BRITISH PHYSICS OLYMPIAD 2003 COMPETITION

Paper 2 - 8th November 2002

3 hours plus 15 minutes reading time.

There are TEN questions.
The marks for each section are given on the right hand side of the page.

- FOUR questions only should be attempted.
- QUESTION 1 IS COMPULSORY. It is expected that students will spend 75 minutes on this question. Students can attempt any, or all, sections, however the maximum total mark possible is 40.
- THREE of the remaining nine questions should be attempted. Students are recommended to spend 35 minutes on each question. The maximum mark for each question is 20. Formulae sheets can be used.

Useful data:

Speed of light in free space	c	3.00×10^{8}	m s^{-1}
Elementary charge	e	1.60×10^{-19}	C
Permeability of a vacuum	μ_o	$4\pi \times 10^{-7}$	Hm^{-1}
Mass of proton	m_p	1.67262×10^{-27}	kg
Mass of neutron	m_n	1.67493×10^{-27}	kg
Mass of electron	m_e	9.11×10^{-31}	kg
Planck constant	h	6.63×10^{-34}	J s
Gravitational constant	G	6.67×10^{-11}	$Nm^2 kg^{-2}$
Acceleration of free fall at Earth's surface	g	9.81	m s^{-2}
Boltzmann constant	k	1.38×10^{-23}	J K^{-1}
Astronomical Unit	AU	1.50×10^{11}	m
Earth-Moon distance	R_{EM}	3.82×10^{8}	m
Radius of the Earth	R_E	6.37×10^{6}	m
Mass of the Earth	M_E	5.97×10^{24}	kg
Mass of the Sun	M_S	1.99×10^{30}	kg
Avogadro constant	N_A	6.02×10^{23}	mol^{-1}

Q1

a) Comment on the following, giving necessary corrections:

(i) The voltage and current across a resistor are 78.46 V and 13.56 mA. One student records the power as 1.1 W and another as 1063.9 mW.

(ii) The dimensions of a rectangular block, measured in mm, are $52.5 \times 35.5 \times 95.0$. It has a density of $4.3 \times 10^3\ kgm^{-3}$. The recorded volume and mass are $0.0001770525\ m^3$ and 0.7665356 kg.

(iii) A student calculates that the current I = 8.03 A in the circuit in Figure 1.1.

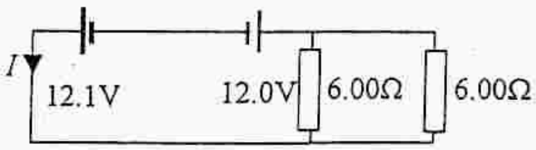


Figure 1.1 [10]

b) Calculate the number of photons produced during a nanosecond (10^{-9}s) pulse from a 0.5 mW laser. The wavelength of the radiation is 639 nm. [6]

c) A rocket, of constant mass 2.00×10^3 kg, is fired vertically upwards with constant acceleration. After one minute it reaches a height of 36.0 km and its engine is switched off. Assuming the rocket's mass and g remain constant, calculate:

(i) the initial acceleration of the rocket
(ii) the maximum height reached
(iii) the total time of flight
(iv) the work done by the rocket

Why is the assumption made unrealistic? [10]

d) The $^{12}C_6$ atom has a mass of 12.00000 amu (atomic mass units), where 1 amu = 1.660566×10^{-27}kg or 9.315088×10^2 MeV.
Determine for the $^{12}C_6$ **nucleus**:

(i) its mass in atomic mass units
(ii) its mass defect in atomic mass units
(iii) binding energy per nucleon in MeV [10]

e) Positive charges of equal magnitude Q are located at three corners of a square, which has sides of length a. What is the electric potential, V, and the electric vector field-strength, $\boldsymbol{E}$, at the fourth corner? [9]

f) A vertical measuring cylinder, of length l, has a base of negligible mass. What is the height of its centre of gravity when it is (i) empty and (ii) full of liquid? Sketch a graph of the height of the centre of gravity, y, against the height, h, of liquid, giving an explanation for the behavior of the graph. [5]

g) A puck of mass m, velocity v, coordinates (x,y), slides under gravity, without friction, on the smooth track shown in Figure 1.2.
Determine, or answer, the following:

