2002 HSC Notes from the Marking Centre Mathematics Extension 1

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2002 HSC NOTES FROM THE MARKING CENTRE
MATHEMATICS EXTENSION 1

Introduction

This document has been produced for the teachers and candidates of the Stage 6 course, Mathematics Extension 1. It is based on comments provided by markers on each of the questions from the Mathematics Extension 1 paper. The comments outline common sources of error and contain advice on examination technique and how best to present answers for certain types of questions.

It is essential for this document to be read in conjunction with the relevant syllabus, the 2002 Higher School Certificate Examination, the Marking Guidelines and other support documents that have been developed by the Board of Studies to assist in the teaching and learning of the Mathematics Extension 1 course.

Question 1

Overall this question produced a good standard of responses from the majority of candidates. Candidates should be aware however, that a question worth only 1 or 2 marks should not require a lengthy solution that takes a disproportionate amount of time. The question consisted of six unrelated parts, most requiring only a couple of steps.

(a) The mark was awarded for the correct answer with correct working. The most common errors were $\sin 3x = 3\sin x$ and $\frac{\sin 3x}{x} = 3$.

(b) This part was correctly answered by a very high proportion of candidates, requiring some simple derivatives and correct use of the product rule.

(c) The use of the standard integral table was specified in the question and most candidates were able to do this, although some unnecessarily included a constant of integration in the following evaluation. A large number of candidates were unable to calculate $\frac{1}{2}\sec\frac{\pi}{3} - \frac{1}{2}\sec 0$. A common mistake was the assumption that $\cos 0 = 0$.

(d) While a good proportion of candidates was able to correctly state the domain and range of this function, the remainder usually showed a lack of understanding of inverse functions. Many gave a domain in radians rather than the range, while some drew the graph but were unable to distinguish between the domain and range. Candidates who gave the range in degrees or omitted the equality signs were not awarded the second mark.

(e) Finding the Cartesian equation caused difficulty for a surprising number of candidates. Many substituted into $x^2 = 4ay$, found ‘a’ and then did not know how to complete the question. Many differentiated and then substituted into some form of tangent equation. A number wrote $3t = 2at$, $2t^2 = at^2$, then finished with two different values for 'a' and could not proceed. Those candidates who correctly found $t = \frac{x}{3}$ had no problems in proceeding.

(f) Again, correctly answered by a high proportion of candidates. Common errors included: reversing the order of new limits to $-\int_{-8}^{-3} du$ without regard to signs, possibly assuming that the smaller number should be on the bottom; failing to simplify signs early and then having problems with the arithmetic at the evaluation step; ignoring the limits and incorrectly converting back to the variable x at the end. Many candidates who had made a minor error early ended with $\ln(-8) - \ln(-3)$ and ignored the fact that the answer was then undefined.

Question 2

Although most candidates scored high marks for this question, they rarely used efficient methods to establish their results, and so many would have spent far too long on the question. The standard of literacy in the geometry section was disappointing.

(a) Generally well done. Those who used logs to base 2 were rarely able to evaluate correctly.

(b) Almost all candidates were able to find one solution correctly but many had difficulty giving a general solution. Answers in degrees or a mixture of degrees and radians were common. Rarely did candidates bother to define 'n' used in their expressions.

(c) The most common method for finding the constant 'a' in the polynomial was via long division. This was not the most efficient method and often resulted in algebraic mistakes.

Another popular method was to write $Q(x)$ as a quadratic and equate coefficients. Setting x equal to -2 and solving $P(-2) = 3$ was not used by many candidates.

(d) The standard of the attempts at this part was disappointing. Many tried to evaluate the integral by quoting a memorized formula rather than using the standard double angle trigonometric substitution. Those who were able to integrate correctly generally did not have any difficulty with the evaluation step.

(e) (i) Some candidates provided a full proof of the Alternate Segment Theorem, oblivious of the fact that the part was only worth one mark. Others wrote a page and still did not manage to clearly communicate knowledge of the Alternate Segment Theorem. 'Property of a circle' was not sufficient to gain the mark!

