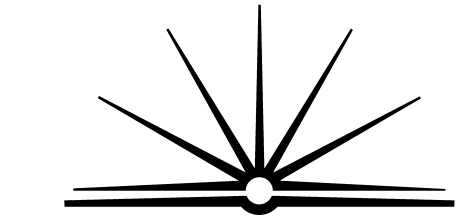

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2000 HIGHER SCHOOL CERTIFICATE EXAMINATION

MATHEMATICS

ENHANCED EXAMINATION REPORT

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General Comments

The three papers for the related HSC Mathematics courses in 2000 provided candidates with appropriately graded opportunities to display the extent to which they had achieved the outcomes of the syllabus. The pattern of distribution of raw marks achieved on the papers was not significantly different from those obtained by candidates on the equivalent papers in earlier years.

The comments in this report are compiled from information supplied by examiners involved in marking each individual question. While they do provide an overview of performance on the 2000 examinations, their main purpose is to assist candidates and their teachers to prepare for future examinations by providing guidance as to the expected standard, highlighting common deficiencies and, in the process, explaining in some detail the criteria that were applied in allocating the marks for each part of each question. Where appropriate, the method of solution is outlined and the merits of different approaches to the question are discussed. This report remains relevant to candidates preparing for examinations under the New HSC arrangements commencing in 2001.

Candidates should be aware of the fact that it is their responsibility to indicate the process by which they have obtained their answer to the examiners. In marking, each individual mark is allocated to a step or process that is essential to a correct solution of that question. Those who provide sufficient evidence that the appropriate step, or its equivalent, has been completed are awarded the mark, which then cannot be lost for a subsequent error. Candidates who give only a single word or figure as their response forego any possibility of earning any marks unless their answer is completely correct. Sometimes, in cases where examiners believe that the correct answer is easily guessed without doing the work required to establish the result, a mere correct answer without any supporting justification may not earn all of the available marks.

It is very important that candidates record their working in the same writing booklet as their answer, even if it is experimental work done to develop an approach to the question. Examiners read everything written by the candidate in an attempt to find evidence that will justify the awarding of a mark. This includes work that the candidate has crossed out, or explicitly requested the examiner not to mark. This is always to the candidate's advantage, as marks are awarded for elements of the solution that are correct and are not deducted for errors that have been made. This means that candidates should take care to make sure that work which has been crossed out is still legible, and should not, in any circumstances, use correction fluid or an eraser. Candidates who wish to distinguish their rough work from their considered answers should use the unruled left hand pages for such work.

Candidates who accidentally answer part of one question in the wrong writing booklet should not waste valuable time transcribing their work from one booklet to another. Instead, they should make a clear note on the cover of both the

writing booklets to the effect that, for example, part of the answer to Question 7 is included in the booklet for Question 5. There are procedures in place at the marking centre to ensure that such misplaced material is brought to the attention of the examiner marking the appropriate question, and no marks are ever deducted for such slips.

Examiners greatly appreciate work which is clearly presented, in which the order of a candidate's work is readily apparent. In particular, candidates are encouraged to avoid setting their work out in two or more columns per page, and to make certain that the parts and subparts of questions are appropriately labelled. It is not essential for the parts within a question to be presented in the same order as they appear in the examination question, but departures from the original order make careful labelling of the responses even more important. On the other hand, the sequence of logic involved is often important, and candidates should be aware that they cannot use results obtained in later parts unless they have been obtained without relying on the result they are attempting to establish. Circular arguments cannot be credited with full marks.

Examiners frequently comment on the need for candidates to provide clearly labelled, reasonably sized and well executed graphs and diagrams. Appropriate use of a ruler and other mathematical instruments is often essential to obtain a diagram of the appropriate standard. In making these comments, examiners are motivated by the assistance such graphs and diagrams provide candidates in the process of answering the question. In particular, candidates ought to realise that instructions on the examination paper asking candidates to reproduce a diagram in their writing booklet are invariably given because the diagram is likely to assist them in the solution of the problem or provide a means for them to explain their solution.

Finally, candidates are reminded that a table of standard integrals appears on the back page of each related Mathematics HSC examination paper. Candidates should become familiar with this table, and be aware of its use for both integration and differentiation.

Mathematics 2/3 Unit (Common)

Question 1

Almost all candidates attempted this question and the majority scored 10 or more marks out of a possible 12 marks. The question consisted of seven parts taken from five separate areas of the syllabus, namely basic arithmetic and algebra, linear functions, logarithmic and exponential functions, trigonometry and probability. While all parts were of a very basic standard, they did require candidates to have an understanding of a broad range of mathematical concepts. The question was well done. The most noticeable problem was with the presentation of graphical solutions in parts (b) and (g).

(a) (2 marks)
The first mark was awarded if it was apparent that the candidate's decimal value was derived from an attempt to evaluate $\log_e 8$. The second mark was awarded for the correct rounding to two decimal places.

Many candidates did not recognise the equivalence of $\log_e$ and ln. Consequently, a common answer was 0.90 which arose from calculating $\log_{10} 8$.

(b) (2 marks)
The first mark was awarded for the algebraic solution and the second for depicting the solution graphically. In order to score the mark for the graph it needed to be consistent with the candidate's working, although a correct graph with no supporting working was awarded 2 marks. A common mistake was to graph the solution on the number plane.

Some candidates treated the inequality as if it was $|x + 7| \geq 3$ and subsequently arrived at two solutions. Many candidates graphed $x > -4$ incorrectly, marking the section of the number line to the left of -4 rather than the section to the right.

(c) (1 mark)
The exact value of $\frac{\sqrt{3}}{2}$ was obtained by the majority of candidates. A significant number of candidates gave decimal approximations which were not accepted.

(d) (1 mark)
This question was well done by the majority of candidates. The answer $\frac{3}{5}$ (or its equivalent) was the only acceptable answer. The fraction $\frac{2}{3}$ was a common answer.

(e) (2 marks)
Candidates were required to display at least some understanding of the process of solving simultaneous equations.

Examiners found far too many examples of transcription errors, in which candidates wrote such things as $3x - 2y = 1$ instead of $3x + 2y = 1$. Candidates were awarded full marks if they successfully solved the pair of simultaneous equations resulting from these mistakes, as they required the same skills and were of the same degree of difficulty.

Poor algebraic skills were common. Many of the candidates using the substitution method incorrectly rearranged the first equation to obtain $y = x + 2$, while those using the elimination method often obtained $x = -3$ as a result of an incorrect attempt to subtract the first equation from the second.

A substantial number of candidates found a solution for either x or y, but did not then proceed to find the value for the remaining variable.

(f) (2 marks)
Most candidates were able to obtain 1 mark for providing the solution $x = 8$. However, many candidates lacked any understanding of absolute value. Candidates whose working contained incorrect statements such as $|2| = -2$ or $|x - 5| = -3$ could gain at most one mark. Solutions involving inequalities and final statements such as $|x| = 8$ received zero marks.

(g) (2 marks)
A graph which was clearly a straight line cutting the y axis at 3 and the x axis at -1.5 was sufficient to gain full marks.

Many candidates presented graphs with incorrect scales or no scale at all and a significant number did not find the x or y intercepts. This meant that it was often difficult to judge the correctness of their answer. Some candidates presented graphs that were not straight lines and these received no marks. There were also a few candidates who indicated that they had not attempted this part because they did not know the meaning of the words 'Cartesian plane'.

Examiners were concerned about the lack of neatness and accuracy of some graphs. In many instances, the sketches could have depicted the algebra more accurately.

Question 2

This question consisted of eight parts taken from the syllabus topic involving applications of geometrical properties. Candidates were initially required to use appropriate formulae to find the gradient and midpoint of an interval, the equation of a line and the distance between two points. In the latter parts of the question candidates were asked to find the equation of a circle, show that the circle passed through a given point and finally they were asked to find the coordinates of the fourth vertex of a rhombus given the coordinates of the other three vertices.

On the whole the question was very well attempted with nearly half the candidates scoring full marks. Most candidates set their working out clearly and sequentially in each step of their solution. It was good to see the number of candidates who attempted later parts of the question despite not knowing how to do earlier parts. These candidates often scored marks in these later sections.

On some occasions, candidates simply restated results which they were asked to 'show'. These candidates could not be awarded the marks without appropriate working or reasoning. Candidates are advised not to cross out (or rub out) work without replacing it, as this can also lead to a potential loss of marks. Finally candidates are encouraged to do a quick check to see if their solution makes sense and thus avoid losing marks for careless errors (for example, in part (h), the common answer $R(2,7)$ is obviously incorrect once placed on a sketch and this should lead candidates to go back and rethink their answers).

(a) (1 mark)
The majority of the candidature were able to substitute into an appropriate formula for gradient. A frequently encountered response was to state that gradient $= \frac{\text{rise}}{\text{run}} = \frac{1}{2}$, followed by an explanation as to why the gradient had to be negative (eg. line slopes down, line leans back, line heads east etc). The marking scheme accepted the response $\frac{2-0}{0-4}$ or equivalent numerical expression for the award of the mark. The answer $\frac{-2}{4}$ was awarded the mark provided it was not preceded by a numerically incorrect statement.

(b) (2 marks)
This was an easy two marks for nearly all candidates. Most successfully applied the midpoint formula to obtain the correct answer $M(2,1)$. The two most common errors were to use the incorrect formula $(\frac{x_1-x_2}{2}, \frac{y_1-y_2}{2})$ or to reverse the order of the coordinates for an answer of $(1,2)$. One mark was awarded for finding the correct x coordinate and one mark for finding the correct y coordinate. Candidates who reversed the correct coordinates were also awarded one mark.

(c) (2 marks)
The overall standard of response for this part was good. Candidates could use the fact that the gradient of PQ had been shown to be $-\frac{1}{2}$ in part (a) to deduce that the gradient of MN was 2, and thus proceed to find the equation of MN. However, a considerable number began by redoing the work of part (a).

The marking scale awarded one mark for a correct gradient of 2 and the second mark for successfully substituting their coordinates of M into a correct general form for the equation of a straight line.

The relationship between the gradients of perpendicular lines was often incorrectly quoted as $m_2 = -m_1$, $m_2 = m_1$ or $m_1 \times m_2 = 1$. A minority of the candidature used the coordinates of N in their working, not realising the circular nature of their argument.

(d) (1 mark)
The majority of the candidature recognised the need to make a substitution into the equation of the line they had found in part (c). Most substituted the value of 0 for x to yield a correct value of -3 for y while others showed that the substitution of 0 for x and -3 for y yielded a correct numerical statement. Alternative approaches involved showing that the gradient of the line through M and $(0, -3)$ was 2, showing that the equation of this line was the same as the one obtained in part (c), or comparing their equation from (c) with the general form $y = mx + b$.

The marking scheme recognised that the onus was on the candidate to 'show' N was on the line. The mark was awarded for making a correct substitution in their equation for MN or showing evidence of their intention to substitute. Numerical errors occurring after the substitution was made were ignored.

(e) (1 mark)
Most candidates were able to apply the distance formula to find the length of NQ correctly, although the negative sign for the coordinate of N caused some problems. Others successfully applied Pythagoras' theorem for the triangle NOQ. A minority did it the long way, first finding the lengths of MN and MQ before using $NQ^2 = MN^2 + MQ^2$. The mark for this part was awarded for a correct substitution in a correct formula. Incorrect statements for distance such as $d = \sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$ or $d = \sqrt{(x_1 + x_2)^2 + (y_1 + y_2)^2}$ resulted in zero marks.

(f) (2 marks)
This was not as well done as earlier parts of the question. A high proportion of the candidates knew something like the formula for the equation of a circle, but the markers encountered many incorrect variations in candidates' scripts. Once again the negative sign for the coordinate of the centre caused problems. Many candidates started with the circle $x^2 + y^2 = 25$ and 'adjusted' this to cater for the new centre. Done correctly, this yielded $(x - 0)^2 + (y + 3)^2 = 25$, but common incorrect answers included $(x - 0)^2 + (y - 3)^2 = 25$, $(x - 0)^2 - (y + 3)^2 = 25$, $x^2 - 9y^2 = 25$ and so on. One mark was awarded for substituting into a formula of the form $(x - p)^2 + (y - q)^2 = k$ using any values of p, q provided $k > 0$. The second mark was awarded for the correct answer. Mistakes in attempting to expand and simplify the equation were ignored.

(g) (1 mark)
Candidates who were successful in part (f) also had a high degree of success with this part. Once again, most realised some form of substitution was required with responses equally divided between replacing x by 0 to find y, replacing y by 2 to find x, or replacing both x and y by the coordinates of P to obtain a correct numerical statement. A good number of candidates realised this part could be done without any reference to part (f). They

simply showed the length of PN was 5 units and pointed out that this was the same as the radius of the circle. Those attempting to solve simultaneously the equation of the circle and the equation of the line PQ met with only limited success. The marking scheme awarded the mark for a correct substitution into the equation of the circle found in part (f) or for correctly using any of the other approaches described above.