(i) the acceleration f along the straight portion of the track
(ii) the total energy E of the puck when located at (x,y)
(iii) comment on the resultant acceleration at 0
(iv) if released from rest at $x = a$, determine its velocity at 0. [5]

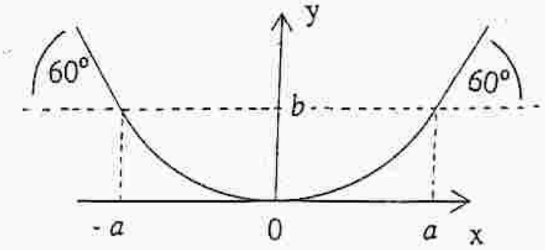


Figure 1.2

h) Two stations on the equator, diametrically opposite each other, communicate by sending, and receiving, radio signals that are tangential to the Earth via two geostationary satellites in circular orbits at 3.59×10^4 km above the Earth's surface. Calculate the time delay between sending and receiving a signal. [10]

i) Two measured quantities, x and y, are thought to satisfy one of the following two theoretical equations, where a, b, c and d are constants:
(i) $1/y = ax + b$ (ii) $y(x^2 + d) = cx$
What straight line relationships determine the best fit?
How could straight line relations be used to evaluate (i) a and b, (ii) c and d? [10]

j) On average 200 MeV of energy is released when a $^{235}_{92}U$ nucleus undergoes fission. A power station using $^{235}_{92}U$ as fuel can operate up to a power level of 800 MW. What is the minimum mass of $^{235}_{92}U$ used each day when operating continuously at this power level? [6]

k) The table gives experimental values of (e/m), in units of 10^{11}C kg^{-1}, for a relativistic electron travelling at speed v in terms of the ratio $(v/c)^2$.

$(v/c)^2$	0.1007	0.1434	0.1833	0.2656	0.4720
(e/m)	1.688	1.647	1.609	1.525	1.293

Show graphically that these results are in accord with the theory of relativity where $m = m_o(1-v^2/c^2)^{-\frac{1}{2}}$, m_o being the rest mass of the electron.
Obtain a value for (e/m_o) and indicate its accuracy. [10]

Q2

Two identical horizontal bars A and B, each of mass M, are situated one above the other, a distance l apart. They are joined by two unstretched springs, each with constant force per unit extension k and natural length l. The lower bar A is initially at a height h above the ground. The system is released from rest, Figure 2.1.

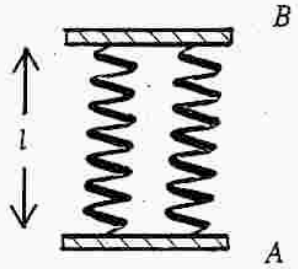


Figure 2.1

a) (i) How does the separation of the bars vary during free fall? What are the velocities of the bars immediately before A impacts with the ground? [2]

(ii) A hits the ground and is brought to rest almost instantaneously. Its kinetic energy is completely dissipated; transformed into heat and sound. Obtain an expression for the remaining energy of the system. [2]

(iii) Express the energy, E, of the system in terms of the subsequent maximum compression of the springs, x_c, which occurs before the spring turns touch each other. [3]

(iv) Deduce an expression for x_c. [5]

b) (i) Immediately after impact the springs extend and A rises above the ground. By considering the forces acting on A determine the value of the extension, x_e, of the springs the instant A leaves the ground. [3]

(ii) Using considerations of energy, deduce an expression for the condition h must satisfy for A to rise above the ground. [5]

(The thickness of the bars can be neglected)

Q3

a) Figure 3.1

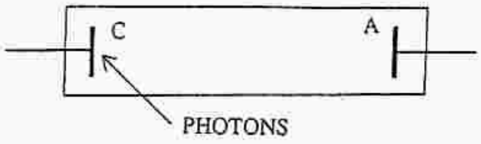


Figure 3.1 shows a photo-electric apparatus. Photons of wavelength λ enter the evacuated glass tube containing an anode A and a cathode C. The photons striking the cathode release photo-electrons. A voltage , V_o, exists across CA so that some photo-electrons just reach A.

