Calculator Permitted

1. Solve algebraically, over the set of real numbers, giving exact values where possible. **(4 marks)**

$$3\sin^2\theta + 5\cos\theta = 1$$

SOLUTION

$$3\sin^2\theta + 5\cos\theta = 1$$

$$3\left(1-\cos^2\theta\right) + 5\cos\theta - 1 = 0 \qquad \leftarrow \textbf{1 mark}$$

$$3 - 3\cos^2\theta + 5\cos\theta - 1 = 0$$

$$-3\cos^2\theta + 5\cos\theta + 2 = 0$$

$$3\cos^2\theta - 5\cos\theta - 2 = 0$$

$$(3\cos\theta + 1)(\cos\theta - 2) = 0$$

$$\tfrac{1}{2}\textbf{ mark} \longrightarrow \quad \cos\theta = -\frac{1}{3} \qquad \cancel{\cos\theta = 2} \leftarrow \tfrac{1}{2}\textbf{ mark}$$

$$\tfrac{1}{2}\textbf{ mark} \longrightarrow \quad 1.91 + 2\pi n, \quad n \in I$$

$$\tfrac{1}{2}\textbf{ mark} \longrightarrow \quad 4.37 + 2\pi n, \quad n \in I$$

$$\uparrow$$

1mk

2. Prove algebraically:

$$\frac{\tan 2\theta(1-\tan\theta)\cos^2\theta}{\sin 2\theta} = \frac{1}{1+\tan\theta}$$

SOLUTION

LEFT SIDE	RIGHT SIDE
$\dfrac{\tan 2\theta(1-\tan\theta)\cos^2\theta}{\sin 2\theta}$	$\dfrac{1}{1+\tan\theta}$
1 mark $\rightarrow$ $= \dfrac{\dfrac{2\tan\theta}{\left(1-\tan^2\theta\right)}\ (1-\tan\theta)\cos^2\theta}{2\sin\theta\cos\theta}$	
1 mark $\rightarrow$ $= \dfrac{\dfrac{2\tan\theta(1-\tan\theta)\cos^2\theta}{(1-\tan\theta)(1+\tan\theta)}}{2\sin\theta\cos\theta}$	
$\frac{1}{2}$ **mark** $\rightarrow$ $= \dfrac{\dfrac{2\sin\theta\cos^2\theta}{\cos\theta(1+\tan\theta)}}{2\sin\theta\cos\theta}$	
$\frac{1}{2}$ **mark** $\rightarrow$ $= \dfrac{\dfrac{2\sin\theta\cos\theta}{1+\tan\theta}}{2\sin\theta\cos\theta}$	
1 mark $\rightarrow$ $= \dfrac{1}{1+\tan\theta}$	

ALTERNATE SOLUTION

$$\frac{\tan 2\theta(1-\tan\theta)\cos^2\theta}{\sin 2\theta} = \frac{1}{1+\tan\theta}$$

Left Side

$\frac{1}{2}$ **mark** $\rightarrow$ $= \frac{\cancel{\sin 2\theta}}{\cos 2\theta}(1-\tan\theta)\cos^2\theta\,\frac{1}{\cancel{\sin 2\theta}}$

$\frac{1}{2}$ **mark** $\rightarrow$ $= \frac{(1-\tan\theta)\cos^2\theta}{\cos 2\theta}$

$\frac{1}{2}$ **mark** $\rightarrow$ $= \frac{(1-\tan\theta)\cos^2\theta}{\cos^2\theta-\sin^2\theta}$

