2005 HSC Notes from the Marking Centre Mathematics Extension 2

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2005 HSC NOTES FROM THE MARKING CENTRE MATHEMATICS EXTENSION 2

Introduction

This document provides candidates and teachers with feedback in relation to the quality of responses provided by candidates to the 2005 Mathematics Extension 2 HSC examination paper. It should be read in conjunction with the 2005 HSC Mathematics Extension 2 examination paper, the marking guidelines and the *Mathematics Stage 6 Syllabus*.

Many parts in the Extension 2 paper require candidates to prove, show or deduce a result. Candidates are reminded of the need to give clear, concise reasons in their answers to convince the examiners in such questions.

Question 1

(a) This part was answered well. Most candidates used the substitution $u = \sin\theta$. However, many left their response as a function of u, not θ, ie $\frac{1}{4}u^{-4} + c$.

(b) (i) This part was answered well. Candidates used the common methods of equating coefficients [$a + b = 0$ and $2a - 3b = 0$], or the substitution of x-values [$x = 3$ and $x = -2$] to obtain $a = 3$ and $b = 2$.

(ii) This part was answered well.

(c) This part was answered reasonably well. The best method appeared to be:

$$u = \ldots \qquad dv = \ldots$$
$$du = \ldots \qquad v = \ldots$$

However, some candidates did not identify u and dv correctly. As well, in using the integration by parts formula, some made errors such as $dv = \ln x \Rightarrow v = \frac{1}{x}$, and the integral of x^7 was given as $7x^6$.

It was noticeable that many candidates had problems with simple algebraic manipulation, writing statements such as $\frac{e^8}{8} - \frac{1}{64}(e^8 - 1) = \frac{e^8}{8} - \frac{e^8}{64} - \frac{1}{64}$, or $\frac{e^8}{8} - \frac{1}{8}(\frac{e^8}{8} - \frac{1}{8}) = \frac{e^8}{8} - \frac{1}{16}e^8 - \frac{1}{16}$.

(d) Many candidates did not realise that they had to manipulate the constant before using the table of integrals. A common incorrect answer was $\sin^{-1}(2x)$.

(e) (i) This part was answered well. There were two main methods used by the candidates:

$$t = \tan\frac{\theta}{2};\quad \frac{dt}{d\theta} = \frac{1}{2}\sec^2\frac{\theta}{2} = \frac{1}{2}(1+\tan^2\frac{\theta}{2}) = \frac{1}{2}(1+t^2),$$

or $$\theta = 2\tan^{-1} t;\quad \frac{d\theta}{dt} = \frac{2}{1+t^2},\quad so\,\frac{dt}{d\theta} = \frac{1+t^2}{2}.$$

(ii) This part was not answered well by many of the candidates. However, there were many correct approaches used by the candidates when proving the result, starting from the data $t = \tan\frac{\theta}{2}$ only.
Many had only memorised the 't' formulae.

(iii) This part was answered well. The candidates were able to use (e)(i) and (e)(ii) to answer this part correctly, even though they may have had problems with part (i) or (ii).

Question 2

(a) This part was done very well with most candidates obtaining correct solutions for each part.

(b) This part was also done well.

(i) Nearly all candidates knew how to find the modulus and argument of β. However, some candidates' responses indicated that they were of the view that answers of the form $r(\cos\theta - i\sin\theta)$ are in the correct modulus–argument form.

(ii) Nearly all candidates found the modulus and argument of β^5 correctly.

(iii) Nearly all candidates converted their answer from (b)(ii) correctly into $x + iy$ form.

(c) Many candidates did not realise that $|2iy| = 2|y|$. $|y| < 1$ was not graphed well. The most common incorrect response was to sketch $y<1$. Most candidates found the correct circle for $|z-1| \geq 1$, but some indicated the wrong region. When sketching graphs, it is important to mark all axes, intercepts or other significant features. When showing the intersection of two graphs, it is advisable to first show each graph separately before sketching the final result.