(i) Explain the polarity of the cathode.
(ii) Why is the photon theory necessary to explain the photo-electric effect?
(iii) Derive an expression, in terms of V_o and λ, for the greatest photon wavelength, λ_c, that results in the net emission of photo-electrons. [8]

b)

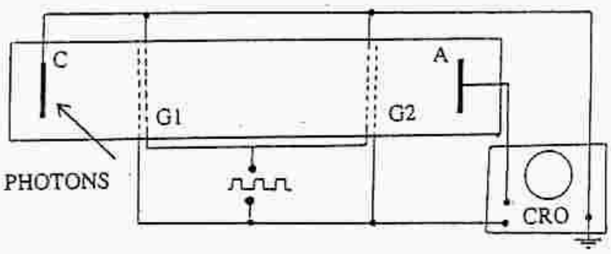


Figure 3.2

Figure 3.2 shows a modified photo-electric apparatus. Photo-electrons emerge from C with kinetic energy 0.750 eV. G1 and G2 are gates, each consisting of two mesh electrodes separated by 5 mm. The two earthed electrodes are 500 mm apart and 40 mm from either C or A. A CRO measures and displays voltages. C is earthed and A is at a base potential of – 0.750 V. A square wave voltage with a period of 3.00 μs, varying between ±1.00 V, is applied to an electrode of G1 and G2, Figure 3.3. The resulting trace observed on the CRO is shown in Figure 3.4.

Figure 3.3

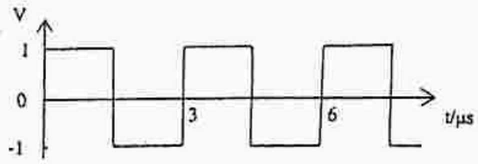
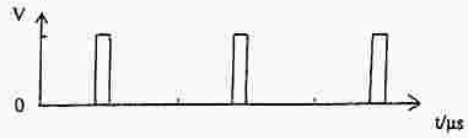


Figure 3.4



The motion of the photo-electrons between C and the final grid of G2 is determined solely by the potentials on the grids. When G1 and G2 are at +1.00 V:

(i) Draw a graph of the electric potential, V, against the distance, x, along the tube from C to the final grid of G2.
(ii) Sketch a graph of the kinetic energy, T, in eV, of the photo-electrons from C to the final grid of G2.
(iii) Determine the speed of a photo-electrons between G1 and G2.
(iv) Explain the shape of the trace in Figure 3.4.
(v) Estimate, in micro-seconds (μs), the time width of the pulse of electrons emerging from G2. [12]

Q4

a)

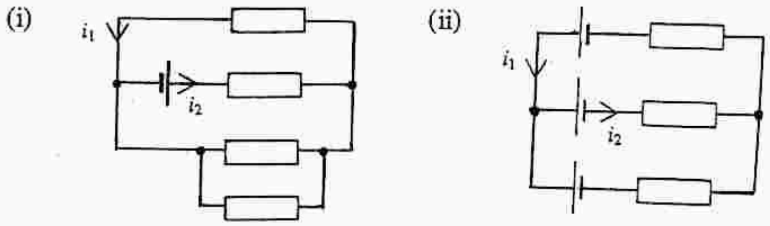


Figure 4.1

Determine the currents, i_1 and i_2, in the circuits in Figure 4.1. The cells have emf E, with negligible internal resistance, and resistors each of resistance R.

