2007 HSC Notes from the Marking Centre Mathematics Extension 2

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2007 HSC NOTES FROM THE MARKING CENTRE
MATHEMATICS EXTENSION 2

Introduction

This document has been produced for the teachers and candidates of the Stage 6 Mathematics Extension 2 course. It contains comments on candidate responses to the 2007 Higher School Certificate examination, indicating the quality of the responses and highlighting their relative strengths and weaknesses.

This document should be read along with the relevant syllabus, the 2007 Higher School Certificate examination, the marking guidelines and other support documents which have been developed by the Board of Studies to assist in the teaching and learning of Mathematics Extension 2.

Many parts in the Extension 2 paper require candidates to prove, show or deduce a result. Candidates are reminded of the need to give clear, concise reasons in their answers.

Question 1

(a) While all but a few responses recognised the primitive as an inverse trigonometric function, many erred in selecting the coefficient.

(b) Candidates choosing a substitution for tan x were able to complete the integration successfully. Where candidates chose to substitute for $\tan^2 x$ or $\sec^2 x$, most were unable to proceed to a correct final answer.

(c) This part was very well done with the method of integration by parts well known. There were, however, a significant number of errors in the evaluation of $\cos \pi$ and $\cos 0$.

(d) There were six different substitutions attempted with the new integrand mostly correct. Most errors occurred through failing to change the limits or successfully negotiate the number of negative signs involved in finding the primitive.

(e) A number of candidates attempted to use partial fractions to prove the first part of the question though no proof was required. The middle integral caused some concern. In the final evaluation, the simplification in exact form of the number of logarithmic expressions caused problems. Approximations to the final answer involving degrees were not uncommon. The better responses simplified $\tan^{-1} 2 - \tan^{-1} (½)$ to $\tan^{-1} (¾)$, although this simplification was not required.

Question 2

(a) (i) All candidates were aware of the relationship between a complex number and its conjugate, although one or two candidates were careless in writing their answer.

(ii) The vast majority of responses were correct. Incorrect responses usually had reversed w and z in the difference.

(iii) A number of responses contained basic errors, either in squaring $4+i$ or failing to correctly subtract i^2 from 16.

(b) (i) Most candidates were able to find the modulus and argument and express $1+i$ in mod-arg form. The most common errors involved calculating the modulus.

(ii) Most candidates correctly applied de Moivre's theorem and were able to transform $17\pi/4$ into a simpler equivalent angle. Errors usually occurred at the final stage of expressing the answer with integer real and imaginary parts.

(c) A significant number of candidates found the correct Cartesian equation of the locus but were not able to complete the square to find the centre and radius of the circle. Common errors included giving a circle with radius $\sqrt{2x}$, a circle with radius i, a locus involving imaginary coefficients, and $z\bar{z} = x^2 - y^2$ leading to a hyperbola as the locus. Very few candidates realised that (0,0) must be excluded from the locus.

(d) This part presented the most challenges for candidates, many of whom struggled to achieve the clarity and logic required for mathematical proof, even when the proof or explanation was relatively straightforward.

(i) Candidates experienced difficulty when attempting to explain the result. A significant number did not use the term *rotation* for the transformation required to move from $R(a)$ to $Q(z_2)$. Those candidates who attempted to show that z_2 and $a\omega$ had the same modulus and argument often omitted key steps.

(ii) The most successful method was to show that $a = z_1\omega$ and hence that $z_1 z_2 = \frac{a}{\omega}\omega a = a^2$.

A common error was writing ω^{-1} as $-\omega$.

(iii) This part was attempted using a wide variety of methods. The most common successful approaches were showing $z_1 + z_2 = a$ and $z_1 z_2 = a^2$, showing that z_1 and z_2 satisfied the equation, and using the quadratic formula to obtain z_1 and z_2 as the roots.

