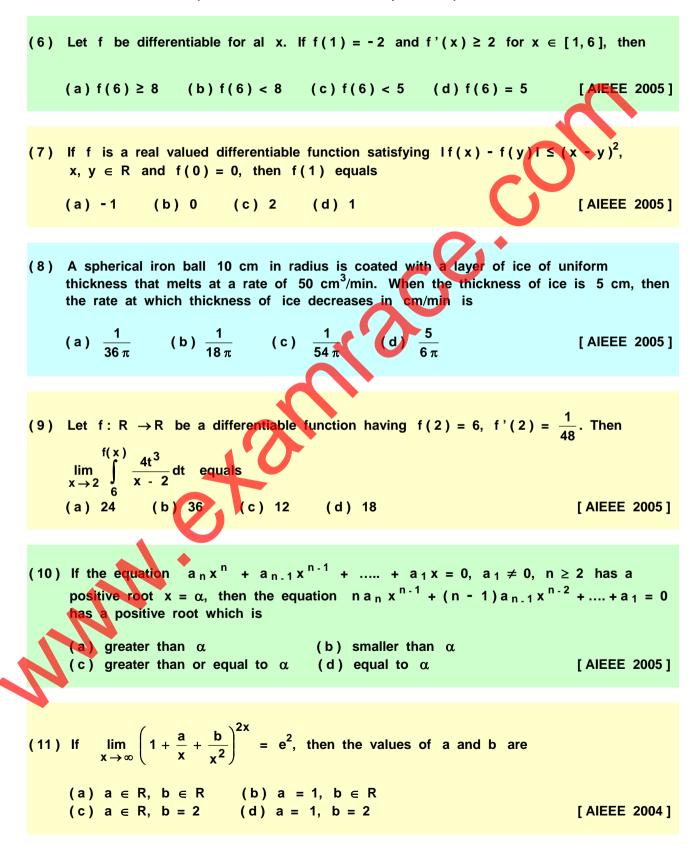
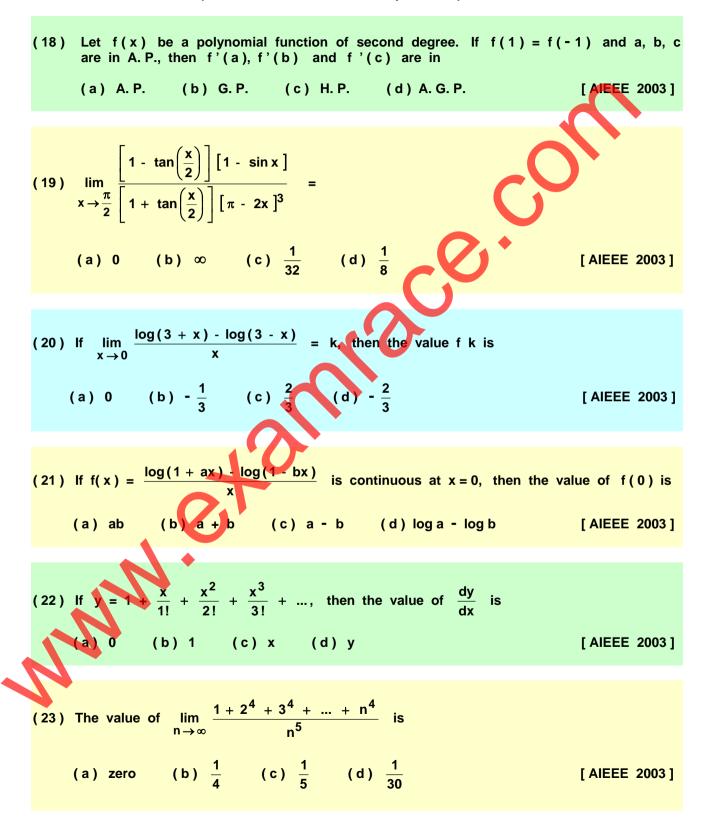
$$\frac{98 - \text{DIFFERENTIAL CALCULUS}}{(\text{Answers at the end of all questions})} \qquad \text{Page 1}$$

$$(1) \lim_{n \to \infty} \left[ \frac{1}{n^2} \sec^2 \frac{1}{n^2} + \frac{2}{n^2} \sec^2 \frac{4}{n^2} + \dots + \frac{1}{n} \sec^2 1 \right] \text{ is}$$

