BOARD OF STUDIES
NEW SOUTH WALES

1999 HSC

Mathematics

Enhanced Examination Report

1999 HIGHER SCHOOL CERTIFICATE EXAMINATION

MATHEMATICS

ENHANCED EXAMINATION REPORT

Contents

Mathematics 3 Unit (Additional) and 3/4 Unit (Common)

Mathematics 4 Unit (Additional)

General Comments

The five mathematics papers in 1999 provided candidates with appropriately graded opportunities to display the extent to which they had achieved the outcomes which are embedded in the various Mathematics syllabuses. Some changes were discernable in the distribution of raw marks achieved on the papers from the pattern which has been established over the past few years. Increases in the mean raw mark for both the 2/3 Unit (Common) paper and the 4 Unit (Additional) paper were the most notable features. These increases reflect the inclusion of slightly more material that was accessible to candidates who just met the minimum requirements of these courses with a corresponding decrease in the number of moderately difficult questions. These variations did not result in any significant impact on the distribution of marks reported to candidates under the Board of Studies' current arrangements.

The comments in this report are compiled from information supplied by examiners involved in marking each individual question. While they do provide an overview of performance on the 1999 examinations, their main purpose is to assist candidates and their teachers to prepare for future examinations by providing guidance as to the expected standard, highlighting common deficiencies and, in the process, explaining in some detail the criteria which were used in the marks for each part of each question. Where appropriate, the method of solution is outlined and the merits of different approaches to the question are discussed.

Candidates should be aware of the fact that it is their responsibility to indicate the process by which they have obtained their answer to the examiners. In marking, each individual mark is allocated to a step or process which is essential to a correct solution of that question. Those who provide sufficient evidence that the appropriate step, or its equivalent, has been completed are awarded the mark, which then cannot be lost for a subsequent error. Candidates who give only a single word or figure as their response forego any possibility of earning any marks unless their answer is completely correct. Sometimes, in cases where examiners believe that the correct answer is easily guessed without doing the work required to establish the result, a mere correct answer without any supporting justification may not earn all of the available marks.

It is very important that candidates record their working in the same writing booklet as their answer, even if it is experimental work done to develop an approach to the question. Examiners read everything written by the candidate in an attempt to find evidence which will justify the awarding of a mark. This includes work which the candidate has crossed out, or explicitly requested the examiner not to mark. This is always to the candidate's advantage, as marks are awarded for elements of the solution which are correct and are not deducted for errors which have been made. This means that candidates should take care to make sure that work which has been crossed out is still legible, and should not, in any circumstances, use correction fluid or an eraser. Candidates who wish to distinguish their rough work from their considered answers should use the unruled

left hand pages for such work.

Candidates who accidentally answer part of one question in the wrong writing booklet should not waste valuable time transcribing their work from one booklet to another. Instead, they should make a clear note on the cover of both the writing booklets to the effect that part of the answer to Question 7 is included in the booklet for Question 5. There are procedures in place at the marking centre to ensure that such misplaced material is brought to the attention of the examiner marking the appropriate question, and no marks are ever deducted for such slips.

Examiners greatly appreciate work which is clearly presented, in which the order of a candidate's work is readily apparent. In particular, candidates are encouraged to avoid setting their work out in two or more columns per page, and to make certain that the parts and subparts of questions are appropriately labelled. It is not essential for the parts within a question to be presented in the same order as they appear in the examination question, but departures from the original order make careful labelling of the responses even more important.

Examiners frequently comment on the need for candidates to provide clearly labelled, reasonably sized and well executed graphs and diagrams. Appropriate use of a ruler and other mathematical instruments is essential to obtain a diagram of the appropriate standard. In making these comments, examiners are motivated by the assistance such graphs and diagrams provide candidates in the process of answering the question. In particular, candidates ought to realise that instructions on the examination paper asking candidates to reproduce a diagram in their writing booklet are invariably given because the diagram is likely to assist the candidate to solve the problem or provide a means for them to explain their solution.

Finally, candidates for the related courses are reminded that a table of standard integrals appears on the back page of each examination paper. Candidates should become familiar with this table, and be aware of its usefulness for both integration and differentiation.

Mathematics in Practice

Questions 1 – 30   Multiple-Choice Questions

For each response to each of the multiple-choice questions, the table lists the percentage of candidates making that response and the mean number of correct answers to all 30 multiple-choice questions given by such candidates. The data for the correct answer is in bold face.

Quest	Omit		A		B		C		D	
No.	%	Mean	%	Mean	%	Mean	%	Mean	%	Mean
1	0.14	10.0	1.67	11.3	22.31	13.0	29.74	13.1	**46.14**	**15.0**
2	0.19	11.0	4.45	11.5	**67.98**	**15.4**	11.96	12.0	15.41	9.8
3	0.10	8.2	**72.25**	**14.9**	4.01	10.2	3.67	10.5	19.97	11.9
4	0.25	12.6	**49.98**	**15.2**	23.79	12.1	18.69	13.4	7.27	12.2
5	0.29	9.2	**78.87**	**15.2**	7.14	9.7	10.59	9.2	3.11	8.8
6	0.61	10.2	13.60	12.9	**37.73**	**16.2**	28.28	12.8	19.78	12.0
7	0.15	8.2	**51.31**	**15.5**	14.90	12.0	10.18	12.2	23.45	12.4
8	0.34	10.0	5.06	10.9	21.36	12.2	38.00	13.8	**35.23**	**15.6**
9	0.41	11.3	13.72	11.6	17.35	11.4	14.14	13.1	**54.38**	**15.6**
10	0.95	12.6	**42.69**	**16.2**	19.66	12.3	24.66	12.2	12.03	12.1
11	0.08	3.4	4.72	9.4	3.50	8.7	**86.52**	**14.6**	5.17	9.8
12	0.32	9.6	5.86	12.1	**38.05**	**15.2**	17.68	12.0	38.09	13.8
13	0.76	9.6	8.46	11.3	**67.39**	**15.5**	14.24	10.0	9.14	10.9
14	0.32	9.9	7.05	12.7	46.75	13.6	**24.01**	**16.5**	21.86	12.4
15	0.49	11.2	**33.80**	**16.9**	24.66	12.2	22.57	12.2	18.47	12.8
16	0.07	2.5	5.98	10.5	27.58	12.3	**58.79**	**15.2**	7.58	12.7
17	0.49	10.0	**21.24**	**15.7**	11.01	11.7	48.05	14.0	19.20	13.1
18	0.19	7.5	20.99	11.7	13.43	11.6	**39.58**	**15.6**	25.82	14.5
19	0.24	8.5	19.71	12.8	47.89	14.0	9.76	10.3	**22.40**	**16.4**
20	0.29	7.8	18.52	10.9	6.70	9.7	**58.07**	**15.7**	16.42	12.8
21	0.20	7.9	14.45	12.4	**52.31**	**15.8**	14.14	10.2	18.90	12.8
22	0.65	12.3	7.73	12.0	**22.35**	**15.1**	25.80	12.4	43.47	14.6
23	0.25	8.8	**57.04**	**15.4**	7.38	11.3	11.01	10.6	24.32	12.9
24	0.42	9.9	17.17	11.6	34.84	13.0	17.59	12.4	**29.98**	**17.3**
25	0.36	9.3	6.92	12.4	37.61	11.9	20.89	12.8	**34.23**	**17.2**
26	0.42	9.6	21.63	12.6	**55.13**	**15.5**	18.22	11.6	4.59	10.3
27	0.36	7.4	7.77	14.1	37.13	13.9	**32.61**	**14.5**	22.13	13.2
28	0.51	10.1	37.90	14.0	**33.55**	**16.4**	18.58	11.4	9.47	9.9
29	0.39	9.9	51.00	12.1	6.05	11.5	**38.22**	**16.8**	4.33	13.0
30	0.71	10.8	2.99	10.8	17.44	10.7	28.45	11.9	**50.41**	**16.4**

Question 31   The Consumer

(a)   (i) (1 mark)
Candidates were asked to calculate the annual cost of a family's health insurance, given the weekly premium. This was well answered. The most common error was to multiply by 12 instead of 52.

   (ii) (1 mark)
Candidates were required to find the reduction in the above yearly premium due to a 30% government rebate.

The question asked how much would be saved. However, a majority of the candidates calculated the new premium.

   (iii) (1 mark)
Candidates were asked to calculate the Medicare levy of 1.5% on a taxable income of \$72 000. This was quite well answered. The most common errors resulted from mistakes in writing 1.5% as a fraction or decimal.

   (iv) (2 marks)
Candidates were told that total health care costs consisted of private health insurance premiums plus the Medicare levy. They were then asked to find the percentage reduction in these costs as a result of the 30% government rebate in part (ii).

This was very poorly answered. Most candidates showed no understanding of the phrase 'percentage reduction' and instead found 30% of the total health care costs.

A mark was awarded if the total health care costs were calculated and an attempt was made to find a percentage.

(b)   (i) (1 mark)
Candidates were given the breakdown in costs for a school formal and were asked to find the total cost. This part was well answered by a majority of the candidates.

The most common incorrect answer was \$1551.30, which resulted from neglecting to multiply the costs which were given 'per student' by 120.

   (ii) (1 mark)
Candidates were asked to find the price per ticket. This was well answered, with most candidates understanding that they needed to divide their answer to part (i) by 120.

   (iii) (1 mark)
Candidates were told that the hire of the function centre increased by 15%. They had to calculate the new price per ticket.

This was reasonably well answered. The most common error was to increase the total cost by 15%.

(c) A table was provided showing monthly repayments per $1000 borrowed for 3 different types of loans.

  (i) 1. (1 mark)
       Candidates had to calculate the monthly repayment on a second mortgage loan of $85 000 over 10 years.

       This was poorly answered. A common wrong answer was $11.61 as candidates neglected to multiply by 85.

       2. (1 mark)
       Following on from the previous question, they then had to calculate how much in total would be repaid over 10 years.

       This part was well answered. The most common error was to multiply by 10 instead of 120.

       3. (1 mark)
       This part involved the calculation of the interest paid, and so required the subtraction of $85 000 from the previous answer.

       Understanding the term 'interest' in context remains a problem for these candidates. Many incorrectly tried to calculate a percentage. This part was not very well answered.

  (ii) (1 mark)
       Candidates were told how much a borrower could afford to pay per month on a first home loan over 20 years and were asked to find the maximum amount that could be borrowed. This was poorly done. The majority multiplied the maximum repayment by 7.20 instead of dividing. Amongst those who did divide, many neglected to multiply by 1000.

