SOLUTIONS TO BPhO 2002 PAPER 2

Q1

		MARK	TOTAL
(a) (i)	$I = 5$ amps	1	
(ii)	$20 = 6I + 8$	1	2
	$I = 2$ amps	1	
(iii)	Balanced Wheatstone Bridge $I = 0$	2	
(iv)	$6\,\Omega$ in parallel with $(6\Omega + 6\Omega) = 12\Omega$ gives $\left(\frac{1}{6}+\frac{1}{12}\right)^{-1} = 4\,\Omega$	2	3
	$4\,\Omega + 6\,\Omega = 10\,\Omega$.		
	$\therefore I = \frac{20}{10} = 2$ amps.	1	
			[8]

Q1 (b)

MARKS

Condition for equilibrium

$$\frac{dN_1}{dt} = \frac{dN_2}{dt} \qquad \text{where} \qquad N = N_i e^{-\lambda_i t} \quad i = 1 \text{ or } 2$$

$$N_1\lambda_1 = N_2\lambda_2 \qquad (1) \qquad 2$$

Now half life $T_i = \ln 2 / \lambda_i \qquad (2)$

Hence from (1) & (2) $\quad N_1 T_2 = N_2 T_1 \qquad 2$

$$T_1 = \frac{N_1}{N_2} T_2$$

Now $\dfrac{N_1}{N_2} = \left(\dfrac{1.00}{238}\right) \Big/ \dfrac{0.300 \times 10^{-6}}{226}$

$$= \left(\frac{226}{238}\right)\frac{10^6}{(0.300)} \qquad 2$$

$$T_1 = \left(\frac{226}{238}\right)\left(\frac{10^6}{0.300}\right) 1602 \text{ years}$$

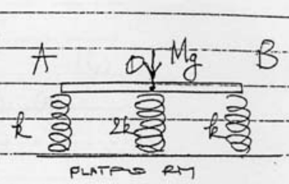
$$= 5.07 \times 10^9 \text{ years} \qquad 2$$

[8]

SOLUTIONS

Mark

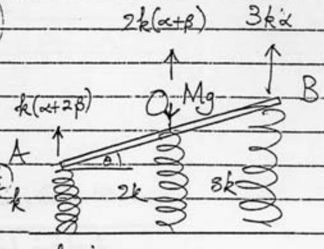
Q1 (c)



i) By symmetry beam horizontal
If compression of springs all x then (1/2)

$$Mg = kx + 2kx + kx = 4kx$$

$$x = \frac{Mg}{4k}$$ (1)



ii) No symmetry, beam inclined
Assume spring B depressed by α
spring O " by $(\alpha+\beta)$
spring A " " $(\alpha+2\beta)$ (1½)
as the compression must be linear — beam linear

Upward compressional forces:

B	$3k\alpha$
O	$2k(\alpha+\beta)$
A	$k(\alpha+2\beta)$

(1)

Resolving forces vertically

$$3k\alpha + 2k(\alpha+\beta) + k(\alpha+2\beta) = Mg \quad (1)$$

$$k(6\alpha + 4\beta) = Mg$$

Moments about O of a beam length $2l$ inclined at angle θ gives
(Alternative moments about A or B acceptable)

$$(l\cos\theta)\,k(\alpha+2\beta) = (l\cos\theta)3k(\alpha)$$

$$2\alpha = 2\beta \quad (1) \qquad \alpha = \beta = \frac{Mg}{10k} \quad (1)$$

~~Check ✓~~

~~Moments about B; $Mg - 2k(\alpha+\beta) = 2k(\alpha+2\beta)$~~

~~$k(4\alpha + 6\beta) = Mg \qquad \alpha = \frac{Mg}{10k}$~~

Compressional forces:

A : $\frac{3Mg}{10}$
O : $\frac{4Mg}{10}$
B : $\frac{3Mg}{10}$ (1)

[8]

Q1 (d) Mark

(i) At ignition weight of rocket balances the thrust

$1.00\times10^4 g = 9.00\times10^2 m$ (1)