(d) (i) Candidates need to take extra care when they are asked to 'explain' a result. In some cases too many steps of explanation were omitted. It was often beneficial to present a labelled diagram. Many candidates did not note or state that the line l bisected $\angle POQ$.

(ii) Candidates need to take extra care when they are asked to 'deduce' a result. Again, too often, too many steps of explanation were omitted.

(iii) Most candidates realised that $z_1 z_2 = |z_1|^2 i$, but did not describe the locus of R.

Question 3

(a) Many candidates provided good answers for this section, particularly in part (ii), which was well done. The best graphs were smooth functions and provided key x-intercept information, which clearly demonstrated the translations and reflections, such as $x = -1$ in part (i), and $x = -2$ in part (iv). In contrast poorer graphs tended to leave out this information.

(b) Many techniques were attempted to construct the graph, including plotting points. Some candidates tried to use calculus, which usually wasted a lot of time with little gain. Some candidates drew the graph by adding the two functions together, which was a good strategy in particular for identifying the $y = x$ asymptote. Some candidates recognised the function as odd and/ or $y \to \infty$ as $x \to \infty$, both of which helped in the sketch. In some cases the x-intercepts were not attempted or candidates could not form or solve the cubic correctly. Generally the limit aspects, apart from the vertical asymptotes, were not done convincingly.

(c) For many candidates this was a routine problem. A significant number of candidates were poor in implicit differentiation, a few even trying to make y the subject. Some candidates used the method of separating the dx and dy for each term and then dividing through by dx. This method led to more steps, and may have caused more errors. Generally this was not the best method. Some candidates substituted (2, 1) immediately after differentiating and before forming an expression for $\frac{dy}{dx}$, which simplified the algebra. Some candidates did not substitute for the gradient at all and incorrectly used an expression in x and y. Overall, differentiating the product and coping with its minus sign caused the most errors.

(d) Many candidates efficiently used the trigonometric identity for sine and cosine squared. Common errors included the introduction of an additional force, which could not be manipulated satisfactorily, or were not able to resolve forces at all. Many candidates missed the squaring identity and found $\tan\theta$ instead, which led to additional manipulation. A common error was for candidates to work backwards from the answer by Pythagoras' theorem without resolving forces at all. In trying to get the final form some candidates incorrectly squared expressions, giving statements such as $(a+b)^2 = a^2 + b^2$.

Question 4

(a) (i) Most candidates demonstrated an understanding of the method of cylindrical shells. Many performed the integration by inspection. Using a formal substitution (eg $u = -x^2$) was also a common correct method. Errors included poor manipulation of the negative signs, and errors during integration. Some candidates attempted to use an inappropriate method such as integration by parts.

(ii) Responses derived from a correct, or near correct, answer to part (i) were usually successful. Responses of zero, or a negative volume, did not always appear to signal to candidates that there may be a problem with such answers.

(b) (i) A common incorrect response was to simply drop one or both of the negative signs ie produce answers of p and r, rather than $-p$ and $-r$. Another common error was to respond with $-p$ and q.

(ii) Apart from carelessness with signs, the most common error was in expanding $(\alpha+\beta+\gamma+\delta)^2$ to get $\alpha^2+\beta^2+\gamma^2+\delta^2+2\sum\alpha\beta\gamma$. Responses including a method using a polynomial equation in $\sqrt{x}$ were usually successful.

(iii) There were many responses that demonstrated a clear understanding of what the question asked for. A significant number of responses did not evaluate p^2-2q.

(iv) Incorrect methods attempted included applications of Newton's method, remainder theorem, factor theorem and halving the interval. After evaluating $P(0)$ and $P(1)$, a good understanding of the question was usually indicated by a statement supporting the existence of a real root, α, such that $0<\alpha<1$. A process of deduction was then required to eliminate all possibilities other than $P(x)$ having two real roots (and two complex conjugate roots). Candidates are reminded that 'deduce' or 'hence' is a signal to use or consider the information from previous parts to answer the question.