(ii) Performance on this part did not always correlate with that on other parts of this question. Many candidates were trying to use circle geometry results and overlooked basic properties of triangles. Finding an expression for every angle in the diagram was not uncommon.

Question 3

The most common source of errors in this question was poorly remembered formulae.

(a) (i) Although this was done well by most candidates, there were still many who could not deal with the fact that the people were seated in a circle.

(ii) Many failed to gain part marks in this part because they gave no explanation for their (incorrect) answers.

(b) (i) This part was generally well done.

(ii) Newton's method was generally handled well. Many candidates did not round their answers to three significant figures, others wasted time evaluating $f(3.71)$, $f(3.72)$ etc.

(c) (i) This was done well, although many tried to solve the differential equation instead of verifying that the given expression was a solution.

(ii) The answers to this part were plagued by arithmetic and transcription errors. Common errors were $A = 80$ and replacing both T and t by 60 in the equation relating to time 10 minutes.

(iii) Candidates should be encouraged not to round early in their calculations. Rounded values for k should not be used in later calculations. Many interpreted 'How long ...' to mean 'How much longer ...' (ie after the first 10 minutes). Correct answers that were not rounded to the nearest minute were not penalised.

Question 4

Parts (a) and (b) were usually done well, but part (c) was not. Many candidates who attempted part (c) seemed to lack understanding of what was required.

(a) (i) This was usually well done. The most common error was for candidates to produce the answer $\left(\frac{2}{3}\right)^9\left(\frac{1}{3}\right)$, rather than the correct $\binom{10}{9}\left(\frac{2}{3}\right)^9\left(\frac{1}{3}\right)$. Perhaps this was due to thinking of a sequence of 9 successes followed by one failure, without realizing that there are 10 different ways of obtaining 9 successes and one failure.

(ii) The majority of candidates understood the concept of complementary events. Errors with parentheses and signs were common, for example many candidates wrote $P[<9] = 1-(P[9]-P[10])$ or $P[<9] = 1-P[9]+P[10]$. Candidates also appeared to misinterpret 'fewer than 9 times' as '9 or fewer times' and so gave their answer as $P[<9] = 1-P[10]$.

(b) (i) and (ii) Most candidates were confident with these parts of the question. Common errors were those of sign and an inability to equate the correct coefficients in $P(x)$ to the terms $\alpha+\beta+\gamma$ and $\alpha\beta\gamma$.

(iii) Many candidates were able to find the value of 2 for the third root, but were unable to go on and correctly find k.

(c) (i) The most common attempted solutions involved use of the equation $v^2 = n^2\left(a^2 - x^2\right)$ or integration of $\ddot{x} = -16x$. In using $v^2 = n^2\left(a^2 - x^2\right)$, the value $a = \sqrt{2}$ often appeared without justification and this was the most frequent error made. The integration approach commonly resulted in the incorrect equation $\dot{x} = -8x + C$. The majority of candidates who attempted the integration approach realised that it was necessary to use the given initial conditions to evaluate the constant of integration. A small minority of candidates attempted to show directly that the equation $|\dot{x}| = 4\sqrt{2-x^2}$ satisfied the given initial value problem $\ddot{x} = -16x$, $x(0) = 1$, $\dot{x}(0) = 4$. None of them succeeded.

(ii) Surprisingly, this was badly done. Candidates were confused by the fact that the initial position was at $x = 1$ and so obtained solutions such as $x = 1+\sqrt{2}$. Others, having seen the words 'greatest displacement' attempted to equate the derivative of $4\sqrt{2-x^2}$ to zero. A very large number of candidates concluded from $x^2 = 2$ that $x = \pm 2$.

(iii) Common approaches here were to start with one of the standard forms $x = a\cos(nt+\alpha)$, $x = a\sin(nt+\alpha)$ or $x = a\cos(nt) + b\sin(nt)$. Candidates who began with a correct standard form were usually successful in obtaining the answer. Very few, however, checked the quadrant of the phase angle α and simply assumed α was in the first quadrant, which was correct if the form $x = a\sin(nt+\alpha)$ was used, but not if the form $x = a\cos(nt+\alpha)$ was used. The other approach was to derive the result. There were many arithmetical slips along the way, but most candidates who attempted this approach showed understanding of the required procedures.