$\underline{m = 1.09\times10^2\ \text{kgs}^{-1}}$ ($\frac{1}{2}$) } ($1\frac{1}{2}$)

(ii) M_R = Mass of rocket = 2.00×10^3 kg Thrust $T = 1.00\times10^4 g$ weight $M_R g$

Now $M_R a_e = T - M_R g = 10^4 g - 2\times10^3 g = 8\times10^3 g$ (1)

$a_e = 8\times10^3 g / 2\times10^3 = 4g = \underline{39.2\ \text{ms}^{-2}}$ ($\frac{1}{2}$) ($2\frac{1}{2}$)

(iii) $M(t) = M(0) - mt$

$= 1.00\times10^4 - 1.09\times10^2 t$

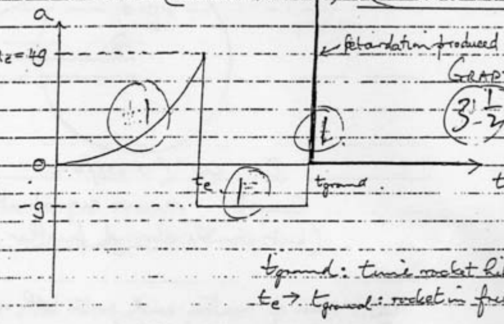
$\underline{M(t) = 1.00\times10^4 - 1.09\times10^2 t}$ } ($1\frac{1}{2}$)

(iv) $M(t)\,a = T - (M - kt)g$ (1) $= 1.00\times10^4 g - (1.00\times10^4 - 1.09\times10^2 t)g$ (1)

$(1.00\times10^4 - 1.09\times10^2 t)a = 1.09\times10^2 t g$

$a = \dfrac{1.09\times10^2\times9.81\,t}{(1.00\times10^4 - 1.09\times10^2 t)} = \dfrac{1.07\times10^3 t}{(1.00\times10^4 - 1.09\times10^2 t)}$ (1) ($2\frac{1}{2}$) (3)

(v)



← Retardation produced by ground

GRAPH ($3\frac{1}{2}$)

t_{ground}: time rocket hits ground

$t_e \rightarrow t_{ground}$: rocket in free fall

[12]

(e) Large raindrop 1cc = 10^{-6} m^3 1

mass $10^{-6}(1000) = 10^{-3}$ kg (1000 kg per m^3) 1

No. of ~~atoms~~ molecules in raindrop $= 6\times10^{23}\left(\dfrac{10^{-3}}{18\times10^{-3}}\right)$ (mol.wt 18) 3

$= \frac{1}{3}(10^{23})$ 1

Mark

Q1 (e) Vol. of Med. $= \frac{1}{3}(10^{23})(10^{-6})m^3 = 3\times10^{16} m^3$ 2

[8]

(f) Stationary above equator 1

Period of Moon 28 days 1

Kepler's law: $(28)^2 = k\,(3.82\times10^8)^3$ 1

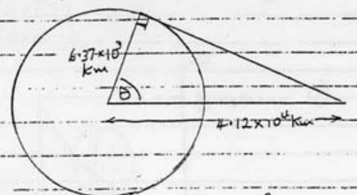
Period of satellite 1 day 1

Kepler's law: $1^2 = kR^3$ 1

Solving for R: $R = 4.12\times10^4$ km 2 1

[Satellite $(R-R_E)$ above Earth's surface $(R-R_E) = 3.84\times10^4$ km]

Satellite signal tangential to Earth's surface prior to extinction



$\theta = \cos^{-1}(0.637/4.12) \approx 81^\circ$

Cannot receive signals at latitudes $> 81^\circ$ directly 2

(but possible through further reflections) 2

[8]

(g) Mass of neutron and proton both m

Mass of deuteron 2m

Conservation of momentum

$mu_n = mv_n + 2mv_d$ ie $u_n = v_n + 2v_d$ ① 2

Conservation of energy

$\frac{1}{2}mu_n^2 = \frac{1}{2}mv_n^2 + \frac{1}{2}(2m)v_d^2$ ie $u_n^2 = v_n^2 + 2v_d^2$ ② 2