Question 3

(a) It is advisable that candidates present large, clear sketches. Sometimes it was not obvious as to what was a preliminary working sketch and what was the final answer. In fact, candidates should label their preliminary graph to assist the examiners in awarding marks if the final graph is incorrect. Candidates should clearly mark axes and label all asymptotes and intercepts. A significant number of candidates sketched $y = |f(x)|$ instead of $y = f(|x|)$.

(b) Many candidates used the method of replacing x with $\frac{x}{2}$ but failed to give a polynomial with integer coefficients. Other candidates confused the polynomial function $x^3 - 5x + 3$ with the polynomial equation $x^3 - 5x + 3 = 0$.

(c) A significant number of candidates were confused about the radius of the cylindrical shell. Instead of x, the examiners often saw $1+x$, $1-x$ or $e-x$ among others. Unfortunately this often led to more difficult integrals to evaluate. Candidates are advised to take care when using the integration by parts technique and also in the final evaluation of the integral.

(d) (i) Some candidates made an error resolving forces horizontally or vertically but tried to manipulate their results to obtain the given result for *N*.

(ii) Some candidates who failed to do (d) (i) made the mistake of not attempting this part of the question.

Question 4

(a) Better responses stated the geometric facts precisely.

(b) (i) The two approaches were to either apply trigonometric expansions or de Moivre's theorem and the binomial theorem. In responses using the former approach, the solution was efficiently arrived at if $\cos 2\theta = \cos^2\theta - \sin^2\theta$ was chosen. Responses using one of the other two expressions for $\cos 2\theta$ were able to arrive at the correct result, although inefficiently.

(ii) In the better responses, $\sin(\theta + \pi/3)$ and $\sin(\theta + 2\pi/3)$ were successfully expanded, and then part (i) was recognised. Responses using formulas with products of trigonometric terms as sums were generally unsuccessful.

(iii) Many candidates recognised that the maximum of $\sin 3\theta$ is 1 but missed the significance of the '4' in the left-hand side of the formula in part (ii). In some responses, candidates applied calculus but often failed to answer the question, and instead found the value of θ at which the maximum occurred.

(c) Better responses recognised the area of the square slice was $(e - e^y)^2$, but careless errors were common in expanding this expression, finding its primitive, substitution of the limits and collection of like terms.

(d) (i) Better responses applied the sum and product of the roots formulas for a cubic polynomial. Occasionally candidates recognised that $P(-q) = 0$ also gave the required result.

(ii) Better responses recognised that β was real from part (i) and hence justified the conjugate pair had to be α and $-\alpha$ and hence showed that α and $-\alpha$ were purely imaginary. Those starting with $\alpha^2 = -r$ often did not recognise that the condition $r > 0$ was necessary for α to be purely imaginary. Many tried to use the result $qr = s$ of part (i) in part (ii).

Question 5

(a) (i) Better responses considered the ways of selecting three red marbles from 12, three yellow marbles from 12 and six marbles from 24 as follows:

$$P(=3\text{ red}) = \frac{{}^{12}C_3 \times {}^{12}C_3}{{}^{24}C_6}$$
$$= 0.36$$

Less successful responses included those that failed to consider the selection of the yellow balls or those that multiplied a series of individual probabilities or numbers, without considering the ways they are selected. For example:

$\frac{12}{24}\times\frac{11}{23}\times\frac{10}{22}\times\frac{12}{21}\times\frac{11}{20}\times\frac{10}{19}$ without multiplying by ${}^6C_3$.

Other less successful responses indicated a possible interpretation of the question as 'with replacement ', rather than 'without replacement'.

ie ${}^6C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^3$.

