(1) Obtain the equations of the line through A(1, 2, 1) and B(2, 3, -1) in vector and Cartesian forms.

[Ans : $\bar{r} = (1, 2, 1) + k(1, 1, -2),\ k \in R,\ \frac{x-1}{1} = \frac{y-2}{1} = \frac{z-1}{-2}$]

(2) Find the angle between $\frac{x-1}{2} = \frac{y-3}{5} = \frac{z+1}{-2}$ and $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z+3}{3}$.

[Ans : $\cos^{-1}\sqrt{\frac{3}{154}}$]

(3) Prove that the lines $\bar{r} = (1, 2, 6) + k(1, 3, 5),\ k \in R$ and $\bar{r} = (-1, 3, 5) + k(2, 1, 1),\ k \in R$ are non-coplanar.

(4) Obtain the perpendicular distance of the line $\bar{r} = (2, 1, 5) + k(1, 0, 1),\ k \in R$ from P(1, 2, 1).

[Ans : $\sqrt{\frac{11}{2}}$]

(5) Show that $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z-2}{5}$ and $\frac{x}{3} = \frac{y-1}{2} = \frac{z-1}{1}$ are skew lines. Find the shortest distance between the.

[Ans : $\frac{2}{\sqrt{3}}$]

(6) Obtain the co-ordinates of the foot of perpendicular from (2, 4, -1) on $\bar{r} = (-5, -3, 6) + k(1, 4, -9),\ k \in R$ and find the distance of the point from the line.

[Ans: (-4, 1, -3), 7]

(7) Obtain the vector and Cartesian forms of equations of the line through A(1, 2, 3) in the direction (-1, 1, 2) and find the co-ordinates of the points on the line at a distance $\sqrt{6}$ from A.

[Ans : $\bar{r} = (1, 2, 3) + k(-1, 1, 2),\ \frac{x-1}{-1} = \frac{y-2}{1} = \frac{z-3}{2},\ k \in R$, (0, 3, 5), (2, 1, 1)]

(8) Determine which among the following pairs of lines are coincident, parallel, intersecting or skew.

(a) $\{(1, 3, 5) + k(-1, 2, 3) \mid k \in R\}$ and $\{(1, 3, 1) + k(1, -2, -3) \mid k \in R\}$

(b) $\bar{r} = (2, 1, 3) + k(1, -1, 1)$, $k \in R$ and $\bar{r} = (3, 0, 4) + k(-1, 1, -1)$, $k \in R$

(c) $\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{5} = \frac{z+2}{2}$

(d) $\{(k+3, -k-1, k+1) \mid k \in R\}$ and $\{(2k, 0, 3k-3) \mid k \in R\}$.

[Ans: (a) parallel (b) coincident (c) skew (d) intersecting]

(9) Find the (perpendicular) distance between the parallel lines $x - 1 = y + 2 = z - 3$ and $x - 2 = y + 3 = z - 5$.

$\left[\text{Ans}: \sqrt{\frac{14}{3}}\right]$

(10) Find the co-ordinates of points on the line $\bar{r} = (1, 2, 1) + k(-1, -2, 1)$, $k \in R$ distant $\sqrt{6}$ units from $(2, 4, 0)$.

[Ans: (1, 2, 1), (3, 6, -1)]

(11) Find equation of the line through (3, -1, 11) perpendicular to $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$.

[Ans: $\bar{r} = (3, -1, 11) + k(1, -6, 4)$, $k \in R$]

(12) Find the shortest distance between $\bar{r} = (2, 1, -2) + k(3, -5, 2)$, $k \in R$ and $\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$.

$\left[\text{Ans}: \frac{1}{\sqrt{3}}\right]$

(13) Show that the angle between any two diagonals of a cube is $\cos^{-1}(1/3)$.

[Note: This angle is 109° 28' which is the angle between any two C - H bonds of methane.]

(14) If a line makes angles α, β, γ and δ with the diagonals of a cube, then show that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma + \sin^2\delta = \frac{8}{3}$.

(15) If the direction cosines l, m, n of two lines satisfy $l + m + n = 0$ and $l^2 - m^2 + n^2 = 0$, then show that the angle between the two lines is $\frac{\pi}{3}$.

(16) Show that the lines whose direction cosines are given by the equations $l + m + n = 0$, $al^2 + bm^2 + cn^2 = 0$ are

(i) perpendicular, if $a + b + c = 0$ and

(ii) parallel, if $a^{-1} + b^{-1} + c^{-1} = 0$.

(17) Show that the lines whose direction cosines are given by the equations $ul + vm + wn = 0$ and $fmn + gnl + hlm = 0$ are perpendicular if $\frac{f}{u} + \frac{g}{v} + \frac{h}{w} = 0$.

(18) Prove that the lines $x = ay + b = cz + d$ and $x = \alpha y + \beta = \gamma z + \delta$ are coplanar if $(\gamma - c)(a\beta - b\alpha) = (\alpha - a)(c\delta - d\gamma)$.

(19) Find the equation of the straight line perpendicular to both the lines $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z+2}{3}$ and $\frac{x+2}{2} = \frac{y-5}{-1} = \frac{z+3}{2}$ and passing through their point of intersection.

[Ans: $\bar{r} = (2, 3, 1) + k(7, 4, -5),\ k \in R$]

(20) Show that the shortest distances between a diagonal of a rectangular parallelopiped whose edges are a, b, c and the edges not meeting it are

$$\frac{bc}{\sqrt{b^2 + c^2}}, \quad \frac{ca}{\sqrt{c^2 + a^2}} \quad \text{and} \quad \frac{ab}{\sqrt{a^2 + b^2}}.$$

(21) If the line L : $x = ay + b = cz + d$ is perpendicular to the line M : $y = a'x + b = c'z + d$, then prove that

$cc'(a + a') + aa' = 0$.