Eliminating v_n from ① & ②

$u_n^2 = (u_n - 2v_d)^2 + 2v_d^2$ 3

$\frac{v_d}{u_n} = \frac{2}{3}$ 1

Fraction of energy acquired by deuteron

$E = \dfrac{\frac{1}{2}(2m)v_d^2}{\frac{1}{2}(m)v_n^2} = \dfrac{2v_d^2}{v_n^2} = 2\left(\frac{2}{3}\right)^2 = \frac{8}{9}$ ie 89% 2

Q1 Mark Total

(g) If n collisions slow neutron from 1MeV to 0.01eV then $\left(\frac{1}{9}\right)$th of energy retained by neutron after each collision. Hence

$$\left(\frac{1}{9}\right)^n < \frac{0.01}{10^6} = 10^{-8}$$ 4

$$n > 8.38$$ 1

$$\therefore \; n = 9$$ 1

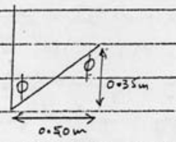
[16]

(h)

$$\sin\phi = \frac{\lambda}{d}.$$

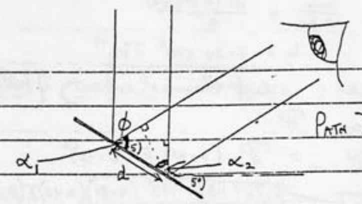
or $d = \lambda / \sin\phi$ } 1

i.e. $\phi = 55.0°$



$$d = \frac{590}{0.819} \quad \left(\sin\theta = \frac{0.50}{\sqrt{(0.50)^2 + (0.35)^2}}\right)$$ 2

$$= \underline{720\ \text{nm}}$$ 1



PATH DIFF $= d\left[\cos\alpha_1 - \cos\alpha_2\right]$

Path difference $= d\cos(90 - \phi + 5) - d\cos(90 - 5)$ 4

$= d\left[\sin(\phi - 5) - \sin(5)\right]$ } 3

$= d\left[\sin(50°) - \sin(5°)\right]$

$\therefore \quad \underline{\lambda = (720)(0.6788) = 489\ \text{nm}}$ 1

[12]

Mark Total

Q2

(a) U : sum of P.E. and K.E. of molecules 1 — not essential for full marks for this part

ΔU : change in internal energy 1

ΔW : work done on system $= -\int p\,dV$ (vol. of system) (V decreases if W +ve) 1

ΔQ : Heat produced $= \Delta U - \Delta W$ (transferred to the system)

$\boxed{\Delta Q = \Delta U - \Delta W} = \Delta U + \int p\,dV$ — not essential to express $\Delta W = -\int p\,dV$ for full marks 2

FULL MARKS ONLY REQUIRES RESULT $\Delta Q = \Delta U - \Delta W$ (or equivalent result if different notation used) [5]

(b) Heat to raise temp. from 20C to 100C (mass m)

$= m(4200)80$ J $\frac{1}{2}$

Energy required to boil water at 100C $= mL$ J $\frac{1}{2}$

Energy supplied per sec P

$$P = m(4200)80/(180) = \frac{m(4200)(4)}{9}$$ $\frac{1}{2}$

In 1200 s energy supplied

$$1200P = mL$$

$$P = \frac{mL}{1200}$$ $\frac{1}{2}$

Hence

$$\frac{mL}{1200} = \frac{m(4200)(4)}{9}$$ $\frac{1}{2}$

$$L = 2.24 \times 10^6 \text{ J kg}^{-1}$$ $\frac{1}{2}$

Assumption: neglect thermal capacity of kettle 1 [4]

(c) (i) Atmospheric pressure $\mathbb{P}$ $\frac{1}{2}$

(ii) $W_{atmos} = -\int \mathbb{P}\,dV = \mathbb{P}\,(0.10)\,\pi\,(0.12)^2$

given $\boxed{1.01 \times 10^5 \text{ Pa}}$

$= \boxed{0.76(1.35)10^4(9.81)}(0.10)\pi(0.12)^2$

$= \boxed{7.6\ (1.35)(9.81)}\pi(1.44)$

$= 4.55 \times 10^2$ J $2\frac{1}{2}$

(iii) 100°C $\frac{1}{2}$

(iv) $Q_c = -\Delta U - \Delta W = L\Delta m - W_{Atmos}$.