Question 32   Travel

(a) Candidates had to answer questions relating to a table showing the cost of return airfares to various cities in different seasons of the year.

  (i) (1 mark)
      Candidates were provided with a table of airfares to a number of cities during different seasons. They had to locate the information in this table corresponding to the planned travel itinerary. Candidates answered this quite well. However, many did not notice that the question asked how much each person would pay, and gave the total cost as their answer.

(ii) (1 mark)

Candidates had to express a child's airfare as a percentage of an adult's airfare.

Quite a few candidates had no idea what was required and did not attempt the question.

As the question did not specify the city or the season, candidates could use any cell in the table, giving a range of correct answers. Many candidates found the difference between the adult and child airfares and incorrectly proceeded to write this as a percentage of one of these fares.

(b) (i) (1 mark)

Candidates were given a table of cancellation charges. They had to read this correctly to determine that the latest date on which the trip could be cancelled without incurring more than a $200 cancellation charge was 15 days before departure. They then had to do a subtraction to determine the date.

The majority of candidates answered correctly. Common errors were

$$18 \text{ Jan} + 15 = 33 \text{ Jan} = 2 \text{ Feb}$$
$$16 \text{ Jan} + 15 = 31 \text{ Jan}$$
$$16 \text{ Jan} - 15 = 1 \text{ Jan}$$

(ii) (1 mark)

Candidates had to find the cancellation fee for each person if the trip was cancelled on 6 January. This required considerable working. Reading the table correctly caused problems as did the actual calculation. As a result, this question was not answered very well.

Common errors included subtracting 40% from the airfare and adding $200, adding the airfare to the cancellation fee and doubling the correct answer to find the total cancellation fee.

(c) (i) (1 mark)

Candidates had to find the distance between 2 cities by reading the information from a table.

A majority of the candidates answered this correctly. The most common error involved candidates adding or subtracting values from the table instead of simply reading the correct value from the table.

(ii) (1 mark)

Candidates had to convert miles to kilometres by multiplying the answer in part (i) by 1.67. This was well answered.

(iii) (1 mark)

An average speed had to be found by dividing the answer in part (ii) by 15.

This was well answered. Common mistakes resulted from candidates computing part (i) ÷15 or part (ii) ÷900.

(d) (1 mark)

Candidates were given an exchange rate and had to convert \$A 3000 to \$US. A majority answered this correctly. Common errors resulted from candidates computing $3000 \div 0.625$ or $3000 \times 0.625 + 3000$.

(e) (i) (1 mark)

Candidates had to find accommodation costs from a table. This was not well answered, with the word 'twin' obviously causing confusion. Many candidates quoted the cost for 1 night rather than 12 nights. Many also quoted the total cost, not the cost per person.

(ii) (1 mark)

Following on from the previous question, candidates were given information about a 'special deal' offering a fourth night free for every 3 nights purchased. Candidates were asked to find how much they would save on the accommodation bill.

This was poorly answered. Most candidates had no idea how to apply the advertised 'buy 3 nights, get the fourth free' to a 12 night stay. Many candidates only applied this once instead of three times. Quite a few worked out the payment not the saving. Many calculated the total for both travellers, but only applied the offer to one traveller in their working.

(f) (i) (1 mark)

Candidates were required to work out the time difference between Pacific time and Central time in the United States, given a map of time zones in the USA. This was very well answered.

(ii) (1 mark)

Given the time in Los Angeles, candidates had to work out the time in New York, again using the information on the map showing time zones.

This was also very well answered. The most common error was to simply read the clock face on the map which displayed 7 am in New York.

Question 33   Accommodation

(a)   (i) (1 mark)
Given a scale drawing of a block of land, candidates were asked to find the actual dimensions.

This was poorly answered and there were many non-attempts.

Candidates often gave the perimeter or area of the block of land. Some used incorrect units. Others simply measured the dimensions of the drawing and gave $4\,\text{cm} \times 6\,\text{cm}$ as their answer.

   (ii) (1 mark)
Candidates had to calculate the area of the land. This was quite well done. Errors arose when candidates tried to convert units.

  (iii) (1 mark)
This part required a calculation of a percentage of the area of the land to determine the maximum floor area for a house.

A majority of the candidates were able to calculate the percentage. However, quite a few then incorrectly went on to subtract this area from the area of the block of land.

(b) A detailed plan was given of a house.

   (i) (1 mark)
A straightforward addition of 3 costs was required here and most candidates answered this part correctly.

  (ii) (1 mark)
Candidates were asked to calculate an area, given the dimensions of rooms on the plan.

The easiest correct approach was to compute $(3.25 \times 3) \times 3 + (3.78 \times 3.5) = 42.48\,\text{m}^2$

This was quite well done. Common errors were to compute the area of 4 rooms of the same size, $(3.25 \times 3) \times 4$, or to add the areas of one room of each size, $(3.25 \times 3) + (3.78 \times 3.5)$.

  (iii) (1 mark)
This involved another area calculation, followed by an application of the cost per square metre and was quite well done.

Candidates often incorrectly rounded in the middle of the calculation, with responses such as $5.63 \times 5.45 = 30$, so the cost is $30 \times \$12 = \$360$ or $5.6 \times 5.5 \times \$12 = \$369.60$. These responses were not awarded the mark.

(iv) (2 marks)

Candidates were given a breakdown of costs involved in outfitting both bathrooms in the house. The quantities of each item needed to be determined by consulting the plan. Candidates had to find the cost of outfitting the bathrooms as a percentage of the total cost of the house.

There was a great deal of confusion evident in responses. Candidates were not clear as to which items should be included in the cost of outfitting the bathrooms. Many did not appear to realise that they had found the total cost of the house in part (i).

Candidates who had correctly determined that the cost of the building was \$108 000 and that the cost of outfitting the bathroom was \$7450 often gave the answer $\frac{7450}{100\,550} \times 100 = 7.4\%$ or $\frac{100\,550}{108\,000} \times 100 = 93\%$, rather than the correct answer which is $\frac{7450}{108\,000} \times 100 = 6.9\%$. Others gave the answer 93.1%, which is $100\% - 6.9\%$, or an answer which was based on the use of only one vanity and one toilet. These answers were all awarded 1 mark.

(c)   (i) (1 mark)

Candidates were given a house price and were told it would increase by 16%. They had to calculate the new price.

This was quite well done. The most common error was to calculate only the amount of the increase.

(ii) (1 mark)

This question required exactly the same type of calculation as part (i). The house price had to be found after a further increase of 7%.

This was not very well answered. Common errors again involved calculating only the increase or mistakenly computing a 23% increase on the original price.

(iii) (1 mark)

Candidates had to calculate by how much the value of the house was predicted to rise over the two-year period. The mark was awarded for correctly subtracting \$180 000 from the answer given to part (ii).

Common wrong answers included 23% and \$14 616, the rise in value over the last year.

(iv) (1 mark)

Candidates had to find the percentage increase over the two-year period, which was 24.12%.

This was poorly done. Common wrong answers included 23%, 9%, $\frac{\text{part (iii)}}{14\,616} \times 100$, $\frac{\text{part (iii)}}{43\,416} \times 100$, $\frac{\text{part (iii)}}{233\,416} \times 100$, $180\,000 \times \frac{43\,416}{100} =$ 24.12% and 24.2%, without any working shown.

Question 34   Design

This question contained a range of constructions and some calculations from various sections of the Design topic. Many candidates disadvantaged themselves by not using mathematical equipment where appropriate. Responses drawn without the aid of a ruler and a pair of compasses were not sufficiently accurate to earn marks in most parts of this question.

(a) (1 mark)
Candidates were required to complete a pattern. Most responses were generally consistent with the given pattern, but many were roughly drawn and inaccurate.

(b) (1 mark)
This question required the recognition of the shapes within a pattern. While most candidates answered this question correctly, a significant number were unable to associate the name pentagon with the 5-sided polygon.

(c) (2 marks)
A diagram had to be enlarged to fit within a circle which had been provided. Candidates who used a pair of compasses were usually successful, while those who only used a ruler were less accurate, typically gaining 1 of the 2 marks. Freehand responses were inappropriate and were not sufficiently accurate to receive any marks in this part.

(d) A small cereal box was to be packed into a larger carton. The dimensions of both boxes were marked on the diagram provided.

(i) (1 mark)
Most candidates were able to correctly calculate the maximum number of cereal boxes that would fit into the carton. This was done either by comparing dimensions or by dividing volumes. The second method received the mark, even though it gives no assurance that the number of smaller boxes will actually fit.

(ii) (2 marks)
Correct answers for the surface area of the carton were relatively rare. One mark was awarded if only one mistake was made. Candidates who assumed that there was no lid on the carton, and those who calculated the surface area of the cereal box instead of the carton, were able to receive both marks if the computation was clear and correct.

(iii) (2 marks)

Candidates were asked to show two different ways of packing the boxes into the carton. The responses ranged from clear and accurate to very untidy and undecipherable. Freehand responses were acceptable, but in many cases they were not clear enough to provide an answer to the question.

(iv) (1 mark)

Candidates were asked to sketch the net of the cereal box. Most drew unscaled diagrams of nets of rectangular prisms, without accounting for relative sizes of the faces of the box. While this was disappointing, these responses were awarded the mark.

A large number of candidates were unable to answer this question.

(e) (2 marks)

A diagram with concentric squares was provided, and candidates were asked to calculate the fraction that was shaded. A common approach was to divide the figure into 25 equal squares.

Most candidates who attempted this question gained at least one mark. Many mistakes were made through carelessness in the counting or calculation, or through misreading the question.

Question 35   Social Issues

(a) (1 mark)

This question involved the calculation of a probability, where choice of appropriate data was required. Many candidates were able to make the appropriate choice and found a correct expression for the probability.

(b)   (i) (1 mark)

Candidates were asked to choose the least humid day from a table. Most did so correctly.

(ii) (1 mark)

Many candidates were not able to provide an acceptable answer by indicating that '° C' stands for Celsius or centigrade.

(iii) (1 mark)

Candidates were asked to find the days on which the maximum temperature was expected to exceed 27° C. Many incorrectly included a day on which the forecast maximum was 27° C, but most recognised that 'exceed' was equivalent to 'greater than'.

(iv) (1 mark)

Candidates were asked to calculate the average 'High' temperature over the four-day period. This was generally well handled, but many failed to divide by 4, and a few added up both the four 'High' temperatures and the four 'Low' temperatures.

(c) A table was provided with population figures, average life expectancy, and population per doctor for 4 different countries. A common mistake was to choose an incorrect figure from the table for the various parts of this question. Another was to transcribe a figure incorrectly.