Right Side

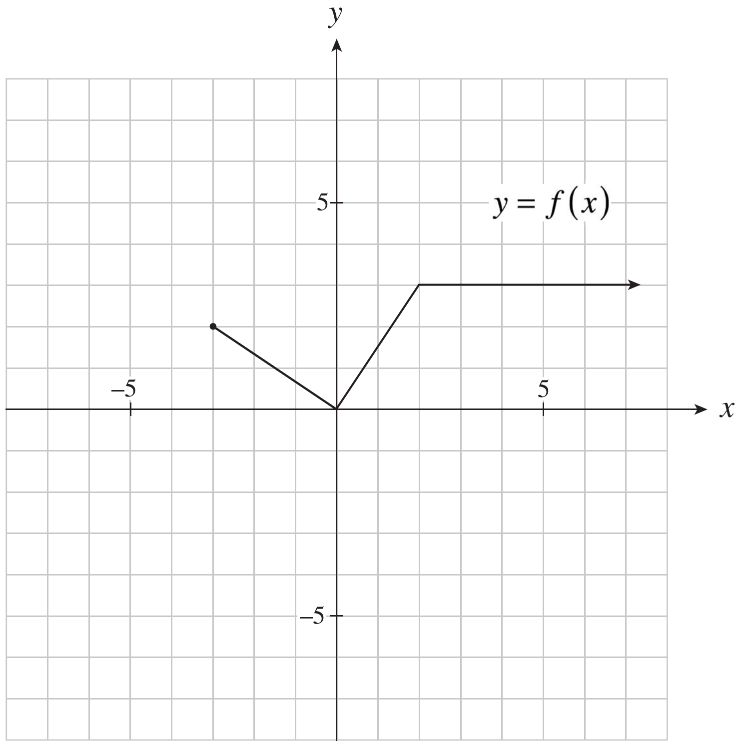
$\left(\frac{1}{1+\tan\theta}\right)\left(\frac{1-\tan\theta}{1-\tan\theta}\right)$ $\leftarrow$ **1 mark**

$\frac{1-\tan\theta}{1-\tan^2\theta}$

$\left(\frac{1-\tan\theta}{1-\frac{\sin^2\theta}{\cos^2\theta}}\right)\left(\frac{\cos^2\theta}{\cos^2\theta}\right)$ $\leftarrow$ **1 mark**

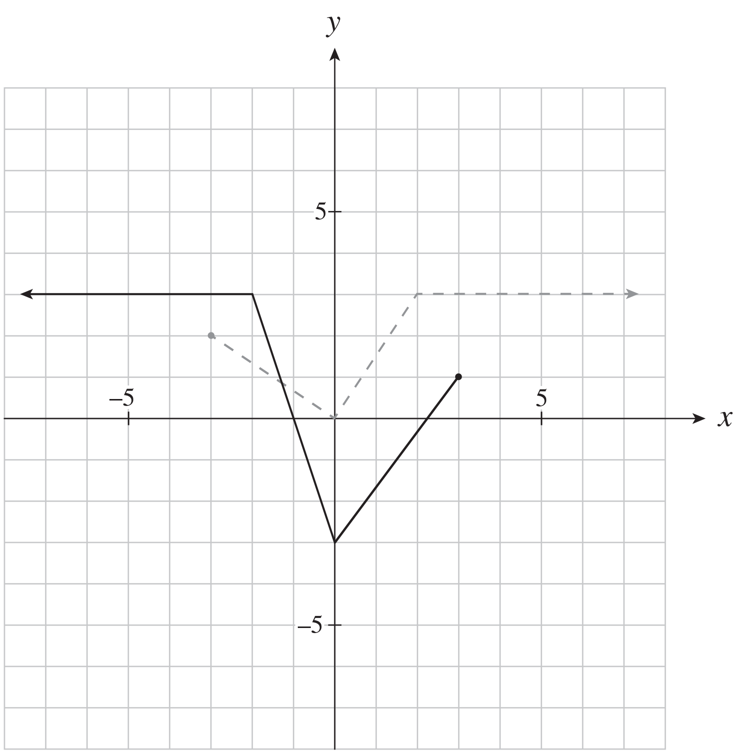
$\frac{(1-\tan\theta)\cos^2\theta}{\cos^2\theta-\sin^2\theta}$ $\leftarrow$ $\frac{1}{2}$ **mark**