$= -(2.24 \times 10^6)\frac{1}{2}(0.37 \times 10^{-3}) - 4.55 \times 10^2$ J $2\frac{1}{2}$

$Q_c = -8.7 \times 10$ J (−ve sign, heat transferred to surroundings) [6]

(d) $\frac{3}{2}kT = \frac{1}{2}mu^2 = mgH$ 1

$$H = \frac{3kT}{2mg} = \frac{3}{2}\,\frac{(1.38 \times 10^{-23})\,283}{32\,(1.67 \times 10^{-27})(9.81)}$$ 1

$= 1.47 \times 10^4$ m $\frac{1}{2}$

(Alternatively one can use $\frac{3}{2}kT = -GM_E m\left[\frac{1}{(R_E+H)} - \frac{1}{R_E}\right]$ with same result)

Mark Total

Escape : $\frac{3}{2}kT_s = \frac{mGM_E}{R_E}$ 1

$T_s = \frac{2}{3k}\left(\frac{mGM_E}{R_E}\right)$

$= \frac{2(32 \times 1.67 \times 10^{-27})(6.67 \times 10^{-11})(5.97) \times 10^{24}}{3(1.38 \times 10^{-23})(6.37 \times 10^{6})}$ $1\frac{1}{2}$

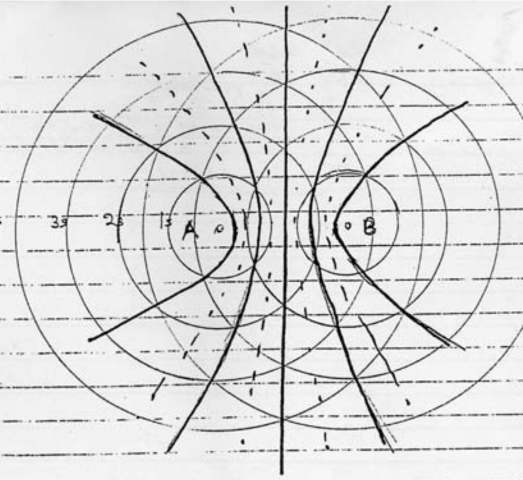
$\underline{T_s = 1.6 \times 10^5 \text{ K}.}$

[5]

MARKS

Q3

(a) (i)



— CONSTRUCTIVE INTERFERENCE (CROSS-OVER PTS)

------ DESTRUCTIVE INTERFERENCE (ANTI-CROSSOVER POINTS)

(i) Drawing of two sets of circles to scale — 2

(ii) Constructive : $x-y = n\lambda$ — 1

Destructive : $x-y = (n+\frac{1}{2})\lambda$ — 1

where n is an integer

(iii) See diagram :

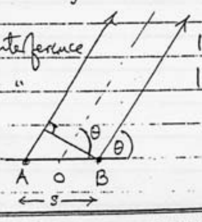
3 full (constructive) curves — 2

2 broken (destructive) curves. — 2

(bonus mark for 2 extra lines ($\frac{1}{2}$ mark for 1) — 1

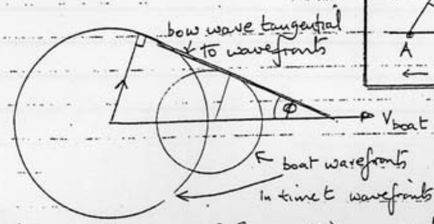
(iv) $s\cos\theta = n\lambda$ constructive interference — 1

$= (n+\frac{1}{2})\lambda$ destructive " — 1



[10]

(b)



EXPLANATION

In time t wavefronts move distance $v_{wave}t$ and boat a distance $v_{boat}t$ — 2