(h) (2 marks)
A variety of approaches were used in attempting a solution for this part. The geometric solution, using a diagram and properties of a rhombus, was by far the easiest. The largest group of candidates realised RQ was a vertical line with R being 5 units above Q and deduced (by inspection) that the coordinates of R were (4, 5). The second most successful approach was to use the property that the diagonals of a rhombus bisect each other. This meant that M was the midpoint of NR, and so the mid-point formula could be applied. In most cases, this was done successfully.

The third common approach proved to be the most difficult for candidates. This involved complicated and long algebraic solutions based on the solution of a variety of pairs of simultaneous equations. This met with varying degrees of success. Candidates did not find the algebra easy and often simplified their equations incorrectly before attempting to solve them. There was also evidence that some candidates failed to read the question carefully, not taking into account the given fact that R was in the first quadrant.

Question 3

The four parts of this question tested the product and chain rules for differentiation, proportionality properties of similar triangles, aspects of integration, and finding the line tangent to a smooth curve at a specified point. The question also tested candidates' knowledge of the logarithmic and exponential functions. Of the twelve marks available, ten were awarded for calculus-based skills.

Most candidates scored between ten and twelve marks; the main sources of error were in parts (b) and (c) (ii). However, it was noticeable that there were centres in which candidates received few marks, and where it appeared that their calculus skills, especially in integration, were confined to polynomial functions. Teachers should ensure candidates work more with logarithmic, exponential and trigonometric functions in the areas of differentiation and integration tested in parts (a), (c) and (d) of this question. Also, it appeared in part (a) (ii) that many candidates believe the derivative of $\sin(f(x))$ is $\cos(f(x))$ or fail to understand the functional notation. Many candidates wrote things such as $\cos(x^2+1)\cdot 2x = \cos 2x(x^2+1) = 2\cos x(x^2+1)$. These problems would be reduced if candidates were instructed to use brackets to indicate the arguments of functions. Candidates should write $\sin(x)$ and $\ln(x)$, and not $\sin x$ and $\ln x$.

Candidates often gained marks in these questions for obtaining the correct answer in the course of their working, even though subsequent errors in their attempts to simplify the expressions they obtained meant that their answer was incorrect. However, many candidates erase working written in pencil, or cross out working so that it is no longer legible. This makes it impossible for the examiners to award any marks unless the final answer is completely correct. Candidates should be reminded that a single neat line is sufficient to indicate that this work is not part of their final answer.

(a) This part required candidates to differentiate $3xe^x$ and $\sin(x^2+1)$.

(i) (2 marks)
The product rule was well done, with the main errors being + replaced with $\times$, or with $-$; the latter often appearing as part of an attempt to apply the quotient rule. The other major error was to quote an incorrect expression for the derivative of e^x such as $\frac{1}{x}$, xe^x or xe^{x-1}.

The form of the product rule $\frac{d}{dx}(uv) = uv[\frac{d}{dx}(\ln u)+\frac{d}{dx}(\ln v)]$ was used by a few centres. While it was clear that candidates were attempting to apply this form of the rule, the success rate was almost zero and their seems to be little merit in introducing this method to 2 Unit Mathematics candidates.

(ii) (2 marks)
Well prepared candidates were able to simply write down the correct answer, $2x\cos(x^2+1)$. The next most successful group of candidates wrote $u = x^2+1$, and then used the chain rule. The examiners were disappointed to see the large number of candidates attempting to use the product rule on sin and x^2+1, applying the distributive law to expand the function as $\sin x^2 + \sin 1$, or integrating to obtain $-\frac{1}{2x}\cos(x^2+1)+C$ as their answer.

(b) (2 marks)
This question required some indication of the reasoning in order to gain both marks. Candidates who simply wrote $x = 5.25$ were only awarded one mark. On the other hand, many candidates proved that triangles ABC and DEC are similar. Clearly, a statement of this fact is all that can possibly be required. The first mark was awarded for a correct ratio statement, with the second mark being awarded for deducing x from this statement.

The most common error was for the candidate to use some form of the ratio statement $\frac{x}{1.5} = \frac{5}{2}$, in place of $\frac{x}{1.5} = \frac{7}{2}$. This error was avoided by those candidates who redrew the diagram as two separate triangles.

Too many candidates assumed that $\angle BAC = \frac{\pi}{2}$ and attempted to find x by the use of trigonometry or Pythagoras' theorem. Those who arrived at $x = 5.25$ by these methods gained one mark.

Interestingly, most candidates who scored eleven for this question lost their mark in this part, (usually having an incorrect ratio statement), while the significant section of the candidature who clearly lacked the basic skills in calculus often scored full marks here.

(c) The first part of this question involved the use of the table of standard integrals and the second involved a logarithmic primitive in a definite integral.

(i) (1 mark)
Most of the candidature who answered this part used the table of standard integrals to obtain $\frac{1}{5}\tan 5x(+C)$. The main errors were $5\tan 5x$, $\frac{1}{5}\tan x$ and $\frac{1}{5}\tan^2 5x$.

(ii) (2 marks)
One mark was awarded for a correct primitive. As expected, the most common one was $2\ln(x+3)$, but $\ln(x+3)^2$ and $\ln(x^2+6x+9)$ occurred regularly. The main error involved misuse of the 2 in the numerator, leading to answers such as $\frac{1}{2}\ln(x+3)$ or $\ln(x+3)$. Those candidates who attempted to use the technique of making the coefficient of x equal to the constant in the numerator invariably arrived at $2\ln(2x+3)$, or $\frac{1}{2}\ln(2x+3)$, or $\frac{1}{2}\ln(2x+6)$ rather than the correct value, $2\ln(2x+6)$. Once again, the examiners do not recommend introducing 2 Unit Mathematics candidates to this method.

The second mark was awarded for the evaluation of $F(1)-F(-2)$. The most common mistakes corresponded to evaluation of $F(1)+F(-2)$ and $F(-2)-F(1)$. Too many candidates thought it necessary to compute $[F(1)-F(0)]+[F(0)-F(-2)]$. While this is not incorrect, many candidates then proceeded to introduce absolute value signs with $|F(0)-F(-2)|$ appearing with great regularity.

(d) (3 marks)
The three distinct steps in this question, finding the derivative of $2\log_e x$, evaluating this at $x=1$ while recognising that the result is the gradient of the tangent line, and calculating the equation of the latter line, were each awarded one mark.

Many candidates used the product rule to calculate $\frac{d}{dx}(2\log_e x)$. This is unneccessarily complicated and often introduced errors as many candidates did not appear to know that $\frac{d}{dx}2=0$. Too many candidates did not know the derivative of $\log_e x$, while others read $\log_e x$ as $\ln(e^x)$. It appears that many candidates are not familiar with the $\log_e$ notation, even though it appears on examination papers with about the same frequency as the ln notation.

A disappointingly large number of candidates who arrived at $m=2$ by legitimate means did not proceed to the third step.

Many candidates by-passed the middle step and substituted their derivative function as the gradient in their tangent line, producing answers such as example $xy = 2x - 2$. These candidates showed a lack of understanding of linear functions.

A few candidates claimed, without any supporting work that $m = 2$ and proceeded to the correct equation of the tangent line, $y = 2x - 2$. However, in the absence of calculus these answers received little credit.

Question 4

This question consisted of two parts each worth 6 marks taken from two separate areas of the syllabus. Part (a) required the candidature to explain and to prove geometrical results about a given composite plane figure. An understanding of how to identify given facts was expected. Similarly, an ability to use acceptable notation when referring to angles and sides of triangles was expected. This part was not generally well done, even though most candidates made an attempt. Part(b) was an application of arithmetic sequences and series although some candidates managed to correctly answer this part using only arithmetic calculation. This part was generally well done.

(a) (i) (1 mark)
Most candidates followed the instruction to "copy the diagram into your Writing Booklet" but many failed to make use of the diagram in establishing their proofs. The majority of candidates did recognise the need to use words in response to the instruction "explain".

A number of candidates made the assumption that the two lines TP and AB were perpendicular without establishing this fact first and, as a consequence, did not earn the mark. However, most candidates avoided this pitfall by referring directly to the properties of an equilateral triangle.

(ii) (3 marks)
In this congruence proof, statements and reasons were required. Some candidates left out the reasons, others left out the statements and a few candidates gave correct statements and reasons but then omitted reference to the congruence test or referred to an inappropriate or incorrect test. The most common omission was failing to draw the conclusion that $\angle PAD = \angle PAT$ from work which established that $\angle PAD = 75°$.

Most candidates who attempted to use parallel lines to establish that the alternate angles were equal, failed to establish that the lines were parallel. Similar problems arose when candidates referred to rhombuses, parallelograms or kites without establishing the figures to which they referred were of these types. The vast majority of correct

responses made no reference to parallel lines, rhombuses, kites or parallelograms.

It seemed that some candidates had studied geometric proofs but were unable to communicate their answer for want of correct notation. Angles were not labelled correctly. Others confused the rhyming letters B, D and P in their answers or had handwriting which made it impossible for the examiner to distinguish between the different letters.

(iii) (2 marks)
Many candidates presented correct and concise proofs. However, many of those candidates who had incorrectly established alternate angles in part (ii) continued to pursue this argument in part (iii), and were again without success. Candidates sometimes used circular arguments, deducing that the triangles were isosceles from the 'fact' that they were isosceles. Such arguments did not receive any marks.

(b) (i) (2 marks)
There was confusion between n^{th} term, T_n, and the sum of the first n terms, S_n. There was also a tendency to write the 'fifteenth' term as T_5 or T_{50} rather than T_{15}.

Many candidates chose to simply add up all terms. This gained the marks, but only if the answer was correct, and it was not possible to gain just one mark by this method.

Arithmetic errors made by candidates who used a correct and appropriate formula were viewed more leniently, and nearly every such candidate scored at least one mark in this part.

Some candidates visualised the depot placed somewhere other than at the start of the highway. This did not result in the loss of marks provided the candidate's answer was expressed in a way which made it clear that this was what had been done.

(ii) (2 marks)
Again there was confusion between T_n and S_n. Those who applied the S_n formula obtained a quadratic equation which they could not deal with in any appropriate fashion.

Most candidates correctly found the simple equation $5000 = 200 + (n-1)150$, but it was disappointing that so many of those who reached this point failed to correctly solve the equation.

There was a problem for the candidates who thought of the problem in terms of return trips. They needed to equate their expression for the length of the n^{th} return trip to 10 km, and most failed to recognise

this. Unfortunately, the use of 5 km in this context results in obtaining a value for n which is not an integer.

(iii) (2 marks)
Either formula for S_n could be used in this part. The most common mistake was the failure to double the single trips calculation. A large number of responses simply added 5 km instead, leading examiners to wonder whether these candidates really understood the question.

Question 5

This question involved trigonometric equations, probability and exponential growth. The majority of the candidature attempted the question and many were able to score nearly full marks on at least one of the topics. A large number of candidates demonstrated an inability to use a calculator correctly. Candidates would be well advised to make sure they can distinguish between the ln and log functions, use the fraction facility on their calculators, and that they know when to select RAD or DEG.

(a) (2 marks)
Candidates were asked to solve the trigonometric equation $\tan x = 2$. They had to give their solutions in radians and solve for the domain $0 < x < 2\pi$. Many candidates found this question difficult. Most who scored two marks went on to score close to full marks for the whole question.

Candidates had difficulty coping with radians. Many started with answers in degrees and then spent a long time attempting to convert to radians. Those candidates who worked with their calculator set in radian mode avoided this source of error and difficulty. A large number of candidates did not consider possible solutions outside the first quadrant.

(b) This part of the question asked candidates to copy a probability diagram into their writing booklets, add the probabilities to each branch and to calculate two probabilities.

(i) (3 marks)
Many candidates did not read the instruction which required them to label each branch with a probability. Many candidates thought completing the diagram involved adding more branches to show all outcomes while a number of others extended the diagram beyond the two selections.

A significant number of candidates misinterpreted the question and treated the question as one without replacement, or with replacement by the same colour. Candidates gained one mark for getting the first selection probabilities correct, a second mark for getting at least two of the second selections correct and three marks for getting all probabilities correct.

(ii) (1 mark)
The majority of students who completed the tree diagram knew to multiply the two probabilities along the WW branch to answer this part of the question. However many candidates were not able to multiply the fractions correctly. In this instance, the mark was awarded for writing down the correct numerical expression, and so arithmetic mistakes were not penalised if the appropriate line of working was shown.

Some candidates omitted answering this part of the question and went directly to part (iii). Even though the correct expression for this part was embedded in their answer to part (iii), the mark for this part could not be awarded unless the candidate explicitly indicated the connection with part (ii).