(i) (1 mark)

Many candidates were able to correctly calculate the number of doctors in Oman. This involved choosing two figures from the table and dividing appropriately.

(ii) (1 mark)

Candidates were asked which country has the greatest number of doctors per person. While many were able to interpret this and use the figures accordingly, many simply chose the larger figure from the population per doctor column.

(iii) (1 mark)

This question required the computation of Australia's population as a percentage of Japan's population. While a large proportion of the candidates successfully completed this, many used incorrect figures from the table (eg. Oman instead of Japan). Quite a number of others used correct figures in incorrect places within the calculations. For example, many performed the calculation

Australian population × Japanese population/100 .

(iv) (1 mark)

Candidates were asked to draw in a missing column on a column graph, using a figure from the table. Candidates whose work displayed some care were usually successful. Careless, freehand responses were common and were often not sufficiently accurate to receive the mark.

(d) The question stated that there are 99 million households with TV sets in the United States, and that random surveys about viewing habits of 50 000 households were conducted.

(i) (1 mark)

This part asked for the percentage of households with TV sets that were surveyed. A high proportion of the candidature approached this question correctly. Quite a number became confused with the position of the decimal place and questions of rounding. Transcription errors,

in particular those involving too many or too few zeros in figures, were common.

(ii) (1 mark)

Candidates were asked to explain why each household has an equally likely chance of being selected for this survey. Most recognised that the concept of randomness of the survey was germane to this question. However, many tried to argue that it was because most houses had TV sets.

(iii) (1 mark)

Candidates were asked to estimate the number of households watching a particular show based on the information that it was watched by 30 000 of the households surveyed. A substantial number obtained the correct answer. However, there was a wide variation in the placement of the numbers 30 000, 50 000 and 99 million within the computation.

Mathematics in Society

Questions 1 – 20   Multiple-Choice Questions

For each response to each of the multiple-choice questions, the table lists the percentage of candidates making that response and the mean number of correct answers to all 20 multiple-choice questions given by such candidates. The data for the correct answer is in bold face.

Quest No.	Omit %	Omit Mean	A %	A Mean	B %	B Mean	C %	C Mean	D %	D Mean
1	0.19	6.2	**70.13**	**11.1**	6.29	6.4	21.06	7.6	2.32	6.4
2	0.13	5.9	19.10	10.6	9.63	6.7	4.40	6.8	**66.75**	**10.4**
3	0.24	7.4	12.99	6.8	13.66	7.5	**57.20**	**11.6**	15.91	8.6
4	0.14	4.9	4.46	5.8	**69.43**	**11.1**	7.28	7.3	18.68	7.7
5	0.24	7.5	21.52	7.7	56.31	10.3	7.23	8.3	**14.70**	**12.8**
6	0.24	6.1	**59.33**	**11.1**	28.36	8.8	9.69	6.8	2.39	6.3
7	0.67	7.8	15.01	7.5	16.81	8.1	**54.79**	**11.8**	12.73	7.2
8	0.16	5.0	7.81	6.8	**79.86**	**10.8**	9.55	6.3	2.63	7.2
9	0.27	7.8	**44.02**	**11.6**	18.07	8.5	12.93	9.5	24.72	8.3
10	0.30	7.4	8.11	6.8	21.46	8.7	28.33	8.8	**41.80**	**12.0**
11	0.15	6.7	8.94	7.7	27.16	7.3	**56.99**	**11.7**	6.76	8.9
12	0.35	7.7	5.82	7.2	32.56	8.1	16.69	7.7	**44.58**	**12.5**
13	0.41	7.5	**30.79**	**12.7**	19.63	9.0	40.52	8.4	8.66	9.4
14	0.17	5.5	12.14	7.9	**47.76**	**11.6**	9.63	7.6	30.31	9.0
15	0.51	7.8	**44.22**	**11.8**	14.08	8.1	10.98	7.9	30.21	8.9
16	0.22	5.9	45.47	8.7	**42.49**	**12.1**	7.60	7.2	4.22	7.0
17	0.31	7.7	5.87	10.9	**21.88**	**12.6**	19.07	9.8	52.87	8.8
18	0.29	7.0	10.11	9.7	13.85	9.5	**45.97**	**11.8**	29.78	7.3
19	0.35	7.0	4.17	7.1	41.36	8.7	8.99	7.3	**45.12**	**11.9**
20	0.26	6.8	14.66	8.0	15.43	7.8	**56.26**	**11.3**	13.40	8.9

Question 21

(a)   (i) (1 mark)

Most candidates attempted this question and the majority used Pythagoras' theorem. Some used trigonometry or scale drawing techniques. The most common error was $\sqrt{14^2 + 6^2} \cong 15$ instead of $\sqrt{14^2 - 6^2}$. There was some confusion over the direction to give the answer 'to the nearest tenth of a metre', with many taking this to mean that they should round their answer to the nearest ten.

(ii) (1 mark)

Most candidates attempted this question by adding the area of a

rectangle and the area of a triangle. Some used the trapezium formula.

Many attempted to use the slant height of the triangle (ie. 14) instead of the perpendicular height found in (i). Candidates should look for the connection between related parts.

(iii) (2 marks)
This was not very well done. Many candidates knew that they needed to make use of the numbers 0.3 and 4, but did not know what to do. Many chose to divide by both numbers.

The context provided in the question required a sufficient number of whole bags to be purchased. This meant that the answer had to be rounded up, but many candidates either rounded down or did not round at all.

(b) (i) (1 mark)
Nearly every candidate correctly answered 1/12.

(ii) (1 mark)
Candidates who drew a tree diagram to help answer this question were generally successful. Those who didn't found this part confusing and had little success. Wrong answers without any working were common, making this the worst section of Question 21.

(iii) (1 mark)
This part was not well done. Many chose to simply subtract the answer to part (b) (ii) from 1 rather than calculate the required lose-lose probability. Again, candidates with a tree diagram had more success.

The most common answer, by far, was 11/12. This was presumably the result of candidates considering only one spin of the wheel.

(c) (i) (1 mark)
Candidates made better attempts at this part as they were assisted by a tree diagram in the question.

The most common error resulted from a failure to adjust the fractions in the second selection to take account of the non-replacement of socks.

Some candidates extended the tree diagram to take into account several selections of socks and became confused.

(ii) (2 marks)
Regardless of the correctness of their answers to part (c) (i), a large number of candidates knew what was required in this part.

Common errors included confusion as to whether to multiply or add the probabilities along the branches, and then whether to add or multiply the results of each branch. Those who showed no working were often unable to receive even one mark as it was not possible to determine how the answer had been obtained.

Many candidates obviously did not use a calculator in adding or multiplying fractions and there were many incorrect answers from correct working. For example, $\frac{2}{15} + \frac{1}{3} = \frac{3}{18}$ was common.

(d) (2 marks)

Candidates were given the benefit of considerable doubt when determining the meaning of their answers, with many dubious answers gaining both marks.

There was a lot of confusion over the units and the transition to hours and minutes from 800 minutes was poorly handled. For example, many wrote 13.33 hours as $13\,\text{hr}\,33\,\text{min}$.

Question 22

A particular concern throughout this question was the incorrect use of calculators with trigonometric functions. There were many problems with the order of operations, no doubt compounded by the different implementation of these functions on approved calculators. On too many occasions, $40\tan 35°$ became $\tan 40° \times 35$ and $13 - 12\cos 135°$ became $\cos 135°$. There were also a few instances where calculators were set in either radian or gradian mode.

(a) (1 mark)

Many candidates were either unable to translate the calculator display of 1.328^{-13} to the correct answer of 1.328×10^{-13} or were unaware of the need to do so. Other common mistakes were rounding of the answer which was inappropriate in the context, or simply performing the wrong computation.

(b) (2 marks)

Many candidates tried squaring both sides but a significant number did so incorrectly, obtaining such expressions as $25x + 1 = 1296$, $5x + 1 = 6$, or $5x = 1225$ via the intermediate step $\sqrt{5x} = 35$.

Even amongst those who correctly obtained $5x + 1 = 1296$, a number made basic arithmetic mistakes which prevented them from obtaining $x = 259$.

(c) (2 marks)

Many convoluted attempts were presented, overlooking the obvious use of tangent ratios in right triangles with angles of 30° and 35°. A number of candidates correctly found the hypotenuse of at least one of these triangles and then used the sine rule correctly in the 5° triangle.

It was disappointing to see so many candidates attempting to use $\tan 5°$ as though the 5° angle were in a right triangle. Others attempted to use either the sine or cosine rule, but used angles and sides from different triangles.

(d) (i) (1 mark)
Candidates generally produced good diagrams which displayed the relative distances with the correct orientation.

(ii) (2 marks)
Many candidates were unable to translate the change of direction from east to north-east into the fact that the angle at this point in the triangle was 135°. This meant that many correct substitutions and calculations were made using the cosine rule with a wide variety of angles.

The most common error was the failure to find the square root at the end of the cosine rule calculation.

(e) (i) (1 mark)
This part was generally well done with most candidates obtaining the correct angle of 3°. However, it was notable that some candidates did not appear to understand the meaning of the symbols $\angle BMC$.

(ii) (2 marks)
Most candidates knew that this was the part in which they should apply the sine rule, and the substitution and calculation were generally well done. However, others did not see the relevance of the sine rule, and attempted to use right triangles, often by incorrectly assuming that $\triangle BMC$ was a right triangle.

(iii) (1 mark)
This was generally well done, particularly by those who made progress with the previous part. One concern was the use of the sine rule by candidates in the right triangle AMC. Obviously this is not incorrect, but it does show that a significant number of candidates were not able to identify the best method of solution.

Question 23

(a) (i) (1 mark)
This was well answered by a majority of the candidature. The most common error was to give the frequency as the answer for the mode. Many gave both, writing either $9 = 21$ or $21 = 9$. A significant number supplied working which showed that they had found the median, and these candidates were not awarded the mark.

(ii) (1 mark)

This was very well answered, although it was not uncommon to see mistakes which apparently arose from incorrect use of the calculator. Many correctly wrote 21×9, but then gave an incorrect answer.

(iii) (1 mark)

Many candidates were able to calculate the mean correctly either by using the statistical features of their calculator or by totalling all the scores and dividing by 30. The most common incorrect answer was 21.5, which resulted from candidates adding the six individual scores and dividing by 6.

(iv) (1 mark)

Many candidates were able to find the standard deviation with the assistance of their calculator. Answers which were consistent with incorrect data obtained in previous parts were awarded the mark, as was 1.67, the sample standard deviation.

(v) (1 mark)

This was well answered by a majority of the candidature. Some candidates did not answer this part, but correctly plotted the cumulative frequency for $23°$ in the graph for the following part.

