BOARD OF STUDIES
NEW SOUTH WALES

2001
HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics

General Instructions
- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 120
- Attempt Questions 1–10
- All questions are of equal value

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Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Marks

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) Evaluate, correct to three significant figures, **2**

$$\sqrt{\frac{3^2+12^2}{231-12^2}}\ .$$

(b) Solve $|x+3|<2$. **2**

Graph your solution on a number line.

(c) Solve $x^2-2x-8=0$. **2**

(d) Find a primitive of $3+\frac{1}{x}$. **2**

(e) Simplify $\frac{x}{x^2-4}+\frac{2}{x-2}$. **2**

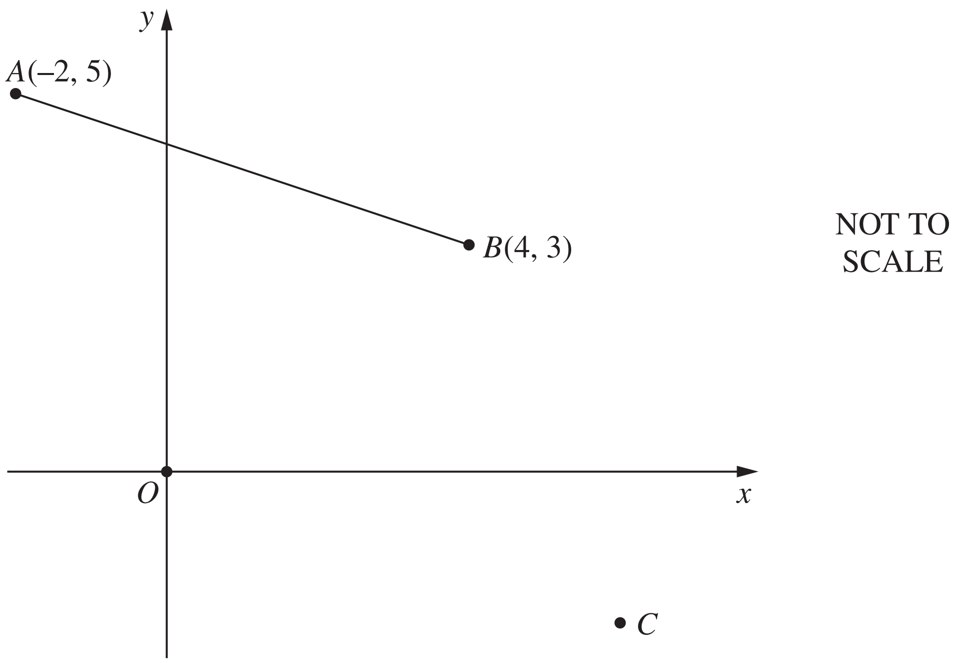
(f) The cost of a video recorder is \$979. This includes a 10% tax on the original price. Calculate the original price of the video recorder. **2**

Marks

Question 2 (12 marks) Use a SEPARATE writing booklet.

(a) Find the equation of the tangent to the curve $y = x^2 + 3x$ at the point $(1, 4)$. **3**

(b)



The diagram shows the points $A(-2, 5)$, $B(4, 3)$ and $O(0, 0)$. The point C is the fourth vertex of the parallelogram $OABC$.

(i) Show that the equation of AB is $x + 3y - 13 = 0$. **2**

(ii) Show that the length of AB is $2\sqrt{10}$. **1**

(iii) Calculate the perpendicular distance from O to the line AB. **2**

(iv) Calculate the area of parallelogram $OABC$. **2**

(v) Find the perpendicular distance from O to the line BC. **2**

Marks

Question 3 (12 marks) Use a SEPARATE writing booklet.

(a) Evaluate $\displaystyle\int_0^1 \frac{dx}{x+4}$. **2**

(b) Assume that the surface area S of a human satisfies the equation **2**

$$S = kM^{\frac{2}{3}}$$

where M is the body mass in kilograms, and k is the constant of proportionality.

A human with body mass 70 kg has surface area 18 600 cm^2.