(iii) (1 mark)
Most candidates who had placed probabilities on their diagram in (i) again demonstrated that they knew to multiply the probabilities along the RW branch and to add their answer to that calculated in part (ii). As in part (ii), the many arithmetic errors associated with the fractions were overlooked.

(c) Here candidates were asked to answer several questions about a population of insects growing exponentially according to the formula $N = 200e^{kt}$.

(i) (2 marks)
Candidates had to calculate the value of the constant k. Those who recognised that they had to solve $400 = 200e^{kt}$ gained the first mark. Those who were able to solve the exponential equation $e^{kt} = 2$ gained the second mark. Most candidates scored at least one mark. However, a number were unable to solve the exponential equation or made mistakes resulting from confusing the ln and log buttons on the calculator.

(ii) (2 marks)
Candidates were required to substitute their value of k and $t = 12$ into the formula to obtain the first mark. In general, this was done well. Not all candidates were then able to go on and obtain the second mark for the calculation.

(iii) (1 mark)
Candidates had to use the fact that $\frac{dN}{dt} = kN$ or differentiate the given formula to find $\frac{dN}{dt} = 200ke^{kt}$. In addition they had to realise that the rate for "after three weeks" meant $t = 3$ or $N = 400$.

A number of candidates did not know whether the context of the question required the word rate to refer to k or the value of $\frac{dN}{dt}$. Others incorrectly thought that the rate was '2 times' or 100% because the population had doubled in the first three weeks.

Question 6

The question dealt with trigonometric functions and applications of calculus to the physical world. More than 25% of the candidature scored at least 10 marks.

(a) (3 marks)
The question required a sketch of the function $1 - \sin 2x$ for the domain $0 \leq x \leq \pi$. Examiners were looking for a sketch with labels and units on the axes and a smooth, up-side-down sine curve which oscillates about $y = 1$.

Part marks were awarded for correct period and amplitude, for correct shape (up-side-down sine curve) and for correct oscillation about $y = 1$.

Approximately 40% of the candidates failed to score any marks. It must be noted that those who used a drawing template usually scored well.

In approaching a less familiar trigonometric function such as this one, a candidate should take into account the period and amplitude of the function and the translation from $y = 0$ to $y = 1$. A successful approach would consider in turn the functions $y = \sin 2x$, $y = -\sin 2x$ and finally $y = 1 - \sin 2x$.

A significant number of candidates mistakenly transformed $1 - \sin 2x$ into $\cos 2x$, presumably confusing it with the relationship $1 - \sin^2 x = \cos^2 x$. It was disturbing to note the number of candidates who showed little understanding of the shape of a sine curve, who drew straight lines with little arcs at the turning points, or who failed to pay attention to the alternating concavity of the sine curve.

(b) More than 80% of the candidates scored full marks for parts (i) and (ii), although in some cases, these were the only marks scored on the whole question.

(i) (1 mark)
This required evaluation of N when $t = 0$ and providing an interpretation of this in the context of the question by writing a response such as "initially there were 175 students logged onto the website".

The most common and worrying error was incorrectly evaluating 0^2 and 0^4 as 1, leading to $175 + 18 \cdot (0^2) - (0^4) = 175 + 18 - 1$.

(ii) (1 mark)
This required the calculation of the value of N when $t = 5$ and providing an interpretation of this answer as meaning that "there were no students logged on at the end of the five hours".

Common amongst the errors were evaluating -5^4 as $+(-5)^4$ and totalling the values of N for $t = 0, 1, 2, \ldots, 5$.

(iii) (3 marks)
Candidates were required to determine the maximum value of the function N. This involved finding the three solutions, $t = 0, \pm 3$ of $N' = 36t - 4t^3 = 0$ and testing the solutions which lie within the prescribed domain for maximum or minimum status. Having discovered that the maximum occurs when $t = 3$, a statement that the maximum value is 256 was required.

Part marks were awarded for finding the correct derivative, solving for when this is zero, and deducing the maximum value.

Common errors included failing to recognise $t = 0$ as a possible solution, getting an incorrect derivative ($32t - 4t^3$ and $36t - 3t^3$ occurred regularly), using solutions for the second derivative instead of the first and simply stating the time when the maximum occurred rather than calculating the maximum value.

A disturbing number of candidates failed to see the need to use calculus to determine when a maximum occurred and simply listed a table of values of N for $t = 0, 1, \ldots, 5$ from which they selected the largest value.

(iv) (2 marks)
This required an understanding that the point where the rate of change was greatest corresponds to the point of inflexion on the graph. Candidates needed to solve $N'' = 0$ to get $t = \sqrt{3}$ and interpret this as meaning that the time when "students were logging onto the website most rapidly" was $\sqrt{3}$ hours or 1h 44min after the start. Again, many candidates failed to see the need to use calculus here.

Part marks were awarded for a correct second derivative and for correctly solving for $N'' = 0$. Only the better candidates were able to score full marks here and approximately half failed to score any mark at all.

A significant number failed to explain why $t = -\sqrt{3}$ was not a valid solution.

The most common errors included an incorrect second derivative, interpreting the answer as an inequality such as $0 \leq t \leq \sqrt{3}$ or $\sqrt{3} \leq t \leq 3$ and solving the inequality $N'' \leq 0$.

(v) (2 marks)
A sketch of the function was required for the domain $0 \leq t \leq 5$. Candidates were expected to show units on each axis and a curve that started with a minimum stationary point on the vertical (N) axis, leading to an inflexion at $t = \sqrt{3}$ followed by a concave down section with a maximum at $t = 3$ sweeping down to the horizontal axis at $t = 5$.

Part marks were awarded for starting the curve from the correct point with correct concavity leading to the inflexion and for a maximum turning point followed by no change in concavity. Where points were joined by straight intervals, no marks were awarded.

Many candidates treated this part of the question in isolation from the earlier parts, working through all the calculus again. Many appeared oblivious to any connection between the maximum and inflexion found here and their answers to parts (iii) and (iv). A reasonable number of candidates gained full marks from a very good sketch which they drew based only on a table of values, without any calculus.

Very few candidates had the inflexion point in an appropriate place, between $t = 1$ and $t = 2$. It was not uncommon to see it just before the turning point.

Overall, attention must be drawn to the need for a clear, smooth curve when sketching a curve. Candidates must avoid feather graphs, crossouts and multiple arcs. Examiners strongly recommend the use of a fine point pencil for any curve sketch so that corrections can be made without the need to re-draw the entire sketch.

Question 7

This question covered the topic of integration with questions involving volume of revolution, Simpson's rule and finding the area between curves. Many candidates were well prepared and scored marks of 11 or 12, while a sizeable number scored only 0 or 1. Non attempts were fairly infrequent for this question.

(a) (4 marks)
The majority of candidates obtained the correct answer, 2π. Common errors were predictable, the inability to properly square y, leaving out π, and placing the limits the wrong way around. A substantial number failed to simplify $\ln e^2$.

(b) (3 marks)
Most candidates handled the structure of Simpson's rule quite well. The most common error was the use of degrees rather than radians in the calculation, leading to the answer 0.023 rather than the correct answer, 0.924. Most candidates who scored 11 out of 12 for this question fell into this error.

Other frequent errors involved using 5 function values (rather than the 3 specified), integrating first and so substituting cosine values into Simpson's rule, and using $x = \frac{1}{3}, \frac{2}{3}, 1$. Poor practice in the use of the calculator was also common, especially rounding off early and using values to only one significant figure.

(c) (i) (2 marks)
This part was done particularly well by a variety of approaches, including even using tables of points. Many candidates who scored zero for the rest of Question 7 managed to gain some marks here. The most common error was the following

$$x(x-1) = 2$$
$$\therefore x = 2 \text{ or } x = 3/x = 1$$

Many candidates unnecessarily found the y coordinates of points P and Q.

(ii) (3 marks)
This question could be done straightforwardly as a difference,

$$A = \int_{-1}^{2} x - (x^2 - 2)\, dx$$

leading to the answer of 4.5.

Not unexpectedly, candidates made the question more difficult by splitting the area up into endless permutations, usually without a supporting diagram. Only a small proportion of these candidates managed to obtain the correct final answer.

One successful strategy employed by candidates who were uncomfortable with a region below the x axis was to translate both curves 2 units up before integration. Very few candidates who obtained a negative answer indicated that they understood that this was not possible.

Question 8

The question consisted of 2 parts which both involved applications of calculus. A substantial number of candidates scored zero for this question, but scores of 12 were not uncommon.

Part (a) concerned the relationship between displacement and velocity. Although many understood tbe basic processes, the number af candidates who could not simplify a numerical statement, solve a simple equation or use logarithms was disturbingly high.

(a) (i) (2 marks)
The majority gave the correct answer. A significant number of candidates were unable to differentiate $3\log_e(t+1)$, but were able to earn 1 mark for correctly differentiating $2t$.

(ii) (1 mark)
Most candidates who attempted this part of the question were able to demonstrate an understanding that the initial velocity refers to $t = 0$. The mark was awarded for substitution of $t = 0$ into the expression for velocity from part (i) and subsequent errors, such as replacing the correct answer, $v = -1$, with $v = 1$ were not penalized on this occasion. Similarly students whose expressions for velocity required that they divide by zero when $t = 0$ was substituted were not penalized, even though their final answer was numerically incorrect. A consequence of this approach to marking was that candidates who showed their working scored much better than those who did not.

Some candidates attempted to find the time taken for the particle to reach zero velocity, whilst others substituted $t = 0$ into the expression for displacement.

(iii) (1 mark)
This was generally well done by those candidates whose answer to part (i) was correct. The mark was awarded to any candidate who substituted $v = 0$ into their velocity expression and then solved the equation. This resulted in some candidates solving quadratics or logarithmic equations to obtain the mark. Candidates whose expression for v involved statements like $v = 2t - 3\log_e + e\log\ldots$ were unable to earn this mark.

(iv) (3 marks)
The vast majority of the candidature gave the displacement of the particle rather than the total distance travelled. That is, they simply substituted $t = 3$ into the expression for x. This earned only the first mark for this part. A second mark was awarded to candidates who recognized that the particle came to rest at $t = 0.5$ secs, and then found its displacement at that time. The third mark was awarded to those who correctly combined this information by realising that

$$\begin{aligned}\text{Distance travelled} = 2&|\text{displacement when } t = 0.5| \\ &+ \text{displacement when } t = 3\end{aligned}$$

Those who drew a diagram faired better than most. A large number of candidates incorrectly used the base 10 logarithm key in their calculation. Candidates who did this were still able to earn the first two marks provided they showed their substitution line, but were unable to earn the third mark. Other common errors were to add the displacement after $t = 0.5$ to the displacement after $t = 3$, or to find the displacement after $t = 0.5$ and multiply it by 6, or add the displacement after $t = 0.5$ to the displacement after $t = 2.5$ secs, or to simply add the displacements at $t = 0$, $t = 1$,

$t = 2$ and $t = 3$. A number of candidates incorrectly interpreted the question to mean 'find the displacement in the third second' ie. $x|_{t=3} - x|_{t=2}$. Candidates who attempted to find the exact answer, $4 + 6\ln(1.5) - 3\ln 4$ were generally less successful than those who worked with numerical approximations found using a calculator.

Another correct approach involved computing $\left|\int_0^{\frac{1}{2}} v\,dt\right| + \int_{\frac{1}{2}}^3 v\,dt$.

Some candidates computed the area under the velocity curve, often using approximation methods such as the trapezoidal rule, with some degree of success.

(b) This part was a maximisation question. It was pleasing to see that many candidates took advantage of the two entry points into this part. Candidates who could not show the shaded area was $A = 117x - \frac{3}{2}x^2$ in part (i) were still able to gain full marks in part (ii) by using the given result. However, a significant number of candidates did not attempt this part at all.

(i) (2 marks)
Responses that included a diagram were more easily undertood than those without. The expression for area could be found in many ways

$$\text{large rectangle} - \text{triangle} = xy - \frac{1}{2}x^2$$

$$\text{small rectangle} + \text{triangle} = x(y - x) + \frac{1}{2}x^2$$

$$\text{trapezium} = \frac{1}{2}x(y + (y - x))$$

Those who were unable to earn this mark usually incorrectly assumed that the small rectangle was a square. Those who recognized that the triangle was right angled and isosceles usually did well in this section. Most candidates who were able to find an expression for area in terms of x and y were then able to deduce the expression in terms of x only by substituting $y = 117 - x$. Those candidates whose expression in terms of x and y was incorrect often went to great lengths to disguise the mistakes in order to arrive at the answer given in the question.

(ii) (3 marks)
A surprisingly large number of candidates deduced that $x = 39$ maximised the area (by replacing y by $2x$ in the constraint $x + y = 117$). By itself, this did not score any marks. To gain the 3 marks for this part a candidate had to solve $A' = 0$, demonstrate that a maximum occurred at this stationary point and show that this point corresponded to $y = 2x$.