(ii) Better responses indicated an awareness of symmetry.

eg $P(>3\text{ red}) = P(<3\text{ red}) = \frac{1}{2}(1-0.36)$

Other successful responses were often of the form:

$$\frac{{}^{12}C_4\,{}^{12}C_2 + {}^{12}C_5\,{}^{12}C_1 + {}^{12}C_6\,{}^{12}C_0}{{}^{24}C_6} \quad \text{or} \quad 1-\frac{{}^{12}C_3\,{}^{12}C_3 + {}^{12}C_4\,{}^{12}C_2 + {}^{12}C_5\,{}^{12}C_1 + {}^{12}C_6\,{}^{12}C_0}{{}^{24}C_6}$$

Some careless responses calculated $P(\geq 3\text{ red})$. Other less successful responses reflected methodology similar to less successful responses in part (i).

(b) (i) Better responses demonstrated competence in deriving the equation of the tangent via implicit differentiation.

(ii) The best responses substituted (x_0, y_0) into the equation of the tangent at both P and Q to produce the two results $\frac{x_1x_0}{a^2}-\frac{y_1y_0}{b^2}=1$ and $\frac{x_2x_0}{a^2}-\frac{y_2y_0}{b^2}=1$, which show that the points (x_1, y_1) and (x_2, y_2) lie on the line $\frac{xx_0}{a^2}-\frac{yy_0}{b^2}=1$, the equation of the chord of contact. Weaker responses did not produce these two results and struggled to demonstrate the necessary understanding.

(iii) The best responses substituted the values ae for x and 0 for y. This produced the necessary result, $x_0=\frac{a}{e}$. Less successful responses included those that failed to show that $x_0=\frac{a}{e}$, often due to incorrect substitution.

(c)(ii) The majority of the responses that made a correct substitution of variable in the integral were successful. Some responses indicated $\frac{3\pi}{2}$ and $\frac{\pi}{2}$ for the lower and upper limits respectively, but then failed to demonstrate a recognition that, for this domain of integration, $\sqrt{4-4\sin^2\theta} = 2\sqrt{\cos^2\theta}$ in the integrand simplifies to $-2\cos\theta$ and not $+2\cos\theta$.

Some responses successfully demonstrated that $y = \sqrt{(x-1)(5-x)} = \sqrt{4-(x-3)^2}$ is the equation of the upper half of the circle with centre $(3,0)$ and radius 2, and hence that the integral is the area of this semicircle (equal to $\frac{1}{2}\times\pi\times 2^2 = 2\pi$).

(d) (i) The better responses made the initial observation that $AP = \cos\frac{\pi}{5}$ and/or that $\angle CAD = \frac{\pi}{5}$ and then proceeded to the correct solution. The responses that used the cosine rule in ΔABC rather than ΔACD were not successful.

(ii) Many responses presented a correct factorisation of the equation $8x^3 - 8x^2 + 1$ to obtain $8x^3 - 8x^2 + 1 = (2x-1)(4x^2-2x-1)$. They then often correctly found the zeros of $4x^2 - 2x - 1$ but did not eliminate the negative zero $x = \frac{1-\sqrt{5}}{4}$ or the zero $x = \frac{1}{2}$ in order to state the unique value for $\cos\frac{\pi}{5}$.

Question 6

(a) (i) Most candidates successfully substituted $a = b = 1$ into the binomial theorem, and identified the appropriate binomial coefficient in the expansion and implicitly or explicitly noticed that the remaining terms were positive. A few attempted unsuccessfully to prove the required result by induction.

(ii) Many responses contained careless mistakes, most commonly resulting in the candidate deducing that part (i) implies that $2^{n-1} > n(n-1)$ for $n > 1$. Of even more concern was the fact that too many candidates began with the required result and deduced that if it held, the result proved in part (i) would be a consequence.

(iii) Better responses demonstrated an understanding of what was required for this part. However, the algebra in the proof of the induction step was rarely accomplished efficiently, increasing the likelihood of candidates not earning the mark associated with this step. In particular, many candidates omitted brackets in places where they were required, or took no notice of them, which inevitably resulted in the signs of some terms occurring in their expression being reversed and so making it impossible for the candidate to complete the proof of the induction step.