(vi) (1 mark)

The responses to this part were quite good, with the most significant problem arising from candidates mis-reading the scale on the partially completed graph. Some candidates omitted the cumulative frequency for $22°$, presumably as a result of the frequency being 0. Others drew the frequency histogram.

The number of candidates who did not use a ruler for this part was a concern.

(vii) (1 mark)

This was poorly answered, with many candidates omitting this part entirely. Candidates did not know where the cumulative frequency polygon should begin and often drew it using the centres or the top left hand corners of the columns. Some candidates who correctly drew the polygon to the top right hand corners of the columns had difficulty with the two columns of equal height corresponding to $21°$ and $22°$.

(viii) (1 mark)

This part was very poorly answered. Many candidates had no idea and failed to answer. Candidates were required to demonstrate that the interquartile range was related to the cumulative frequencies of 7.5 and 22.5, and read the corresponding interquartile range from their polygon.

Some were able to begin, but failed to clearly indicate a range in any form. Another common error was to mis-read the lower quartile as 20.5 instead of 20.1.

(b) (1 mark)

Candidates often displayed poor language skills in justifying their conclusion. Examiners were looking for an indication of the difference between the surveyed dentists and all dentists or a correct statement concerning the sample size.

Common answers that did not receive the mark were literal translations such as '2 out of 3' is the same as 2/3 and direct quotation or simple restatement of the claim in the question. Candidates who answered yes or no, without justification, were not awarded the mark.

It appears that many candidates did not understand what this question was about.

(c)   (i) (1 mark)

This part was generally well answered. The most common error was to state the probability of the complementary event. Many counted all 48 squares, rather than the 36 squares which represented the sums of the two rolls. As a result, 1/36, 34/35 and 47/48 were common incorrect responses.

  (ii) (1 mark)

Many candidates answered correctly without showing any working, demonstrating some ease in handling the table in the question. The fraction was rarely reduced to the simplest form.

Many did not use the lattice diagram and attempted working which did not follow from the information given. Some showed incorrect working leading to the correct answer. Such answers were not awarded the mark.

 (iii) (1 mark)

This part was poorly answered. Candidates who attempted this used many combinations of numbers to arrive at an answer. A large number attempted to draw a tree diagram without success.

Simple techniques such as shading the 11 squares corresponding to events in which the second roll gives a number which is greater than that obtained on the first roll made this question quite easy. However, very few candidates seemed to be aware that such techniques enabled the table to be used for a different purpose.

Very few answers had any working, and the most common response was 1/2 or 50%.

Question 24   Space Mathematics

This question contained parts dealing with units of measurement, eccentricity and its significance, the average distances of planets from the sun (Kepler's third law) and the escape velocity from planets. On the whole, the question was answered reasonably well. The main weaknesses were the failure of candidates to read the question carefully enough to extract all the relevant information, not answering what was asked, poor calculator work with large numbers and poor handling of formulae.

(a) (2 marks)

A reasonable number of the candidates scored full marks. Many were let down because they did not know the meaning of 10^{11}. Typically, they interpreted it as 10×10^{11}, repeating the common error in Question 22 part (a).

(b)   (i) (1 mark)

This question was reasonably well answered. A significant number of candidates measured from the diagram, with varying degrees of accuracy, encouraged by the absence of a 'not to scale' label. It is not clear how many of these would have been able to compute b/a directly from the information $\dfrac{QM}{PM} = \dfrac{2}{3}$ if the diagram had been labelled 'not to scale'.

(ii) (1 mark)

This was well done. Almost all those who had written down b/a correctly had no problem in solving the equation for e.

(iii) (1 mark)

As in part (ii), success in the previous part usually ensured success here. Many simply computed the length of CS and did not proceed to write down the coordinates of S.

(iv)   1. (1 mark)

Those who realised that $e = \dfrac{CT}{CA}$ scored the mark. Many were puzzled by the negative 1 and gave the answer as -1/3. This showed both a lack of understanding of eccentricity and of the fact that the formula involves distances.

In general, this part was poorly done. Many confused the b/a ratio with the eccentricity, e.

2. (1 mark)

Candidates generally gave good descriptions. The best answers used the fact that the second focus T is closer to the centre than S to deduce that the second ellipse is outside the first, but

it was noticeable that there were a number of centres in which almost every candidate worked out the b/a ratio for the second ellipse and compared it with the first ellipse.

Many incorrectly believed a circle has $e = 1$, which led them to the wrong conclusion from a correct value for e.

(c) (i) (1 mark)
This was very well done. Quite a number of candidates had trouble with scientific notation. Once again, the differences between mathematical notation and calculator notation led candidates to enter 1.5×10^8 into their calculator by keying $1.5 \times 10 \exp 8$, which gives 1.5×10^9, instead of $1.5 \exp 8$.

(ii) (1 mark)
This part was poorly answered, and many candidates with correct answers to the rest of this question did not receive this mark. Many had no idea of the meaning of a light year, while others made errors such as using the wrong value for the number of days in a year.

(iii) (1 mark)
Many candidates did not realise R must be in AU and T in years, even though this was explicitly stated in the question. Others substituted a value for T to find R instead. Almost all those who realised that $R = 30$ had little trouble obtaining T correctly, with only a small number unable to go beyond $T^2 = 27\,000$.

(iv) (2 marks)
Most candidates were able to substitute values into the formula correctly, but could not proceed further, usually because they were not able to square both sides correctly. These candidates received 1 mark.

Only a very small percentage of the candidates received both marks, and some of these had succeeded by a process of trial and error.

Question 25   Mathematics of Chance and Gambling

This question contained four parts dealing with the language of chance, Pascal's triangle, fairness and percentage margin.

On the whole, the question was not well answered. Less than 1% of the candidature scored full marks. Many were unable to tell the difference between 'odds on' and 'odds against' and were unable to express them as probabilities. Failure to understand the significance of Pascal's triangle made part (c) (ii) difficult to answer.

(a) (i) (1 mark)
This part was very well done. A significant number of candidates answered as if Susan had shown her card and returned it to the pack before Stephen's card was dealt.

(ii) (1 mark)
This part was reasonably well done, although many candidates believed that only 12 cards (Jack, Queen and King) were above 10 and so ended up with an answer of 12/51.

(b) (i) (1 mark)
This part was poorly done. Common answers were 1/100 or 1/12.

(ii) (1 mark)
This was reasonably well done, with many candidates realising that the total value of the prizes must be $500. A number of candidates believed that as the first prize was $300 and the total amount for third prize was $100, the value of the second prize must be $200.

(c) (i) (1 mark)
Approximately half of the candidature obtained the mark. Many omitted the 1 from one or both ends of the line.

(ii) (1 mark)
Less than 1% of the candidates scored this mark. Most tried unsuccessfully to complete a tree diagram but found that 5 choices with 32 outcomes was just too large for them to handle.

(d) (i) (1 mark)
This part was also poorly done. Many were under the impression that the team with the shortest odds was the least likely to win and so answered antelopes or, ignoring the word 'on', camels.

(ii) (1 mark)
The language of chance was poorly understood by the candidates. Many calculated the winning amount ($20), rather than the amount collected from the bookmaker ($30).

(iii) (1 mark)
This was very badly done. Very few candidates knew the term 'odds on', and so the answer $45 was much more common than the correct answer, $20.

(iv) (1 mark)
This was reasonably well done, although a lack of understanding of the difference between winning and collecting led many candidates to give the answer $20.

(v) (2 marks)

This part was left unanswered by most candidates as they had no understanding of 'percentage margin'. Less than 1% of the candidates were awarded 2 marks for this part.

Question 26   Land and Time Measurement

This question was very straightforward and followed the style of questions from previous years. As a consequence, there were relatively few candidates who received marks less than 4 marks.

(a)   (i) (1 mark)

This was poorly done by a significant number of candidates. Many had great difficulty in expressing the ratio in its correct form and it was often oversimplified. For example, 1 cm : 10 m frequently became 1 : 10. Many had difficulties changing a ratio such as 1 cm : 10 m to an expression without units.

Some candidates seemed to be confused with the conversion between units of length and gave answers such as 1 m = 100 cm. It was sometimes difficult to distinguish these answers from correct answers. A question designed so that the correct answer was 1 : 50 or 1 : 200 might give a better indication of the level of real understanding of these concepts.

(ii) (1 mark)

This was quite well done as most candidates were able to find the correct value. It was obvious that candidates did not use the scale given as their answer to part (i). Many who lost the mark in part (i) still obtained the correct answer here by using the correct scale. The most common mistake was to use Pythagoras' theorem to find the missing side.

(iii) (1 mark)

Those who had a protractor answered this question easily. Candidates who used the cosine rule to find the angle were also very successful, especially considering that the formula was not given to them here. The most common mistake was to use right-angle trigonometry to find the angle. The use of the word 'measure' in the question may have encouraged greater use of a protractor.

(iv) (1 mark)

This part was generally well done, with most candidates able to substitute in and evaluate the expression correctly. Once again, it was apparent that some candidates had their calculator set in radian or gradian mode.

(b) (2 marks)

This was easily the most successfully completed part of the question. Most were able to attempt this and gained either 1 or 2 marks. Common mistakes included mixing up the values of D_M and D_L, forgetting to multiply by 4 or using $h = 180$ instead of 90. Errors in calculator use such as ignoring the brackets and mis-reading the final display were also common. For instance, 24 900 often became 2490.

(c)   (i) (2 marks)

The greatest problem encountered by candidates was in finding the correct angle to use. Candidates added or subtracted latitude and longitude angles seemingly at random, while others only used one of the longitude angles.

Many made transcription errors, writing $105°W$ longitude as $150°W$ in their writing booklets. Others made careless mistakes such as $105° + 150° = 225°$. Once an angle was found, candidates were able to calculate the corresponding time difference. However, candidates who had an incorrect angle often demonstrated that some confusion still exists between decimal time and hours and minutes.

   (ii) (1 mark)

Candidates had a great deal of difficulty in answering this part. Many did not provide all the details that the question required, leaving out either the day or an indication of am or pm. A significant number subtracted the time instead of adding it, while others had difficulty carrying out the actual addition or subtraction of time, giving answers that were entirely inconsistent with their answer in part (i).

(d)   (i) (1 mark)

This question was well done, in part because 34° was emphasised in the wording of the question. A common mistake was to use a wrong angle, such as one obtained by adding or subtracting the two longitudes.

   (ii) (2 marks)

This part was poorly done. Many candidates gained 1 mark here as a mark was awarded if the computation involved multiplication by 132/360 or the circumference of the relevant circle had been calculated. Both marks were available for the few candidates who worked on the basis that the plane flew west along the parallel of latitude.

