differentiable at $x = 1$
(d) differentiable in $(-1, 1)$ (e) none of these [IIT 1986]

(94) If $f(x) = \dfrac{\sin[x]}{[x]}, \quad [x] \neq 0$

$\qquad\quad = 0, \qquad [x] = 0,$

where x] denotes the greatest integer less than or equal to x, then $\lim\limits_{x \to 0} f(x)$ equals

(a) 1 (b) 0 (c) - 1 (d) none of these [IIT 1985]

(95) If $f(x) = x(\sqrt{x} - \sqrt{x+1})$, then

(a) f (x) is continuous but not differentiable at x = 0
(b) f (x) is differentiable at x = 0
(c) f (x) is not differentiable at x = 0 (d) none of these [IIT 1985]

(96) $\lim\limits_{n \to \infty} \left\{ \dfrac{1}{1 - n^2} + \dfrac{2}{1 - n^2} + \dots + \dfrac{n}{1 - n^2} \right\}$ is equal to

(a) 0 (b) $-\dfrac{1}{2}$ (c) $\dfrac{1}{2}$ (d) none of these [IIT 1984]

(97) If $x + |y| = 2y$, then y as a function of x is

(a) defined for all real x (b) continuous at x = 0

(c) differentiable for all x (d) such that $\dfrac{dy}{dx} = \dfrac{1}{3}$ for x < 0 [IIT 1984]

(98) If $G(x) = -\sqrt{25 - x^2}$, then $\lim\limits_{x \to 1} \dfrac{G(x) - G(1)}{x - 1}$ has the value

(a) $\dfrac{1}{24}$ (b) $\dfrac{1}{5}$ (c) $-\sqrt{24}$ (d) none of these [IIT 1983]

(99) If f (a) = 2, f ' (a) = 1, g (a) = - 1, g' (a) = 2, then the value of $\lim\limits_{x \to a} \dfrac{g(x)f(a) - g(a)f(x)}{x - a}$ is

(a) - 5 (b) $\dfrac{1}{5}$ (c) 5 (d) none of these [IIT 1983]

(100) The function $f(x) = \dfrac{\ln(1 + ax) - \ln(1 - bx)}{x}$ is not defined at x = 0. The value which should be assigned to f at x = 0, so that it is continuous at x = 0, is

(a) a - b (b) a + b (c) ln a + ln b (d) none of these [IIT 1983]

(101) The normal to the curve $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$ at any point 'θ' is such that

(a) it makes a constant angle with the X-axis (b) it passes through the origin
(c) it is at a constant distance from the origin (d) none of these
[IIT 1983]

(102) If $y = a\ln x + bx^2 + x$ has its extremum values at $x = -1$ and $x = 2$, then

(a) $a = 2,\ b = -1$ (b) $a = 2,\ b = -\frac{1}{2}$
(c) $a = -2,\ b = \frac{1}{2}$ (d) none of these [IIT 1983]

(103) There exists a function $f(x)$ satisfying $f(0) = 1$, $f'(0) = -1$, $f(x) > 0$ for all x and

(a) $f''(x) > 0$ for all x (b) $-1 < f''(x) < 0$ for all x
(c) $-2 \le f''(x) \le -1$ for all x (d) $f''(x) < -2$ for all x [IIT 1982]

(104) For a real number y, let $[y]$ denote the greatest integer less than or equal to y. Then the function $f(x) = \dfrac{\tan[\pi(x - \pi)]}{1 + [x]^2}$ is

(a) discontinuous at some x
(b) continuous at all x, but the derivative $f''(x)$ does not exist for some x
(c) $f'(x)$ exists for all x, but the derivative $f''(x)$ does not exist for some x
(d) $f''(x)$ exists for all x [IIT 1981]

(105) If $f(x) = \sqrt{\dfrac{x - \sin x}{x + \cos^2 x}}$, then $\lim\limits_{x \to \infty} f(x)$ is

(a) 0 (b) ∞ (c) 1 (d) none of these [IIT 1979]

Answers

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
c	d	c	a	c	a	b	b	d	b	b	c	c	d	b	a	a	a	c	c

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
b	d	c	d	c	c	d	b	d	a	a	d	d	d	d	a	a	c	a	b

41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
b	d	b	a	c	d	a	d	c	c	d	d	a	a	b	a	d	b	c	d

61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
d	d	b	b	b	d	c	b,d	d	a	a,c,d	a,c	c	b	d	b	d	b	c	b

81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96
a,c	b	a,b,c,d	d	b,c	b	c	a,b,c	a	a	c	b,d,e	a,d	d	a	b

97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113
a,b,d	d	c	b	c	b	a	d	c								