It is also worth mentioning that the concluding statements provided by many candidates show that they incorrectly think that a proof by induction is actually an iterative proof, in which you imagine that the recipe should be repeated as many times as necessary in order to verify the statement for whichever positive integer is of interest. In fact, the Principle of Mathematical Induction is that every set with the property that, for each integer n in the set, n+1 is also in the set and which also contains 1 contains all positive integers. So, having established that the statement is true for 1 and, if true for some integer, is also true for the next integer, the correct conclusion is to simply state that, by induction, the statement is true for all positive integers.

(iv) Few candidates noticed the connection with part (iii). A number of candidates wrote that some part of the limit was $\frac{\infty}{\infty}$, which was then often evaluated to be 0 or 1 or ∞, without any justification. Such responses did not earn the mark, even if the limiting sum obtained in this way happened to be correct. Others attempted to determine the limiting sum using geometric series, usually without success, although a few students rewrote the sum as $(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+...)+(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+...)+(\frac{1}{4}+\frac{1}{8}+...)+...$ and were able to obtain the correct limiting sum in this way. It should be noted that this method only works because the sum above is absolutely convergent, and that, in other examples, such methods can lead to incorrect conclusions.

(b) (i) Most candidates succeeded in differentiating the given expression for x to obtain the result stated on the examination paper, and provided sufficient working to demonstrate that this was how the result had been obtained.

(ii) Many candidates were able to square the result in part (i) to obtain an expression for v^2 but were then often unsuccessful in their attempts to convince the examiner that the expression they had obtained in this way was equal to the one given on the examination paper. Candidates who also expanded the expression for v^2 given on the examination paper were generally more successful.

(iii) A number of candidates missed a – sign in the course of differentiating, and then made another error in substituting, possibly in order to get the answer on the examination paper. Unfortunately, this meant that they did not earn either of the marks for this part. Candidates should be advised to never intentionally make an error. Examiners know that it is not possible to legitimately obtain the correct answer from an incorrect expression, and attempts to recover from mistakes by making further errors usually result in the candidate missing out on additional part marks that might otherwise have been earned.

(iv) Most responses referred to resistance, a force opposing the fall, friction or wind resistance. Each of these was awarded the mark. It was pleasing to note that, even among the small number of candidates who had not attempted the earlier parts of (b), it was quite common to find correct responses to parts (iv) and (v).

(v) Many different approaches were used, which was not surprising as the answer can be obtained from part (i), part (ii) or part (iii). The most common incorrect answer was 0 m/s, obtained by substituting $x = 0$, even though the question states that x is the distance the raindrop has fallen.

Question 7

(a) (i) In responses that were awarded marks for this part, candidates either observed that $\sin x$ and x agree at $x = 0$ and that the gradient of $y = \sin x$ is always less than or equal to the gradient of $y = x$, or noted that $x - \sin x$ is a non-decreasing function whose value at 0 is 0. Sometimes these statements were made without the justification of a derivative, relying instead on sketches.

A small number of candidates observed that for $x > 1$ it is clear that $x > \sin x$ as 1 is the maximum value of $\sin x$. Only a very few then went on to deal with the remaining values of x.

A small number of candidates attempted a geometric proof involving either the area or side length of a suitable triangle compared to the area or arc length of a sector of a circle. Only a very few then went on to deal with the case $x > \pi$ or $\frac{\pi}{2}$ as appropriate.

In many responses, candidates felt that a sketch was a sufficient justification, neglecting the problem of what happens near the origin. A significant number of these responses included sketches that contradicted the question.

(ii) Better responses used a correct approach for this part with incorrect differentiation being the major problem. The remaining responses either omitted this part or attempted to use the first derivative to decide concavity.

(iii) The typical response to this part was to claim, without justification, that $f(x)$ was increasing and so was positive for $x > 0$ or to claim, using part (ii), that as $f(x)$ was concave up for $x > 0$ it was positive for these values of x. In both cases the need to consider the situation at $x = 0$ was ignored.