Some did not see the link between this part and the previous part. Errors arose from not being able to determine the relevant angle of 132°, use of $1° = 60$ nautical miles (which is only appropriate for great circles) and the omission of the 2 or the π in the formula for the

circumference of a circle. Other candidates incorrectly used 6400 km as the radius of the small circle.

Question 27   Personal Finance

This question required candidates to read and interpret tables of information in three different contexts and then use calculators to process the given information. The question was ususally well done, with many candidates scoring full marks.

However, many candidates lost marks by making careless errors or by not showing all necessary working. Without working, it is impossible to award part marks. Candidates should be aware that the full marks are sometimes awarded for finding a correct numerical expression, even if the evaluation is incorrect, but only if the numerical expression is recorded in the writing booklet. Candidates should also examine their answers to see that they are reasonable.

(a) This part involved reading a table, calculating percentages of amounts and finding an average.

(i) (1 mark)
This part was answered correctly by about 90% of the candidature. The most common error was finding 6% rather than 4% of $130 000. A significant number of candidates found the correct answer, $5200 and then continued to calculate the amount which the sales representative would receive.

(ii) (1 mark)
This part was generally well done. A number of candidates misinterpreted the question and found the commission as if the table had presented the fees as a sliding scale, leading to the answer $14 000. Other common errors included finding 5%, 6% or 9% of $340 000.

(iii) (1 mark)
A majority of the candidature completed this part correctly. Candidates with incorrect responses in part (ii) were often able to find the correct answer here, although answers based on the previous answer were also accepted. The most common error involved including the entire amount of the agency's commission in Joyce's pay. Many added $450 instead of $150.

(iv) (1 mark)
A significant number of candidates simply divided the answer to part (iii) by 3, forgetting to add $300, while a large number of candidates just found the total income of $1392 for three weeks. Another common error was to add $450 to the answer in part (iii), forgetting that $150 had already been included.

(b) This part involved using a premium table where the rates were given in $ per $1000.

   (i) (1 mark)
   A large number of candidates calculated either $90\,000 \times 1.65$ or $90\,000 \div 100 \times 1.65$. Apparently, a premium of \$148\,000 or \$14\,800 for insurance cover of \$90\,000 did not seem excessive.

   (ii) (1 mark)
   Errors that occurred in part (a) were often repeated in this part. Common incorrect answers were \$6.93, \$693 and \$69\,300.

   (iii) (1 mark)
   This was poorly done, with only a slight majority of the candidature answering this part correctly. Common errors included $\$84.70 \div 0.77\ (= \$110)$ and $\$84.70 \times 0.77 \times 1000\ (= \$65\,219)$.

(c) This part involved the use of an income tax table, with the additional complication of the Medicare levy.

   (i) (1 mark)
   This was answered successfully by a large number of candidates. The most common error was to subtract $39\,401$ instead of $39\,400$. A number of candidates also calculated 5600×44 instead of 5600×0.44. Once again, candidates must consider whether answers are reasonable.

   (ii) (1 mark)
   A large number of candidates found 1.5% of the answer in part (i) instead of 1.5% of \$45\,000. Many candidates calculated the Medicare levy on $\$45\,000 - \$11\,610$. Answers of \$6.75, \$67.50 and \$6750 were also common.

   (iii) (1 mark)
   This part was poorly done. Although the mark was awarded even if candidates omitted the Medicare levy from their calculations, only 60% of the candidature was awarded this mark. Many candidates responded by stating that \$11\,960 was owed. It was disappointing to see that a significant number of candidates did not know the number of fortnights in a year. Many used 12, 24 or 28 in their calculations.

   (iv) (2 marks)
   This part was very badly answered. The most common incorrect answers were $22\,\text{c}$ or $13\,800 \div 4560 = 30\,\text{c}$. Another common response was $32\,\text{c}$, the average of $20\,\text{c}$ and $44\,\text{c}$. A significant number of candidates also wrote $13\,800 \div 4560$ instead of $4560 \div 13\,800$, yet still managed to obtain the correct answer of $33\,\text{c}$.

   Many candidates wrote $0.33\,\text{c}$ instead of $33\,\text{c}$ or \$0.33. The trial and error method was common with candidates substituting various values

for A into the formula $3140 + A \times 13\,800$ in order to find a value close to 7700. A very small number of candidates used the figures in the table, calculating $(9146 - 3140) \div (39\,400 - 21\,200) = 6006 \div 18\,200 = 33\,\mathrm{c}$.

Question 28   Mathematics in Construction

This question provided opportunities to gain some easy marks, particularly in part (a) where most of the questions were straightforward. Unfortunately, the ability to read and interpret building plans was poor amongst this year's candidates, as was their understanding of technical terms.

Part (b) also provided similar opportunities for candidates with good spatial awareness, so it was surprising to find that 10% of the candidature left out this entire part. Examiners could only conclude that they failed to read the instructions given on both page 23 and 25 indicating that there were more parts of this question on page 26.

(a)   (i)   (1 mark)

This part was reasonably well answered and should have been obvious from the plan of the house. Many candidates tried to measure and then convert the units, with 300 mm being the most common answer. The required answer was 270 mm (or equivalent) which could be read directly from the plan. Some candidates gave unreasonable answers for the thickness of a wall such as 270 cm or 270 m, indicating poor skills in the appropriate use of units.

    (ii)   (1 mark)

Many candidates expected to find the scale on the plan, sometimes writing 'what scale?' in response to this question. Those who measured usually came up with the right answer provided they had selected an appropriate length on the plan to use as the basis for their computation.

(iii)   (1 mark)

Many candidates were not able to correctly interpret the phrase 'the height of the pergola above the paved play area'. Many confused the level of the paved areas with the level of the balcony wall enclosing the play area and the walkways.

The height of the pergola was also confused, with some candidates measuring to the top of the cross-beam rather than the bottom. Most candidates were able to measure to within the 1 mm tolerance allowed by the examiners and use the scale to convert to a measurement which fell within the accepted range. Again, some answers were simply unreasonable.

(iv) (1 mark)

This part was generally well answered. Most candidates could clearly show that the area would result from $4100\,\text{mm} \times 2600\,\text{mm}$ but some had trouble converting back to square metres. Typical incorrect answers were $1066\,\text{m}^2$ and $10\,660\,\text{m}^2$. Some candidates were careless when reading the width of the bedroom from the plan and used the length of the bathroom ($2150\,\text{mm}$) instead in their calculations.

(v) (1 mark)

Very poor responses were typical in this part, with most of the candidates interpreting the question incorrectly. Many did not understand that the carpet was sold by the lineal metre and continued to make calculations based on the answer to part (iv) as if the carpet cost $155 per square metre. A significant number of candidates pursued calculations based on two strips of carpet despite the statement that the carpet was to have no joins.

(vi) (1 mark)

Markers expected candidates to find this part very easy, but it was not answered very well at all. The most popular answer was the family room, with few candidates giving the correct answer which was the rumpus room.

(vii) (1 mark)

This was generally well answered. Incorrect answers usually resulted from attempts to count all of the windows or, in some cases, all of the glass panes in the timber-framed doors.

(b) (i) (1 mark)

This part was generally well answered, perhaps because there were so many correct possibilities. Incorrect answers were usually pairs of parallel lines.

(ii) (1 mark)

This question was by far the most difficult to mark due to the variety of responses. Candidates who did not interpret vertical section correctly usually gave a front, side or top view of the house. Others gave a section perpendicular to the line EF. With diagrams that were not labelled it was not possible to distinguish whether the candidate was describing the correct section $EGHF$ or the part of the roof $EBCF$. Several candidates indicated that their answer was on the diagram in their question paper. Unfortunately, this was not available to the markers, and so it was not possible for these candidates to gain the mark.

(iii) (1 mark)

The extremely poor response to this part was largely due to mis-interpretation of the question. The required answer was $\angle EGH$.

Many candidates tried to describe the type of angle rather than name the angle. Incorrect responses included acute, obtuse, hipped, gable, pitch, EGB, EBG and G.

(iv) (2 marks)

This part was very well answered. Most correct answers were the result of applying Pythagoras' theorem, although some candidates succeeded by using the cosine rule or some other trigonometric method. The most common error was to use the length AB in places where the length of BG was required. Some candidates who chose to use trigonometric methods failed to find angle EBG correctly or simply assumed it was 45°. This simplified the question and did not attract any marks.

Mathematics 2/3 Unit (Common)

Question 1

As expected, this question was attempted by almost all candidates and the overall standard was quite pleasing. A good proportion of the candidature scored 10 or more marks.

Two important general points need to be made. Firstly, many candidates seemed to think that the whole question needed to be answered on a single page. The cluttered setting out and errors that this strategy leads to made marking this question more difficult. Secondly, a substantial number of candidates began working on Question 2 in the same booklet, and then wasted time copying their answers to another booklet.

(a) (2 marks)

A surprising proportion of the candidature did not know the midpoint formula. The most common variant was $(\frac{x_2 - x_1}{2}, \frac{y_2 - y_1}{2})$. The gradient formula was also used regularly. Graphical attempts to find the midpoint were very rare.

(b) (2 marks)

Candidates were asked to find the value of e^3, correct to three significant figures. A lack of understanding of significant figures was apparent, with many candidates who answered the rest of Question 1 correctly giving their answer to three decimal places.

It was also common to find that candidates had given the value of $\ln 3$ instead of e^3. Those who used scientific notation often gave the wrong power of 10.

(c) (2 marks)

As would be expected, many failed to reverse the inequality sign when dividing by -2 in the course of solving this linear inequality. However, markers were more surprised to discover the number of candidates who treated the question as if it was the absolute value inequality $|3 - 2x| \geq 7$ and obtained the solution $x \leq -2$ or $x \geq 5$. These candidates were awarded 1 mark.

Another common approach was to first solve the equation $3 - 2x = 7$, and then test the inequality on either side of the solution $x = -2$. When this was done correctly, candidates were awarded full marks.

(d) (2 marks)

Both the elimination and substitution methods were used by candidates to solve the pair of simultaneous equations.

Examiners found far too many examples of transcription errors, in which candidates wrote such things as $x - y = 1$ instead of $x + y = 1$, $2x - y = 1$ instead of $2x - y = 5$ or $2x + y = 5$ instead of $2x - y = 5$. Candidates were awarded full marks if they successfully solved the pair of simultaneous equations resulting from these mistakes, as they required the same skills and were of the same degree of difficulty.