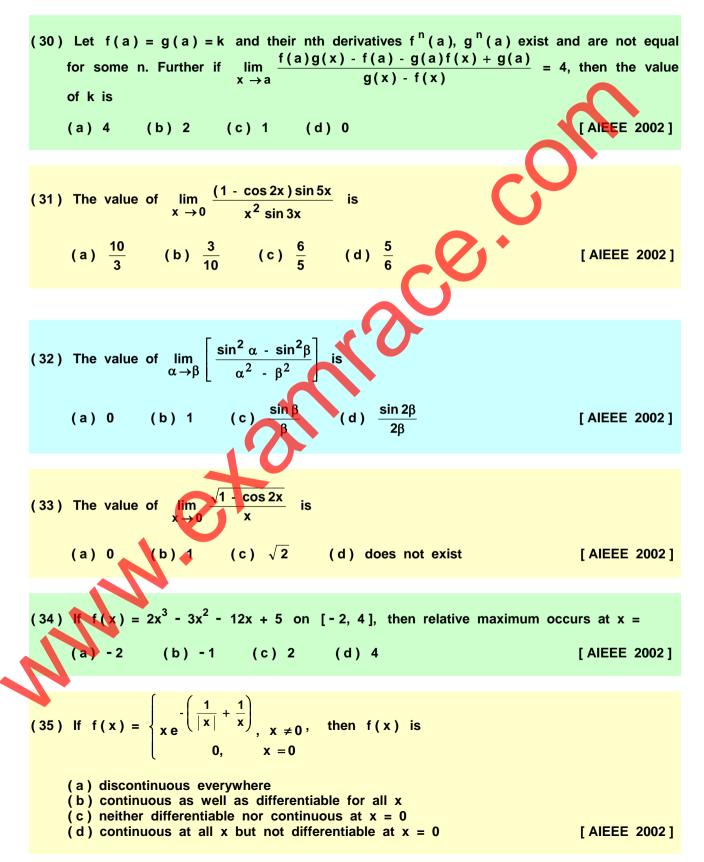
$$(a) \frac{1}{2} \sec^2 (1 - b) \frac{1}{2} \csc^2 (1 - c) \tan^2 (1 - b) \frac{1}{2} \tan^2 (1 - b) \frac{1}{2} \cos^2 (1 - c) \tan^2 (1 - c) \frac{1}{2} \tan^2 (1 - b) \frac{1}{2} \tan^2 (1 - c) \frac{1}{2}$$



$$\frac{08 - DIFFERENTIAL CALCULUS}{(Answers at the end of all questions)} Page 3$$
(12) Let  $f(x) = \frac{1 \cdot \tan x}{4x - \pi}$ ,  $x \neq \frac{\pi}{4}$ ,  $x \in \left[0, \frac{\pi}{2}\right]$ . If  $f(x)$  is continuous in  $\left[0, \frac{\pi}{2}\right]$ , then  $f\left(\frac{\pi}{4}\right)$  is
  
(a) 1 (b)  $\frac{1}{2}$  (c)  $-\frac{1}{2}$  (d)  $-1$  [AIEEE 2004]
(13) If  $x = e^{y} + e^{y + \dots, \infty}$ ,  $x > 0$ , then  $\frac{dy}{dx}$  is
  
(a)  $\frac{x}{1 + x}$  (b)  $\frac{1}{x}$  (c)  $\frac{1 - x}{x}$  (d)  $\frac{r + x}{x}$  [AIEEE 2004]
(14) A point on the parabola  $y^2 = 18x$  at which the ordinate increases at twice the rate of the abscissa is
  
(a)  $(2, 4)$  (b)  $(2, -4)$  (c)  $\left(-\frac{9}{8}, \frac{9}{2}\right)$  (d)  $\left(\frac{9}{8}, \frac{9}{2}\right)$  [AIEEE 2004]
(15) A function  $y = f(x)$  has a second order derivative  $f''(x) = 6(x - 1)$ . If its graph passes through the joint (2, 1) and at that point the tangent to the graph is  $y = 3x - 5$ , then the unction is
  