Question 5

While this question was quite well done by the average student, very good candidates were frequently able to score 11 or 12 marks. Candidates were required in different parts of the question to display ease with mathematical notation and reasoning. Candidates need to be reminded to show all working, particularly substitution into algebraic expressions.

(a) Candidates were required to use the principle of mathematical induction to show that a statement was true. Although the factorial notation involved in the question proved beyond the understanding of some candidates, the first two marks were easy to attain if they understood the structure of mathematical induction proofs. The first mark was gained by showing that the statement was true for $n = 1$. It was necessary to show the substitution into both sides of the statement and not just state that $2 = 2$. The second mark was gained for the assumption statement for $n = k$ and the LHS of the statement for $n = k + 1$. Presentation and notation were often unclear with candidates using S_k, $S(k)$ etc without any clear explanation for their notation. Candidates who introduced sigma notation mostly did so unsuccessfully and usually did not gain the second mark. The use of LHS and RHS was similarly unclear with it difficult to ascertain whether candidates were referring to their statement for $n = k$ or $n = k + 1$. The last mark was gained by correctly manipulating the LHS of the statement for $n = k + 1$ to show that the statement was true. Lack of understanding of factorial notation marred many solutions. Candidates need to be reminded of the futility of 'fudging' their algebra.

(b) (i) Candidates were required to show that $r = \frac{1}{3}h$. Many candidates gave no indication of similar triangles but were still awarded the mark for correct proportion statements, such as $\frac{r}{h} = \frac{4}{12}$. The most common error was for candidates to substitute the values $r = 4$

and $h = 12$ into $r = \frac{1}{3}h$. This approach did not gain the mark. Some candidates ignored the diagram and tried to use the formula for volume to show the result.

(ii) Generally, this section was well done with candidates demonstrating a clear understanding of the use of a chain rule, and, at times, some creative approaches to manipulating r, h and V. The most common error, however, was to differentiate the formula for volume in terms of r and h and simply treat one of the variables as a constant. This approach only allowed candidates to gain 1 mark out of 3.

Many candidates were let down by poor arithmetic/algebraic skills. When using a chain rule poor handwriting and/or use of notation made it very difficult to decipher what candidates were meaning, with dV, dr and dh being indistinguishable.

(c) (i) Candidates were required to show that the derivative of a function involving inverse trigonometric expressions equalled zero. Although many candidates understood how to differentiate inverse sine, they often showed little understanding of differentiating the function of a function. In this case, candidates were usually unable to gain any marks. The first 2 marks were gained for correct evaluation of each inverse trigonometric expression in the function. The final mark was allocated to showing that the derivative equalled zero. Many candidates undertook time-wasting pages of algebra trying to show this, although to little avail when their differentiation was incorrect. Other candidates 'fudged' their algebra. It is essential in a 'show that' question not to make too many leaps in the algebra, but to write down clear and logical solutions.

(ii) Candidates were required to graph the function given in (i). Many candidates successfully did so although they were unable to show that the derivative equalled zero in part (i). It was essential to graph the function correctly over the given domain to gain the two marks. Some candidates attempted to graph the function by subtracting ordinates, not realising the significance of the gradient being zero. Although their approach was quite sophisticated they missed the point of the entire question. Candidates need to be reminded to look for the linkages between different parts of the question.

Question 6

This question contained two unrelated parts, each with related subparts. There were few non-attempts, most candidates scoring some marks in the first part of the question.

(a) Candidates are reminded that projectile motion equations need to be derived for the given conditions, not just stated as formulae used in Physics. Those quoting and attempting to use formulae rarely gained full marks for this part.

(i) This was generally well done, with most candidates scoring two marks.

(ii) Many candidates understood that the conditions $x = 60$ and $y = 0$ needed to be used, however the majority had difficulty correctly eliminating 't' to find the velocity. Those using the 'range' formula often scored zero, as they did not allow for the correct initial conditions.