(c) (i) Many responses included either manipulating or deriving the equation of the normal. Once the simple substitution was made leading to $b^3x_1=(a^2-b^2)x_1y_1$, it was vital for the response to indicate that there are two distinct cases: (1) if $x_1\neq 0$ then $y_1=\dfrac{b^3}{a^2-b^2}$, or (2) if $x_1=0$ then $y_1=\pm b$. Most responses ignored the possibility that $x_1=0$.

(ii) The subtlety of $y_1\leq b$ was absent in most attempts. Those few candidates who attempted this step generally progressed to the complete solution.

Question 5

(a) (i) Candidates are advised to write full explanations when the question asks to show a result is true. In this question, candidates would be advised, for example, to explain why $ad=bc$ by considering the area of ΔABC as suggested in the question, or by appealing to similar triangles, and state that $a^2=b^2+c^2$ holds by Pythagoras' theorem.

(ii) Candidates were most successful when they used the result in part (i). The use of LHS = RHS setting out is recommended.
Some candidates assumed incorrectly that $\dfrac{1}{x}=\dfrac{1}{y}+\dfrac{1}{z}$ is equivalent to $x=y+z$.

(b) (i) Candidates who listed the five outcomes were most successful. Those who explained in a sentence needed to make it clear that there were six goals in the competition and that Mary scored the last goal.

(ii) This part was done poorly. Those who listed the cases were most successful. Candidates needed to recognise that either Mary or Ferdinand won in a competition where 5, 6, 7, 8 or 9 goals were scored.

(c) (i) This part was done poorly. The fact that the graph of $y = f'(x)$ is the reflection of $y = f(x)$ in the line $y = x$ was a distraction. Better responses used a diagram, clearly labelling the two areas in which the graph $y = f(x)$ divides the rectangle with dimensions *a* by *b*. Some ignored the fact that $f(0) = 0$ in their diagram. Candidates who attempted the proof via integration by parts often claimed $f'(x) = f^{-1}(x)$ and did not justify the change in limit from *a* to *b*.

(ii) Many candidates could not find the inverse of $f(x) = \sin^{-1}\frac{x}{4}$ correctly or evaluate $\sin^{-1}\frac{2}{4}$ to find *b*. Candidates should be careful of the signs when evaluating the definite integral. Those who chose to use integration by parts often had difficulty differentiating $\sin^{-1}\frac{x}{4}$.

(d) (i) This part was done well. Candidates are advised to draw a diagram and label it clearly.

(ii) Most candidates were successful in writing the correct definite integral. However, substitution in the integration was poorly executed, either through not changing the limits, or not expressing *dx* in terms of the new variable.

Question 6

(a) (i) Many who attempted this question verified the case for *n* = 1 rather than for *n* = 0. Some candidates integrated by parts 'upwards', that is, integrated rather than differentiated the t^k term, but few who did this were able to re-arrange successfully to complete the proof. Many candidates started with the right-hand side of the desired statement and tried to work backwards. Others successfully worked on both sides until they came to the same point.

The setting out of the induction warrants comment. A very large number of candidates who successfully completed the question (and many who attempted it) ended the induction proof with some version of the following:

'The statement is true for *n* = 0 and hence is true for *n* = 1. The statement is true for $n = 1$ and hence is true for $n = 2$. The statement is true for $n = 2$ and hence is true for $n = 3$ and so on. Hence the statement is true for all integers $n \geq 0$ (by induction).'

In a large number of cases the words 'by induction' were omitted. Much time is wasted writing such a lengthy final statement and it would be better if candidates ended induction proofs with a simple statement like:

'Hence the statement is true for all $n \geq 0$ by induction.'

(ii) Only a minority of candidates were able to do this part correctly. Typically, candidates tried to use integration by parts, integrating the t^n term, or a graphical approach using rectangles. Neither approach was successful.

(iii) This part was done quite well. Many candidates, who were unable to do much on the earlier parts, could successfully complete this part. There was some confusion between the substitution $x = 1$ and $n = 1$.

(iv) Many candidates showed that the series converges to *e*. There was a lot of confusion between the series and the terms of the series, resulting in the claim that the series (or positive quantities) tended to zero as *n* became large.