(a)  $(x - 1)^2$  (b)  $(x - 1)^3$  (c)  $(x + 1)^3$  (d)  $(x + 1)^2$  [AIEEE 2004]
(16) The uprimal to the curve  $x = a(1 + \cos \theta)$ ,  $y = a \sin \theta$  at ' $\theta$ ' always passes through the fixed point
  
(17) If  $2a + 3b + 6c = 0$ , then at least one root of the equation  $ax^2 + bx + c = 0$  lies in the interval
  
(a)  $(0, 1)$  (b)  $(1, 2)$  (c)  $(2, 3)$  (d)  $(1, 3)$  [AIEEE 2004]

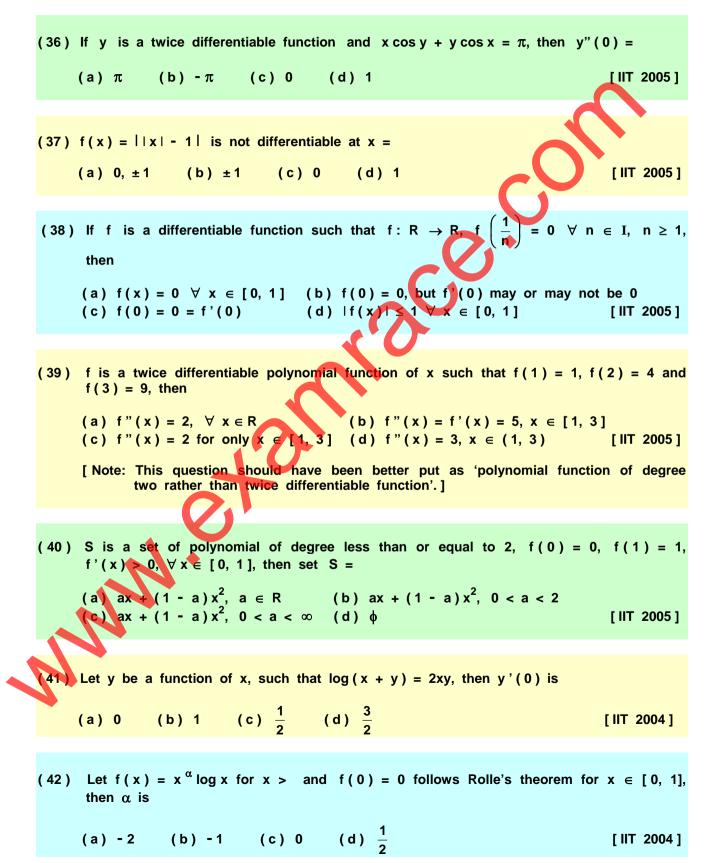


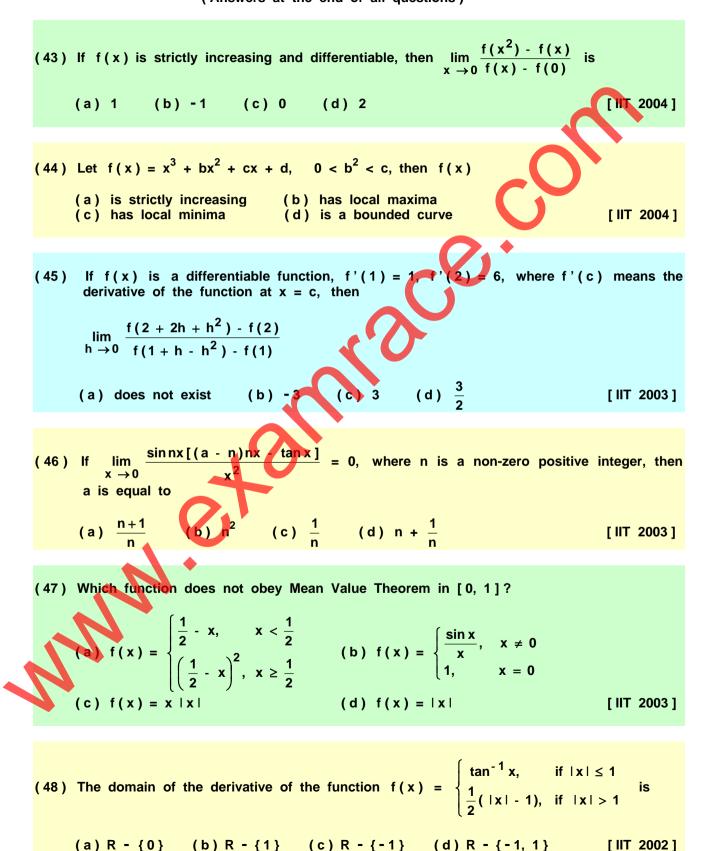
(24) If 
$$f: \mathbb{R} \to \mathbb{R}$$
 satisfies  $f(x + y) = f(x) + f(y)$ , for all  $x, y \in \mathbb{R}$  and  $f(1) = 7$ ,  
then the value of  $\sum_{r=1}^{n} f'(r)$  is  
(a)  $\frac{7n}{2}$  (b)  $7n(n+1)$  (c)  $\frac{7(n+1)}{2}$  (d)  $\frac{7n(n+1)}{2}$  [AIEEE 2003]  
(25) The real number  $x$  when added to its inverse gives the minimum value of the sum at  
 $x$  equal to  
(a) 2 (b) -2 (c) 1 (d) -1 [AIEEE 2003]  