[5]

b)

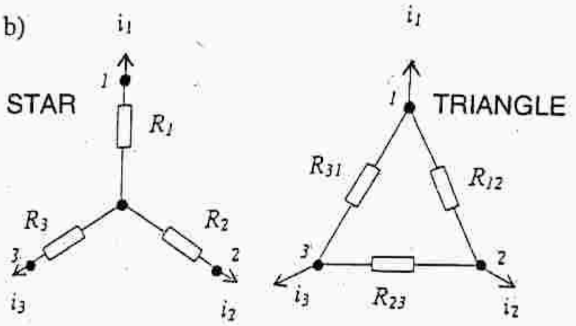


Figure 4.2

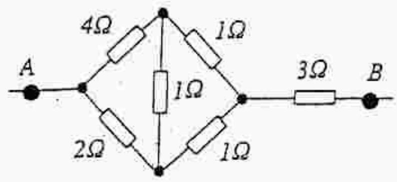


Figure 4.3

Figure 4.2 contains two circuits, each with points labelled 1,2 and 3, and each containing three resistors:

(i) the star circuit with resistances R_1, R_2 and R_3

(ii) the triangular circuit with resistances R_{12}, R_{23} and R_{31}
(the subscripts indicate the resistances between the labelled points).

The two circuits have identical currents i_1, i_2 and i_3 and identical potential differences between sets of points (1,2), (2,3) and (3,1).
For $i_1 = 0$ calculate the potential difference across (2,3) for (i) star and (ii) triangular networks.
By equating potential differences in the two circuits, show that

$$(R_2 + R_3)(R_{31} + R_{12} + R_{23}) = R_{23}(R_{31} + R_{12}).$$ [5]

c) Deduce two further relations by setting (i) $i_2 = 0$ and (ii) $i_3 = 0$. [2]

d) Figure 4.3 contains a triangular arrangement of three 1Ω resistors. Verify that the resistors in the equivalent star circuit are all ⅓ Ω. [3]

e) Hence determine the resistance, R_{AB}, between AB. [5]

Q5

a) The magnitude of the magnetic flux density **B** at a distance r from an infinite straight wire carrying a current I is given by

$$B = \frac{\mu_o I}{2\pi r} \quad , \qquad \text{where } \mu_o \text{ is the permeability of free space.}$$

Show that the force per unit length, F, between two infinite parallel wires carrying currents I_1 and I_2, a distance r apart, is given by

$$F = \frac{\mu_o I_1 I_2}{2\pi r} \ .$$

Indicate the direction of F in a diagram. [2]

b) Sketch the field lines due to two parallel wires, each with current I
(i) in the same direction and (ii) in opposite directions.
Draw a graph of the resultant magnetic flux density, B, against distance, x, along the infinite line through the two wires, in a plane perpendicular to the wires, for cases (i) and (ii). [9]

c) Power line conductors are arranged as a set of three parallel wires, each carrying a current of 200 A in the same direction, and separated from each other by 0.50 m, Figure 5.1.
Determine the force per metre on each wire. Draw a force diagram. [5]

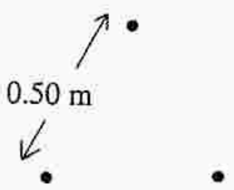


Figure 5.1

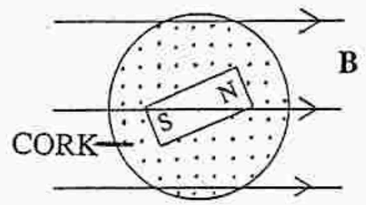


Figure 5.2

d) Figures 5.2 shows a cork, with a magnet resting on it, which is floating on water in a constant horizontal magnetic field of flux density B. When perturbed from equilibrium it oscillates with angular frequency f given by

$$f^2 \doteq \alpha B \ , \qquad \text{where } \alpha \text{ is a constant.}$$

When the magnet is suspended by a thread in the magnetic field

$$f^2 = \gamma B + \lambda \ , \qquad \text{where } \gamma \text{ and } \lambda \text{ are constants.}$$