(b) In a small but significant group of responses, candidates seemed to think that the definition of a conic implied that the ratio of the distance from a point to the focus and from the point to the directrix was constant for *any* point on the directrix. For example, in part (i), some attempted to argue that as we have an ellipse the ratios $\frac{PS}{PR}$ and $\frac{QS}{QR}$ are equal as R is on the directrix.

(i) In the better responses, candidates observed that appropriate triangles are similar and so the ratios of corresponding sides are equal. A few applied trigonometry using the common angle in the given right triangles.

(ii) Better responses successfully argued using the properties of an ellipse.

(iii) In most responses, candidates used the sine rule and the facts deduced in parts (i) and (ii) to conclude that $\sin(\theta) = \sin(\phi + \theta)$, but could not then find the required solution. A large number of candidates wrote down the general solution and proceeded to reject the solution $\phi = 0$ and so show that $\phi = \pi - 2\theta$.

(iv) Many candidates successfully found the limiting value without attempting the earlier parts. A significant group of candidates did much of the work to find the limiting value but stopped short of an answer.

(c) (i) The most common mistake in this part was to use the wrong ratio for the eccentricity, in many cases using a ratio at odds with the one used in part (b)(ii), for example stating $\frac{PS}{PN} = e$ in this part having asserted that $\frac{PU}{PS} = e$ in part (b)(ii).

(ii) Better responses observed that $\frac{PS'}{PW} = e\cos\beta$ and used either trigonometry or similar triangles to finish. The common mistakes were to use the wrong trigonometric ratio or to make no attempt to justify why the triangles were similar.

Question 8

(a) (i) Many candidates who did not attempt this part were able to do part (ii). Candidates are advised to indicate clearly which side of an equation they are working on and to show clearly that the LHS becomes the RHS or vice versa – all steps must be shown. Also, candidates are reminded that it is not a proof to let $f(x)$ be a particular function, for example using x or x^2 to establish a result.

(ii) A variety of successful approaches were demonstrated. However, a common error when integrating the identity satisfied by f was the omission of the integration of $f(a)$, ie $f(x) + f(a-x) = f(a) \quad \therefore \int_0^a f(x)\,dx + \int_0^a f(a-x)\,dx = \text{"} f(a)\text{"}$. Another error was to state:

as $f(x) = f(a-x)$ hence $f(x) = \frac{1}{2}f(a)$ then $\int_0^a f(x)\,dx = \frac{1}{2}af(a)$ or $\frac{1}{2}f(a)$.

Candidates are advised not to omit the *dx* or the *du* in their working, as it was frequently absent in their setting out and it is crucial for change of variables concept.

(b) (i) A common error was not recognising that the LHS was a geometric series. It was evident that from those who got $\frac{z^{2n}-1}{z^2-1}$, many had difficulty manipulating this to the final RHS, with only the better responses demonstrating this correctly. It was also evident some candidates were trying to use some obscure factorising 'identities' to produce the result. They were not convincing in their use of these identities and/or did not justify them.

(ii) Better responses demonstrated fully how to show the given result. Many candidates were not able to derive/show how the LHS became the RHS with clear setting out. Just stating 'using de Moivre's theorem' and writing down what was given in the question is neither sufficient nor appropriate to gain marks. Basic steps appeared to be skipped in producing the RHS. Candidates are advised to be explicit and not implicit when attempting these types of questions.

(iii) Taking the imaginary part was not interpreted well. The crucial step of taking the imaginary parts on both sides in the result of part (b) (ii) must be made. Some candidates did not see this and believed the whole expression in part (ii) for $\theta = \frac{\pi}{2n}$ somehow reduced to the expressions in this part. Typically, the real part was incorrectly claimed to be 0. In taking the imaginary parts no simplification steps should be omitted, so the simplification of $\sin\left[(n-1)\frac{\pi}{2n}\right]$ should show $\sin\left[\frac{\pi}{2} - \frac{\pi}{2n}\right] = \cos\left(\frac{\pi}{2n}\right)$ or equivalent.