(26) If the function  $f(x) = 2x^3 - 9ax^2 + 12a^2 + 14$  where  $a > 0$ , attains its maximum and  
minimum at  $p$  and  $q$  respectively such that  $p^2 = q$ , then  $a$  equals  
(a) 3 (b) 1 (c) 2 (c) 4 [AIEEE 2003]  
(27) If  $f(x) = x^n$ , then the value of  $Y(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} + \frac{f'''(1)}{3!} + ... + \frac{(-1)^n f^n(1)}{n!}$   
is  
(a)  $2^n$  (b)  $2^{p-1}$  (c) 1 (d) 0 [AIEEE 2003]  
(28) If  $x = t + t + 1$  and  $y = sin(\frac{\pi}{2}t) + cos(\frac{\pi}{2}t)$ , then at  $t = 1$ , the value of  $\frac{dy}{dx}$  is  
(a)  $\frac{a}{2}$  (b)  $-\frac{\pi}{6}$  (c)  $\frac{\pi}{3}$  (d)  $-\frac{\pi}{4}$  [AIEEE 2002]  
(29) If  $x = 3 cos \theta - 2 cos^3 \theta$  and  $y = 3 sin \theta - 2 sin^3 \theta$ , then the value of  $\frac{dy}{dx}$  is  
(a)  $sin \theta$  (b)  $cos \theta$  (c)  $tan \theta$  (d)  $cot \theta$  [AIEEE 2002]



# 08 - DIFFERENTIAL CALCULUS

(Answers at the end of all questions)