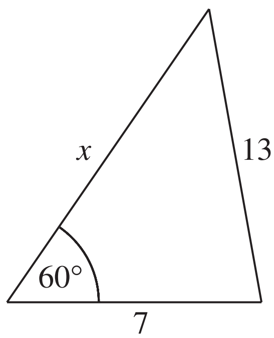
Find the value of k, and hence find the surface area of a human with body mass 60 kg.

(c) Differentiate with respect to x:

(i) $\ln\left(x^2 - 9\right)$ **2**

(ii) $\dfrac{x}{e^x}$. **2**

(d) **4**



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The diagram shows a triangle with sides 7 cm, 13 cm and x cm, and an angle of 60° as marked.

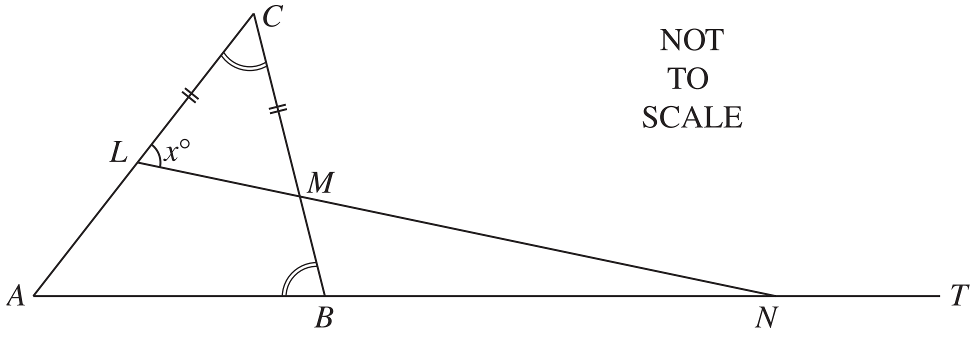
Use the cosine rule to show that $x^2 - 7x = 120$, and hence find the exact value of x .

Marks

Question 4 (12 marks) Use a SEPARATE writing booklet.

(a) Find the values of k for which the quadratic equation $3x^2 + 2x + k = 0$ has no real roots. **2**

(b)



In the diagram, ABC is an isosceles triangle with $\angle ABC = \angle ACB$. The line LMN is drawn as shown so that $CL = CM$, and $\angle CLM = x°$.

Copy or trace the diagram into your Writing Booklet.

(i) Show that $\angle ABC = 180 - 2x°$. **2**

(ii) Hence show that $\angle TNL = 3x°$. **2**

(c) (i) Sketch the curve $y = 3\sin 2x$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. **2**

(ii) On your diagram for part (i), sketch the line $y = \frac{1}{4}x$, and shade the region represented by **2**

$$\int_0^{\frac{\pi}{4}} \left(3\sin 2x - \frac{1}{4}x\right) dx .$$

(iii) Find the exact value of the integral in part (ii). **2**

Marks

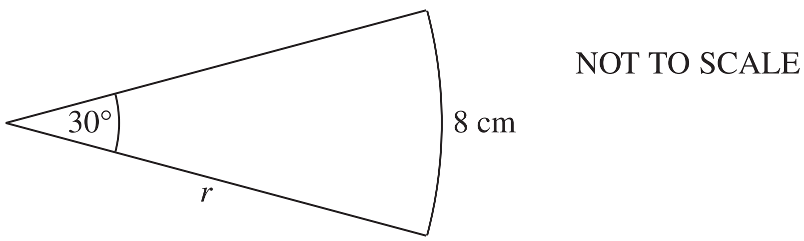
Question 5 (12 marks) Use a SEPARATE writing booklet.

(a) State the domain and range of the function $y = 2\sqrt{25 - x^2}$. **3**

(b) (i) Find $\log_{10}(2^{1000})$ correct to 3 decimal places. **2**

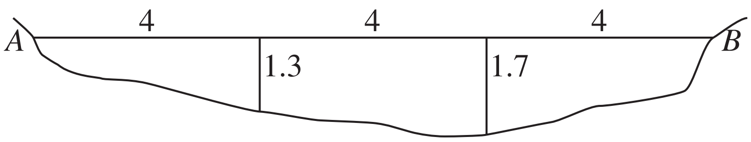
(ii) We know that $2^{10} = 1024$, so that 2^{10} can be represented by a 4 digit numeral. How many digits are there in 2^{1000} when written as a numeral? **1**

(c) **2**



Find the length of the radius of the sector of the circle shown in the diagram. Give your answer correct to the nearest mm.