(c) (i) This was handled well in the best responses. The first step was merely to produce:

$$d_1 + d_2 + \ldots + d_{n-1} = \frac{1}{\sin \frac{\pi}{n}} \left(\sin \frac{\pi}{n} + \sin \frac{2\pi}{n} + \ldots + \sin \frac{(n-1)\pi}{n} \right) = \frac{\cot \frac{\pi}{2n}}{\sin \frac{\pi}{n}}$$ from part (b) (iii).

The next step was to simplify this new RHS by converting the cotangent and using a double angle formula on the denominator.

(ii) The better responses stated why $p = n$ by various means and incorporated part (c) (i) to complete this part.

(iii) Many responses gave an answer of 0 and frequently 2 by incorrect arguments. The candidates did not appear to be very experienced in applying the result $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$.

For example, as $n \to \infty$, $\sin \frac{2\pi}{n} \to 0$, so $n \sin \frac{\pi}{2n} \to \infty.0 = 0$ hence $\frac{p}{q} \to 0$.

Mathematics Extension 2

2007 HSC Examination Mapping Grid

Question	Marks	Content	Syllabus outcomes
1 (a)	2	4.1	E8
1 (b)	2	4.1	HE6, E8
1 (c)	3	4.1	E8
1 (d)	4	4.1	HE6, E8
1 (e)	4	4.1	E8
2 (a) (i)	1	2.1	E3
2 (a) (ii)	1	2.1	E3
2 (a) (iii)	1	2.1	E3
2 (b) (i)	2	2.2	E3
2 (b) (ii)	3	2.2	E3
2 (c)	3	2.5	E2, E3, E9
2 (d) (i)	1	2.2	E3
2 (d) (ii)	1	2.2	E3
2 (d) (iii)	2	2.2, 2.3, 7.5	E2, E3
3 (a) (i)	1	1.3	E6
3 (a) (ii)	2	1.3	E6
3 (a) (iii)	2	1.2	E6
3 (b)	2	7.5	E4
3 (c)	4	5.1	E7
3 (d) (i)	3	6.3	E5
3 (d) (ii)	1	6.3	E5
4 (a)	2	8.1	E2, E9
4 (b) (i)	2	8.0	E2, E9
4 (b) (ii)	2	8.0	E2, E9
4 (b) (iii)	1	8.0	E9
4 (c)	3	5.1	E7
4 (d) (i)	3	7.5	E4
4 (d) (ii)	2	7.4	E2, E4, E9
5 (a) (i)	1	8.0	HE3
5 (a) (ii)	2	8.0	HE3
5 (b) (i)	2	3.2	E4

Question	Marks	Content	Syllabus outcomes
5 (b) (ii)	2	3.2	E4
5 (b) (iii)	1	3.2	E2, E4
5 (c) (i)	1	8.0	E4
5 (c) (ii)	2	4.1	E8
5 (d) (i)	2	8.0	E2, E9
5 (d) (ii)	2	7.4, 8.0	E2, E4, E9
6 (a) (i)	1	8.3	E9
6 (a) (ii)	2	8.3	E2, E9
6 (a) (iii)	3	8.2	HE2
6 (a) (iv)	1	8.2	E9
6 (b) (i)	2	6.1	E5
6 (b) (ii)	2	8.0	E5
6 (b) (iii)	2	6.1	E5
6 (b) (iv)	1	6.2	E5
6 (b) (v)	1	6.2	E9
7 (a) (i)	2	8.0, 8.3	E2, E6
7 (a) (ii)	2	8.0, 8.3	E2, E6
7 (a) (iii)	2	8.0, 8.3	E2, E6
7 (b) (i)	1	3.1	E2
7 (b) (ii)	1	3.1	E4
7 (b) (iii)	2	8.0	E2, E9
7 (b) (iv)	1	3.1, 8.0	E9
7 (c) (i)	2	3.1	E2, E4
7 (c) (ii)	2	8.0	E2, E4
8 (a) (i)	1	4.1	E8, E9
8 (a) (ii)	2	4.1	E8, E9
8 (b) (i)	2	8.0	E2, E4
8 (b) (ii)	3	2.4	E2, E4, E9
8 (b) (iii)	2	2.1, 8.0	E2, E4, E9
8 (c) (i)	2	8.0	E2, E4, E9
8 (c) (ii)	2	8.0	E2, E4, E9
8 (c) (iii)	1	8.0	HE2, E3, E4, E9