Poor algebraic skills were common. Many candidates using the substitution method incorrectly rearranged the first equation to obtain $y = x - 1$, while those using the elimination method often obtained $x = 4$ as a result of an incorrect attempt to subtract the first equation from the second.

A substantial number of candidates found a solution for either x or y, but did not then proceed to find the other solution.

(e) (2 marks)

Finding the correct expansion of $(5 - \sqrt{2})^2$ was the first difficulty, with answers such as $25 - 10\sqrt{2} + 4$, $25 + 10\sqrt{2} + 2$, $25 - 2$ being quite common. Candidates who correctly obtained $27 - 10\sqrt{2}$ were awarded the first mark.

For the second mark, candidates needed to state the appropriate integer values for a and b from their expansion. Many thought that $b = -10\sqrt{2}$.

Many candidates altered their correct expansion $27 - 10\sqrt{2}$ to $-27 + 10\sqrt{2}$. Presumably, this was the result of candidates believing that the expression $a + b\sqrt{2}$ required a positive value for b.

(f) (2 marks)

The formula for the area of a sector was not well known, with a sizeable proportion of the candidature proceeding by calculating a fraction of the area of the complete circle.

The formula for the area of a segment, $A = \frac{1}{2}r^2(\theta - \sin\theta)$ and the incorrect formula, $A = \pi r^2\theta$ were used frequently.

Many candidates did not understand the importance of using radians rather than degrees. Some found the correct answer, 40π, but could not be awarded the second mark because they then proceeded to claim that the area was 7200, apparently believing that $\pi = 180$. Others left their answer in unsimplified form as $200\pi/5$.

Another common error arose from a lack of attention to detail. Many wrote $A = \frac{1}{2}r^2\theta = \frac{1}{2} \times 20 \times \pi/5 \ (= 2\pi)$. These candidates were awarded 1 mark.

Question 2

This question consisted of two parts, involving integration and coordinate geometry.

Most candidates handled this question well, with a large number scoring full marks. The small proportion who received no marks on this question consisted almost entirely of candidates who had not attempted the question at all.

(a) Candidates were asked to find two indefinite integrals. Most were able to score marks for one or both integrals but there were many notational inaccuracies such as writing the integral sign as part of the answer and omitting the brackets in writing $\sin(2x + 1)$. A large proportion of the candidature did not include the constant of integration in their answers.

(i) (2 marks)

Candidates were required to find $\int \dfrac{1}{x^2} + \dfrac{1}{x}\,dx$. Most recognised the need to write $1/x^2$ as x^{-2}, and most were then able to integrate this term correctly. However, many tried to treat $1/x$ in the same way, failing to recognise this as one of the standard integrals and seemingly unaware of the problem with their answer, $x^0/0$.

On the other hand, some candidates correctly found $\ln x$ as the integral of the $1/x$ term, but tried to treat the term $1/x^2$ in the same fashion, obtaining a variety of incorrect answers.

(ii) (2 marks)
This part asked candidates to find $\int \cos(2x+1)\,dx$. Those who saw the connection with the standard integral $\int \cos ax\,dx$ given in the table of standard integrals were usually able to gain full marks.

Common errors included attempts to differentiate or expand the function. An incomplete understanding of integration commonly led to answers such as $-2\sin(2x + 1)$, $\sin(2x + 1)$ and $\dfrac{1}{2x + 1}\sin(2x + 1)$.

(b) Candidates were given a diagram showing two lines forming a triangle with the x axis. Those candidates who copied the diagram into their writing booklet were able to avoid most of the errors with coordinates of points and lengths that were common in answers which were not accompanied by a copy of the diagram.

(i) (1 mark)
Almost all candidates attained this mark. A small percentage of candidates did not know the gradient formula $\dfrac{y_2 - y_1}{x_2 - x_1}$. These candidates usually used $\dfrac{x_2 - x_1}{y_2 - y_1}$, and were presumably reassured when they obtained the value stated in the question.

Candidates who arrive at an answer which is different from the one stated in the question need to understand that they have almost certainly made at least one error and make some attempt to find and correct it.

(ii) (1 mark)

Once again, most candidates scored this mark. A few failed to handle the -2 when substituting the coordinates of A into the point-gradient form of the equation of the line, while others interchanged the x and y coordinates for their chosen point.

(iii) (1 mark)

There were several possible approaches to this part. Two short solutions were $m = 1$ so $\tan\theta = 1$ and so $\theta = 45°$ or $\triangle ABD$ is isosceles and $\angle D = 90°$ so $\angle BAC = 45°$.

Candidates following one of these approaches were nearly always successful. Some candidates did pages of unnecessary work, finding lengths of sides and using either the sine or cosine rule.

(iv) (1 mark)

Those candidates who recognised that Pythagoras' theorem could be applied usually gained the mark. A common mistake was for candidates to use the x coordinate of B as the height of the triangle, rather than the y coordinate.

Candidates who did not recognise the application of Pythagoras' theorem often ended up writing many pages of unnecessary work.

(v) (2 marks)

The simple approach using $A = \frac{1}{2}bh$ nearly always led to full marks. Candidates who used $A = \frac{1}{2}ab\sin C$ often used an incorrect combination of sides and angle. Some candidates did not realise that the length of AD was 5 units.

(vi) (2 marks)

Once again, many approaches were possible. Candidates using the cosine rule or $\angle DBC + \angle ABD$ or $180° - (\angle BAD + \angle BCA)$ obtained the correct answer without having to consider whether the angle was obtuse or acute. Many candidates who first found $\sin\angle ABC$, either by applying the sine rule or using the formula $\frac{1}{2}ac\sin B$ for the area of a triangle, did not realise that there is both an acute and an obtuse angle corresponding to this value. These candidates almost invariably gave the size of an acute angle as their answer, but the correct answer was an obtuse angle.

Question 3

This question was based on differentiation, the cosine rule and simple deductive geometry. The question was answered well, with approximately 30% gaining full marks and about 5% no marks. A majority of the candidature gained at least 10 marks.

(a)   (i) (2 marks)

Most candidates used the product rule correctly, and correctly wrote $\sec^2 x$ when the derivative of $\tan x$ was required. However, a significant number showed their lack of understanding of trigonometrical functions by attempting to simplify their correct answer. Mistakes such as $x \sec^2 x$ becoming $\sec^2 x^2$ and $\tan x + x \sec^2 x$ becoming $x(\tan + \sec^2 x)$ were common.

 (ii) (2 marks)

Most candidates used the quotient rule correctly or were able to rewrite the expression as $e^x(1 + x)^{-1}$ and apply the product rule.

Many candidates displayed poor algebraic skills. Those using the quotient rule frequently wrote $1 + xe^x$ where $(1 + x)e^x$ was intended. This was accepted if the intention was made clear by a subsequent correct expansion. Others simplified $\dfrac{(1 + x)e^x - e^x}{(1 + x)^2}$ to $\dfrac{e^x - e^x}{(1 + x)}$, showing a lack of understanding of the basic properties of fractions. Similar errors were mentioned in the 1998 examination report on the corresponding question.

(b) (4 marks)

Many candidates, including those who did well in the rest of the question, did not seem to appreciate that the gradient of the tangent (and normal) to a curve depends on the value of x. These candidates correctly found that the slope of the normal was given by $-2\sqrt{x + 2}$ but did not evaluate this expression at $x = 7$ before substituting into the point-gradient form of the equation of a line.

Most candidates were able to apply the chain rule successfully, although many made mistakes in simplifying $\frac{1}{2}(7 + 2)^{-\frac{1}{2}}$ or thought that a decimal approximation would be sufficient. Others solved the equation to determine where the derivative was equal to zero, obtaining $x = -2$, and concluded that the gradient of the normal was (always) $1/2$.

Some candidates incorrectly rewrote $\sqrt{x + 2}$ as $\sqrt{x} + \sqrt{2}$ or somehow produced a 'gradient' without differentiating. Unsupported statements claiming the gradient of the tangent was 1 or that the slope of the normal was -1 were common.

Some candidates mis-read the question and either found the equation of the tangent or just the gradient of the normal.

(c)   (i) (2 marks)

This part was not done well as a significant number of candidates were unable to correctly state and use a form of the cosine rule. Those who started with a correct formula such as $c^2 = a^2 + b^2 - 2ab \cos C$ were often unable to make $\cos C$ the subject of the formula. This was

particularly common amongst candidates who attempted to change the subject before substituting for the lengths of the sides. Others had $\sin C$ as the subject of an otherwise correct formula or correctly wrote $\cos C = 22/42$ but then calculated the angle as if they had written $\sin C = 22/42$.

Other candidates, who had memorised the cosine rule in one of the other forms such as $a^2 = b^2 + c^2 - 2bc \cos A$, were unable to adapt it to this situation and found either $\angle BAC$ or $\angle ABC$. Candidates need to appreciate the pattern involved in the cosine rule, distinguishing between the pair of adjacent sides and the side opposite a particular angle.

A significant number of candidates assumed ABC was a right triangle and used 'trigonometric ratios' such as $\cos C = 3/7$.

(ii) (2 marks)
Most candidates were able to gain at least one mark and a small percentage of the candidature gained their only marks for the question here. By far the most successful were those who recognised that $\angle DBC$ and $\angle ACB$ were co-interior angles formed by parallel lines and hence were supplementary. The next most successful group of candidates used the fact that $\angle BAC$ and $\angle DBA$ were alternate angles between parallel lines. This approach required the calculation of either $\angle BAC$ or $\angle ABC$.

Among those who used the sine rule to find $\angle ABC$, there was not a single candidate who appreciated the fact that there were two possible solutions. These candidates all found $\angle ABC = 84°$ and not the correct value which was $96°$. However, candidates were awarded both marks if this was their only error.

Other successful candidates extended either side BC or line DB and used either alternate or corresponding angles.

Unfortunately some candidates referred to additional points such as E or X in their answers, but did not supply a diagram or written statement indicating the location of these points. This invariably meant that the candidate was not able to gain full marks.

A common mistake was to state that $\angle DBC$ and $\angle ACB$ or $\angle DBA$ and $\angle ACB$ were alternate and therefore equal.

Question 4

This question contained parts on sequences and series as well as parts on integration and inequalities.

On the whole, the question was well done, with a majority of the candidature scoring a reasonable mark. Clear working, set out sequentially, was a feature of the responses, especially amongst those who obtained most of the available marks.

(a) (2 marks)

This was generally done very well, although candidates used a variety of incorrect equations for the limiting sum. The most common mistake was to write $\frac{a}{r-1}$ as the limiting sum, leading to the answer $r = 5/3$ which is impossible since $|r| > 1$.