(d)



The diagram shows the cross-section of a creek, with the depths of the creek shown in metres, at 4 metre intervals. The creek is 12 metres in width.

(i) Use the trapezoidal rule to find an approximate value for the area of the cross-section. **2**

(ii) Water flows through this section of the creek at a speed of $0.5\ \mathrm{m\,s^{-1}}$. Calculate the approximate volume of water that flows past this section in one hour. **2**

Marks

Question 6 (12 marks) Use a SEPARATE writing booklet.

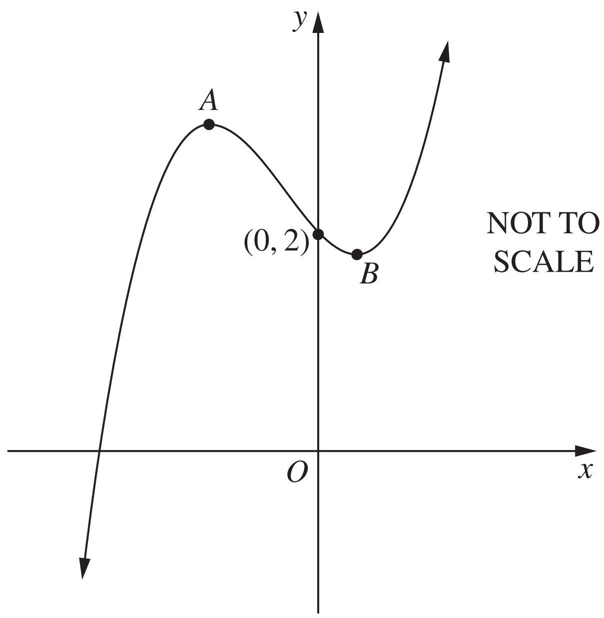
(a) The first three terms of an arithmetic series are $-1+4+9+\ldots$

(i) Find the 60th term. **2**

(ii) Hence, or otherwise, find the sum of the first 60 terms of the series. **2**

(b) Find α so that the equation $P = 100(1.23)^t$ can be rewritten as $P = 100e^{\alpha t}$. Give your answer in decimal form. **2**

(c) The graph of $y = x^3 + x^2 - x + 2$ is sketched below. The points A and B are the turning points.



(i) Find the coordinates of A and B. **3**

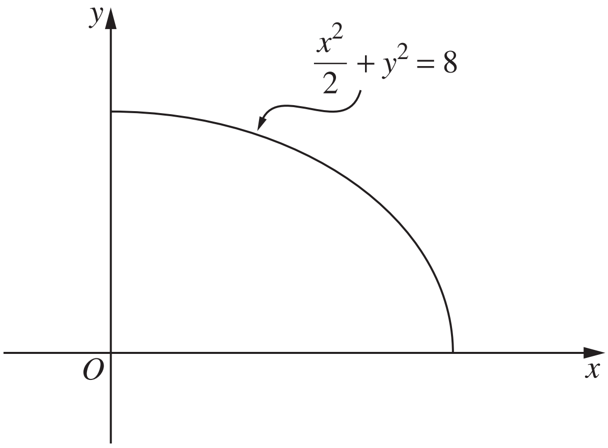
(ii) For what values of x is the curve concave up? Give reasons for your answer. **2**

(iii) For what values of k has the equation $x^3 + x^2 - x + 2 = k$ three real solutions? **1**

Marks

Question 7 (12 marks) Use a SEPARATE writing booklet.

(a) **3**



The part of the curve $\frac{x^2}{2} + y^2 = 8$ that lies in the first quadrant is rotated about the x axis.

Find the volume of the solid of revolution.