(b) (i) Some (brief) comment was needed as to why the sum $(1 + \omega + \omega^2 + ... + \omega^{n-1})$ was zero, which many candidates overlooked.

(ii) This part was generally done well, with candidates showing a good understanding of inverses and de Moivre's theorem.

(iii) Many candidates found the real part, although not always from correct working. Many did not see the relationship between this and the previous part and this it made it more difficult to get the answer.

(iv) While this part was attempted by a large number of candidates, setting out, which was crucial to success, was often rather poor. Many tried to simply put $\omega = \cos\frac{2\pi}{5}$. Another common error was to claim that the real part of $\frac{1}{\omega - 1} = -\frac{1}{2}$ implied that the real part of $\omega - 1$ was –2.

(v) There were many attempts at this part. It was accessible even to those who had managed to do little else in part (b). A reasonable number of those who attempted the question managed to write the terms in the sum in terms of $\cos\frac{2\pi}{5}$ and $\cos\frac{\pi}{5}$ or $\cos\frac{4\pi}{5}$ and $\cos\frac{2\pi}{5}$. Many were then able to form a quadratic in either $\cos\frac{2\pi}{5}$ or $\cos\frac{\pi}{5}$. There were a number of candidates who correctly reached this stage but then made careless errors in solving the quadratic equation.

Question 7

(a) (i) The absence of a diagram was noted in many cases. A common error was to state the opposite 'sides' add to 180°.

(ii) Some candidates did not accompany valid statements with valid reasons. Candidates who did not include a figure often named angles incorrectly. The use of algebra with named angles, rather than with pronumerals, frequently created unnecessary work. Good solutions were achieved in four lines, though many candidates took up to two pages to arrive at the relevant conclusion.

(iii) The use of properties associated with parallel lines and a transversal was the most efficient method used, though multiple similar triangles were more commonly used.

(iv) Any simple reason, using Pythagoras' theorem or the hypotenuse, was the minimum required.

(b) (i) Many candidates struggled with the concepts of force, acceleration and angular velocity, while a smaller number attempted to integrate too early. The inclusion of the mass, *m*, on one side of an equation for acceleration was common.

(ii) Candidates who replaced acceleration with $\frac{d}{dx}(\frac{1}{2}v^2)$ were able to proceed most successfully. Frequent errors included the incorrect (or lack of) evaluation of the constant for indefinite integration, or having the limits in the wrong order for a definite integral approach. Candidates are reminded of the necessity of showing steps leading to a final answer, particularly in a 'show' question where the final answer is given in the paper.

(iii) Only a very few candidates recognised the need for the velocity to be negative.

Question 8

(a) (i) Many candidates applied the quotient rule to differentiate *f(x),* but errors in applying the chain rule when differentiating the expression in the denominator were common. The most successful approach was to rewrite *f(x)* as $\frac{1}{3\sqrt[3]{ab}}\left[(a+b)x^{-\frac{1}{3}}+x^{\frac{2}{3}}\right]$, and candidates who then expressed the derivative as $\frac{2x-(a+b)}{9\sqrt[3]{ab}x^{\frac{4}{3}}}$ were most likely to obtain full marks, as this made it much easier to convince the examiners that the stationary point was a minimum by reference to the sign of the derivative on either side of the stationary point. Again, candidates need to provide evidence of their understanding in a 'show' question. It is not sufficient to assert that *f(x)* has a minimum at $x=\frac{a+b}{2}$ because $f''(\frac{a+b}{2})>0$ if $f''(x)$ has not been calculated, or has been written in a form which makes it difficult to see that the value at $\frac{a+b}{2}$ is, in fact, positive.