On what physical property does λ depend ?
Describe how to use the latter arrangement to (i) verify that B, due to an infinite straight wire, falls off with distance r as $(1/r)$ and (ii) determine λ. [4]

Q6

a) Explain, with suitable ray diagrams, the eclipses of the Sun and the Moon. Why do these eclipses not occur once a month? [5]

b) Assuming a circular orbit, show that the orbital period T of the Moon around the Earth is related to the radius R by

$$T^2 = \frac{4\pi^2}{GM_E} R^3, \qquad [4]$$

where M_E is the mass of the Earth and $R = R_{EM}$, the distance between the centres of the Earth and the Moon.

c) This relation is valid for any possible elliptical orbit providing R is interpreted as the semi-major axis of the ellipse (half the major axis); a circle is a special case of an ellipse when both foci coincide at the centre of the ellipse.

In the limiting case of a "thin narrow' ellipse the foci are at the extremities of the ellipse. This corresponds to the Moon falling to Earth from an initial stationary position (the major axis being the distance between the centres of the Earth and the Moon); a conceivable possibility in the early formation of the solar system. Neglecting the radii of the Earth and the Moon, calculate:

(i) the time, τ, taken for the Moon to strike the Earth
(ii) the average speed of the Moon during its fall to Earth
(iii) the Moon's speed, V_M, at a distance equal to the radius of the Earth, R_E, from the centre of the Earth
(iv) the time taken for a stationery asteroid, at 2.00 AU from the Sun, to fall into the Sun; the stationary asteroid could result from a collision with another asteroid. [11]

Q7

a) Explain, with diagrams, the differences between longitudinal and transverse waves. Give an example of a polarized wave. [3]

b) Explain how the following acoustical phenomena arise:

(i) beats due to two sound waves
(ii) Doppler effect. [4]

c) If a source of sound, emitting a note of frequency f_o, is moving towards an observer with velocity v_s, show that, providing there is no wind, the frequency f detected by the observer is given by

$$f = \frac{f_o}{(1 - v_s / c)},$$

where c is the speed of sound in air. [5]

d) How is the result in (c) modified if the source is moving away from the observer?
What occurs when the velocity, v_s, towards the observer equals c?
Why are transverse acoustic waves not propagated in air? [3]

e) A twin-engined aeroplane flies low over a stationary observer. The beat frequency from the two engines is observed to change from 8.00 s^{-1} to 2.00 s^{-1} as it passes overhead.
If there is no wind, deduce the speed of the aeroplane.

(Speed of sound in air c = 330 ms^{-1}.) [5]

Q8

a) A frequency band of electromagnetic radiation, such as present in white light, is transmitted through a medium. By observing the transmitted radiation, how can one conclude that the refractive index of the medium varies with wavelength? [2]

b)

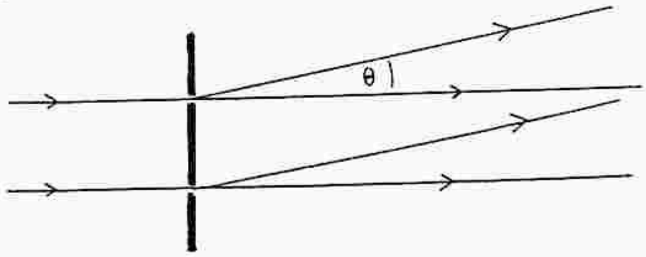


Figure 8.1

A laser beam, wavelength λ, impinges normally on two slits, a distance d apart, producing a diffracted beam at angle θ to the incident beam, Figure 8.1.

(i) What are the conditions for constructive and destructive interference?

(ii) Show that for the first order beam, where θ less than 0.1 radians, constructive interference occurs when $\theta = \lambda / d$.