(b) Onslo tries to connect to his internet service provider. The probability that he connects on any single attempt is 0.75.

(i) What is the probability that he connects for the first time on his second attempt? **2**

(ii) What is the probability that he is still not connected after his third attempt? **1**

(c) A particle moves in a straight line so that its displacement, in metres, is given by

$$x = \frac{t-2}{t+2} \quad \text{where } t \text{ is measured in seconds.}$$

(i) What is the displacement when $t = 0$? **1**

(ii) Show that $x = 1 - \frac{4}{t+2}$. **3**

Hence find expressions for the velocity and the acceleration in terms of t.

(iii) Is the particle ever at rest? Give reasons for your answer. **1**

(iv) What is the limiting velocity of the particle as t increases indefinitely? **1**

Marks

Question 8 (12 marks) Use a SEPARATE writing booklet.

(a) In November 1923, 18 koalas were introduced on Kangaroo Island. By November 1993, the number of koalas had increased to 5000. **5**

Assume that the number N of koalas is increasing exponentially and satisfies an equation of the form $N = N_0 e^{kt}$, where N_0 and k are constants and t is measured in years from November 1923.

Find the values of N_0 and k, and predict the number of koalas that will be present on Kangaroo Island in November 2001.

(b) Five candidates, A, B, C, D and E, are standing for an election. Their names are written on pieces of cardboard that are placed in a barrel and are drawn out randomly to determine their positions on the ballot paper.

(i) What is the probability that A is drawn first? **1**

(ii) What is the probability that the order of the names on the ballot paper is that shown below? **2**

$\boxed{A}$

$\boxed{B}$

$\boxed{C}$

$\boxed{D}$

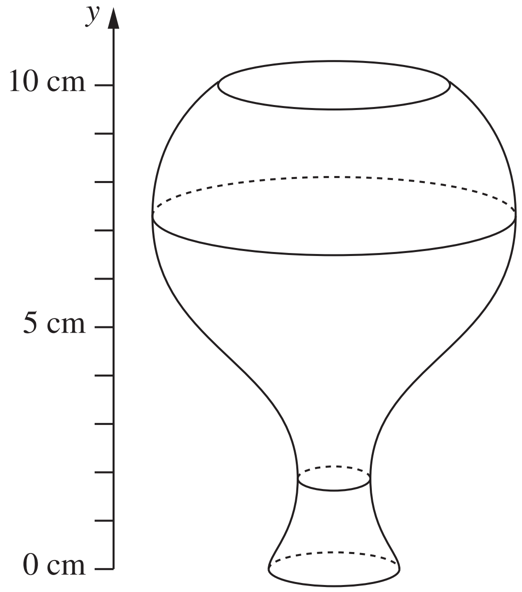
$\boxed{E}$

Question 8 continues on page 11

Marks

Question 8 (continued)

(c)



The diagram shows a ten-centimetre high glass that is being filled with water at a constant rate (by volume). Let $y = f(t)$ be the depth of water in the glass as a function of time t.

(i) Find the approximate depth y_1 at which $\dfrac{dy}{dt}$ is a maximum. **2**

Find the approximate depth y_2 at which $\dfrac{dy}{dt}$ is a minimum.

(ii) Assume that the glass takes 5 seconds to fill. **2**

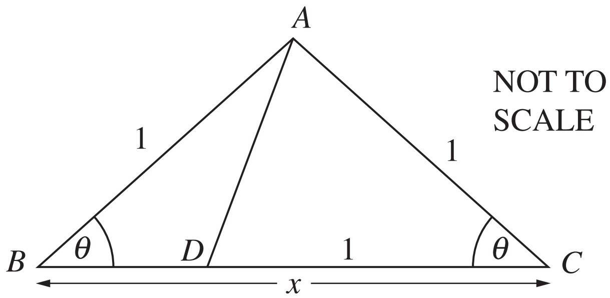
Graph $y = f(t)$ and identify any points on your graph where the concavity changes.

End of Question 8

Marks

Question 9 (12 marks) Use a SEPARATE writing booklet.