Most candidates found $x = 39$ from $A' = 0$,and then showed that $A'' = -3$. They deduced that a maximum occurs when $x = 39$ and then demonstrated that $y = 117 - 39 = 78 = 2x$.

A particularly good answer, which was not uncommon, went as follows. A stationary point occurs when $A' = 117 - 3x = 0$ so $117 = 3x$. But $117 = x + y$, so $y = 2x$. Moreover, $A'' = -3$, so this stationary point is a maximum.

Fortunately for many candidates, the curve $A = 117x - \frac{3}{2}x^2$ is always concave down. Many candidates used test points that were a long way from $x = 39$, providing working such as

x	30	39	40
A'	+	0	−

to justify their conclusion that the stationary point was a maximum.

Question 9

The question consisted of three parts. Part (a) required the sketch of $y = \log_e x$ and the addition of a straight line to their graph to determine the number of solutions to the equation $\log_e x - x = -2$. Part (b) required a sketch of the primitive of a function whose graph was provided. Part (c) required knowledge of the cosine rule, trigonometric identities, solution of an equation reducible to a quadratic and a correct deduction to eliminate 3 of 4 solutions of the quartic equation.

A small number of candidates managed to obtain full marks, with roughly the same number scoring no marks. It was pleasing to see that many of the less able candidates were able to score some marks by correctly applying the cosine rule in part (c) (i).

Many candidates made simple arithmetic mistakes and so gained answers which were different to those given in parts (c) (i), (ii) and (iii). Candidates should check their calculations when they differ from the given answers, and make the necessary corrections.

Candidates would also be advised to use a ruler to draw straight lines such as the axes.

(a) (i) (1 mark)
Successful candidates knew the shape of the curve and provided a smooth curve for $x > 0$. Those who sketched by plotting coordinates gained from their calculator, usually only considered integer values of x, $x \geq 0$ and were not awarded the mark. Other common errors involved sketches which crossed the y-axis or which were in fact the sketch of $y = e^x$.

(ii) (2 marks)
This part was not very well done and few candidates gained 2 marks.

Most candidates did not appreciate the fact that they could use their existing sketch, with the addition of the line $y = x - 2$. Instead many ignored the $-x$ and graphed the line $y = -2$.

Others attempted to graph $y = \log_e x - x$ and $y = -2$ with varying degrees of success. Common errors with this method involved considering only positive integer values of x or drawing conclusions based on the intersection of $y = \log_e x$ and $y = \log_e x - x$. A small group successfully sketched $y = \log_e x - x + 2$ and counted intersections with the x axis or drew $y = \log_e x + 2$ and counted intersections with $y = x$. Other candidates had several straight lines on their graph and it was difficult to determine the reasoning for the given answer.

Candidates need to read questions carefully. "Find, graphically" means that interpretation from their graphs is necessary and that an algebraic approach is not appropriate.

(b) (2 marks)
Many candidates were able to gain at least one mark and understood what the question required. There were 4 main aspects of the function to be considered. The curve passes through the origin with a positive slope, there is a horizontal point of inflexion at $x = 1$, there is another point of inflexion at $x = 2$ and the curve has a maximum turning point at $x = 3$.

Common mistakes included not making the point of inflexion at $x = 1$ horizontal (often in spite of candidates explicitly writing that there is a horizontal point of inflexion at $x = 1$) or having the same shape at $x = 1$ and $x = 2$. A substantial number of candidates sketched the derivative, $y = f'(x)$.

(c) (i) (2 marks)
Candidates were more likely to gain marks in this part than any other parts of Question 9. However, candidates should note that "show questions" provide the answer, and so it is necessary to show intermediate steps such as

$$\cos\alpha = \frac{x^2 + 2^2 - 6^2}{2 \cdot x \cdot 2} = \frac{x^2 - 32}{4x}$$

in order to gain the marks.

Candidates were less successful when they started with the equation $6^2 = x^2 + 2^2 - 2 \cdot x \cdot 2\cos\alpha$ as it provided extra opportunity for mistakes in making $\cos\alpha$ the subject.

(ii) (2 marks)
About one third of the candidates who were successful in part (i) gained marks here, with marks of 1 or 2 being equally common.

Candidates who gained 1 mark usually noted the similar form of the answer to part (i) and started by writing $\sin\alpha = \ldots$. However, examiners were not always confident that the candidates understood why it was appropriate to use $\sin\alpha$ in the cosine rule here, and the other mark, which was awarded for indicating the equality of $\sin\alpha$ and $\cos(90^\circ - \alpha)$ was not awarded unless convincing evidence was provided that the candidate had made this connection.

(iii) (1 mark)
Few candidates were able to recognise the link between sine and cosine. Candidates who used $\cos^2\alpha + \sin^2\alpha = 1$ and showed the substitution

$$\left(\frac{x^2-32}{4x}\right)^2 + \left(\frac{x^2-16}{4x}\right)^2 = 1$$

were able to gain the mark. Another successful approach applied Pythagoras' theorem in a right triangle with sides x^2-16 and x^2-32 and hypotenuse $4x$.

(iv) (2 marks)
Many candidates gained one mark for solving the quartic equation. However, few candidates were able to provide a satisfactory reason to gain the second mark. Many were able to eliminate the negative solutions, but the elimination of $x = 4$ was much rarer. Those gaining the mark usually showed that the substitution of $x = 4$ into either the $\sin\alpha$ or $\cos\alpha$ formulae, produces an angle of either 0 or 180 degrees. Others showed that if $x = 4$ the diagonal of the square is $\sqrt{32}$ which is less than 6.

Many candidates incorrectly reasoned that the length of BC must be less than the length PC in triangle PBC. A few candidates realised that if $x = 4$ then the sides of the 'triangle' would be 2, 4 and 6, which violates the triangle inequality.

Question 10

This question consisted of two parts. The first was concerned with a loan repayment, whilst the second tested an application of rates of change.

The question was not done well. There were a large number of zeros and non attempts. Candidates gained most marks in part (a) and very few received more than 2 marks out of 6 for part (b).

(a) Candidates had to answer 4 parts, culminating with a calculation of the value of M, the regular monthly repayment on a loan of \$5000 on which interest was payable at a rate of 1% per month over the last 33 of 36 months.

(i) (1 mark)
While most candidates were able to find $A_3 = 5000 - 3M$, the amount owing at the end of 3 months, many did not understand that there was an interest-free period during the first 3 months and so tried to write something with powers of 1.01.

(ii) (2 marks)
Candidates were asked to show that $A_5 = (5000 - 3M)1.01^2 - M(1 + 1.01)$. This was usually well done, but some candidates fudged and lost marks. Candidates who had obviously learned how to develop these types of expressions did best, but an alarmingly high number of them made fundamental mistakes with grouping symbols.

(iii) & (iv) (3 marks)
Candidates were asked to find an expression for A_{36} and evaluate M. Again candidates who had practised these kinds of questions did best. Many had difficulty with the pairs of indices in forming the expression $A_{36} = (5000 - 3M)1.01^{33} - M(1 + 1.01 + \ldots 1.01^{32})$ with many candidates obtaining indices of 33 and 33 or 36 and 35 in place of 33 and 32. Many either left out the $3M$ or wrote $34M$ instead.

Numerous errors occurred in the attempts to sum the series, with many candidates obtaining an index 1 less than required. The final calculation proved difficult because candidates made mistakes in making M the subject of their equation. Better candidates managed well and correctly gave \$161.34.

(b) (i) (2 marks)
Candidates were puzzled by the wording and did not know how to start. Many just restated the question or reworded it. Others wrote an aimless collection of true or false equations. Although many candidates obtained 1 mark, very few were awarded 2 marks. Those who were awarded 2 marks generally had a very clear idea of direct proportion and of the role of constants.

(ii) (4 marks)
Only a handful of candidates correctly gained all 4 marks. Most candidates who managed a start were unable to supply appropriate limits of integration. They typically used $t = 0$ and 2 (or 6 and 8) rather than T and $T + 2$ in their substitution into the expression $x = k \ln t + c$ for the distance travelled.

Correct solutions usually resulted from establishing $k \ln \left(\frac{T+2}{T}\right) = 1$ and $k \ln \left(\frac{T+5.5}{T+2}\right) = 1$ or an equivalent pair. Eliminating k allowed candidates to form a quadratic equation from which T could be found. This then allowed candidates to deduce that the snow started to fall at 3:20 am.

Mathematics 3 Unit (Additional) and 3/4 Unit (Common)

Question 1

While this question was generally well done, with over 25% of candidates earning 12 marks, candidates need to be reminded to show all working, to take care with even the simplest working, to learn formulae correctly and not to make the solution unnecessarily difficult. Familiarity with the standard integral tables would have assisted many candidates in this question.

(a) (2 marks)
Candidates were required to differentiate $x \sin^{-1} x$. Most candidates recognised the product rule and earned one mark for applying it correctly, but many showed little understanding of the derivative of $\sin^{-1} x$. Candidates need to be familiar with the use of the standard integral tables. A common incorrect answer was $\frac{d}{dx} \sin^{-1} x = \cos^{-1} x$. Many candidates interpreted $\sin^{-1} x$ as $\frac{1}{\sin x}$ and had little success in differentiating $\frac{x}{\sin x}$.

(b) (2 marks)
Candidates were required to find the acute angle between two lines in the form $y = mx + b$. It is recommended that candidates learn the formula correctly. Many candidates stated the formula $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$ correctly but then made arithmetic mistakes. Many candidates could not evaluate $2 - \frac{1}{3}$ correctly and others misinterpreted their own notation as $2 \cdot \frac{1}{3} = 2\frac{1}{3}$. Candidates who did not show their substitution were disadvantaged as these arithmetic errors made it difficult to award them marks. Most candidates could gain the second mark by correctly finding the value of θ. Some candidates increased the level of difficulty of the question by differentiating the linear functions to find their gradients or by finding the common point on the two lines and then using the distance formula and cosine rule to find the angle between the lines.

(c) (2 marks)
Candidates were required to find a coefficient of a polynomial given a factor of the polynomial. Candidates who attempted this part using the factor theorem usually gained 2 marks easily and quickly. However, one common error was to evaluate 3^3 as 9. Candidates who attempted long division were usually unsuccessful, even after a number of attempts and an investment of a considerable amount of time. A minority of the candidature attempted the question using the relationship between the roots and the coefficients. Again few of these candidates were successful.

(d) (3 marks)
Candidates were required to evaluate a definite integral resulting in an inverse trigonometric function. A significant number of candidates were unable to use the table of standard integrals. The constant, 4, proved to

be a difficulty for a number of candidates, resulting in variations on the product $a\tan^{-1}\frac{x}{3}$, with a being $\frac{1}{12}$, $\frac{3}{4}$, $\frac{1}{3}$ etc. As the final mark was for the evaluation of a definite integral of the form $a\tan^{-1}\frac{\sqrt{3}}{3}$ in radians, candidates with this error were still able to earn 2 of the 3 marks.

Candidates should also take care with their notation as those writing the integral as $\frac{4}{3}\tan\frac{\sqrt{3}}{3}$ were often unable to earn any marks.

(e) (3 marks)
Candidates were required to solve an inequality with the unknown in the denominator. A number of methods were used with varying success. To gain all three marks candidates had to state that $x \neq -2$ and solve the inequality correctly. Candidates were generally awarded one mark for an attempt at a correct method. It is recommended that candidates identify values which are excluded because they would result in division by zero as the first step of any solution. Many candidates incorrectly stated that $x \neq 2$. Lack of care with inequality signs, errors in both arithmetic and algebra, and incomplete solutions hampered many candidates' work.

The three most common methods were: multiplying by the square of the denominator; dividing into two cases; and the method of testing regions. Candidates who attempted the first method were usually successful, although many did not identify that $x \neq -2$. Some candidates had difficulties solving quadratic inequalities, particularly if their quadratic was concave down. Again care needed to be taken with notation as their solutions often showed confusion between 2 and -2 as zeros of their quadratic.

Consideration of the two cases was generally not well handled, with incomplete solutions common among candidates who attempted this approach. The method of testing regions showed little middle ground. Candidates either applied it very well or very badly. However, it seems that many candidates view this method merely as a recipe, and have little understanding of what they are doing.

A few candidates attempted a graphical solution and usually were very successful, gaining three marks quickly and easily. Another method which was often very successful was the following:

$$\begin{aligned} \frac{5}{x+2} - 1 &\leq 0 \\ \frac{3-x}{x+2} &\leq 0 \\ (3-x)(x+2) &\leq 0 \end{aligned}$$

(with some candidates supplying the justification $\frac{a}{b} \leq 0$ when $ab \leq 0$ for the last line).

Common incorrect solutions included treating the inequality as an absolute value inequality, or simply solving $5 \leq x+2$. Overall, candidates display a

poor understanding of "and" and "or", with solutions often being written as $-2 > x \geq 3$.