(iii) How does the condition in (ii) alter if the system is in a medium of refractive index μ ? [6]

c)

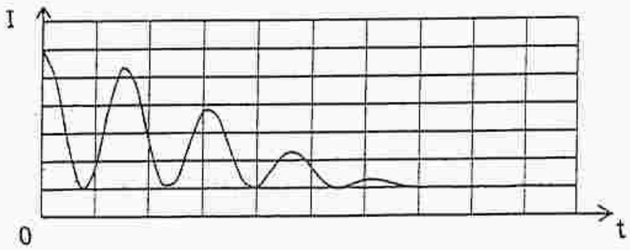


Figure 8.2

A radio telescope has two aerials 50 m apart on an east-west line around the equator. On March 21st at noon, when the Sun is directly overhead , it records the radio signals from the Sun at a wavelength of 0.75 m. The amplitudes of the signals received by the aerials are fed through equal lengths of cable, added together, and amplified; coherence of the signals being maintained. Figures 8.2 shows the resultant intensity, I, as a function of time, t, elapsed after 12 noon.

(i) Determine the time interval between adjacent maxima around noon.

(ii) How would the recording be modified if the observations were performed near Cambridge, England, at latitude 52°N? [6]

d) A pulsar is a rotating neutron star which emits a band of radio frequencies in short pulses. The frequency of the pulses is equal to the frequency of rotation of the star.

(i) The radius of a neutron star can contract. How could one detect this?

(ii) When a cloud of ionised gas passes in front of the star, the refractive index of the interstellar medium changes.
How could an observer detect this cloud? [6]

Q9

a) An electron in a hydrogen atom makes a transition from its ground state, energy $E_0 = -13.60\ eV$, to the first excited state, energy $E_1 = -3.40\ eV$.

(i) Deduce that a photon has been emitted or absorbed?
(ii) Determine the wavelength of the photon.
(iii) What is the ionization energy of the atom? [4]

b) How do the energy levels of a bound electron in the hydrogen atom differ from those of a free electron? [2]

c) A stationary positron, with rest energy $m_e c^2$, decays into two photons of equal wavelength λ.

(i) Why are the wavelengths equal?
(ii) What differences arise if the positron is not stationary?
(iii) Calculate λ. [6]

d) An X-ray of wavelength λ is scattered by a stationary electron producing a directly back scattered X-ray of wavelength λ_b. The electron acquires a, non-relativistic, energy of 5.00 keV.
Determine the wavelength λ.

(The momentum of a photon, p, is related to its energy, E, by $E = pc$.) [8]

Q10

a) An insulated closed vessel contains a block of ice at –20°C. A constant wattage heater and a temperature sensor are embedded in the ice. The heater is switched on and the system slowly heats up so that it is always at a uniform temperature, at all times.

Sketch graphs showing the variation, from -20°C to 120°C, of the following:

(i) temperature, T, with time, t

(ii) density, ρ, with temperature, T

Explain the arrangement and motions of the molecules in the different temperature regions. [9]

b) A Dewar flask contains a liquefied gas into which has been inserted an electrical heater. The heater has a resistance of 2.00 Ω and takes a current of 0.90 A. The flask rests on a balance. The balance reading is recorded at two minute intervals, before and after the heater is switched on. The table below gives the recorded results.

Determine the specific heat of vaporization of the liquid. [5]

Time/*minutes*	0.00	2.00	4.00	6.00	8.00	10.00	12.00	14.00	16.00
Mass/*grams*	73.0	72.5	72.0	71.5	70.5	68.5	66.5	64.5	62.5

c) Black coffee at 70.0°C, with a mass of 225 grams and specific heat capacity of 4200 J $kg^{-1}°C^{-1}$, is poured into a porcelain mug which is at 20.0°C, with mass 200 grams and specific heat capacity of 1200 J $kg^{-1}°C^{-1}$. This is followed by the addition of 15 grams of cream at 5.0 °C, with specific heat capacity of 4000 J $kg^{-1}°C^{-1}$. Determine the final equilibrium temperature of the white coffee assuming no heat is lost.

Explain, without any calculation, why more black coffee, at 70°C, is required to raise the equilibrium temperature of the white coffee by 1°C when it is at temperature T_1°C compared with that at temperature T_2°C, T_1 being greater than T_2. [6]