(iii) Most candidates understood that the maximum height occurred when the vertical component of the velocity was zero, but some had difficulty correctly evaluating the maximum height from their velocity. Careless arithmetic errors often resulted in candidates not gaining the second mark in this part.

(b) (i) The majority of candidates who attempted this part of the question did not see the link between the integral and the area under the curve. Among the few who did see this link, many confused the areas of the rectangles with the function values at $x = n$ and $x = n+1$. Candidates needed to indicate clearly that they were comparing the area under the curve with the area of the upper and lower rectangles.

(ii) Many candidates gained the first mark for correctly evaluating the integral, but very few gained the full three marks in this part. Care needs to be taken with all inequality signs during the deduction. Candidates are reminded that the substitution of specific numerical values does not constitute a proof and, as such, does not gain any marks.

Question 7

It could have been fairly easy to score two or three marks on this question but exam fatigue had obviously set in as quite a number of candidates did not complete the question. However, many did at least attempt (a), even though it was not all that well done.

(a) (i) Most candidates scored at least 1 mark for this part. The sketch was generally done satisfactorily, even though slightly sloppily: some candidates did not show the y-intercept, or labelled it as 1 instead of 2; some did not clearly illustrate the symmetry about the y-axis.

In answering why $y = g(x)$ did not have an inverse, most candidates said 'the horizontal line test fails'. Other accepted responses included: not a 1–1 function; each value of y has more than 1 value of x; even function; not monotonic increasing or decreasing for all x. The biggest problem here was the candidates' confusion regarding the x and y values: some saying there were two y values for every x; and then others not being clear in whether they were talking about $g(x)$ or the inverse function.

(ii) This was not that well done. Many candidates had difficulty in reflecting in $y = x$, while some who did manage this forgot to restrict the domain.

(iii) The majority of the candidature managed to exchange the x and y (gaining 1 mark), at some stage in finding the inverse function, but many candidates were very confused about correct notation. Some candidates managed to use the quadratic formula correctly to find e^y but did not make the right decision in selecting the correct root. Those who used 'completing the square' generally forgot to take both positive and negative square roots, thus only being able to score 1 mark.

(b) (i) Many candidates managed to gain 1 mark for this part, either for differentiating the expansion of $(1+x)^n$ correctly or for substituting $x = 1$ into its expansion. Some candidates were trying differentiation or integration but not making any connection with the question. Many candidates tried to 'show' by using induction and this scored no marks.

(ii) Those few candidates who did attempt this question generally did it quite well. They at least gained 1 mark by integrating $(1+x)^n$ or $(1-x)^n$ correctly. Some integrated twice but found no constants, thus finding the sum to be zero. A couple of candidates successfully found the sum by pattern detection. The most common incorrect attempts involved using the sum of a G.P., or dividing the expression by $(n+1)(n+2)$.