Question 2

On the whole, this question was quite well done. Most candidates demonstrated good knowledge of the concepts covered by this question, but were sometimes let down by arithmetic or algebraic errors in simple tasks such as solving quadratics. Many candidates scored 12 marks and very few earned no marks, although there were a surprising number of candidates who only scored 2 or 3 marks.

(a) (2 marks)
This was generally well done with most candidates managing to score at least 1 mark. Demonstrating awareness of the fact that there were 9! arrangements before accounting for repetitions, or that each arrangement was repeated 3! times enabled the candidates to score 1 mark. The most common errors were neglecting to divide by 3!; some divided by 3; writing 7 different letters and 3 the same, therefore $\frac{7!}{3!}$; ${}^9C_3$ or even ${}^9P_3$.

(b) (3 marks)
This was also generally well done but there was much evidence to suggest that candidates have a lot of trouble with indices and do not really understand the meaning of coefficient. Many candidates chose the 6^{th} term ($k = 6$) or the term $(x^2)^4$ as the required term and in some cases it was hard to see any logic in their choice of term. However, if they had everything else correct for their chosen term they were able to score 2 marks. Some responses only gave the coefficient as ${}^7C_3$, or 2 raised to their power, or the product of 2 and 5 raised to the appropriate powers. Most of these alternatives were able to gain a mark. Many candidates neglected to raise 2 to their power (giving answers such as ${}^7C_3 5^4 2$) and so were not able to score full marks. A significant number of candidates wrote out the correct binomial expansion but, sadly, neglected to indicate which term they wanted to consider. This was only able to score 1 mark. Candidates who tried to expand by multiplying were generally not able to earn any marks as few proceeded far enough to clearly indicate which term in the expansion was relevant to the question.

(c) (4 marks)
Most candidates were able to score at least 2 marks, with a surprising number losing a mark because they were unable to solve their quadratic. There were also many examples of algebraic errors involving trigonometric functions such as:

$$\cos 2\theta = \sin \theta$$
$$2\cos \theta = \sin \theta$$
$$\tan \theta = 2\,.$$

Most candidates were able to replace $\cos 2\theta$ correctly with $1 - 2\sin^2\theta$. Of those who did solve the quadratic correctly, some discarded their solution $\sin\theta = -1$ or incorrectly deduced that $\theta = \frac{3\pi}{4}$ from this case.

A significant number gave solutions in degrees which meant that they could score at most 3 marks.

Candidates who used the t-method found it very difficult and generally did not score more than 2 marks.

Those who tried to solve graphically were usually successful provided they could read their scale. Most identified 3 solutions but many then incorrectly guessed the first two solutions to be $\frac{\pi}{4}$ and $\frac{3\pi}{4}$.

Those who used trial and error usually did not obtain all 3 solutions but were usually able to score some marks for correct solutions identified.

Those who used the complementary angle method

$$\cos 2\theta = \sin\theta$$
$$\therefore \sin(\frac{\pi}{2} - 2\theta) = \sin\theta$$

found only one solution, but were awarded 2 marks.

(d) (3 marks)
Many candidates were unable to substitute successfully. The x in the numerator caused many problems and there was often no attempt to substitute for dx. Once again, errors with indices were common in the course of simplifying the expression before integration. Many made mistakes integrating fractional or negative indices and it was not clear if these were the result of careless errors or lack of skill. A significant number of candidates did not rewrite their correct integral in terms of x and only scored 2 marks.

Question 3

This question involved differentiation from first principles, inverse trigonometric functions and 3-dimensional trigonometry. The question was generally well done, with most candidates scoring marks in the range from 9 to 12.

(a) (2 marks)
Although candidates recognised the given formula as relating to differentiation from first principles, very few had the algebraic skills to complete the substitution and simplification. Confusion reigned over the expansion of $(x + h)^3$ which was frequently mistaken for $x^3 + h^3$ or $x^3 + h$. Many candidates did not know how to handle the function notation. Failed expansion attempts often involved the use of $(x+h)(x^2-xh+h^2)$; coefficients of 2 instead of 3; the omission of h^3 and the confusion of x^2h with xh^2.

The majority of the candidature were unable to score full marks on this part. Even though most knew that the answer should be $3a^2$ or $3x^2$, very few were able to derive it from the formula.

In attempting to find the limit many candidates divided by powers of h, hoping for relevant cancellations from their incorrect method. It was also difficult to discern whether the h^2 term, after factorisation, was being dropped from the expression because candidates knew that powers of h were infinitesimally small or because their algebra was careless. Justification for such a step should always be stated in the proof.

The quality of responses to this part were unusually dependent on the candidate's centre, with many centres completing this part well while other centres had almost no attempts for this part.

(b) Candidates were required to state the range and draw a sketch of $3\tan^{-1} x$. They also were asked to evaluate the gradient of the curve at a specified point. Most candidates knew the basic shape of an inverse tangent curve but had great difficulty in determining how the multiplication by 3 affected the range and sketch.

(i) (1 mark)
Many candidates were unable to express their range as a single set of inequalities and confusion between domain and range was evident. Many divided by 3 to get endpoints for the range as $\pm\frac{\pi}{6}$, while others left the limits at $\pm\frac{\pi}{2}$ without any attempt to accommodate the coefficient of 3.

(ii) (2 marks)
Marks were awarded for correct shape at the origin and correct behaviour of the curve at either the correct asymptotes, or the asymptotes implied from part (i). Labelling of the asymptotes was commonly omitted, which made the award of full marks difficult in the absence of a clear indication of the range of the curve. Lack of care also meant that many attempts did not indicate an understanding of asymptotic behaviour with curves regularly touching, crossing or never in the vicinity of the relevant lines.

(iii) (2 marks)
Candidates were mostly competent in finding the correct derivative to get their first mark. Many subsequently stumbled in their substitution of $\frac{1}{\sqrt{3}}$ either by using $\sqrt{3}$ or getting $\frac{1}{9}$ after squaring. Clear deficiencies were shown in the many poor attempts to evaluate the expression.

Many candidates assumed that they were required to find the equation of the tangent and wasted a considerable amount of time in the process.

(c) This part asked candidates to apply trigonometry to a 3-dimensional diagram. It was generally well done.

(i) (1 mark)
Either the use of trigonometry to get $h\sqrt{3}$ (or an equivalent trigonometric expression) or an application of Pythagoras to get $\sqrt{h^2 + 100^2}$ were sufficient to obtain this relatively easy mark. A number of candidates could not successfully deal with a fraction in the denominator when trying to make OB the subject of their expression.

(ii) (2 marks)
Candidates must not leave out steps when asked to "show" a result. There were many attempts to convince examiners that the correct answer followed from incorrect expressions, and others where too many steps had been omitted and examiners were not convinced that the correct approach had been taken.

Equating the expression from trigonometry to the one derived from Pythagoras was the usual way of obtaining the result. Although applications of ratios of sides in similar triangles could also produce the desired outcome, it regularly led to cumbersome working and error.

The most common error was to incorrectly state the Pythagorean relationship in triangle AOB, mistaking AB for the hypotenuse.

(iii) (2 marks)
The most common successful method of finding the bearing was to use trigonometry to evaluate angle AOB and subtract it from 180°. Another method involved finding angle OBA and adding 90°, but candidates who used this method regularly subtracted from 180° to produce the wrong bearing.

Examiners were surprised at the inability of candidates to state a bearing correctly, even when they had found the correct relevant angle.

The number of attempts which incorporated the cosine rule or sine rule instead of right-angled trigonometry was of concern. These methods, while correct, open further possibilities for error and are more time consuming.

Question 4

This question was reasonably well done, with many candidates scoring 10 or more marks. Overall, parts (b) and (c) were better done than parts (a) and (d).

(a) (3 marks)
This question asked candidates to use mathematical induction to prove a result concerning the sum of a series. The question was well done by those who understood the concept of induction. However, it was clear that many

candidates did not understand this concept and had attempted to learn the steps by rote.

One mark was awarded for proving the result for $n = 1$ and stating the induction hypothesis that the result holds for $n = k$. Examiners were concerned by the number of candidates who revealed their lack of understanding by writing "Assume $n = k$, ie $\frac{1}{2}(k+1)(k+2) = \frac{1}{6}k(k+1)(k+2)$". Many of these candidates apparently believe that they are attempting to deduce that $n = k+1$ from the hypothesis that $n = k$, and this confusion was reflected in their later work.

A further mark was awarded for correct use of the inductive hypothesis in an attempt to prove the result true for $n = k+1$. Most candidates who gained this mark also went on to gain the third mark, which was given for completing the proof that the result would be true for $n = k+1$ if it were true for $n = k$.

(b) (3 marks)
This question required one application of Newton's method to find an approximation to a root of an equation. There was a disappointingly large number of simple algebraic or arithmetic mistakes. Candidates lost marks through an inability to differentiate $(1+r)((1+r)^{24}-1) - 50r$ or by erroneously simplifying this expression to $(1+r)^{25} - (1-r) - 50r$. Full marks were awarded for the result

$$r_2 = 0.06 - \frac{1.06 \times (1.06^{24} - 1) - 50 \times 0.06}{25 \times 1.06^{24} - 51},$$

or its numerical equivalent. Candidates who did not get this result scored at most 2 marks. These could be gained for any of the following, up to a maximum of 2 marks:

(i) Correctly identifying the function $f(r) = (1+r)((1+r)^{24}-1) - 50r$ whose approximate zero was to be found.

(ii) Correctly stating Newton's rule $r_2 = 0.06 - f(r_1)/f'(r_1)$, having previously correctly identified f.

(iii) Correctly differentiating the function f.

Common mistakes included not identifying the function f correctly, incorrect differentiation, incorrect substitution, confusing 0.06 and 1.06 and not removing grouping symbols correctly.

A handful of candidates used first principles by finding the intercept of the tangent to the curve $y = f(r)$ with the r-axis. This approach was usually successful. Candidates who attempted to find the result by trial and error calculations received at most one mark (for identifying f).

(c) (4 marks)
Candidates approached this question in a variety of ways. The most common was to use the standard formulae for the sum of roots, sum of the products of the roots taken two at a time and the product of roots. The most common error with this method involved the omission of negative signs, with subsequent fudging to get the correct result.

The second most common approach involved the expansion of the expression $(x-\sqrt{k})(x+\sqrt{k})(x-\alpha)$ and equating it to x^3+px^2+qx+r. This turned out to be the simplest way of doing the problem and most candidates who did not make algebraic errors scored full marks.

Part (c) was marked as a whole. Candidates who used the first approach earned one mark each for the routine expressions obtained for part (i) and part (ii), with part (iii) accounting for the remaining 2 marks. Candidates using the second approach earned one mark for expanding the expression $(x-\sqrt{k})(x+\sqrt{k})(x-\alpha)$ and the additional marks for deducing each of the three required results.

(d) (2 marks)
Candidates adopted a variety of methods of solution. Generally, candidates who used the formula $v^2 = n^2(a^2-x^2)$ did better than those who began with $x = 3\cos(t/2)$, $x = 3\sin(t/2)$ or $x'' = -x/4$. One mark was awarded for correctly identifying numerical values for n and the amplitude a in a correct formula. A further mark was awarded for finding $v = 3/2$.

Common errors included not finding n correctly, incorrectly memorised formulas, and incorrect integration when using $x'' = -x/4$. A fair number of candidates made unfortunate changes of variables, obtaining answers such as $y = \sin(x/2)$. Worse still, many wrote $x = \sin(x/2)$.

Question 5

This question had two parts. Part (a), worth four marks, was a proof involving circle geometry. Part (b), worth eight marks, involved a question on a function and its inverse.

(a) (i) (2 marks)
This was a fairly straightforward proof which involved two steps of reasoning, each worth one mark. Most candidates were awarded the two marks, although those who did not state 'alternate segment theorem' often had difficulties with their reasoning for $\angle PBA = \angle PCB$. Candidates should take care to ensure that they correctly name the angles to which they are referring.

(ii) (2 marks)
Many candidates did not attempt this part of the question. Those

who could not easily see that the proof involved the ratio of corresponding sides in similar triangles PAB and PCB found it difficult to gain any marks.

Candidates who attempted to use the sine rule in the correct triangles needed correct statements involving the sides and angles in triangles PAB and PCB and also had to realise that $\angle PAB = \angle PBC$ before they could gain any marks. This method often wasted a lot of time without reaching the correct conclusion.

A few candidates reasoned that PB was a tangent to the circle through B, A and C and hence $PB^2 = PA \times PC$. This method was only awarded marks when the candidate provided correct justification for their reasoning.

(b) (i) (1 mark)
This was generally well done. Most candidates gained the mark for this part, finding the correct derivative in simplified form. Candidates who did not use the quotient rule often had difficulty expressing the derivative in a form that was clearly positive.