2007 HSC Mathematics Extension 2
Marking Guidelines

Question 1 (a)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correct answer	2
• Answer of the form $A\sin^{-1} Bx$ or equivalent merit	1

Question 1 (b)

Outcomes assessed: HE6, E8

MARKING GUIDELINES

Criteria	Marks
• Correct answer	2
• Attempts an appropriate substitution	1

Question 1 (c)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correction solution	3
• Correct primitive	2
• Attempts to use integration by parts	1

Question 1 (d)

Outcomes assessed: HE6, E8

MARKING GUIDELINES

Criteria	Marks
• Correct solution	4
• Correct primitive with appropriate limits	3
• Correctly makes an appropriate substitution	2
• Attempts an appropriate substitution	1

Question 1 (e)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correction solution	4
• Correct primitive or equivalent merit	3
• Correct primitive of two terms	2
• Correct primitive of one term	1

Question 2 (a) (i)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 2 (a) (ii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 2 (a) (iii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 2 (b) (i)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	2
• Correct modulus or argument	1

Question 2 (b) (ii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Correct modulus and argument	2
• Correct modulus or argument	1

Question 2 (c)

Outcomes assessed: E2, E3, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Obtains the circle on which the locus lies or equivalent progress	2
• Some progress towards finding a correct Cartesian equation for the locus	1

Question 2 (d) (i)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct explanation	1

Question 2 (d) (ii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 2 (d) (iii)

Outcomes assessed: E2, E3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Considers the sum or product of z_1 and z_2	1

Question 3 (a) (i)

Outcomes assessed: E6

MARKING GUIDELINES

Criteria	Marks
• Correct sketch	1

Question 3 (a) (ii)

Outcomes assessed: E6

MARKING GUIDELINES

Criteria	Marks
• Correct sketch	2
• Some indication of symmetry about the y-axis	1

Question 3 (a) (iii)

Outcomes assessed: E6

MARKING GUIDELINES

Criteria	Marks
• Correct sketch	2
• Indicates the three zeros of $f(x) - x$ or equivalent merit	1

Question 3 (b)

Outcomes assessed: E4

MARKING GUIDELINES

Criteria	Marks
• Correct answer	2
• Attempts to apply the sum and product formulae or equivalent merit	1

Question 3 (c)

Outcomes assessed: E7

MARKING GUIDELINES

Criteria	Marks
• Correct solution	4
• Correct primitive or equivalent merit	3
• Attempts to use integration by parts	2
• Correct integrand	1

Question 3 (d) (i)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Resolves correctly in both directions	2
• Resolves correctly in one direction	1

Question 3 (d) (ii)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 4 (a)

Outcomes assessed: E2, E9

MARKING GUIDELINES

Criteria	Marks
• Correct proof. Justifications (abbreviated or otherwise) which indicate the appropriate geometric fact are acceptable	2
• Shows $\angle ALB = \angle APB$ with justification or equivalent merit	1

Question 4 (b) (i)

Outcomes assessed: E2, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Correctly applies the angle sum formula	1

Question 4 (b) (ii)

Outcomes assessed: E2, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Applies the angle sum formula to the LHS or equivalent merit	1

Question 4 (b) (iii)

Outcomes assessed: E9

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 4 (c)

Outcomes assessed: E7

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Correct integral expression in terms of y only	2
• Obtains $\int_0^1 (e - x)^2 \, dy$ or equivalent merit	1