Mathematics Extension 1

2002 HSC Examination Mapping Grid

Question	Marks	Content	Syllabus outcomes
1(a)	1	The trigonometric functions	HE4
1(b)	2	The tangent to a curve and the derivative of a function Logarithmic and exponential functions	P7, H5, PE5
1(c)	2	The trigonometric functions	H5
1(d)	2	Inverse functions and the inverse trigonometric functions	HE4
1(e)	2	The quadratic polynomial and the parabola	PE3
1(f)	3	Integration	HE6
2(a)	2	Logarithmic and exponential functions	H3
2(b)	2	Trigonometric ratios	HE7
2(c)	2	Polynomials	PE3
2(d)	3	The trigonometric functions; Integration	HE6
2(e)(i)	1	Plane geometry	PE3
2(e)(ii)	2	Plane geometry	PE2, PE3
3(a)(i)	1	Permutations, combinations and further probability	PE3
3(a)(ii)	2	Permutations, combinations and further probability	PE3
3(b)(i)	1	Polynomials	H3, PE3
3(b)(ii)	3	Polynomials	H3, PE3
3(c)(i)	1	Applications of calculus to the physical world	HE3
3(c)(ii)	2	Applications of calculus to the physical world	HE3
3(c)(iii)	2	Applications of calculus to the physical world	HE3
4(a)(i)	1	Permutations, combinations and further probability	HE3
4(a)(ii)	2	Permutations, combinations and further probability	HE3
4(b)(i)	1	Polynomials	PE3
4(b)(ii)	1	Polynomials	PE3
4(b)(iii)	2	Polynomials	PE3
4(c)(i)	2	Applications of calculus to the physical world	HE3
4(c)(ii)	1	Applications of calculus to the physical world	HE3
4(c)(iii)	2	Applications of calculus to the physical world	HE3
5(a)	3	Series and applications	HE2
5(b)(i)	1	Plane geometry	P4
5(b)(ii)	3	Applications of calculus to the physical world	HE5
5(c)(i)	3	Inverse functions and the inverse trigonometric functions	HE4
5(c)(ii)	2	Inverse functions and the inverse trigonometric functions	HE4
6(a)(i)	2	Applications of calculus to the physical world	HE3

Question	Marks	Content	Syllabus outcomes
6(a)(ii)	3	Applications of calculus to the physical world	HE3
6(a)(iii)	2	Applications of calculus to the physical world	HE3
6(b)(i)	2	Integration	H8, PE6
6(b)(ii)	3	Integration	HE7
7(a)(i)	2	Inverse functions and the inverse trigonometric functions	H9, HE4
7(a)(ii)	1	Inverse functions and the inverse trigonometric functions	H9, HE4
7(a)(iii)	3	Inverse functions and the inverse trigonometric functions	H9, HE4
7(b)(i)	3	Binomial theorem	HE3, HE7
7(b)(ii)	3	Binomial theorem	HE3, HE7

2002 HSC Mathematics Extension 1
Marking Guidelines

Question 1 (a)

Outcomes assessed: HE4

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer	1

Question 1 (b)

Outcomes assessed: P7, H5, PE5

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer	2
• Gives correct derivatives for $3x^2$ and $\ln x$ OR • Correctly uses product rule	1

Question 1 (c)

Outcomes assessed: H5

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer	2
• Correctly uses standard integrals OR • Correctly substitutes limits into incorrect expression for the integral	1

Question 1 (d)

Outcomes assessed: HE4

MARKING GUIDELINES

Criteria	Marks
• Gives correct domain and range	2
• Gives correct domain OR • Gives correct range	1

Question 1 (e)

Outcomes assessed: PE3

MARKING GUIDELINES

Criteria	Marks
• Gives correct equation	2
EITHER • Obtains $t = \frac{x}{3}$ or $t = \sqrt{\frac{y}{2}}$ OR • Substitutes expression for t in $y = 2t^2$ or in $x = 3t$	1

Question 1 (f)

Outcomes assessed: HE6

MARKING GUIDELINES

Criteria	Marks
• Correctly evaluates integral showing relevant working	3
• Correctly uses substitution showing $du = -2x\,dx$ and correctly changes limits of integration OR • Correctly evaluates the integral apart from failure to change the limits	2
• Correctly substitutes $u = 1 - x^2$ and $du = -2xdx$ OR • Correctly changes limits of integration	1

Question 2 (a)

Outcomes assessed: H3

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer (Ignore rounding errors)	2
• Writes $\log 2^x = \log 3$ OR • Writes $\ln 2^x = \ln 3$	1

Question 2 (b)

Outcomes assessed: HE7

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer	2
• Finds a particular solution eg $x = \dfrac{\pi}{6}$	1

Question 2 (c)

Outcomes assessed: PE3

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer	2
• Substitutes $x = -2$ into the polynomial or equivalent	1

Question 2 (d)

Outcomes assessed: HE6

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer	3
• Correctly evaluates the integral apart from one minor error	2
• Uses the substitution $2\sin^2 4x = 1 - \cos 8x$ or its equivalent OR • Correctly integrates a non-trivial expression used to replace $\sin^2 4x$	1