Too many candidates who correctly reached $12 = \dfrac{8}{1-r}$ were not able to solve this equation.

(b) (i) (1 mark)

A majority of the candidature correctly used $T_n = a + (n-1)d$ to determine the answer. A substantial number used a tedious approach listing the number of cabbages in each row to eventually find the correct number for the 12th row.

A common error was to incorrectly use $S_n = \frac{n}{2}(2a + (n-1)d)$ to determine the number of cabbages in the 12th row. Other mistakes included the use of the incorrect formula $T_n = a(n-1)d$ or the inappropriate formulae $T_n = ar^{n-1}$ or $S_n = \frac{n}{2}(a+l)$.

(ii) (2 marks)

Once again, a majority of the candidature used the correct formula for T_n to evaluate the number of rows required. Many candidates followed a similar approach to the one which was common in Question 1 (c) and solved the equality $T_n = 200$ to find $n = 42.25$, and then correctly concluded that 43 rows were needed. A significant number of students, however, rounded their solution to the equality and stated that 42 rows were required.

Other candidates used the inequality $T_n > 200$ directly to generate their solution. Quite a few used the incorrect inequality sign and wrote $T_n < 200$.

As in part (i), the use of incorrect or inappropriate formulae was common. A number of candidates made arithmetic errors, with the line $35 + 4n - 4 > 200$ in the solution often followed by $4n + 39 > 200$. It is also important to note that many candidates tediously listed the numbers of cabbages in the first 43 rows, often making many mistakes in the process.

(iii) (2 marks)

This part was well done. Most candidates who had made progress in

the first two parts correctly used $S_n = \frac{n}{2}\left(2a + (n-1)d\right)$ to give the solution required.

However, a number of candidates continued to use T_n and found the first row which would contain at least 945 cabbages. Once again, a small number of candidates engaged in the time consuming task of listing the total number in the first n rows for enough values to arrive at the correct answer.

Once again, a variety of incorrect expressions for S_n were used, with the most common being $S_n = \frac{n}{2}(2a + nd)$. The formula, $S_n = \frac{n}{2}(a+l)$ was also used. While this is correct, it was of little use in solving the question. Candidates who scored one mark in this part usually obtained the required quadratic equation, but were then unable to solve it correctly.

(c) (i) (1 mark)
This was generally very well done. The vast majority of the candidature correctly solved $4x - x^2 = 0$ and then singled out the required solution. A large number of candidates wrote down both coordinates of the point B rather than just the x coordinate as had been asked in the question. Candidates who wrote $(4, 0)$ as their answer were viewed in this light, and awarded the mark, but those whose final answer was $(0, 4)$ were deemed to be listing both solutions to the quadratic equation and did not receive the mark.

Very few algebraic mistakes were encountered in the solutions. Of those that did occur, the most common was

$$4x - x^2 = 0$$
$$(2 - x)(2 + x) = 0$$
$$\therefore x = 2 \text{ or } -2$$

leading to the conclusion that the x coordinate of B was 2.

It was difficult to know whether or not candidates who wrote $4x = x^2$ and therefore $x = 4$ had intentionally discarded the solution $x = 0$ because B was not on the y axis. A number of candidates appeared to think that the question required them to find the x coordinate of the turning point which occurs at $(2, 4)$.

(ii) (2 marks)
This part was very well done. The great majority of candidates were able to find the appropriate definite integral and evaluate it correctly. There were very few errors in the integration although some candidates made mistakes such as $\displaystyle\int_0^4 4x - x^2\, dx = \left[x^5 - \frac{x^3}{3}\right]_0^4$.
A small number attempted to find $\int_0^4 y^2\, dx$ or $\pi \int_0^4 y^2\, dx$.

Some candidates differentiated instead of integrating, scoring zero. The fact that Simpson's rule leads to a correct answer for linear functions, quadratics and cubics was fortunate for some candidates.

(iii) (2 marks)

This part asked candidates to indicate a pair of inequalities that describe the shaded region. Many were able to write down at least one of $y \geq 0$ or $y \leq 4x - x^2$, and each of these earned a mark. Candidates were also awarded the marks if these inequalities were replaced by the strict inequalities $y > 0$ or $y < 4x - x^2$.

Unfortunately, a number of candidates viewed the problem as one of finding x for which $4x - x^2 \geq 0$ and gave $0 \leq x \leq 4$ as their answer. Still others thought that they had been asked to give the range and domain of $y = 4x - x^2$.

Many candidates scoring zero in this part did not appear to know the meaning of the word inequality and gave ordered pairs as their answer. Others did not clearly understand the use of test points and so were led to make mistakes such as rejecting $y \geq 0$ because $(0, 1)$ is not within the shaded region.

Question 5

The first part of this question concerned the sketching of a cubic function, the second tested logarithms while the third tested candidates' understanding of the definite integral.

On the whole the question was well done. It was clear that curve sketching had been well prepared and many candidates from each centre scored the 8 marks available in this part of the question. However, it was apparent that candidates from some centres were inadequately prepared to answer questions involving logarithms and an understanding of the definite integral.

(a) (i) (1 mark)

Almost every candidate could differentiate $x^3 - 6x^2 + 9x + 1$ and so earned this mark.

(ii) (3 marks)

Many had difficulty factorising the derivative to obtain $3(x-1)(x-3)$. Amongst those who succeeded, many could not handle the common factor of 3 in the quadratic equation $3(x - 1)(x - 3) = 0$ and gave solutions 0, 1, 3. Candidates who used the quadratic formula to solve $3x^2 - 12x + 9 = 0$ enjoyed similar levels of success.

Having found the x coordinates of the stationary points, many attempted to find the y coordinates by substituting into the first

derivative. This error created a difficulty for these candidates in attempting to draw the sketch in part (iv), since it meant that both stationary points were supposed to be on the x axis.

(iii) (2 marks)

Testing the nature of the stationary points was well done by candidates who used the second derivative test, but was often poorly handled when approached by testing the sign of the first derivative on either side of the stationary point. Many candidates drew the wrong conclusion from their evidence and then reversed their answers after sketching their curve.

Some candidates seemed to think that the instruction asking them to determine the nature of the stationary points meant that they had to find points of inflexion.

(iv) (2 marks)

Many candidates had placed themselves in an impossible position by this stage, having made mistakes which meant that no curve could fit their evidence. It is not possible to draw a continuous curve where the only two stationary points are both local maxima or where the only stationary points are a local maximum and a local minimum which both lie on the x axis. Hardly any of these candidates went back and checked their work. Most drew curves which looked like quartics or other irregular curves and simply ignored the inconsistencies.

Candidates who did well in this part knew they were sketching a 'positive' cubic and knew it passed through $(0, 1)$. They labelled their axes clearly and drew a smooth curve between stationary points. Most of these candidates plotted a few extra points as a check on the accuracy of their calculus.

In the context of this question, it was not necessary to mark the point of inflexion. Sketches which extended onto the left hand side of the y axis were accepted, with markers taking the view that this restriction had been placed on the sketch in order to prevent candidates from thinking that they needed to find the coordinates of the point where the curve crosses the x axis.

(b) (2 marks)

In this part, candidates either knew the properties of logarithms or struggled to gain even 1 mark. It was not well answered, with responses such as $\log_a 12 = \log_a 2 \cdot \log_a 2 \cdot \log_a 3 = x^2 y$ being quite common.

Candidates seemed especially confused by the use of $\log_a$, rather than $\log_e$ or $\log_{10}$. Many knew that they had to change from $\times$ to $+$ somewhere, but appeared to have little understanding of how they should go about this.

(c) (2 marks)

Many candidates did not attempt this part at all. Others, because no formula for $f(x)$ was given, tried some form of integration using a range of made up functions. Simpson's rule featured occasionally.

The relationship between the area under the curve and the definite integral does not appear to be well understood. The majority of the candidature showed that the existence of such a relationship was known and gained 1 mark by simply adding the 3 areas to obtain an answer of 12. Those who understood the relationship better knew that they should perform the calculation $8 - 3 + 1 = 6$, although it was interesting to note that most of these candidates went on to give their answer as $6 \, \text{units}^2$.

Question 6

The first part of this question was on exponential decay and involved computations using logarithms and exponentials. The second linked coordinate geometry with Euclidean geometry in the process of proving that a given quadrilateral was a parallelogram.

With many candidates scoring at least 10 marks, the overall impression was that the question was well answered. However, it was noticeable that there were some centres where virtually the entire candidature either did not do one of the parts, or did it badly.

(a) On reading the stem, almost half of the candidature immediately calculated k, only to discover that part (i) required other information. Many of these then wasted valuable time repeating their k calculation in part (ii). A reference indicating that k had been computed in part (i) would have been sufficient.

Many otherwise excellent responses appeared to have overlooked one or other of parts (iii) and (iv). Such unneccessary losses of marks can be avoided if each part is marked off on the question paper as it is completed.

The candidates who changed the equation in the question to $M = 10e^{+kt}$, and so obtained negative values for k, were not penalised for doing this.

(i) (1 mark)

Most candidates realised the reference to 'initial' meant $t = 0$, but approximately 10% of them evaluated $e^{-k \cdot 0}$, or e^0, as e rather than 1.

A number of candidates first computed k and then substituted $t = 1$ into the formula to find the 'initial' mass. These candidates did not receive the mark.

Many candidates 'knew' that the 10 in the formula $M = 10e^{-kt}$ was the answer.

(ii) (2 marks)

The first mark was awarded for the correct substitution of $M = 5$ and $t = 100$ into the given equation. Those who then divided through by 10 and wrote $5/10$ as 0.5 before changing from exponential to logarithmic form succeeded in arriving at $-100k \ln e = \ln 0.5$. Only a handful of those candidates who took logarithms immediately after the substitution converted $\ln(10e^{-kt})$ to $-kt \ln e + \ln 10$. A small percentage of the candidature did not recognise that $\ln e = 1$.

The process of making k the subject of $-100k = \ln 0.5$ and the numerical computation to arrive at $k = 0.006\,93\ldots$ left much to be desired. Many candidates could not interpret the value shown on their calculator, and simply copied it literally as $6.93\cdots - 03$ or wrote 0.000693. On the other hand, it was pleasing to see that many of these candidates stored the value for k in their calculator's memory and used the stored value as necessary in the later parts, resulting in the correct values being found in the later parts despite these errors.

A significant percentage of the candidature rounded their answer at this stage, even as far as to 0.007, and used this rounded value in subsequent calcuations. Such indiscriminate rounding should be discouraged, but it was not penalised when the candidate provided sufficient documentation in their answer for the examiner to see that this was the source of the error.