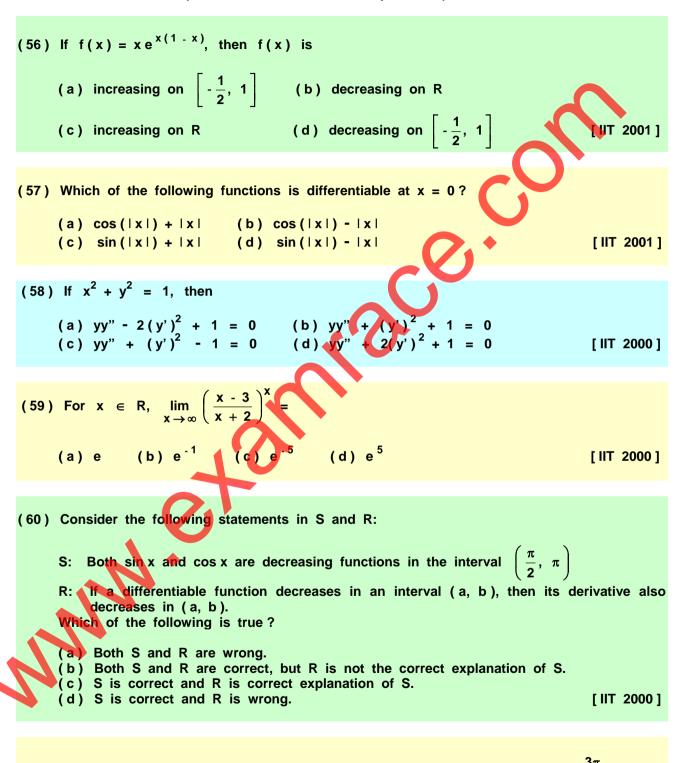
(49) The integer n for which  $\lim_{x \to 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$  is a finite non-zero number is 2002 ] (a) 1 (b) 2 (c) 3 (d) 4  $\lim_{x \to 0} \left( \frac{f(1+x)}{f(1)} \right)^{\frac{1}{x}}$ (50) If f:  $R \rightarrow R$  be such that f(1) = 3 and f'(1) = 6, then equals (a) 1 (b)  $e^{\frac{1}{2}}$  (c)  $e^{2}$  (d)  $e^{3}$ [IIT 2002] (51) The point (s) on the curve  $y^3 + 3x^2 = 12y$  where the tangent is vertical, is/(are) (a)  $\left(\pm\frac{4}{\sqrt{3}}, -2\right)$  (b)  $\left(\pm\sqrt{\frac{11}{3}}, 0\right)$  (c) (0, 0) (d)  $\left(\pm\frac{4}{\sqrt{3}}, 2\right)$ [ IIT 2002 ] (52) Let f: R  $\rightarrow$  R be a function defined by f(x) = {x, x<sup>3</sup>}. The set of all points where f(x) is not differentiable is (b) {-1,0} (c) {0,1} (d) {-1,0,1} [IIT 2001] (a) {-1, 1} (53) The left hand derivative of  $f(x) = [x] \sin(\pi x)$  at x = k, where k is an integer, is (a)  $(-1)^{k} (k-1)\pi$  (b)  $(-1)^{k-1} (k-1)\pi$ (c)  $(-1)^{k} k\pi$  (d)  $(-1)^{k-1} k\pi$ [IIT 2001] The left hand derivative of  $f(x) = [x] \sin(\pi x)$  at x = k, where k is an integer, is (a)  $(-1)^{k}(k-1)\pi$  (b)  $(-1)^{k-1}(k-1)\pi$ (c)  $(-1)^{k}k\pi$  (d)  $(-1)^{k-1}k\pi$ [IIT 2001] (55)  $\lim_{x \to 0} \frac{\sin(\pi \cos^2 x)}{x^2}$  equals