Question 4 (d) (i)

Outcomes assessed: E4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Finds an expression for two of q, r, and s in terms of α and β	2
• Finds an expression for one of q, r, and s in terms of α and β	1

Question 4 (d) (ii)

Outcomes assessed: E2, E4, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Observes that two of roots are complex conjugates	1

Question 5 (a) (i)

Outcomes assessed: HE3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 5 (a) (ii)

Outcomes assessed: HE3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	2
• Recognises the symmetry or equivalent merit	1

Question 5 (b) (i)

Outcomes assessed: E4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Obtains correct gradient	1

Question 5 (b) (ii)

Outcomes assessed: E4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Observes that $\frac{x_1 x_0}{a^2} - \frac{y_1 y_0}{b^2} = 1$	1

Question 5 (b) (iii)

Outcomes assessed: E2, E4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 5 (c) (i)

Outcomes assessed: E4

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 5 (c) (ii)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Makes some progress towards the substitution	1

Question 5 (d) (i)

Outcomes assessed: E2, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Shows $\angle CAD = \dfrac{\pi}{5}$ or $AP = u$ or equivalent merit	1

Question 5 (d) (ii)

Outcomes assessed: E2, E4, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Factors $8x^3 - 8x^2 + 1$	1

Question 6 (a) (i)

Outcomes assessed: E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 6 (a) (ii)

Outcomes assessed: E2, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Applies the result of part (i)	1

Question 6 (a) (iii)

Outcomes assessed: HE2

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Verifies the case $n = 1$ and attempts to prove the inductive step OR • Proves the inductive step	2
• Verifies the case $n = 1$ or equivalent merit	1

Question 6 (a) (iv)

Outcomes assessed: E9

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 6 (b) (i)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Makes some progress	1

Question 6 (b) (ii)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Expresses $e^{\frac{x}{5}}$ in terms of t or equivalent merit	1

Question 6 (b) (iii)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Applies $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ or equivalent merit	1

Question 6 (b) (iv)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 6 (b) (v)

Outcomes assessed: E9

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 7 (a) (i)

Outcomes assessed: E2, E6

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Observes $x - \sin x = 0$ for $x = 0$ or equivalent merit	1

Question 7 (a) (ii)

Outcomes assessed: E2, E6

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Correct second derivative	1

Question 7 (a) (iii)

Outcomes assessed: E2, E6

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Shows either that $f(0) = 0$ or $f'(0) = 0$	1

Question 7 (b) (i)

Outcomes assessed: E2

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 7 (b) (ii)

Outcomes assessed: E4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 7 (b) (iii)

Outcomes assessed: E2, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Finds an expression for $\sin\alpha$ in both triangles	1

Question 7 (b) (iv)

Outcomes assessed: E9

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 7 (c) (i)

Outcomes assessed: E2, E4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Applies the focus directrix property	1

Question 7 (c) (ii)

Outcomes assessed: E2, E4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• States $\frac{PS'}{PW} = e\cos\beta$ or equivalent merit	1

Question 8 (a) (i)

Outcomes assessed: E8, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 8 (a) (ii)

Outcomes assessed: E8, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Applies the result of part (i)	1

Question 8 (b) (i)

Outcomes assessed: E2, E4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Recognises that the left-hand side is a geometric series	1

Question 8 (b) (ii)

Outcomes assessed: E2, E4, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Makes substantial progress	2
• Applies de Moivre's theorem	1

Question 8 (b) (iii)

Outcomes assessed: E2, E4, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Appropriately equates the imaginary parts of both sides of the expression in part (ii)	1

Question 8 (c) (i)

Outcomes assessed: E2, E4, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Applies the result of part (b) (iii)	1

Question 8 (c) (ii)

Outcomes assessed: E2, E4, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Observes that $p = n$	1

Question 8 (c) (iii)

Outcomes assessed: HE2, E2, E4, E9

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1