(a)



In the diagram, ABC is an isosceles triangle where $\angle BAC = \frac{3\pi}{5}$ and $AB = AC = 1$. The point D is chosen on BC such that $CD = 1$.

Let $BC = x$, and let $\angle ABC = \theta$, and note that $\theta = \frac{\pi}{5}$.

(i) Show that $\angle ADC = 2\theta$ and hence show that triangles DBA and ABC are similar. **3**

(ii) From part (i) deduce that $x^2 - x - 1 = 0$. **1**

(iii) By using the cosine rule, deduce that **2**

$$\cos\frac{\pi}{5} = \frac{1+\sqrt{5}}{4}.$$

(b) When a valve is released, a chemical flows into a large tank that is initially empty. The volume, V litres, of chemical in the tank increases at the rate

$$\frac{dV}{dt} = 2e^{t} + 2e^{-t}$$

where t is measured in hours from the time the valve is released.

(i) At what rate does the chemical initially enter the tank? **1**

(ii) Use integration to find an expression for V in terms of t. **2**

(iii) Show that $2e^{2t} - 3e^{t} - 2 = 0$ when $V = 3$. **1**

(iv) Find t, to the nearest minute, when $V = 3$. **2**

Marks

Question 10 (12 marks) Use a SEPARATE writing booklet.

(a) Helen sets up a prize fund with a single investment of \$1000 to provide her school with an annual prize valued at \$72. The fund accrues interest at a rate of 6% per annum, compounded annually. The first prize is awarded one year after the investment is set up.

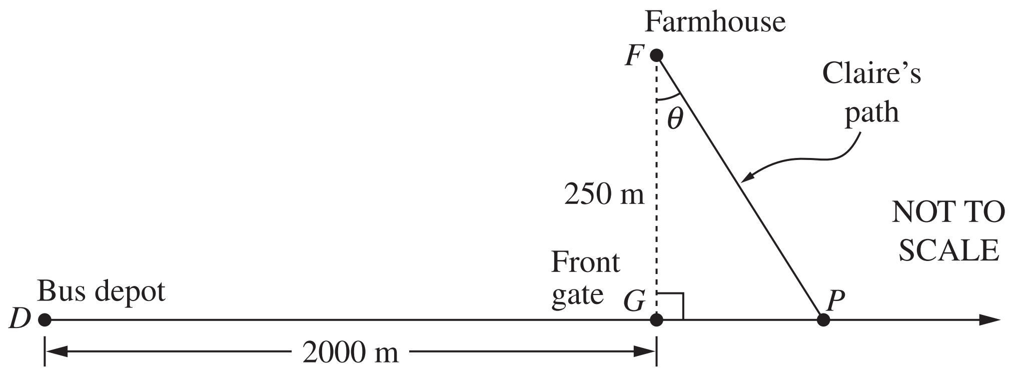
(i) Calculate the balance in the fund at the beginning of the second year. **1**

(ii) Let $\$B_n$ be the balance in the fund at the end of n years (and after the nth prize has been awarded). Show that $B_n = 1200 - 200 \times (1.06)^n$. **2**

(iii) At the end of the tenth year (and after the tenth prize has been awarded) it is decided to increase the prize value to \$90. **3**

For how many more years can the prize fund be used to award the prize?

(b)



The diagram shows a farmhouse F that is located 250 m from a straight section of road. The road begins at the bus depot D, which is situated 2000 m from the front gate G of the farmhouse. The school bus leaves the depot at 8 am and travels along the road at a speed of 15 m s^{-1}. Claire lives in the farmhouse, and she can run across the open paddock between the house and the road at a speed of 4 m s^{-1}. The bus will stop for Claire anywhere on the road, but will not wait for her.

Assume that Claire catches the bus at the point P on the road where $\angle GFP = \theta$.

(i) Find two expressions in terms of θ, one expression for the time taken for the bus to travel from D to P and the other expression for the time taken by Claire to run from F to P. **2**

(ii) What is the latest time that Claire can leave home in order to catch the bus? **4**

End of paper

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STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax\,\tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$