(d) 1

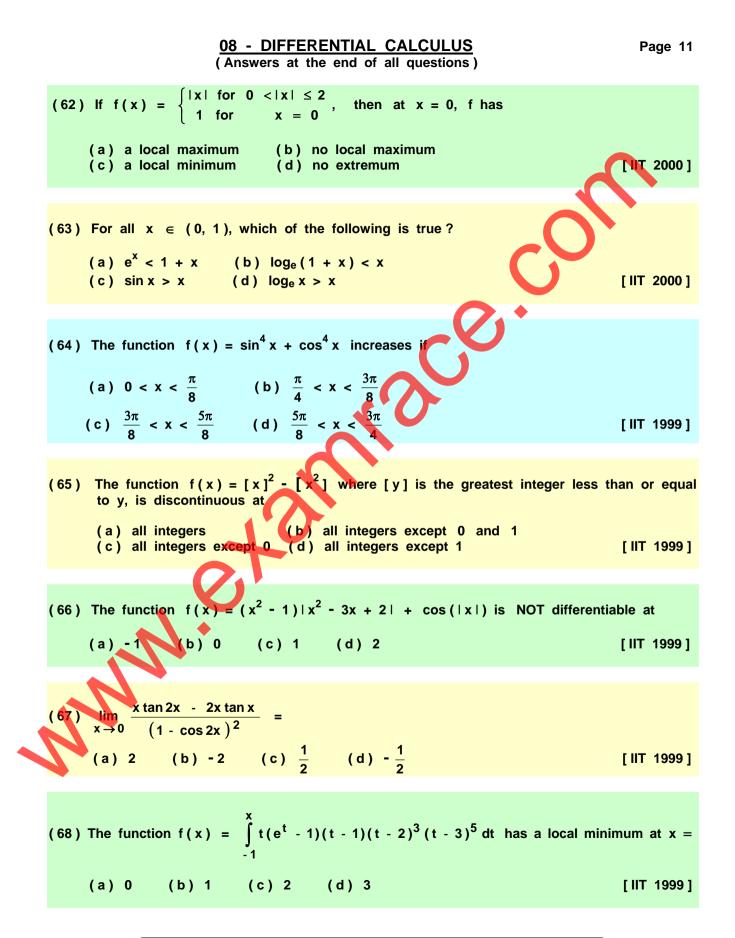
(a) -  $\pi$  (b)  $\pi$  (c)  $\pi/2$ 

Page 9

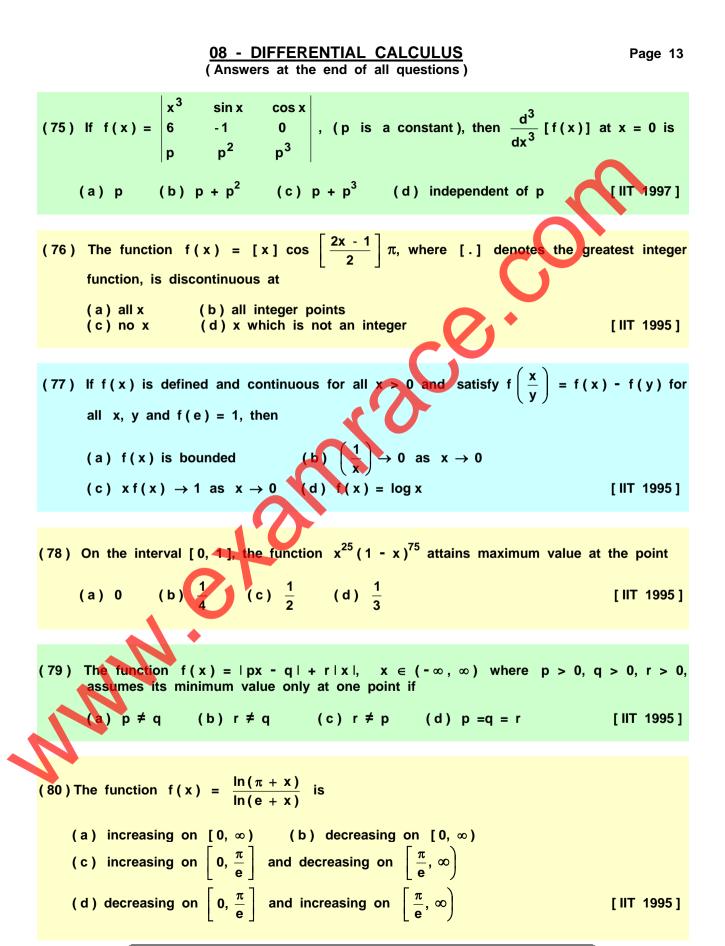
[IIT 2001]

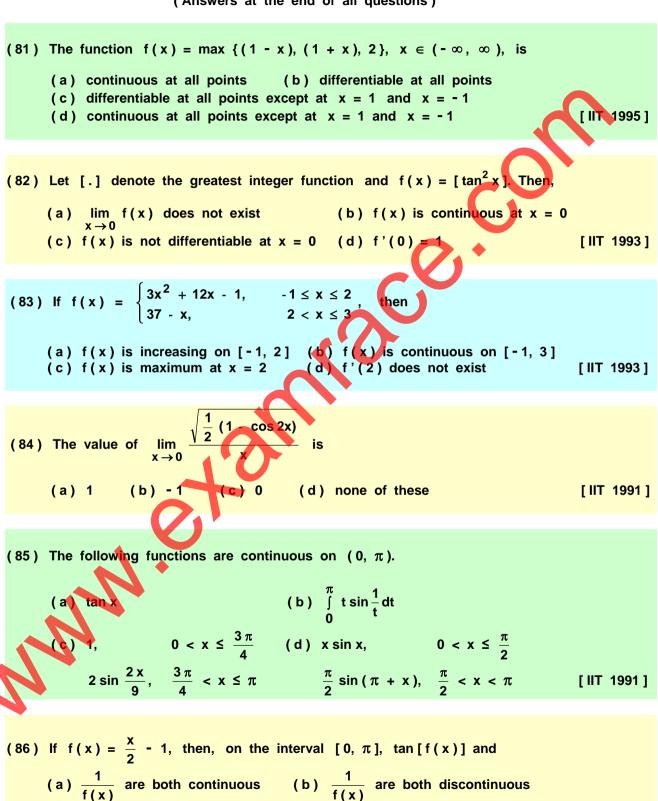


(61) If the normal to the curve y = f(x) at the point (3, 4) makes an angle  $\frac{3\pi}{4}$  with the positive X-axis, then f'(3) =