$\sin\phi = \frac{v_{wave}}{v_{boat}} = \frac{2.5}{5.0} = 0.5$

$\phi = 30°$

CALCULATION 2

[4]

Q4

(a) (i) Velocity of charged particle constant as it is perpendicular to B field which provides a constant force perpendicular to velocity, producing circular motion — $1\frac{1}{2}$

Direction of magnetic field must be out of the diagram perpendicular to plane of diagram — $\frac{1}{2}$

[4]

(ii) For circular motion

$$BQv = \frac{Mv^2}{R} \qquad 1\tfrac{1}{2}$$

As $v \neq 0$,

$$\underline{BQR = Mv} \quad (A) \qquad \tfrac{1}{2}$$

(b)(i) $E = \frac{1}{2}Mv^2 = VQ$ (B) or $v = \sqrt{\frac{2VQ}{m}}$ $(V = 500V)$ — 1

Let

$$v_0 = \sqrt{\frac{2VQ}{m_p}} = \sqrt{\frac{2(500)(1.60 \times 10^{-19})}{1.67 \times 10^{-27}}} = \underline{3.095 \times 10^{5}\, ms^{-1}} \qquad 1$$

For $^{39}_{19}K$. $\quad v_{39} = \frac{1}{\sqrt{39}} v_0 = \underline{4.96 \times 10^4\ ms^{-1}}$ — $\frac{1}{2}$

For $^{41}_{39}K$ $\quad v_{41} = \frac{1}{\sqrt{41}} v_0 = \underline{4.83 \times 10^4\ ms^{-1}}$ — $\frac{1}{2}$

(ii) Speed at which ions hit film same as injection speed v_{39} and v_{41} (given above) as speed constant for circular motion — 1

(iii) $OP = 2R$ as trajectories are semi circles

$\therefore OP = \frac{2Mv}{BQ}$ from (A) — 1

Substituting $M = 39m_p$ and $M = 41m_p$, the values of v_{39} and v_{41} and $B = 0.7T$

$(OP)_{39} = 5.76$ cm — 1

$(OP)_{41} = 5.91$ cm — 1

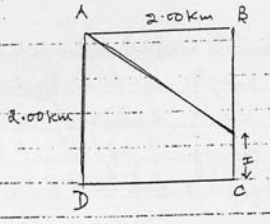
(NB $\Delta OP = (OP)_{41} - (OP)_{39} = 0.15$ cm) (C)

[7]

Mark | Tota

Q(3)

(c)



(i) $\lambda = \frac{3\times10^{8}}{1\times10^{7}} = 30\,m$

(ii) At B $\quad 2000 = n\lambda = n30$ $\frac{1}{2}$

$n = \frac{2000}{30} = 66.7$ $\frac{1}{2}$

Using units of km

(ii) $(2\sqrt{2} - 2) = n\lambda = \left(\frac{30}{1000}\right)n$ $\frac{1}{2}$

$n = \frac{200}{3}(\sqrt{2}-1) = 27.6$ $\frac{1}{2}$

Path difference $\sqrt{2^2+(2-x)^2} - (2-x) = 28\lambda = 28\left(\frac{30}{1000}\right)$ 1

$= \frac{21}{25}$ 2

EITHER Substitute $x = 0.039$ km and verify that LHS = RHS of equation

OR

$$\sqrt{2^2+(2-x)^2} = \frac{21}{25} + (2-x)$$

$$2^2+(2-x)^2 = \left(\frac{21}{25} + (2-x)\right)^2$$

$$= \left(\frac{21}{25}\right)^2 + 2\frac{21}{25}(2x) + (2-x)^2$$

$$\text{ie} \quad 2^2 = \left(\frac{21}{25}\right)^2 + \frac{42}{25}(2-x)$$

$$x = \left[\left(\frac{21}{25}\right)^2 - 2^2 + \frac{84}{25}\right]\frac{25}{42}$$

$$= \frac{25}{42}\left[\left(\frac{21}{25}\right)^2 - \frac{16}{25}\right]$$

$$= \frac{25}{42}\left(\frac{1}{25}\right)^2[441-400]$$

$$= \frac{41}{(42)(25)}$$

$$= 0.039 \text{ km}$$ 2

$$= 39\,m$$

[6]