(ii) (1 mark)
About half the candidates did not indicate that the horizontal asymptote was $y = 1$. For some, this was because they were unsure as to the meaning of the word 'horizontal', giving $x = -2$ in their answer. Common mistakes made by other candidates were $y = 0$ or giving their answer simply as 1.

(iii) (2 marks)
Candidates needed to sketch a rectangular hyperbola with a vertical asymptote indicated to obtain any marks. Most candidates correctly indicated $x = -2$ as the vertical asymptote but some then omitted one branch of the hyperbola or failed to indicate the horizontal asymptote. Those candidates whose attempt relied on plotting points often failed to indicate the appropriate features and as a result could not obtain either mark.

(iv) (1 mark)
Many candidates stated a correct reason for the existence of an inverse function. However, a significant number then proceeded to elaborate on their answer using incorrect reasoning. Candidates are encouraged to make sure that if multiple reasons are stated they are all correct and not in conflict.

(v) (2 marks)
Most candidates realised that an interchange of x and y is required in order to find the equation of the inverse function. Candidates are advised to do this before attempting to change the subject, as a

mark is often awarded for this step. Candidates who used function notation often had problems correctly interchanging the variables and therefore had difficulty gaining any marks. Examiners were generally impressed with the algebraic skills shown in this part.

(vi) (1 mark)
Candidates were awarded this mark for the correct domain for the inverse function they supplied in their answer to part (v).

Candidates whose answer to part (v) was not a function (or who had not answered part (v)) were awarded the mark if they deduced the correct answer from their solutions to parts (ii) and (iii). Candidates need to ensure that their solutions to different parts of inter-related questions are consistent with each other.

Question 6

The question comprised two unrelated parts. Part (a) involved finding the route which would result in the fastest journey across a circular lake. Part (b) was a counting problem involving unordered selection from a standard pack of playing cards. While most candidates were able to obtain some marks in the question, high marks were not common. Part (b) was not well done. However, examiners were left with the impression that this was the result of candidates deciding to allocate the available time to less demanding parts of the paper.

(a) (i) (3 marks)
The distance AP could be found in many ways: using elementary circle geometry and simple right-angle trigonometry, via the sine rule, or by applying the cosine rule. The sine and cosine rule methods were longer, more difficult and required facility with trigonometric identities. Many who chose this approach got lost in the algebraic manipulation, being unable to deal successfully with identities such as $\cos(\pi - 2\theta) = -\cos 2\theta$, $\cos 2\theta = 2\cos^2\theta - 1$. Others treated $\sin(\pi - 2\theta)$ as if it were $\sin(\frac{\pi}{2} - \theta)$. Some candidates gave up; others tried to disguise their errors to make it appear as if they had found the required expression.

Determining the arc length PB was generally done better, as was the determination of the total time of the journey, although some believed that time = distance × speed. Candidates should be aware that questions which state the answer require some explanation of the steps involved, even if it seems obvious. It is especially important to note that these explanations must be written in the writing booklet as examiners cannot see any rough working which may appear on the diagram on the examination paper.

(ii) (2 marks)
The differentiation was generally well done, although the 3θ term was often treated as a constant. Despite the wording of the question, many answers were given in degrees. Given the context of the question, examiners decided that this was acceptable in this instance. Obtuse angle solutions showed a lack of appreciation of the context, which clearly constrains θ to the range $0 \leq \theta \leq \pi/2$.

(iii) (3 marks)
Candidates were asked to find the point that minimised the time taken. In this question, the stationary point provided a local maximum and not a minimum. Both endpoints of the domain had to be investigated to obtain the correct conclusion in order for candidates to be awarded full marks.

Many candidates were surprised by the sign of the second derivative. Some gave up at this point, as though suffering a sense of betrayal. Those who merely guessed that the stationary point was a minimum and contrived their working to support this conclusion were not awarded marks. Some concluded that there was no minimum and proceeded no further. Others found a minimum corresponding to an obtuse value of θ, but this lies outside the range of values appropriate to this context.

Quite a few seemed to believe that P had to be between the points A and B and could not actually be one of them.

In fact, it is not necessary to compute the second derivative at all. Computation of the value of T at the endpoints and at the stationary point is all that is required to establish which of these three points corresponds to the absolute minimum. It appears that the distinction between absolute minimum and local minimum is not widely appreciated.

(b) In both parts of this question there was much evidence of uncertainty. A surprisingly large section of the candidature treated this as a probability question. Many of the responses included pages of fractions and even tree diagrams. There was also much confusion between permutations and combinations. Multiple crossed out answers were common.

(i) (2 marks)
Full marks were awarded for the expression ${}^{13}C_2 \times {}^{13}C_4$. Some candidates added rather than multiplied, or were not sure whether to do either. Other factors such as ${}^{52}C_4$ or ${}^{6}C_4$ found their way into some answers.

(ii) (2 marks)
Only a minority of the candidature successfully treated "at least 5"

by adding the two separate cases of "exactly 5" and "exactly 6" to obtain the correct response $4 \times {}^{13}C_5 \times 39 + 4 \times {}^{13}C_6$. Nevertheless, there were quite a number of candidates who were able to obtain a mark for a partial solution, such as by correctly obtaining the number of possibilities for one of these cases.

A popular wrong answer, which was awarded 1 mark, was $4 \times {}^{13}C_5 \times 47$. This method which chooses 5 cards from 1 suit and then any 1 from the remaining 47 cards shows a fair degree of understanding, but distinguishes the sixth card when it comes from the same suit and so is not completely unordered.

It was common to forget the factor of 4 (for the four different suits). As in part (i), stray factors and terms indicating choices from the whole 52 cards were fairly common.

Question 7

Very few candidates managed to finish this question, with parts (a) (ii) and (b) (iv) usually being left unfinished or not attempted at all.

(a) Even though *per hour* was emphasised by the use of italics, most candidates did not realise that F is proportional to $\frac{1}{t}$.

(i) (2 marks)
Many candidates earned their only mark for question 7 by correctly differentiating F and deducing that $\left(\frac{B}{3A}\right)^{1/4}$ was the speed at which a stationary point occurred. Most candidates failed to determine the nature of this stationary point. Many correctly differentiated F, noted that A, B and u were positive but then claimed $F'' < 0$ and concluded that they had found the maximum. If this was the case, they would have established that this corresponded to the maximum rate of fuel consumption per hour, which would result the minimum flight time.

A significant number of candidates had difficulty differentiating F twice. It should be noted that a '+' corrected to a '−' by overwriting the horizontal line often leaves the candidate's intention unclear to the examiner, and so may result in the examiner being unable to award a mark. It should also be emphasized that it is virtually impossible to correctly determine the nature of a stationary point of this form by looking at the gradient to the left and right of the point. Since A and B are unknown, this approach is quite complicated, and merely choosing particular values for A and B gives no general information.

Candidates who correctly deduced that $F'' > 0$ implied that this stationary point for F was a minimum were usually not confused

by this. Almost all of these candidates realised that maximum flight time must occur at the speed which provides for minimum fuel consumption per hour. Indeed, many candidates stated this fact as the introduction to their answer.

(ii) (2 marks)
Only a very small minority of the candidature even attempted this part, and most of those merely evaluated F or F'' at the stationary point found in part (i) hoping that 1.32 would somehow appear. Other candidates evaluated F at $1.32 \times \left(\frac{B}{3A}\right)^{1/4}$.

Amongst the few who deduced that the distance flown was proportional to u/F, a large proportion could not then differentiate their expression. Of those who succeeded in finding the stationary point for this expression, only a small number were able to determine its nature.

(b) Many candidates had learned how to derive the formulae related to projectile motion and proceeded to do so before starting the question. This was simply a waste of time, as the question stated that candidates were to assume the given equations for the motion.

(i) (1 mark)
Most candidates attempting Question 7 tried this part, with many gaining a mark. An inordinate number of candidates had difficulty with the algebra in this part. A small but significant number of candidates began with the equation $y = x\tan\theta - x^2\sec^2\theta$ and attempted to work back towards the parametric equation.

(ii) (3 marks)
For a large number of candidates, this was the last part of Question 7 attempted. Some candidates began this part several times, often without any change of approach between successive attempts. Given the time that candidates had obviously devoted to this part, it was fortunate that these long repeated attempts usually earned some marks.

The most common mistake was to assume that the range is found by setting y equal to 0. Correct solutions proceeded by either solving the cartesian equations using $y = x\tan\alpha$ and Pythagoras, or by the quicker method using the polar coordinates of the point T, namely $(r\cos\alpha, r\sin\alpha)$. A small group unfortunately used $(r\sin\alpha, r\cos\alpha)$ instead.

(iii) (2 marks)
Most of the candidates attempting this part did not complete their working, presumably because of a lack of time. Many assumed that the maximum range occurred when $\theta = \pi/4$. Others had difficulty

differentiating their expression for r, with some treating α as a variable. Even when the correct expression $2\theta - \alpha = \pi/2$ was found, many candidates could not simplify their expression for r. Some used the hint given to obtain a term of the form $\sin(2\theta - \alpha)$ but then immediately expanded this to get $\sin 2\theta \cos\alpha - \cos 2\theta \sin\alpha$.

Candidates who applied the hint at the beginning of their work found the quickest solution. As $r = (\sin(2\theta - \alpha) - \sin\alpha)/(2\cos^2\alpha)$ it can be noted that r has its maximum value when $\sin(2\theta - \alpha) = 1$ and the result follows in a line or two.

Candidates who computed the value of r at the stationary point usually were able to show that this value was a maximum.

(iv) (2 marks)
Very few candidates attempted this part. Examiners were disappointed to note that many of those who did obtain full marks in this part did not attempt or complete part (a) (ii).

Two candidates nearly found the elegant solution involving symmetry. This solution notes that the maximum range along a slope, with angle α to the horizontal, occurs when $\theta = (\pi/2 + \alpha)/2$. Looking at the point T and reversing time the maximum range must again be R and occur when $\theta = (\pi/2 - \alpha)/2$. The rest is geometry.

Mathematics 4 Unit (Additional)

Question 1

This question was generally well done, with over half of the candidates earning at least 14 of the 15 available marks. However, some candidates showed poor algebraic skills in handling negative indices and fractions, indicating the need for additional practice of these skills.

(a) (2 marks)
One mark was awarded for the substitution $u = \sin x$, $du = \cos x\, dx$ or equivalently, an answer of the form $a(\sin x)^n$; the second mark was awarded for the correct values of the constants a and n. Common mistakes included the wrong sign in the derivative of $\sin x$ and differentiating instead of integrating u^{-4}.

(b) (2 marks)
One mark was awarded for $(x + 3)^2$ and the second mark for the correct answer $4\tan^{-1}(x+3)$. A few candidates did not know the difference between $\tan u$ and $\tan^{-1} u$ while others obtained various multiplying constants.

(c) (i) (2 marks)
One mark was awarded for approaching this part by a correct method, such as writing $9 = (ax + b)(3 - x) + cx^2$ and the second mark was awarded for the correct values of all three constants a, b and c.

(ii) (2 marks)
One mark was awarded for $\int \frac{dx}{3-x} = -\ln|3 - x|$ or $-\ln(3 - x)$ or $\ln(\frac{1}{3-x})$; the minus sign in front of the logarithm (or the logarithm of the reciprocal) was essential for this mark. The other mark was awarded for the correct evaluation of $\int \frac{ax+b}{x^2}\, dx$ using the values of a and b the candidate had obtained in part (i) provided $b \neq 0$.

(d) (3 marks)
One mark was awarded for the correct choice of u and v and the corresponding expressions for $\frac{du}{dx}$ and $\frac{dv}{dx}$. Many candidates asserted that the integral of $\ln x$ was $\frac{1}{x}$; others had various incorrect ways of handling $\sqrt{x}$. The second mark was awarded for correct substitution into the integration by parts formula, even if the expressions for u and v were wrong. Some candidates memorised the formula with a plus sign; others were confused about whether to use the function or its derivative in the formula. The final mark was awarded for a correct final integration; the final integration was the same as that required to construct v so candidates who repeated the earlier mistake here were awarded two marks provided this was the only mistake in their answer.

(e) (4 marks)
One mark was awarded for substituting the correct formulae for both $\sin\theta$ and $\cos\theta$; another mark was awarded for using the correct formula for $d\theta$. The third mark was awarded for correct algebra to simplify the integrand provided this was set out clearly. The final mark was awarded for correct integration (even if the integrand was wrong).

Many candidates had memorised the formulae incorrectly and ended up with difficult integrations. Other candidates attempted to use the double angle formulae to work out the correct substitutions to use. They were generally successful in obtaining the correct formulae for both $\sin\theta$ and $\cos\theta$ but had problems finding $d\theta$.

Question 2

This question tested a wide range of knowledge about complex numbers. Every candidate gained at least one mark, and over half the candidature gained at least 12 of the 15 available marks.