(ii) A number of candidates attempted to use an algebraic technique. This is possible. The need to show that $(\frac{a+b+c}{3\sqrt[3]{abc}})^3 \geq (\frac{a+b}{2\sqrt{ab}})^2$ suggests studying

$$\frac{(a+b+c)^3}{27abc}-\frac{(a+b)^2}{4ab}=\frac{4(a+b+c)^3-27c(a+b)^2}{108abc}=\frac{(4a+4b+c)(a+b-2c)^2}{108abc}$$

which is clearly non-negative and equal to zero if and only if $c=\frac{a+b}{2}$. However, it is doubtful that any candidate could have found the factorisation of the trinomial in the numerator. The majority of candidates did not see the connection with (a) (i), in which it was established that $f(c)\geq f(\frac{a+b}{2})$. Many were able to deduce the arithmetic-geometric mean inequality ($\frac{a+b+c}{3}\geq\sqrt[3]{abc}$ from $(\frac{a+b+c}{3\sqrt[3]{abc}})^3 \geq (\frac{a+b}{2\sqrt{ab}})^2$. A few candidates provided independent algebraic proofs of the arithmetic-geometric mean inequality by first showing that $u^2+v^2+w^2\geq uv+vw+wu$, and then multiplying both sides of this inequality by $u+v+w$ to show that, for non-negative *u*, *v* and *w*, $u^3+v^3+w^3\geq 3uvw$, from which the desired result easily follows.

(iii) This part was generally done well.

(iv) While most candidates applied the result in (iii), very few realised that this only established that the polynomial did not have three positive roots. Candidates also needed to observe that $f(x) < 0$ for $x \le 0$ in order to establish that the polynomial does not have three real roots. A substantial number of candidates successfully established the required result using calculus, and one candidate solved this part by showing that the three roots of this equation, α, β and γ, satisfy $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 = -4$ which is inconsistent with all three roots being real. Many candidates tried to find the real root of the equation, presumably with the intention of then factoring the polynomial and showing that the remaining quadratic factor had no real zeros.

(b) (i) This was generally answered correctly by those who attempted it. Some had difficulty finding the coordinates of *A*, and it was surprising how many candidates used the distance formula to calculate *AP* and *PB*.

(ii) Most candidates did not attempt this part. Those who did make some attempt often had difficulty in finding the size of $\angle PAC$, or did not apply the sine rule.

(iii) Most attempts at this part were successful.

(iv) Those who attempted this part rarely saw the connection with part (iii).

(v) Candidates had more success with this part, with most attempts indicating that the result followed from part (iv) by examining what happens as *P* is moved closer to *T*. A very small number of candidates established the result independently, by showing that if *T* is the point $(a\sec\varphi, b\tan\varphi)$, the tangent through *T* is $(b\sec\varphi)x - (a\tan\varphi)y = ab$, and so *U* is the point $\left(\dfrac{a}{\sec\varphi - \tan\varphi}, \dfrac{b}{\sec\varphi - \tan\varphi}\right)$, while *V* is the point $\left(\dfrac{a}{\sec\varphi + \tan\varphi}, \dfrac{-b}{\sec\varphi + \tan\varphi}\right)$. It was then easy to deduce that *T* is the mid-point of *UV*.

Mathematics Extension 2

2005 HSC Examination Mapping Grid

Question	Marks	Content	Syllabus outcomes
1 (a)	2	4.1	E8
1 (b) (i)	2	7.6	E4
1 (b) (ii)	1	4.1	E8
1 (c)	3	4.1	E8
1 (d)	2	4.1	E8
1 (e) (i)	1	4.1, 8.0	PE5
1 (e) (ii)	2	4.1, 8.0	H5
1 (e) (iii)	2	4.1	HE6, E8
2 (a) (i)	1	2.1	E3
2 (a) (ii)	1	2.1	E3
2 a) (iii)	1	2.1	E3
2 (b) (i)	2	2.2	E3
2 (b) (ii)	2	2.2	E3
2 (b) (iii)	1	2.2	E3
2 (c)	3	2.5	E3
2 (d) (i)	2	2.2	E2, E3
2 (d) (ii)	1	2.3	E3
2 (d) (iii)	1	2.5	E3, E9
3 (a) (i)	1	1.3	E6
3 (a) (ii)	1	1.3	E6
3 (a) (iii)	2	1.7	E6
3 (a) (iv)	2	1.3	E6
3 (b)	4	1.2, 8.0	E6
3 (c)	3	1.8, 8.0	E6
3 (d)	2	6.3.4	E5
4 (a) (i)	3	5.1	E7
4 (a) (ii)	1	1.4	E7, E9
4 (b) (i)	2	7.5	E4
4 (b) (ii)	2	7.5	E2, E4, E9
4 (b) (iii)	1	7.5	E2, E4, E9
4 (b) (iv)	2	7.4	E4
4 (c) (i)	2	3.1	E3
4 (c) (ii)	2	3.1	E3
5 (a) (i)	1	2.6 (Mathematics/Extension 1), 8	H5, HE7, E2, E9