Question 2 (e) (i)

Outcomes assessed: PE3

MARKING GUIDELINES

Criteria	Marks
• Includes ‘angle in the alternate segment’ in response	1

Question 2 (e) (ii)

Outcomes assessed: PE2, PE3

MARKING GUIDELINES

Criteria	Marks
• Gives appropriate proof with reasons	2
• Establishes one relevant fact about ΔAXY with reasons eg $\angle AXY = \alpha + \beta$ OR $\angle AYX = \alpha + \beta$	1

Question 3 (a) (i)

Outcomes assessed: PE3

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer	1

Question 3 (a) (ii)

Outcomes assessed: PE3

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer	2
• Evaluates number of arrangements where Kevin and Jill sit next to each other OR • Makes sensible attempt at enumeration	1

Question 3 (b) (i)

Outcomes assessed: H3, PE3

MARKING GUIDELINES

Criteria	Marks
• Demonstrates change of sign of function between $x = 3.7$ and $x = 3.8$	1

Question 3 (b) (ii)

Outcomes assessed: H3, PE3

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer (Ignore rounding)	3
• Gives any two of the one-mark components	2
• Obtains $f'(x) = e^x - 6x$ OR • Uses correct formula for Newton's method OR • Correctly evaluates $f(3.8)$ and their $f'(3.8)$	1

Question 3 (c) (i)

Outcomes assessed: HE3

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer	1

Question 3 (c) (ii)

Outcomes assessed: HE3

MARKING GUIDELINES

Criteria	Marks
• Gives correct values for A and k	2
• Gives correct value of A OR • Writes a correct equation from each of $t = 0$, $T = 80$ and $t = 10$, $T = 60$	1

Question 3 (c) (iii)

Outcomes assessed: HE3

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer (Ignore rounding)	2
• Correctly substitutes their A and k, and $T = 30$, to obtain an expression for t	1

Question 4 (a) (i)

Outcomes assessed: HE3

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer	1

Question 4 (a) (ii)

Outcomes assessed: HE3

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer	2
• Correctly identifies the complementary event OR • Finds $1-\left(\frac{2}{3}\right)^{10}$ OR • Writes the answer as P(target is hit 0 times) + P(target is hit once) +…+ P(target is hit 8 times)	1

Question 4 (b) (i)

Outcomes assessed: PE3

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer	1

Question 4 (b) (ii)

Outcomes assessed: PE3

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer	1

Question 4 (b) (iii)

Outcomes assessed: PE3

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer	2
EITHER • Finds value of third root OR • Substitutes their value of γ into polynomial to find k	1

Question 4 (c) (i)

Outcomes assessed: HE3

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer	2
• Finds a correct expression for $\frac{1}{2}\dot{x}^2$ OR • Uses $\dot{x}=4$ when $x=1$ to obtain a value for C in their expression for $\dot{x}$	1

Question 4 (c) (ii)

Outcomes assessed: HE3

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer	1

Question 4 (c) (iii)

Outcomes assessed: HE3

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer	2
• Uses $n=4$ and $A=\sqrt{2}$ in $x(t)=A\cos(nt+\alpha)$ OR • Evaluates α from incorrect values of n or A OR • Derives an expression for x directly from (i) or solves quadratic for e^x	1

Question 5 (a)

Outcomes assessed: HE2

MARKING GUIDELINES

Criteria	Marks
• Gives correct proof (Ignore incorrect or omitted final statement)	3
• Gives proof that includes both of the one-mark components	2
• Gives proof for $n = 1$ OR • Gives correct assumption statement for $n = k$ and substitution into the expression for $n = k + 1$ (LHS)	1

Question 5 (b) (i)

Outcomes assessed: P4

MARKING GUIDELINES

Criteria	Marks
• Gives correct argument via similar triangles	1

Question 5 (b) (ii)

Outcomes assessed: HE5

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer	3
• Gives solution that includes both of the one-mark components OR • Correctly calculates $\frac{dV}{dh}$ and gives correct evaluation when $h = 9$	2
• Gives correct expression for V as a function of h OR • Substitutes into chain rule with correct $\frac{dV}{dt}$ value or expression	1