(a) (3 marks)
Most candidates found the solutions, $x = 5$, $y = 1$, and $x = -5$, $y = -1$, to the equation $(x + iy)^2 = 24 + 10i$. However, many wrote this in the ambiguous form, $x = \pm 5$, $y = \pm 1$, which cost them a mark. Only a few candidates used the restriction that x and y must be integers to go to the solution directly from the equation $xy = 5$, obtained by comparing imaginary parts; most chose instead to find all real solutions for x and y by solving the quartic equation obtained by eliminating one variable. This greatly increased the chance for error and loss of marks.

(b) (2 marks)
The largest group of candidates chose the simplest approach, substituting i for z to obtain $i^2 + ai + (1 + i) = 0$ for the first mark, and solving correctly for a to gain the second mark.

A more complicated approach, which many candidates saw to a successful conclusion, was to solve the pair of equations obtained by considering the sum and product of the roots of the polynomial.

An even more complicated approach was to divide $z^2 + az + (1 + i)$ by $z - i$ by long division and set the remainder to zero. Only a few candidates succeeded by this method.

The most complicated approach was to apply the quadratic formula and then attempt to solve $i = \frac{-a \pm \sqrt{a^2 - 4(1+i)}}{2}$ for a. Only one or two candidates obtained the right answer in this way.

Many candidates appeared unwilling to accept that the correct value for a, -1, was a complex number.

The most common error involved assuming that $-i$ was also a root of the equation. The conjugate root theorem does not apply as the equation does not have real coefficients.

(c) (i) (2 marks)
The bulk of the candidature applied de Moivre's theorem correctly to deduce that $z^6 = \cos\pi + i\sin\pi$, gaining the first mark. Most then correctly evaluated this to -1.

(ii) (2 marks)
Most candidates correctly plotted $\frac{\sqrt{3}}{2} + \frac{1}{2}i$ as one of the solutions of $z^6 = -1$ for the first mark. Many went on to plot the other 5 vertices of a regular hexagon centered at the origin for the second mark. The most common error involved a diagram which was a rotation of the correct hexagon. This diagram in fact depicted the solutions of $z^6 = 1$ and was most easily distinguished by having vertices on the axes at 1 and -1, rather than the correct diagram which has vertices on the axes at i and $-i$.

(d) (3 marks)
Most candidates replaced z by $x + iy$. This quickly led to the inequality $x^2 + y^2 + 4x \leq 0$, for which the first mark was awarded. Many were able to rewrite this as $(x+2)^2 + y^2 \leq 4$ and then to recognize this as describing the interior and boundary of the circle of radius 2 centered at $(-2, 0)$. The second mark was for the correct centre and radius, and the third mark for indicating that the inside of the circle was intended.

An astonishing number of candidates wrote $x^2 + y^2 + 2x \leq 0$, and so lost the first mark.

(e) (i) (1 mark)
Most candidates recognized that the line segment OC could be obtained from the line segment OA by a rotation about O through $\pi/2$ radians anticlockwise (corresponding to multiplication of ω by i), followed by a doubling of the length, yielding the answer $2i\omega$.

(ii) (2 marks)
Many candidates noted that the complex number corresponding to D is half the complex number corresponding to B. This observation was sufficient to earn one mark. The question could then be completed by noting that B corresponds to $\omega + 2i\omega$, the sum of the numbers corresponding to C and A.

Others gained their first mark by noting that D is the midpoint of the line segment AC. It follows that D corresponds to the average of ω and $2i\omega$.

A sizable contingent elected to compute the argument of the number corresponding to D by trigonometric considerations involving the

right-triangle OAB and then applied Pythagoras' theorem to find its modulus. This occasionally led to unnecessarily complicated correct answers such as $\frac{\sqrt{5}}{2}|\omega|\,(\cos(\arg\omega + \tan^{-1}2) + i\sin(\arg\omega + \tan^{-1}2))$. Those following this method were awarded a mark for the correct modulus and a mark for the correct argument.

Many candidates first wrote $\omega = x + yi$, and then expressed their answers in terms of x and y, often without stating explicitly that x and y were the real and imaginary parts of ω. Some fell into the error of giving their answer as $(\frac{1}{2}x - y, x + \frac{1}{2}y)$. This could not be awarded full marks as the problem specifically asked for a number. Worse yet were answers, such as $(\frac{x-2y}{2}, \frac{2x+y}{2}i)$ or $(\omega/2, i\omega)$, that demonstrate serious confusion about the meaning of the notation.

Question 3

The first part of this question was concerned with curve sketching. This part was well done, with many candidates gaining full marks, although there were a considerable number who had little idea of how to proceed. The second part, involving volumes, was a simple exercise for about half of the candidature, with the rest making the question far more difficult than it need have been. The last part, involving the use of Stirling's formula to approximate 100! in scientific notation, indicated that many candidates had a slavish dependence on their calculators and were unable to proceed when it would not deal with large numbers. Fewer than 10% were able to gain full marks on this part.

(a) (i) (1 mark)
In the main the graph of $y = |f(x)|$ was well drawn, although a number of candidates incorrectly dealt with the portions of $f(x)$ which were to the left of the y axis or below the x axis. The mark was awarded to those candidates who convinced the examiners that they knew that the segment below the x axis had to be reflected in the x axis.

(ii) (2 marks)
Many candidates recognised that $x = 4$ and the x axis were asymptotes, but omitted a branch or added an asymptote on the y axis. Some candidates confused $\frac{1}{f(x)}$ with $f^{-1}(x)$. Candidates with incorrect sketches were awarded one mark provided they showed the asymptote at $x = 4$ and drew one of the two branches correctly.

(iii) (2 marks)
This was the most poorly done of the four sketches. Many candidates were confused about the location of the y intercepts, placing them at 9 and -9, or at 81 and -81. Many candidates drew only the top half of the graph, and so earned only one mark.

(iv) (2 marks)
Relatively few candidates failed to gain the two marks on offer for this graph. Many of those who failed to do so correctly stated that a function and its inverse were reflections of each other in the line $y = x$, but despite this drew the reflection of the graph of $y = f(x)$ in the x axis.

(b) (i) (2 marks)
The major step in this part of the question was to recognise that $a = 2\sqrt{h}$ (worth one mark) and then to proceed to multiply $\frac{1}{2} \times \pi \times 2\sqrt{h} \times \sqrt{h}$ to achieve the required result. Many candidates determined every possible relationship among x, y, a, b and h in the process of finding the correct result. This gained both marks, but wasted a good deal of time. A significant number ignored the formula which was given for the area of an ellipse and attempted to prove this result.

(ii) (2 marks)
About half the candidates had no problem in determining the volume of the solid S. Many of the remainder were confused as to what the expression for the volume should be, often writing $\int \pi h \, dx$ and then treating h as a constant. A number of others failed to perform the relatively simple integration correctly, with answers of 2π, 4π and 8π occurring regularly. The first mark for this question was awarded for simply writing down the expression $\int_0^4 \pi h \, dh$, with the second mark awarded for arriving at the correct value, 8π cubic units.

(iii) (2 marks)
Those candidates who correctly found the volume of T by integration and correctly stated its ratio to the volume of S obtained in (ii) easily gained the two marks. Some candidates attempted to reason from the relative heights of the two solids, but often forgot that in the case of the solid T the rotation involved a complete revolution about the x axis. Many candidates stated an incorrect ratio with no supporting work, thus depriving themselves of any marks for this part. A number of the others could not express the ratio correctly, so that the ratio of $S(\frac{64\pi}{3})$ to $T(32\pi)$ was often expressed as $3 : 2$. Those whose answer was "the volume of T is larger than the volume of S" were awarded one mark as the question did not explicitly state that the ratio of the two volumes was required.

(c) (2 marks)
This was the most difficult part of this question. The first mark was obtained either for writing 100! in scientific notation or for writing a correct numerical expression for the number of years needed. Once candidates discovered that the numbers involved were outside the computational range of their calculators (one candidate wrote of trying three calculators with no success), most simply gave up trying to write it in scientific notation.

The manipulation of indices was a cause for concern. Those who attempted to convert $100^{100.5}$ to a power of 10 often arrived at 10^{1005}, whilst the division of $100^{100.5}$ by 10^{12} frequently yielded $100^{88.5}$. Of equal concern is the fact that the expression 10^{12} was frequently entered in the calculator as 10 EXP 12. This actually gives 10^{13}, since the calculator notation means 10×10^{12}. Other simple errors, such as using 256, 265 or 356 for the number of days in a year occurred regularly. Such errors meant that the answers obtained varied enormously, from millionths of a second to over 10^{1000} years.

Question 4

This question proved challenging to the average candidate, but almost 10% of the candidature were able to score full marks. There were very few candidates who did not attempt this question or who scored zero, mainly due to part (a) (i). Many candidates used two booklets and made several attempts to answer this question, often leaving it unclear as to which attempt was their considered answer.

(a) (i) (3 marks)
Most candidates gained full marks for this part.

(ii) (1 mark)
Candidates were awarded the mark for substituting $y = c^2/x$ into the normal or showing that both $x = cp$ and $x = -c/p^3$ satisfied the given equation. This was well done.

(iii) (4 marks)
About 30% of the candidates received no marks for this part. One mark was awarded for finding the coordinates of Q, another mark for finding the coordinates of R or the gradient of PO or PR. The remaining two marks were awarded for a correct method and conclusion.

Most candidates found the gradients of PR and QR, a few used Pythagoras' theorem and very few found $\tan\theta$. Many were not able to find Q correctly. Candidates could see that they were wrong, since $m_1 m_2 \neq -1$, and made many attempts to find the correct coordinates. Unfortunately algebraic errors using the quadratic formula made this difficult. A minority of the candidature noticed that they could use the sum of roots and quickly found Q.

(b) This part was difficult for many candidates. They were clearly confused by the presence of two temperatures T_1 and T_2 that change with time, and the presence of so many t's caused additional confusion in the course of integration and differentiation.

(i) (1 mark)
Candidates were required to use the given information to show that $\frac{3}{4}\frac{dT_1}{dt} + \frac{dT_2}{dt} = 0$ and therefore C is a constant. Many candidates wrote '$0 = C \therefore$ true'. Unfortunately many persisted by using $C = 0$ in the following parts.

(ii) (3 marks)
Approximately half of the candidature did not attempt this part or received no marks. About 20% gained full marks. One mark was awarded for obtaining $\frac{dT_1}{dt} = -k(\frac{7}{4}T_1 - C)$ or an equivalent expression. The remaining 2 marks could be gained in two ways.

The easier way involved taking the given expression for T_1 and showing that it satisfied the differential equation. The more difficult method involved separating the variables and integrating to derive the given expression for T_1. Using this method, candidates needed to be careful with their choice of notation for the constant of integration, as there was already a C in the expression. Many B's appeared miraculously in the final line or as a result of considerable fudging of constants.

(iii) (3 marks)
Half the candidature gained 2 or 3 marks for this part. There were many simple algebraic errors, possibly reflecting the time pressure at this point in the examination. Many candidates did not realise that C was a constant and so recalculated it as T_1 changed, leaving T_2 constant at 22°. Some tried, without success, to fit the given information to a result derived from Newton's law of cooling which they had memorised.

Question 5

Part (a) of this question resulted in good attempts by most candidates. Candidates who recognised the sequential nature of the various questions within part (b) were able to provide answers to many of these parts by relying on information given in earlier parts, even if they were unable to establish these intermediate results. Examiners also noted that a good understanding of the question was revealed by the use of force diagrams in appropriate sections of part (b).

(a) (i) (2 marks)
Most candidates substituted α into $P(x) = 0$ to gain their first mark. The result was then obtained by writing $\alpha(a\alpha^3 + b\alpha^2 + c\alpha + d) = -e$.

Another method which was often successful involved writing $P(x)$ as $(x - \alpha)(fx^3 + gx^2 + hx + j)$ and comparing the constant terms. This gives $\alpha j = -e$. However, establishing that j is an integer

then requires some skillful reasoning, best approached by arguing in succession that f, g and h are integers. Those who proceeded by performing the division algorithm usually made an algebraic error, gave up, or came to an invalid conclusion.

(ii) (2 marks)
Those who saw the theoretical connection with the previous part were able to show that the only possible integer roots were ± 1 and ± 3 to gain their first mark. The second mark was obtained by evaluating $P(x)$ at each of these four integers to show that none of these were zeros of the polynomial.

(b) (i) (2 marks)
Use of the arc length formula, $s = l\theta$, and the consequent relation $ds = l\, d\theta$ were necessary to establish the given result. There were many different approaches. The more successful candidates began with $\frac{1}{l}\frac{d}{d\theta}(\frac{1}{2}v^2)$ and proceeded to show this equal to $\frac{d^2s}{dt^2}$. Multiple attempts were common and much time consumed trying to introduce $\frac{1}{2}v^2$ or $\frac{1}{l}$ into the working. The most efficient method was a modification of the memorised proof that $\frac{d^2x}{dt^2} = \frac{d}{dx}(\frac{1}{2}v^2)$.