3. The graph of $y = f(x)$ is shown below.



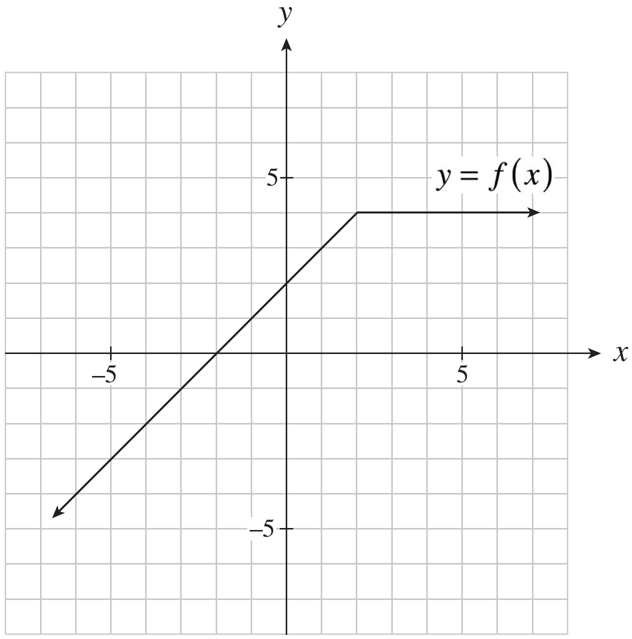
On the grid provided, sketch the graph of $y = 2f(-x) - 3$. **(4 marks)**

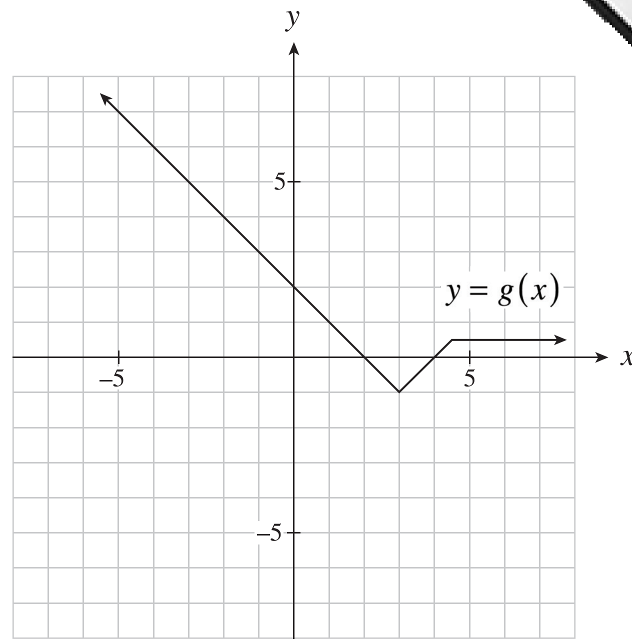
SOLUTION



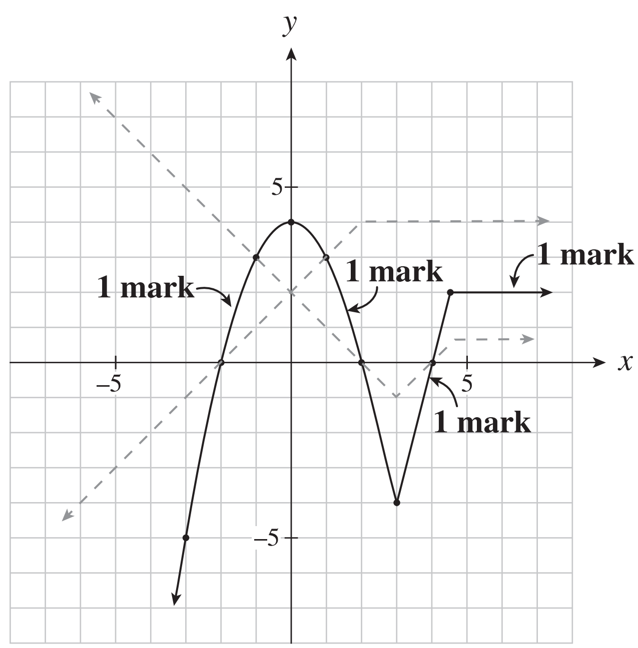
1 mark: vertical expansion
1 mark: reflection
1 mark: down 3
1 mark: ends/shape

4. Use the graphs of $y = f(x)$ and $y = g(x)$ shown to sketch the graph of $y = f(x)g(x$
on the grid provided. Clearly indicate a sufficient number of points to get an accurate representation of your graph. **(4 ma**





SOLUTION



> ☞ *Students should notice which portions of the graph resulting from product function $y = f(x)g(x)$ are parabolic and linear.*