Question	Marks	Content	Syllabus outcomes
5 (a) (ii)	2	5.6 (Mathematics/Extension 1), 8	H5, HE7, E2, E9
5 (b) (i)	1	18.1 (Mathematics/Extension 1), 8	PE3, E2, E9
5 (b) (ii)	2	18.1 (Mathematics/Extension 1), 8	PE3, HE7
5 (c) (i)	1	15 (Mathematics/Extension 1), 8	HE3, E2, E9
5 (c) (ii)	3	15 (Mathematics/Extension 1), 8	HE4, HE7, E9
5 (d) (i)	2	5.1	H5, E2
5 (d) (ii)	3	5.1	E7
6 (a) (i)	4	7.4 (Mathematics/Extension 1), 4.1, 8	HE2, E2, E8, E9
6 (a) (ii)	1	4.1	E2, E8, E9
6 (a) (iii)	1	8.3	E2, E9
6 (a) (iv)	1	8	E2, E9
6 (b) (i)	2	2.4	E3
6 (b) (ii)	1	2.4	E3
6 (b) (iii)	1	2.4	E3
6 (b) (iv)	1	2.4	E3
6 (b) (v)	3	2.4	E3
7 (a) (i)	1	8.1	PE3, E2
7 (a) (ii)	3	8.1	PE3, E2
7 (a) (iii)	2	8.0, 8.3	E2, E9
7 (a) (iv)	1	8.0, 8.3	E2, E9
7 (b) (i)	2	6.3.2	E5
7 (b) (ii)	3	6.1.2	E5
7 (b) (iii)	3	6.1.2	E5
8 (a) (i)	3	8.0	PE5, E2, E6
8 (a) (ii)	2	8.3	E2, E9
8 (a) (iii)	1	7.5	E2, E4, E9
8 (a) (iv)	2	7.5	HE7, E4, E9
8 (b) (i)	1	3.2, 8	E3, E9
8 (b) (ii)	2	3.2, 8	E3, E9
8 (b) (iii)	1	3.2, 8	E3, E9
8 (b) (iv)	2	3.2, 8	E3, E9
8 (b) (v)	1	3.2, 8	E3, E9

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2005 HSC Mathematics Extension 2
Marking Guidelines

Question 1 (a)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correct primitive	2
• Attempts to make an appropriate substitution or equivalent merit	1

Question 1 (b) (i)

Outcomes assessed: E4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Uses an appropriate technique to attempt to evaluate a and b	1

Question 1 (b) (ii)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correct primitive	1

Question 1 (c)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Correct primitive or equivalent merit	2
• Demonstrates some knowledge of the technique of integration by parts	1

Question 1 (d)

Outcomes assessed: E8

MARKING GUIDELINES

Criteria	Marks
• Correct primitive	2
• Rewrites the integral in the form $\int \frac{k\,dx}{\sqrt{x^2 - \frac{1}{4}}}$ or equivalent merit	1

Question 1 (e) (i)

Outcomes assessed: PE5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 1 (e) (ii)

Outcomes assessed: H5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Writes $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$ or equivalent progress	1

Question 1 (e) (iii)

Outcomes assessed: HE6, E8

MARKING GUIDELINES

Criteria	Marks
• Correct primitive	2
• Shows substantial understanding of the method of substitution	1

Question 2 (a) (i)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 2 (a) (ii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 2 (a) (iii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 2 (b) (i)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Correct modulus OR argument OR equivalent merit	1