Mark Total

Q4

(c)(i) One can obtain the OP variation for $V = (500 \pm 5)V$ by taking the limiting cases 505V and 495V. However it is simpler to apply calculus – both methods obtain full marks

CALCULUS METHOD
$$\Delta OP = 2\Delta R = \frac{\sqrt{2ME}}{QB}\frac{\Delta E}{E} \quad \text{from (B)}$$

Substituting $\frac{\Delta E}{E} = \pm\frac{5}{500} = \pm 0.01$ and the 'mean' values of M and E

$$\Delta OP = 0.06 \text{ cm}$$

OR

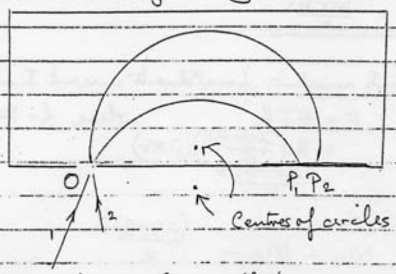
LIMITING CASE METHOD will give $\Delta OP = 0.06$ for each ion

5

Thus each ion has an "uncertainty" of ± 0.06cm. In worst situation when displacements in opposite directions, due to (500 ± 5)V, this leads to a separation of the two ions of $(0.15-0.12)$cm $= 0.03$cm (equation (C)). Consequently the spectrometer is capable to distinguishing the two species

2

(ii)



Paths remain circular, with the same radius, but centres of circles no longer along OP

2

[9]

Mark Total

Q5

(i) Resultant vertical downward force on student = 5g N $\frac{1}{2}$

Vertical downward acceleration = $\frac{5}{80}g = \frac{9.81}{16} = 0.613\,ms^{-2}$ $1\frac{1}{2}$

(ii) $\frac{1}{16}g$ is the vertical component of acceleration down slope

Thus acceleration down slope is

$$\frac{1}{16}g\ \frac{1}{\cos 60^\circ} = \frac{1}{8}g = 1.23\ ms^{-2}$$ 3

(iii) Resolving forces down slope, where R is normal reaction,

$$\frac{Mg}{8} = Mg\cos 60 - \mu R.$$ $2\frac{1}{2}$

$$= \frac{1}{2}Mg - \mu R$$ $\frac{1}{2}$

$$\therefore\ \mu R = \frac{3}{8}Mg$$ $\frac{1}{2}$

But $R = Mg\cos 30^\circ = Mg\frac{\sqrt{3}}{2}$ $\frac{1}{2}$

So

$$\mu = \frac{3}{8}\frac{2}{\sqrt{3}} = \frac{\sqrt{3}}{4} = 0.433$$ 1

[10]

(b) (i) $I = \frac{d\Phi}{dt}\frac{1}{R}$ where Φ flux. 1

Now $\frac{d\Phi}{dt} = 2\pi r Bv$ 1

$I = \frac{2\pi r Bv}{R}$ $\frac{1}{2}$

(ii) Vertical resistive force F due to current I in ring given by

$F = BIl$ where $l = 2\pi r$. 1

Sub^n for I, $= B\left(\frac{2\pi r Bv}{R}\right)(2\pi r)$ $\frac{1}{2}$

$= \frac{(2\pi r B)^2 v}{R}$ $\frac{1}{2}$

$\Rightarrow$ Equation of motion

$$Ma = Mg - \frac{(2\pi r B)^2 v}{R}$$ (A) $\frac{1}{2}$

(iii) Neglecting "v" force in (A) $Ma = Mg$

$a = g$ (constant accn. motion) $\frac{1}{2}$

$\therefore\ v = gt$ for small t 1

From (A) terminal velocity occurs when $a = 0$ ie $V_\infty = \frac{RMg}{(2\pi r B)^2}$ $1\frac{1}{2}$