Calculator NOT Permitted

1. Solve algebraically: $\log_{15}(3-x)+\log_{15}(1-x)=1$.

   Justify the validity of each solution. **(4 marks)**

SOLUTION

$\log_{15}(3-x)(1-x)=1 \quad \leftarrow \tfrac{1}{2}$ **mark**

$(3-x)(1-x)=15 \quad \leftarrow \tfrac{1}{2}$ **mark**

$3-4x+x^2=15$

$x^2-4x-12=0$

$(x-6)(x+2)=0$

$x=6\,, \quad x=-2 \quad \leftarrow \tfrac{1}{2}$ **mark** $+\ \tfrac{1}{2}$ **mark**

$x=6$ is rejected because either $3-x$ or $1-x$ will result in a negative number.
The log of a negative number is not defined. $\leftarrow$ **1 mark**

$x=-2$ is accepted because both $3-x$ and $1-x$ result in a positive number.
The log of a positive number is defined. $\leftarrow$ **1 mark**

Alternate validation

$D:\ x<1$

$x=6$ is rejected because it is not the domain.

$x=-2$ is accepted because it is in the domain.

2. Consider the graphs of $f(x)=\dfrac{x^2-x-6}{x^2-9}$ and $g(x)=\dfrac{x}{x^2-9}$. (4

Use your knowledge of rational functions to outline the similarities and differences between these two graphs. You will be evaluated on the concepts expressed, the organization and accuracy of your work, and your use of language.

Scoring Rationale:

Concepts and Connections	***Curricular Reference*:** **B14.2 Analyze the graphs of a set of rational functions to identify common characteristics** **B14.4 Determine if the graph of a rational function will have an asymptote or a hole for a non-permissible value**
Problem Solving and Reasoning	**Students may:** **1) factor numerator and denominator** **2) cancel common factors**
Procedures	**The student was able to accurately determine vertical asymptotes, *x*-intercepts, domain and the existence of a hole in the graph**
Representation and Communication	**The reader is able to easily understand that the student has used the proper terminology and has connected the similarities and differences in a clear, complete and organized manner**

Criteria:

- **Consider domain, asymptotes, holes and intercepts**
- **Consider similarities and differences**

Pre-Calculus Mathematics 12 Scoring Rubric

	1 Below Expectations	2 Minimal	3 Proficient	4 Excellent
Snapshot	*Does not meet basic requirements of the problem*	*Partially solves the problem and meets some basic requirements but solution is incomplete or flawed*	*Solution is reasonable and complete for most parts of the task. All requirements met (may be minor flaws)*	*Solution is well-developed and justified. Thoroughly satisfies requirements; may be insightful or innovative*
Concepts and Connections [CN] *Recognizes the mathematics needed; explanation shows understanding of concepts*	• Does not recognize the mathematics; shows little/no understanding (may misunderstand)	• Recognizes/applies some concepts needed; shows partial understanding (often vague/incomplete)	• Recognizes/applies concepts needed; shows understanding of most relevant concepts	• Recognizes/applies concepts needed (may make insightful connections); shows thorough understanding
Problem-solving and reasoning [PS] [R] [V] [ME] *Uses appropriate strategies to solve the problem* *Verifies and justifies that results are reasonable*	• Does not use appropriate strategies • Does not verify results or solutions	• Uses appropriate strategies for some parts • Attempts to verify or justify results or solutions but is not fully successful	• Uses appropriate strategies for all parts • Verifies and justifies results or solutions (may be imprecise)	• Selects and uses highly effective, and often innovative, strategies • Verifies and justifies results or solutions with precision
Procedures [ME] [T] *shows accuracy and precision (e.g., in recording, substitutions, calculations, units, and symbols); efficient*	• Limited accuracy; major errors or omissions	• Follows procedures with partial accuracy; some errors or omissions	• Follows procedures accurately with minor errors or omissions	• Follows procedures accurately; very few if any minor errors/omissions; highly efficient
Representation and Communication [C] [V] *Clear, complete, organized using words, pictures and/or numbers* *Includes appropriate graphics; representations (e.g., charts, tables, graphs, diagrams; sketches)*	• Unclear; confusing and/or incomplete • Omits required graphics or representations and/or does not construct them appropriately; many omissions; serious flaws	• Presents parts of the process and solution; parts are omitted or unclear • Constructs most required graphics and/or representations; parts are omitted or inappropriate	• Presents process and solution clearly • Work is generally clear; easy to follow • Constructs required graphics and/or representations appropriately; may have minor errors or flaws	• Presents process and solution clearly and effectively • Work is detailed, precise and logically organized • Constructs required graphics and/or representations effectively and accurately

