

# BRITISH MATHEMATICAL OLYMPIAD

Round 1 : Wednesday 18th January 1995

**Time allowed** *Three and a half hours.*

**Instructions** • *Full written solutions are required. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then draft your final version carefully before writing up your best attempt. Do not hand in rough work.*

- *One complete solution will gain far more credit than several unfinished attempts. It is more important to complete a small number of questions than to try all five problems.*
- *Each question carries 10 marks.*
- *The use of rulers and compasses is allowed, but calculators and protractors are forbidden.*
- *Start each question on a fresh sheet of paper. Write on one side of the paper only. On each sheet of working write the number of the question in the top left hand corner and your name, initials and school in the top right hand corner.*
- *Complete the cover sheet provided and attach it to the front of your script, followed by the questions 1,2,3,4,5 in order.*
- *Staple all the pages neatly together in the top left hand corner.*

Do not turn over until **told to do so**.

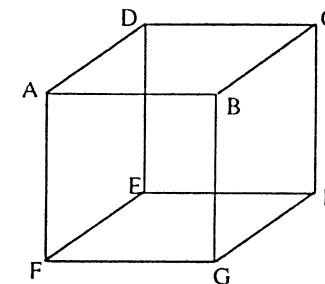
# BRITISH MATHEMATICAL OLYMPIAD

1. Find the first positive integer whose square ends in three 4's. Find all positive integers whose squares end in three 4's. Show that no perfect square ends with four 4's.

2.  $ABCDEFGH$  is a cube of side 2.

(a) Find the area of the quadrilateral  $AMHN$ , where  $M$  is the midpoint of  $BC$ , and  $N$  is the midpoint of  $EF$ .

(b) Let  $P$  be the midpoint of  $AB$ , and  $Q$  the midpoint of  $HE$ . Let  $AM$  meet  $CP$  at  $X$ , and  $HN$  meet  $FQ$  at  $Y$ . Find the length of  $XY$ .



3. (a) Find the maximum value of the expression  $x^2y - y^2x$  when  $0 \leq x \leq 1$  and  $0 \leq y \leq 1$ .

(b) Find the maximum value of the expression

$$x^2y + y^2z + z^2x - x^2z - y^2x - z^2y$$

when  $0 \leq x \leq 1$ ,  $0 \leq y \leq 1$ ,  $0 \leq z \leq 1$ .

4.  $ABC$  is a triangle, right-angled at  $C$ . The internal bisectors of angles  $BAC$  and  $ABC$  meet  $BC$  and  $CA$  at  $P$  and  $Q$ , respectively.  $M$  and  $N$  are the feet of the perpendiculars from  $P$  and  $Q$  to  $AB$ . Find angle  $MCN$ .

5. The seven dwarfs walk to work each morning in single file. As they go, they sing their famous song, "*High - low - high - low, it's off to work we go ...*". Each day they line up so that no three successive dwarfs are either increasing or decreasing in height. Thus, the line-up must go *up-down-up-down-...* or *down-up-down-up-...*. If they all have different heights, for how many days they go to work like this if they insist on using a different order each day?

What if Snow White always came along too?