(69) 
$$\lim_{x \to 1} \frac{\sqrt{1 - \cos 2(x - 1)}}{x - 1}$$
  
(a) exists and is equal to  $\sqrt{2}$  (b) exists and is equal to  $-\sqrt{2}$   
(c) does not exist because  $x - 1 \to 0$   
(d) does not exist because left hand limit  $\neq$  right hand limit  
(70) If  $\int_{0}^{x} f(t) dt = x + \int_{1}^{x} tf(t) dt$ , then the value of  $f(1)$  is  
(a)  $\frac{1}{2}$  (b) 0 (c) 1 (d)  $-\frac{1}{2}$  [IIT 1998]  
(71) Let  $h(x) = \min [x, x^{2}]$ , for every real number x, then  
(a) h is continuous for all x (b) h if differentiable for all x  
(c) h'(x) = 1 for all x > 1 (d) h is not differentiable at two  
values of x [IIT 1998]  
(72) If  $h(x) = f(x) - [f(x)]^{2}$  for every real number x, then  
(a) h is increasing whenever f is decreasing  
(b) h is increasing whenever f is decreasing  
(c) h h is decreasing with ever f is decreasing  
(d) nothing can be said in general [IIT 1998]  
(73) If  $f(x) = \frac{x}{\sin x}$  and  $g(x) = \frac{x}{\tan x}$ , where  $0 < x \le 1$ , then in this interval  
(h) both  $f(x)$  and  $g(x)$  are increasing functions  
(c)  $f(x)$  is an increasing function  
(d)  $g(x)$  is an increasing function  
(f) both  $f(x)$  and  $g(x)$  are increasing functions  
(c)  $f(x)$  is an increasing function  
(d)  $g(x)$  is an increasing function  
(d)  $g(x)$  is an increasing function  
(f)  $y(x)$  is an increasing function  
(f)  $y(x)$  is an increasing function  
(g)  $y(x)$  is an increasing function  
(h)  $y(x)$  is an increasing fun





(c)  $f^{-1}(x)$  are both continuous (d)  $f^{-1}(x)$  are both discontinuous [IIT 1989]

08 - DIFFERENTIAL CALCULUS Page 15 (Answers at the end of all questions) (87) If  $y^2 = P(x)$ , a polynomial of degree 3, then  $2 \frac{d}{dx} \left( y^3 \frac{d^2 y}{dx^2} \right)$ equals (a) P'''(x) + P'(x)
(b) P''(x)P'''(x)
(c) P(x)P'''(x)
(d) a constant [ **IIT** 1988 ] (88) The function  $f(x) = \begin{cases} x - 3 & x \ge 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4} & x < 1 \end{cases}$ is (a) continuous at x = 1 (b) differentiable at x = 1(c) continuous at x = 3 (d) differentiable at x = 3[IIT 1988] (89) The set of all points where the function f(x) =is differentiable is (c)  $(-\infty, 0) \cup (0, \infty)$ (b) (0, ∞) (a)  $(-\infty, \infty)$  (b)  $(0, \infty)$ (d)  $(0, \infty)$  (e) none of these (a)  $(-\infty, \infty)$ [IIT 1987] (90) Let f and g be increasing and decreasing functions respectively from (0,  $\infty$ ) to  $(0, \infty)$ . Let h(x) = f[g(x)]. If h(0) = 0, h(x) - h(1) is (a) always zero (b) always negative (c) always positive [IIT 1987] (d) strictly increasing (e) none of these Let P(x) =  $a_0 + a_1x^2 + a_2x^4 + ... + a_nx^{2n}$  be a polynomial in a real variable x with (91)  $0 < a_0 < a_1 < a_2 < \dots < a_n$ . The function P(x) has (a) neither a maximum nor a minimum (b) only one maximum c) only one minimum (d) only one maximum and only one minimum [IIT 1986] (e) none of these The function  $f(x) = 1 + |\sin x|$  is (a) continuous nowhere (b) continuous everywhere (c) differentiable (d) not differentiable at x = 0 (e) not differentiable at infinite number of points [IIT 1986] (93) Let [x] denote the greatest integer less than or equal to x. If  $f(x) = [x \sin \pi x]$ , then f(x) is