Question 2 (b) (ii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Correct modulus OR argument OR equivalent merit	1

Question 2 (b) (iii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 2 (c)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct region	3
• Graph indicates the region corresponding to $\left\|\mathrm{Im}(z)\right\| < 1$ or equivalent merit	2
• Graph shows circle centre 1 and radius 1 or equivalent merit	1

Question 2 (d) (i)

Outcomes assessed: E2, E3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Recognition that ℓ bisects the $\angle POQ$ or equivalent progress	1

Question 2 (d) (ii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 2 (d) (iii)

Outcomes assessed: E3, E9

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 3 (a) (i)

Outcomes assessed: E6

MARKING GUIDELINES

Criteria	Marks
• Correct sketch	1

Question 3 (a) (ii)

Outcomes assessed: E6

MARKING GUIDELINES

Criteria	Marks
• Correct sketch	1

Question 3 (a) (iii)

Outcomes assessed: E6

MARKING GUIDELINES

Criteria	Marks
• Correct sketch	2
• Correct domain OR correct asymptote	1

Question 3 (a) (iv)

Outcomes assessed: E6

MARKING GUIDELINES

Criteria	Marks
• Correct sketch	2
• Sketch has correct shape, but asymptote and intercepts are not indicated or are incorrect OR equivalent merit	1

Question 3 (b)

Outcomes assessed: E6

MARKING GUIDELINES

Criteria	Marks
• Correct odd shape with the three correct x intercepts and all three asymptotes in the correct positions	4
• Odd shape, vertical asymptotes in the correct position and $y \to \infty$ as $x \to \infty$ or equivalent merit	3
• Some understanding of the asymptotic behaviour and three correct intercepts or equivalent merit	2
• Shows some understanding of the asymptotic behaviour or equivalent merit	1

Question 3 (c)

Outcomes assessed: E6

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Finds the slope of the normal at (2, 1) or equivalent progress	2
• Attempts to use implicit differentiation to find $\frac{dy}{dx}$ or equivalent progress	1

Question 3 (d)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Attempts to resolve forces in the horizontal plane and the vertical direction OR equivalent merit	1

Question 4 (a) (i)

Outcomes assessed: E7

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Substantially correct primitive or equivalent merit	2
• Correct integrand or equivalent merit	1

Question 4 (a) (ii)

Outcomes assessed: E7, E9

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 4 (b) (i)

Outcomes assessed: E4

MARKING GUIDELINES

Criteria	Marks
• Correct values	2
• One correct value	1

Question 4 (b) (ii)

Outcomes assessed: E2, E4, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Attempts to apply an appropriate technique	1

Question 4 (b) (iii)

Outcomes assessed: E2, E4, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 4 (b) (iv)

Outcomes assessed: E4

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Deduces that there is at least one real root or equivalent merit	1

Question 4 (c) (i)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Substitutes B into the equation of the normal	1

Question 4 (c) (ii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Observes that $y_1 \le b$ or equivalent progress	1

Question 5 (a) (i)

Outcomes assessed: H5, HE7, E2, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 5 (a) (ii)

Outcomes assessed: H5, HE7, E2, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Observes the connection with part (i) or equivalent progress	1

Question 5 (b) (i)

Outcomes assessed: PE3, E2, E9

MARKING GUIDELINES

Criteria	Marks
• Correct explanation	1

Question 5 (b) (ii)

Outcomes assessed: PE3, HE7

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Some further progress towards enumerating the outcomes	1

Question 5 (c) (i)

Outcomes assessed: HE3, E2, E9

MARKING GUIDELINES

Criteria	Marks
• Correct explanation	1

Question 5 (c) (ii)

Outcomes assessed: HE4, HE7, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Explicitly rewrites the integral using part (i) or equivalent progress	2
• Calculates the appropriate inverse function or equivalent progress	1

Question 5 (d) (i)

Outcomes assessed: H5, E2

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Calculates either the height or the width of the rectangle	1

Question 5 (d) (ii)