Code 0

- Data simply recopied form the question
- Picture, work or solution does not relate to the problem
- Incorrect solution with no work shown
- Everything erased

Code NR

- No response (answer page is blank)

Resource A: Written-Response Question 2 (calculator NOT permitted) Exemp

Consider the graphs of $f(x)=\frac{x^2-x-6}{x^2-9}$ and $g(x)=\frac{x}{x^2-9}$.

Use your knowledge of rational functions to outline the similarities and differences between these two graphs. You will be evaluated on the concepts expressed, the organization and accuracy of your work, and your use of language.

D
R
Asy.
x int
y int

$f(x)$

$\frac{x^2-x-6}{x^2-9}$

$= \frac{(x+2)(x-3)}{(x-3)(x+3)}$

$= \frac{x+2}{x+3}$

x-int: -2, $\cancel{3}$

y-int: $\frac{2}{3}$

$D: \{x \mid x \neq 3, -3\}$

discontinuous

$R = \{y \mid y \neq 1\}$

Hori. Asy $= y = 1$

Vert. Asy $= x = -3$

$g(x) = \frac{x}{(x-3)(x+3)}$

x-int $= 0$

y-int $= 0$

$D = \{x \mid x \neq 3, -3\}$

$R = \{y \mid y \neq 0\}$

Hori. Asy $= y = 0$

Vert Asy $= x = 3, -3$

Both graphs have domains $x \neq 3, -3$. $G(x)$ has vert. Asymptote as $x = 3, -3$. $f(x)$ only has vert Asy. $x = -3$, because in $f(x)$, $x = 3$ is a point of discontinuity. $f(x)$'s horizontal asymptote is at $y = 1$, whereas $g(x)$'s is at $y = 0$. $f(x)$ x int is $x = -2$, y int is $y = \frac{2}{3}$, and $g(x)$'s x/y-int. is both at 0. The range of $f(x)$ is restricted to $y \neq 1$, and $g(x)$ is restricted to $y \neq 0$. $f(x)$ is a discontinuous function, while $g(x)$ is a regular function.

Exemplar #1: Score 4

- shows thorough understanding
- uses highly effective algebraic strategies
- highly efficient with one minor error (range) and one minor omission (*y*-coordinate of hole)
- presents solution clearly and efficiently

Resource A: Written-Response Question 2 (calculator NOT permitted) Exemp

Consider the graphs of $f(x)=\frac{x^2-x-6}{x^2-9}$ and $g(x)=\frac{x}{x^2-9}$.

Use your knowledge of rational functions to outline the similarities and differences between these two graphs. You will be evaluated on the concepts expressed, the organization and accuracy of your work, and your use of language.

$$f(x)=\frac{(x-\cancel{3})(x+2)}{(x\cancel{-3})(x+3)} \qquad g(x)=\frac{x}{(x+3)(x-3)},\quad x\neq\pm3$$

$$=\frac{x+2}{x+3}\quad x\neq\pm3$$

	$f(x)$	$g(x)$	Similarity? (Y/N)
y-int:	$\frac{2}{3}$	0	N
x-int:	-2	0	N
domain:	$x\neq\pm3$	$x\neq\pm3$	Y
range:	$\mathbb{R}$	$\mathbb{R}$	Y
vertical asymptote:	$x=-3$	$x=3$, $x=-3$	they share one asymptote alike, $x=-3$
horizontal asymptote:	$y=1$	$y=0$	N
points of discontinuity	$(3, \frac{5}{6})$	none	N

Exemplar #2: Score 4

- shows thorough understanding
- uses highly effective algebraic strategies
- follows procedures accurately with one minor error (range)
- work is detailed, precise and logically organized with appropriate use of vocabulary

Resource A: Written-Response Question 2 (calculator NOT permitted) Exemp

Consider the graphs of $f(x) = \frac{x^2 - x - 6}{x^2 - 9}$ and $g(x) = \frac{x}{x^2 - 9}$.