Mark Total

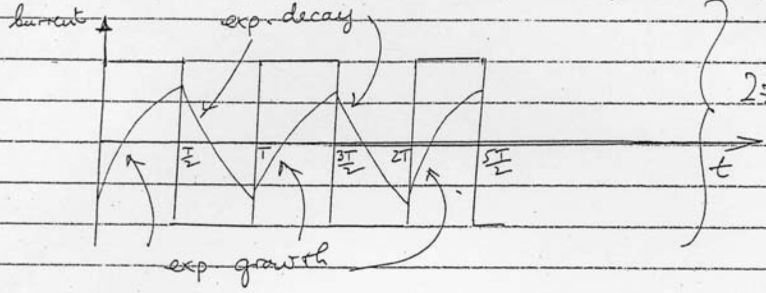
Q6 (d) Time constant $\tau = CR = 300 \times 1.5 \times 10^{-9}$

$\tau = 0.45 \times 10^{-6}$ s

1 MHz has a period T of, $T = 10^{-6}$ s

$2\frac{1}{2}$

As T and τ comparable the resistance will produce an appreciable decay of the current and an exponential growth, giving a <u>degraded</u> square wave signal.



$2\frac{1}{2}$

[5]

) I is ~~the~~ charge flowing per second
Charge on capacitor $Q = CV$
This discharges through ammeter n times per sec
Total charge flowing through ammeter in 1s
$\underline{I = nCV}$ } 3

(i) Capacitor would not charge fully to CV in one cycle
Current would consequently be reduced 2
(ii) Capacitor would not fully discharge in one cycle
Current would be reduced 2

[7]

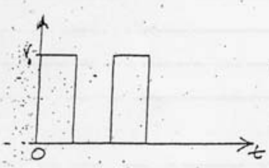
b) $\frac{4}{\pi}\cos(2\pi f t) - \left(\frac{4}{\pi}\right)\frac{1}{3}\cos(6\pi f t) + \left(\frac{4}{\pi}\right)\left(\frac{1}{5}\right)\cos(10\pi f t)$

Amplitude A	$\left(\frac{4}{\pi}\right)$	$\left(\frac{4}{3\pi}\right)$	$\left(\frac{4}{5\pi}\right)$
Frequency ν	f	$3f$	$5f$
Phase θ	0	$\pm\pi$	0
Period T	$\frac{1}{f}$	$\frac{1}{3f}$	$\frac{1}{5f}$

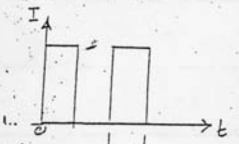
$\frac{1}{2}\times 12 = 6$

[4]

c) $V_F(t)$

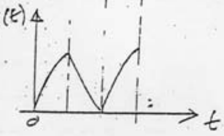


(i)



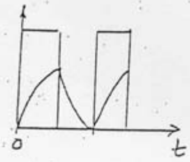
Current through resistor [$1\frac{1}{2}$]

(ii) $Q(t)$



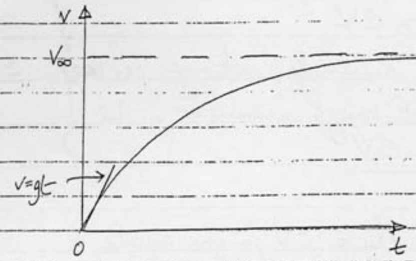
Charge Q on the Capacitor [2]

(iii)

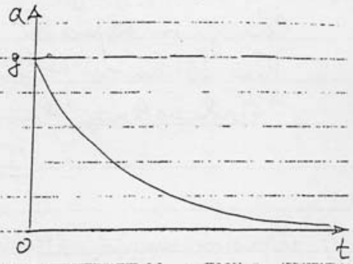


Superimpose current and charge [$\frac{1}{2}$]

Assumptions: (a) resistor has no stray capacitance
(b) capacitor is a perfect component
(no time constant given)



GRAPH 1



GRAPH 2

	Mark
GRAPH 1	1
GRAPH 2	1

[10]

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