(a) continuous at x = 0 (b) continuous in (-1, 0) (c) differentiable at x = 1(d) differentiable in (-1, 1) (e) none of these [IIT 1986]

(94) If 
$$f(x) = \frac{\sin[x]}{|x|}$$
,  $[x] \neq 0$   
= 0,  $[x] = 0$ ,  
where x] denotes the greatest integer less than or equal to x, then  $\lim_{x\to 0} f(x)$  equals  
(a) 1 (b) 0 (c) -1 (d) none of these [IIIT 1985]  
(95) If  $f(x) = x(\sqrt{x} - \sqrt{x+1})$ , then  
(a)  $f(x)$  is continuous but not differentiable at  $x = 0$   
(b)  $f(x)$  is indifferentiable at  $x = 0$  (d) none of these [IIT 1985]  
(96)  $\lim_{n\to\infty} \left\{ \frac{1}{1-n^2} + \frac{2}{1-n^2} + \dots + \frac{n}{1-n^2} \right\}$  is equal to  
(a) 0 (b)  $-\frac{1}{2}$  (c)  $\frac{1}{2}$  (d) none of these [IIIT 1984]  
(97) If  $x + |y| = 2y$ , then y as a function of x is  
(a) defined for all real x (b) continuous at  $x = 0$   
(c) differentiable of all x (d) such that  $\frac{dy}{dx} = \frac{1}{3}$  for  $x < 0$  [IIT 1984]  
(98) If  $G(x) = -\sqrt{25-x^2}$ , then  $\lim_{x\to 1} \frac{G(x) - G(1)}{x-1}$  has the value  
(a)  $\frac{1}{24}$  (b)  $\frac{1}{5}$  (c)  $-\sqrt{24}$  (d) none of these [IIT 1983]  
(98) If  $G(x) = 2$ ,  $f'(a) = 1$ ,  $g(a) = -1$ ,  $g'(a) = 2$ , then the value of  $\lim_{x\to a} \frac{g(x)f(a) - g(a)f(x)}{x-a}$  is  
(a) -5 (b)  $\frac{1}{5}$  (c) 5 (d) none of these [IIT 1983]  
(100) The function  $f(x) = \frac{\ln(1+ax) - \ln(1-bx)}{x}$  is not defined at  $x = 0$ . The value which should be assigned to f at  $x = 0$ , so that it is continuous at  $x = 0$ , is

(a) a - b (b) a + b (c) Ina + Inb [IIT 1983] (d) none of these

(101) The normal to the curve  $x = a(\cos \theta + \theta \sin \theta)$ ,  $y = a(\sin \theta - \theta \cos \theta)$  at any point  $\theta'$  is such that (a) it makes a constant angle with the X-axis (b) it passes through the origin (c) it is at a constant distance from the origin (d) none of these [ IIT 1983 1 (102) If  $y = a \ln x + bx^2 + x$  has its extremum values at x = -1 and x = 2, then (a) a = 2, b = -1 (b)  $a = 2, b = -\frac{1}{2}$ (c) a = -2,  $b = \frac{1}{2}$  (d) none of these [IIT 1983] (103) There exists a function f(x) satisfying f(0) = 1, f'(0) = -1, f(x) > 0 for all x and (a) f''(x) > 0 for all x -1\_< f"(x) < 0 for all x (c)  $-2 \le f''(x) \le -1$  for all x (d) 1 (x) < -2 for all x [IIT 1982] (104) For a real number y, let [v] denote the greatest integer less than or equal to y. Then the function  $f(x) = \frac{\tan[\pi(x - \pi)]}{1 + [x]^2}$  is (a) discontinuous at some x (b) continuous at all x, but the derivative f"(x) does not exist for some x (c) f'(x) exists for all x, but the derivative f"(x) does not exist for some x (d) f<sup>w</sup>(x) exists for all x [IIT 1981]  $\frac{x - \sin x}{x + \cos^2 x}$ , then lim f(x) is  $x \to \infty$ f(x) =105) (b)  $\infty$  (c) 1 (d) none of these (a) 0 [IIT 1979]

Page 18