Use your knowledge of rational functions to outline the similarities and differences between these two graphs. You will be evaluated on the concepts expressed, the organization and accuracy of your work, and your use of language.

$f(x) = \frac{x^2-x-6}{x^2-9}$

$= \frac{(x-3)(x+2)}{(x+3)(x-3)}$

$D = \{x \mid x \in \mathbb{R},\ x \neq 3, -3\}$

$R = \{y \mid y \in \mathbb{R},\ y \neq 1\}$

asymptotes $= x \neq -3$
$y \neq 1$

x-int $=$ $x = 3, -2$

y-int $=$ $y = \frac{2}{3}$

$g(x) = \frac{x}{x^2-9}$

$= \frac{x}{(x-3)(x+3)}$

$D = \{x \mid x \in \mathbb{R},\ x \neq 3, -3\}$

$R = \{y \mid y \in \mathbb{R},\ y \neq 0\}$

asymptotes $:$ $x \neq 3, -3$
$y \neq 0$

x-int $= x = 0$

y-int $=$ $y = 0$

Both graphs have asymptotes at $x=-3$, however $g(x)$ has another at $x=3$, while $f(x)$ simply can't touch (restriction) $x=3$, can go past it. $f(x)$ has a y asymptote of 0, while $g(x)$ has one at $y=1$. $f(x)$ has 2 x-intercepts, $x=3,-2$, while $g(x)$ has one at $x=0$. Both graph's have 1 y-int, $f(x)$ at $y=\frac{2}{3}$, $g(x)$ at $y=0$

Exemplar #3: Score 3

- shows understanding of most relevant concepts
- uses appropriate algebraic strategies for some parts
- follows procedures accurately with minor errors (x-intercepts)
- solution is generally clear with some incorrect notation and use of language

Resource A: Written-Response Question 2 (calculator NOT permitted) Exemp

Consider the graphs of $f(x)=\dfrac{x^2-x-6}{x^2-9}$ and $g(x)=\dfrac{x}{x^2-9}$.

Use your knowledge of rational functions to outline the similarities and differences between these two graphs. You will be evaluated on the concepts expressed, the organization and accuracy of your work, and your use of language.

$f(x)=\dfrac{(x-3)(x+2)}{(x-3)(x+3)}$

D: $x \neq \pm 3$

R: $y \neq 1$

x-int: 3, -2

y-int: $\frac{2}{3}$

hor. asymptote : $y=1$

vert. asymptote : $x=-3$

$g(x)=\dfrac{x}{(x-3)(x+3)}$

D: $x \neq \pm 3$

R: $\mathbb{R}$

x-int: 0

y-int: 0

hor asymptote : $y=0$

vert asymptote : $x=\pm 3$

Similarity of these two graphs is that they both lie in the same domain.

The difference is they have differen shape. g(x) has an additional asymptote because in f(x), +3 becomes a hollow dot.

Exemplar #4: Score 3

- shows understanding of most relevant concepts
- uses appropriate algebraic strategies for some parts
- follows procedures accurately with minor errors and omissions
- parts of the presentation are omitted or unclear

Resource A: Written-Response Question 2 (calculator NOT permitted) Exemp

Consider the graphs of $f(x)=\dfrac{x^2-x-6}{x^2-9}$ and $g(x)=\dfrac{x}{x^2-9}$.

Use your knowledge of rational functions to outline the similarities and differences between these two graphs. You will be evaluated on the concepts expressed, the organization and accuracy of your work, and your use of language.

$f(x)=\dfrac{x^2-x-6}{x^2-9}$ $\quad$ $g(x)=\dfrac{x}{x^2-9}$ $\quad x\neq\pm3$

$x\neq\pm3.$

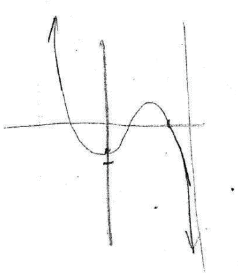
$=\dfrac{(x-3)(x+2)}{(x-3)(x+3)}.$ $\quad$ $=\dfrac{x}{(x-3)(x+3)}$

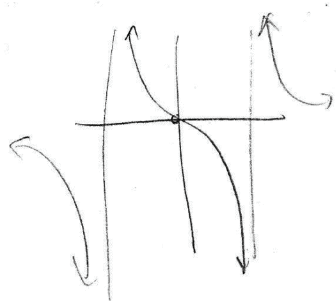
$=\dfrac{x-2}{x+3}$

y-int $=\dfrac{-2}{3}$. $\quad$ y-int $=0$

x-int $=2$ $\quad$ x-int $=0$

Y: A.