Question 5 (c) (i)

Outcomes assessed: HE4

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer with correct working	3
• Correctly differentiates both parts	2
• Gives correct derivative for one of $2\sin^{-1}\sqrt{x}$ and $\sin^{-1}(2x-1)$	1

Question 5 (c) (ii)

Outcomes assessed: HE4

MARKING GUIDELINES

Criteria	Marks
• Correctly sketches graph	2
EITHER • Sketches horizontal line with domain $0 \le x \le 1$ OR • Evaluates $f(x)$ at some point in the domain	1

Question 6 (a) (i)

Outcomes assessed: HE3

MARKING GUIDELINES

Criteria	Marks
• Correctly integrates with evaluation of constants of integration	2
• Gives correct integration, with exception of constants of integration	1

Question 6 (a) (ii)

Outcomes assessed: HE3

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer	3
• Expresses y in terms of x, eliminating θ OR • Substitutes $y = 0$ and $x = 60$ to obtain V in terms of θ	2
EITHER • Substitutes $t = \dfrac{x}{V\cos\theta}$ into $y = Vt\sin\theta - 5t^2 + 5$ OR • Uses $y = 0$ and $x = 60$ to attempt to obtain a value of V in terms of θ	1

Question 6 (a) (iii)

Outcomes assessed: HE3

MARKING GUIDELINES

Criteria	Marks
• Gives correct answer (Ignore units) OR • Uses correct method but with incorrect value of V	2
• Finds a value for t or x when y is a maximum or equivalent	1

Question 6 (b) (i)

Outcomes assessed: H8, PE6

MARKING GUIDELINES

Criteria	Marks
• Gives appropriate proof of result	2
• Associates the integral with the area under the graph of $y = \frac{1}{x}$ between $x = n$ and $x = n + 1$	1

Question 6 (b) (ii)

Outcomes assessed: HE7

MARKING GUIDELINES

Criteria	Marks
• Gives appropriate proof of result	3
• Evaluates $\int_n^{n+1} \frac{dx}{x}$ and establishes one of the inequalities $e < \left(1 + \frac{1}{n}\right)^{n+1}$ OR $e > \left(1 + \frac{1}{n}\right)^n$	2
• Obtains $\int_n^{n+1} \frac{dx}{x} = \ln\left(\frac{n+1}{n}\right) = \ln\left(1 + \frac{1}{n}\right)$	1

Question 7 (a) (i)

Outcomes assessed: H9, HE4

MARKING GUIDELINES

Criteria	Marks
• Correctly sketches $g(t)$ and gives reason why $g(x)$ has no inverse function	2
EITHER • Correctly sketches function OR • Gives reason why $g(x)$ has no inverse function	1

Question 7 (a) (ii)

Outcomes assessed: H9, HE4

MARKING GUIDELINES

Criteria	Marks
• Correctly sketches inverse function (correct from (i))	1

Question 7 (a) (iii)

Outcomes assessed: H9, HE4

MARKING GUIDELINES

Criteria	Marks
• Gives correct function	3
• Solves the quadratic equation for e^y	2
• Obtains an expression relating x and y only	1

Question 7 (b) (i)

Outcomes assessed: HE3, HE7

MARKING GUIDELINES

Criteria	Marks
• Gives correct derivation	3
• Differentiates expansion of $(1+x)^n$ term by term and substitutes $x = 1$ into $(1+x)^n$ and its derivative	2
• Correctly differentiates $(1+x)^n$ expansion OR • Substitutes $x = 1$ into the expression for $(1+x)^n$	1

Question 7 (b) (ii)

Outcomes assessed: HE3, HE7

MARKING GUIDELINES

Criteria	Marks
• Gives correct derivation	3
• Correctly integrates $(1+x)^n$ twice, handling constants OR • Equivalent	2
• Integrates the expression for $(1+x)^n$ with respect to x OR • Splits result into two terms using $\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$	1