Outcomes assessed: E7

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Substantially correct primitive or equivalent merit	2
• Writing down the correct definite integral or equivalent merit	1

Question 6 (a) (i)

Outcomes assessed: HE2, E2, E8, E9

MARKING GUIDELINES

Criteria	Marks
• Establishes the initial case and proves the inductive step	4
• Proves the inductive step or gives proof with minor algebraic errors and verifies the initial case	3
• Attempts to relate the case when $n = k + 1$ to the case when $n = k$	2
• Verifies the case $n = 0$ or equivalent merit	1

Question 6 (a) (ii)

Outcomes assessed: E2, E8, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 6 (a) (iii)

Outcomes assessed: E2, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 6 (a) (iv)

Outcomes assessed: E2, E9

MARKING GUIDELINES

Criteria	Marks
• Correct answer	1

Question 6 (b) (i)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2

Question 6 (b) (ii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 6 (b) (iii)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 6 (b) (iv)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 6 (b) (v)

Outcomes assessed: E3

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Obtains a quadratic equation for $\cos\frac{\pi}{5}$	2
• Correctly rewrites in terms of $\cos\frac{\pi}{5}$ and $\cos\frac{2\pi}{5}$	1

Question 7 (a) (i)

Outcomes assessed: PE3, E2

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 7 (a) (ii)

Outcomes assessed: PE3, E2

MARKING GUIDELINES

Criteria	Marks
• Correct proof	3
• Shows both $\angle BNM = \angle BPM$ and $\angle BPM = \angle BAP$ or equivalent merit	2
• Shows that $\angle BNM = \angle BPM$ or $\angle BPM = \angle BAP$ or equivalent merit	1

Question 7 (a) (iii)

Outcomes assessed: E2, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Observes that $\frac{s}{u} = \frac{r}{p+q}$ or equivalent merit	1

Question 7 (a) (iv)

Outcomes assessed: E2, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 7 (b) (i)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Indicates that $V = \frac{2\pi R}{T}$ or $\lvert \ddot{x} \rvert = \frac{V^2}{R}$ or equivalent merit	1

Question 7 (b) (ii)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Obtains $v^2 = \frac{2k}{x} + c$ and attempts to evaluate the constant or equivalent merit	2
• Writes down $\frac{1}{2}v^2 = \int \frac{-k}{x^2}\,dx$ or equivalent merit	1

Question 7 (b) (iii)

Outcomes assessed: E5

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Obtains $t = -\int \frac{T}{2\sqrt{2}\pi R}\sqrt{\frac{x}{R-x}}\,dx$ or equivalent merit	2
• Writes $\frac{dx}{dt} = \frac{-2\sqrt{2}\pi R}{T}\sqrt{\frac{R-x}{x}}$ or equivalent merit	1

Question 8 (a) (i)

Outcomes assessed: PE5, E2, E6

MARKING GUIDELINES

Criteria	Marks
• Correct solution	3
• Shows $f(x)$ has a stationary point at $x = \frac{a+b}{2}$	2
• Calculates $f'(x)$	1

Question 8 (a) (ii)

Outcomes assessed: E2, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Observes that $f(c) \geq f\left(\frac{a+b}{2}\right)$ or equivalent merit	1

Question 8 (a) (iii)

Outcomes assessed: E2, E4, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 8 (a) (iv)

Outcomes assessed: HE7, E4, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Either observes that the cubic cannot have three positive real roots OR that it cannot have any negative real roots	1

Question 8 (b) (i)

Outcomes assessed: E3, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 8 (b) (ii)

Outcomes assessed: E3, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Shows that $\angle ACP = \alpha - \beta$ and $\angle CAP = \beta + \frac{\pi}{2}$ or equivalent progress	1

Question 8 (b) (iii)

Outcomes assessed: E3, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1

Question 8 (b) (iv)

Outcomes assessed: E3, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	2
• Observes that $PC \times PD = QC \times QD$	1

Question 8 (b) (v)

Outcomes assessed: E3, E9

MARKING GUIDELINES

Criteria	Marks
• Correct solution	1