The 2 graphs f(x), g(x), both have restrictions that x cannot be equal to ±3. g(x) graph goes through the origin (0,0) while f(x) does not, instead, it's been shifted up $\frac{2}{3}$ and right 2.

Exemplar #5: Score 2

- shows partial understanding of concepts
- uses appropriate algebraic strategies for some parts
- some procedural errors
- presents part of the solution

Resource A: Written-Response Question 2 (calculator NOT permitted) Exemp

Consider the graphs of $f(x) = \dfrac{x^2 - x - 6}{x^2 - 9}$ and $g(x) = \dfrac{x}{x^2 - 9}$.

Use your knowledge of rational functions to outline the similarities and differences between these two graphs. You will be evaluated on the concepts expressed, the organization and accuracy of your work, and your use of language.

$$f(x) = \frac{(x+2)(x-3)}{(x-3)(x+3)} \qquad g(x) = \frac{x}{(x-3)(x+3)}$$

- These two graphs will have the same asymptotes at $x = 3$ and $x = -3$.
- The domain will be the same for both graphs. $x \in \mathbb{R},\ x \neq \pm 3$
- The range will be the same. $y \in \mathbb{R}$
- Y-intercepts will be different. For $f(x)$, y-intercept $= \frac{2}{3}$, for $g(x)$, y-intercept $= 0$ (Plug in $x=0$ for both equations)

Exemplar #6: Score 2

- shows partial understanding of concepts
- uses appropriate algebraic strategies for some parts
- follows procedure with partial accuracy, major errors and omissions
- presents part of the solution

Resource A: Written-Response Question 2 (calculator NOT permitted) Exemp

$(x-3)(x+2)$

Consider the graphs of $f(x) = \frac{x^2 - x - 6}{x^2 - 9}$ and $g(x) = \frac{x}{x^2 - 9}$.

$(x+3)(x-3)$

Use your knowledge of rational functions to outline the similarities and differences between these two graphs. You will be evaluated on the concepts expressed, the organization and accuracy of your work, and your use of language.

- both graphs have the same asymptotes
- the y-int of graph g(x) is 0
- $x \neq \pm 3$
- the y int of f(x) $x = -2$

Exemplar #7: Score 1

- shows little or no understanding of concepts
- uses appropriate algebraic strategies for some parts
- major errors and omissions
- incomplete

Resource A: Written-Response Question 2 (calculator NOT permitted) Exemp

Consider the graphs of $f(x)=\dfrac{x^2-x-6}{x^2-9}$ and $g(x)=\dfrac{x}{x^2-9}$.

Use your knowledge of rational functions to outline the similarities and differences between these two graphs. You will be evaluated on the concepts expressed, the organization and accuracy of your work, and your use of language.

$$\begin{array}{c|c} 1 & -3 \\ 1 & 2 \end{array}$$

$$f(x)=\frac{x^2-x-6}{x^2-9}=\frac{(\cancel{x-3})(x+2)}{(x+3)(\cancel{x-3})}=\boxed{\frac{x+2}{x+3}}$$

$$g(x)=\frac{x}{x^2-9}=\boxed{\frac{x}{(x+3)(x-3)}}$$

- Both the denominator is $(x+3)(x-3)$, or (x^2-9)
- There's a x-intercept for the graph f(x), but no x-intercepts for g(x).
- After factoring, you can cancel out $(x-3)$ for f(x), but nothing cancels out for g(x).
- They both have asymptotes at ± 3 due to the similarity of the denominator.

Exemplar #8: Score 1

- shows little or no understanding of concepts
- uses appropriate algebraic procedures for all parts
- major errors and omissions
- parts of the process are omitted or inappropriate