(ii) (1 mark)
Candidates were required to explain where the expression $-g\sin\theta$ came from. This was best done by constructing a force diagram involving the tangent to the circle at P. Most candidates did this, but many had an incorrect position for the right angle by not making their force vector parallel or perpendicular to the tangent. Some care was needed if relocating θ.

(iii) (2 marks)
This part was very well done. Candidates needed to recognise that they should begin by integrating both sides of the expression given in the previous part with respect to θ. One mark was gained for finding $+g\cos\theta$ as a primitive of $-g\sin\theta$. The second mark was gained by evaluating and substituting the constant of integration. It was good to observe the number of candidates who were able to detect and correct errors which they had made with the sign. Those whose method of solution involved finding definite integrals of both sides sometimes confused the limits, interchanging V with v or 0 with θ.

(iv) (2 marks)
A good way to begin an answer was to state an intention to resolve forces radially. A force diagram similar to that used in section (ii) showing $mg\cos\theta$ gained one of the marks. The expression $\frac{1}{l}mv^2$ was sometimes written as $\frac{1}{2}mv^2$, presumably as a result of confusion with the well-known physics formula.

(v) (3 marks)
One mark was awarded for using $V^2 = 3gl$ in $V^2 = v^2 + 2lg(1-\cos\theta)$. Another mark was awarded for using the equation $T - mg\cos\theta = \frac{mv^2}{l}$ and letting $T = 0$. The final mark was obtained by combining these two equations and finding a value for θ. Poor algebraic technique, particularly in paying attention to detail, was the main cause for concern. For example, on many occasions $3gl - 2gl$ was not simplified until the final line. When $T = 0$, the result was often $mg\cos\theta = \frac{mv^2}{l}$ with the negative sign omitted. $2lg(1-\cos\theta)$ often became $2lg - \cos\theta$ on removal of brackets. The expression $\cos\theta = -\frac{1}{3}$ was sometimes followed by $\theta = 70°32'$. Errors such as these cost the final mark but, for the majority, the overall level of skill shown was quite pleasing.

(vi) (1 mark)
By contrast, this part was the most poorly answered within Question 5. Many candidates stated that the string would break and the mass would fly off at a tangent. Other common responses asserted that the mass then describes the path of a pendulum or that the mass falls vertically until $T > 0$.

If a correct answer was gained in part (v), it can be shown that $v = \sqrt{\frac{lg}{3}}$ when $T = 0$ by substitution back into $-mg\cos\theta = \frac{mv^2}{l}$. Thus the particle will adopt projectile motion taking as its path a parabolic arc inside the circle. At the point where $T = 0$, the circle and the parabolic arc share a common tangent. This motion will continue until the particle reaches the point where this parabolic arc touches the circle again.

The mark was awarded for answers which provided the examiner with sufficient detail to infer a general understanding of this parabolic arc immediately after the point where $T = 0$. Candidates who wrote that the particle would proceed along the circle or move outside it were not awarded the mark.

Question 6

Part (a) of this question was a relatively straightforward circle geometry question. Most candidates attempted at least some parts. It was not a difficult question, but very few candidates were able to score full marks. Many candidates obviously knew some geometric facts, but were not able to construct a logical argument.

Part (b) involved a standard integral, after which candidates were asked to explain an inequality involving $\int_0^{1/2} \frac{dx}{\sqrt{1-x^n}}$. This latter part was very poorly done.

(a) (i) (1 mark)
Almost all candidates scored this mark. Some candidates wasted

time proving various triangles congruent, rather than simply stating the result about tangents from an external point.

(ii) (2 marks)
Candidates using the suggested method were required to show where both sides of the equation came from in order to gain the 2 marks. Those who wrote nothing more than $RS^2 = 2r_1 \times 2r_2$ scored zero. Candidates using either Pythagoras' theorem or similar triangles in the right-angled triangle ARB were generally successful.

(iii) (4 marks)
Many candidates made surprisingly hard work of this part. Those who realised that the first result followed immediately from the fact that O_1M and O_2M bisect angles PMS and QMS, respectively, were the most successful. Other approaches were sometimes successful, although many attempts were very hard to follow. Some candidates had obviously marked new labels on the diagram on the examination paper, but failed to say what these new labels represented in their answer contained in the writing booklet. The notation of the question, using O_1, O_2 and Q, required candidates to write clearly and carefully so that they would not be confused when reading their own handwriting.

A reasonably large number of candidates attempted to show that the sides of triangle O_1MO_2 satisfied Pythagoras' theorem. These candidates generally assumed $MS = RS/2$, and some very circular reasoning followed.

Deducing the second result of this part, that $MS^2 = r_1r_2$, was handled somewhat better. Candidates used the intersecting chords theorem again, or Pythagoras' theorem or similar triangles, and these methods were equally successful.

(iv) (2 marks)
The simplest method was to use parts (ii) and (iii) to show that $MS = RS/2$, and so the diagonals of $PSQR$ bisect each other, and are equal. Many candidates asserted that it was a rectangle simply because the diagonals bisect each other. Candidates who assumed that the points A, P and R are collinear were also unsuccessful.

(b) (i) (2 marks)
Almost all candidates were able to gain these two marks.

(ii) (4 marks)
This part was not handled at all well, and many candidates did not attempt it. Very few candidates realised that they should consider values of x between 0 and $1/2$, and that for these values of x, $1 \leq \frac{1}{\sqrt{1-x^n}} \leq \frac{1}{\sqrt{1-x^2}}$, from which the result follows immediately.

Many incorrect statements were made, such as the claim that if $n \geq 2$ then $\frac{1}{\sqrt{1-x^n}} \leq \frac{1}{\sqrt{1-x^2}}$ for all $x < 1$. A large number of candidates attempted to explain the left hand side of the inequality by considering $\lim_{n\to\infty} \frac{1}{\sqrt{1-x^n}}$. Such an approach cannot possibly establish that an inequality holds for all $n \geq 2$.

Question 7

The first part of this question required candidates to prove identities involving the summation of values of logarithm functions. The second required candidates to prove the reflection property of the ellipse by a geometric argument. This question was found to be very difficult by the vast majority of the candidature.

(a) (i) (2 marks)

This part was reasonably well done, with many candidates gaining 2 marks by accurately sketching $y = \ln x$ and its tangent at $x = 1$. The tangency at (1, 0) needed to be clear.

Many approached the question by considering the stationary point of the function $y = \ln x - x + 1$. Those who used the second derivative test were more likely to be correct than those who just considered $\frac{dy}{dx} = \frac{1}{x} - 1$.

A common mistake was to say that $y' < 0$ for $x > 0$. There was also confusion between the concept of a function being concave down and a decreasing function.

A strange feature was the large number of candidates who tried to use mathematical induction.

(ii) (3 marks)

Very few students were able to make the connection with part (i) and write down $\sum_{i=1}^{n} x_i \ln \frac{y_i}{x_i} \leq \sum_{i=1}^{n} x_i(\frac{y_i}{x_i} - 1)$. Those who did were able to gain at least 2 marks for this section. Very few were able to note that $\frac{y_i}{x_i} = 1$ for equality to occur.

(iii) (3 marks)

Despite the question saying that the y_i were equal and $\sum_{i=1}^{n} y_i = 1$, hardly any candidates were able to write down $y_i = \frac{1}{n}$. Those who did went on to score 2 or 3 marks for this section.

(iv) (1 mark)

In order to be awarded this mark candidates needed to realise that $\log_2 x = \frac{\ln x}{\ln 2}$, and so the working for part (iii) is the same apart from a constant factor of $\frac{1}{\ln 2}$.

(b) (i) (2 marks)
Those candidates who used the triangle inequality on triangle QRS were usually successful in gaining the two marks, but there were relatively few who did this. An argument based on the fact that $S'Q + QS \neq S'R + RS$ (which is a fixed constant for any point R on the ellipse) gained one mark.

(ii) (2 marks)
Candidates gained the 2 marks for stating that $S'P + PS = S'R + RS$ and connecting this with the result in part (i). A candidate could gain 1 mark for stating the constant property of the ellipse.

(iii) (2 marks)
One mark was obtained by identifying $\angle S^*PT$ and $\angle S'PQ$ as vertically opposite angles and also giving clear reasons why $\angle S^*PT = \angle TPS$. Acceptable reasons included congruent triangles and properties of isosceles triangles. A large number of candidates gained this mark.

In order to gain the second mark candidates needed to give a reasonably convincing argument that $S'PS^*$ was a straight line. The easiest way to do this was to argue that if the points S', P and S^* were not collinear, it would contradict the result proved in part (ii). Very few candidates were able to gain 2 marks for this part.

Question 8

This question consisted of two parts. The first asked candidates to prove a trigonometrical identity from complex numbers and the second involved an application of probability to investigating relationships when tossing a coin.

The candidates did not score highly in this question, but it was clear that candidates had used good techniques to ensure that they earned as many marks as possible from this question.

(a) (i) (3 marks)
Many correct methods were used to show that the nested summation of the left hand side was equal to the expression on the right. However, many candidates were only awarded the first mark which was obtained for summing the inner geometric series on the left. It was very common for candidates to evaluate this inner sum as $\frac{z(1-z^n)}{1-z}$ rather than the correct value which is $\frac{z(1-z^k)}{1-z}$. This inevitably led to futile efforts to manipulate one side or both sides to try and obtain equivalent expressions. Some even tried to multiply their left hand side by $\frac{1-z}{1-z}$ in order to obtain the expression on the right.

The remaining two marks were awarded for showing where the term $\frac{nz}{1-z}$ came from and for deriving the second term on the right.

(ii) (3 marks)
This was quite demanding, and many candidates had problems with the level of sophistication required to answer this question. Their skill and experience were found wanting when it came to applying the given formula $\frac{z}{1-z} = \frac{i}{2\sin\frac{\theta}{2}}\operatorname{cis}\frac{\theta}{2}$ and substituting $z = \operatorname{cis}\theta$ in the right hand side. At times poor setting out was a contributing factor, as candidates had difficulty understanding their own convoluted expressions.

Many did not understand the correct order of operations when taking the imaginary parts of the terms. They thought, for example, that $\operatorname{Im}(z_1 z_2) = \operatorname{Im}(z_1)\operatorname{Im}(z_2)$, $\operatorname{Im}(\frac{z_1}{z_2}) = \frac{\operatorname{Im}(z_1)}{\operatorname{Im}(z_2)}$ or $\operatorname{Im}(z^n) = (\operatorname{Im}(z))^n$. These mistakes frequently led to expressions such as

$$\cdots - \frac{\sin^2\theta}{1-\sin\theta}(1-\sin^n\theta) \quad \text{or} \quad \ldots \frac{\sin^2\theta}{1-\sin\theta}(1-\sin n\theta) \quad \text{or}$$
$$\cdots - \frac{\sin 2\theta}{1-\sin\theta}(1-\sin n\theta)\,.$$

Marks were awarded for applying de Moivre's theorem, taking the imaginary part of the right hand side of part (i) correctly, and for generating the required result.

(b) (i) (3 marks)
This part was not well done at all, and candidates had great difficulty in constructing suitable explanations. The stem of the question was particularly difficult to understand, and proved too daunting for almost all candidates. Only a handful showed any understanding of the subtlety in this question.

The examiners were looking for a response such as:

> For r, $s > 0$, r heads will appear before s tails appear in either of the following cases:
>
> 1. the first toss is a head with probability $P(H) = \frac{1}{2}$ and thereafter $(r-1)$ heads appear before s tails with probability $P(r-1, s)$ or
> 2. the first toss is a tail with probability$P(T) = \frac{1}{2}$ and thereafter r heads appear before $(s-1)$ more tails with probability $P(r, s-1)$.
>
> Hence we obtain the stated result
>
> $$P(r,s) = \frac{1}{2}P(r-1,s) + \frac{1}{2}P(r,s-1)\,.$$

(ii) (3 marks)
Many candidates gained 1 mark for using the relationship from part (i) but most of these then became lost in the maze of numerical expressions that followed. Candidates who took care and were methodical were rewarded with the correct answer.

One mark was awarded for using the relationship, a second mark for getting to $P(0, s) = 1$ or $P(r, 0) = 0$, and the third mark for correctly deducing that $P(2, 3) = \frac{11}{16}$.

(iii) (3 marks)
Similar difficulties to those which were evident in part (i) meant that this part was also not well done. Some were able to establish the initial case for the induction with $n = r$ or $n = s$.

Marks were awarded for establishing this initial case, for applying the relationship from part (i) on $P(r + 1, s)$ or $P(r, s + 1)$ to provide an avenue to apply the induction hypothesis and then for using Pascal's relationship to complete the proof.

While the examiners were aware of a number of approaches which did not involve induction, these did not occur in any of the work presented by the candidates.