Many of the candidates who made substitutions other than 5 for M and 100 for t gained one mark for correctly solving their equation for k. The most common incorrect substitution here was $t = 5$.

(iii) (2 marks)

Candidates found this to be the most difficult part on exponential decay.

The first mark was awarded for the substitution of $t = 1000$ and the value for k found in part (ii) into $10e^{-kt}$, and the second for the numerical evaluation of the resulting exponential expression. Although a majority of the candidature arrived at $M = 0.0097\ldots$, it was quite common for the numerical answer to be inconsistent with the values the candidate claimed to have used in their computation. Regardless of these inconsistencies and the variations introduced by candidates who had replaced $10e^{-kt}$ by $10e^{kt}$, 2 marks were awarded if the numerical value given as an answer was consistent with $10e^{-|1000k|}$ using the value for k found in part (ii).

Many candidates had little idea of exponential evaluation on their calculator. A small percentage of the candidature failed to realise that the mass must decrease with the passing of time. A few candidates, realising the significance of the half-life, arrived at the answer by

successive halving of the previous answer.

(iv) (2 marks)

The first mark was awarded for the correct substitution of $M = 8$ and the value for k found in part (ii) into the given equation. The second mark was for the solution of their equation, provided the value found for t was positive.

Most of the candidates who arrived at a negative numerical value for t reported $|t|$ as their answer. Only a few tried to attach some meaning to the negative sign. Candidates who had experienced difficulty in changing the exponential equation into a logarithmic equation in part (ii) suffered the same difficulty here.

(b) This part laid bare the inadequate comprehension skills of the many candidates who believed from the beginning that $OABC$ was a parallelogram and that deductions based on this information were being solicited in the subparts. Another significant fraction of the candidature successfully answered parts (i) and (ii) before deciding that part (iii) required them to use the fact that $OABC$ is a parallelogram to prove triangles AOB and CBO congruent.

Many candidates did not transfer the diagram to their writing booklet, and only a small percentage of those who did showed useful working on it in relation to equal angles or sides. The use of such diagrams and markings does not replace the need for a proper argument, but greatly assists the examiner (and the candidate) in understanding what has been written.

On the other hand, the use of non-standard symbols and abbreviations, such as $''AB$ or $\overline{AB}$ rather than AB to denote the length of the line segment joining A to B, should be discouraged as they detract from the clarity of the answer. The syllabus gives clear guidelines on the appropriate notational conventions for Euclidean geometry.

(i) (2 marks)

This part was well done by most candidates, who calculated the gradients of the two lines from the given information and indicated that the fact that the gradients were equal meant that the lines were parallel.

Some candidates made dealing with $y = 2x + 10$ unneccessarily complicated, using differentiation or two points on the line to establish that the slope of the line was 2, while others thought it too trivial to mention and so did not gain full marks. Candidates who could not establish the gradient of OC from the coordinate information usually found the equation of the line OC by assuming that the gradient was 2. These candidates were only able to gain full marks if they then verified that their line did indeed go through both O and C.

Full marks were obtained by the few candidates who first showed that O and C were the same perpendicular distance from the line AB.

Apart from the common error of beginning with the assumption that $OABC$ was a parallelogram, there were a number of common sources of error. Some tried to show that $m_1 m_2 = -1$, while others believed the common gradient to be $2x$. A few candidates thought 'show' meant 'put arrow heads on the diagram'.

(ii) (1 mark)
Very few candidates did not attempt this subpart. The answer should be a single statement, indicating that the named angles are equal because they are alternate angles between the parallel lines AB and OC. The many well-prepared candidates did this.

While the bare 'alternate angles' and 'AB is parallel to OC' both gained the mark, the bare 'corresponding, etc ... angles' did not. The candidates who believed 'state' meant 'prove' were awarded the mark only when they indicated the connection to the lines AB and OC in the course of their 'proof'.

(iii) (2 marks)
The first mark was awarded for the correct use of congruence to equate sides or angles of the triangles AOB and CBO. Two congruences are possible, though part (ii) attempts to lead candidates into $\triangle AOB \equiv \triangle CBO$. There are at least four distinct ways to then prove that $OABC$ is a parallelogram (and one of these has three different routes). The setting of this question after parts (i) and (ii) provides a bias towards deducing that $\angle AOB = \angle OBC$, from which it follows that $OA \parallel BC$ etc. To obtain the second mark, candidates had to logically link the significant parts of their congruence data with one of the tests for a parallelogram.

Although almost one quarter of the candidature obtained full marks, this number would have been much reduced had the examiners insisted on a precise statement of any one (of the four relevant) tests. Too many candidates believed they had to prove all of the opposite sides equal and parallel, and opposite angles equal. Too many candidates were not awarded the second mark when they gave poorly worded reasons such as 'sets of equal sides'. Too many candidates did not know at least one test for proving a quadrilateral a parallelogram.

All too few earned the second mark with a succinct statement such as '$AB = OC$ (corresponding sides of congruent triangles) $\therefore OABC$ is a parallelogram (one pair of opposite sides equal and parallel)'.

The majority of the candidature tried to prove either opposite angles equal or opposite sides parallel. Many of the first group failed to gain full marks when they named angles incorrectly, or only proved

one pair of the opposite angles equal. Similarly, many of the second group failed to gain full marks when they claimed that one pair of lines parallel and a pair of equal transversals implies that the transversals are automatically parallel. The most common roof shape provides a counter-example to this claim.

A significant fraction of the candidature wrote down every piece of information that could be extracted from the diagram but did not link any of it to $OABC$ being a parallelogram. For these candidates, it would have been better to first work out where they were going, preferably on a copy of the diagram drawn in their writing booklet.

Unfortunately, too many candidates believed ' ... OB divides the quadrilateral ... ' meant that OB bisects $\angle ABC$ or $\angle AOC$. Too many candidates used properties of the parallelogram $OABC$ to prove either that $OABC$ was indeed a parallelogram or that the triangles OAB and OBC were congruent. Finally, too many candidates purported to prove $OABC$ was a parallelogram without reference to two congruent triangles.

Question 7

The first part of this question was a time payments problem. It consisted of 3 sections which led the candidates through the process of determining an initial investment. Unfortunately there were a significant number of candidates who took the phrase 'at the end of each of the next six years' to mean 'at the end of each 6 year period' rather than 'at the end of each year for the next 6 years'.

Many candidates were unable to make the necessary adjustments to their reasoning when faced with regular withdrawals as opposed to the conventional time payment problem requiring a regular instalment.

The second part of the question came from the topic 'Applications of Calculus to the Physical World'. The first two questions concerning the motion of the particle provided almost all candidates with the opportunity of securing 1 or 2 marks, while the remaining questions rewarded those who took the time to think more carefully about the question.

Nearly all candidates attempted this question. However, there were a large number with marks in the 0–2 range, and few candidates were able to earn full marks.

(a) (i) (1 mark)
This part was quite well done, with most candidates displaying a sound understanding of compound interest. Common mistakes involved the interchange of P and A and the use of incorrect growth factors, such as 1.08^6.

(ii) (1 mark)

Most were able to write $A_2 = A_1 \times 1.08 - 3000$, which was sufficient to earn the mark here. Many began their prepartion for part (iii) here, by substituting the expression for A_1 obtained in the previous part. Errors in the process of simplification were common.

(iii) (3 marks)

This part was generally well handled by those who recognised the GP, but there were many who were unable to generalise from the pattern of part (ii). Amongst those who recognised the GP, there were many who made errors in writing an appropriate expression for the sum of the GP. These errors were associated with a failure to choose the appropriate value for the initial term a or to correctly count the number of terms in the series.

(b) (i) (1 mark)

Although this was reasonably well done, it was a concern to note that many candidates were unable to differentiate a simple trigonometric function correctly. Common mistakes included a failure to differentiate the term t, integrating instead of differentiating, and misuse of the product rule.

(ii) (1 mark)

A variety of answers were given which managed to convey the information that the particle is moving in a positive direction. A significant number seemed unaware that the direction is determined by the velocity and instead, substituted $t = 0$ into $x = 2\sin t - t$. Another common mistake was made by candidates who thought that the fact that $v = 1$ when $t = 0$ meant that the particle is moving upwards or north-east.

Many candidates were apparently able to guess the answer.

(iii) (1 mark)

A large number of candidates knew that the particle comes to rest when $v = 0$, but failed to gain the mark because they gave an answer for t in degrees, instead of using radians. However, many knew to convert the 60° to $\pi/3$. Many attempted to find t by letting x or a equal zero.

(iv) (2 marks)

Most were able to find an expression for the acceleration, but were unable to determine when it was negative. Those who used a graph to solve $-2\sin t < 0$ enjoyed greater success than many of those who attempted to solve it algebraically.

Quite a few gave the answer $t = \pi/2$. This is the time when the acceleration $a = -2\sin t$ is most negative. Others solved $-2\sin t = 0$ and gave the answer $t = 0$, π, 2π.

As in the previous part, many wrote their answer in degrees and did not receive the mark.

(v) (2 marks)

This section was a good discriminator in that it rewarded those who had an appreciation of the physical significance of the mathematics. Unfortunately, most did not understand that their answers to the previous parts meant that the particle moved in the positive direction for the first $\pi/3$ seconds and in the negative direction for the remainder of the first π seconds.

Most incorrect answers were the result of a computation of the displacement of the particle rather than the total distance travelled. Others found the distance travelled in the first $\pi/2$ seconds and then doubled this answer.

Candidates applying the area under the curve approach to the velocity function were the most successful.

Question 8

This question consisted of three parts taken from two separate areas of the syllabus, namely calculus and probability. Part (a) required candidates to obtain an exact expression (in terms of an integral) for the volume of a solid formed when a shaded region was rotated around the y axis. They were then asked to find an approximation for the volume by applying the trapezoidal rule to this integral.

Part (b) was a probability question involving an experiment in which two cards were drawn from a box containing five cards (0, 3, 3, 5, 5) without replacement. In part (c) candidates were given a sketch of the gradient function of the curve $y = f(x)$ (cutting the x axis in two places), and asked to identify, with justification, the x coordinate of the local minimum for $y = f(x)$.

The question was answered reasonably well, with the majority of candidates obtaining more than six marks. There was evidence of candidates not reading the question carefully or not understanding the language of the questions. Words and phrases such as 'show', 'with replacement', 'at least 8' and 'the gradient function of the curve' all caused considerable difficulty. Working was not shown on a significant number of occasions, particularly in the probability section of the question. Candidates with incorrect answers cannot be awarded part marks for the steps they have done correctly if there is no evidence of these steps in their writing booklet.

(a) (i) (1 mark)

This part was generally quite well done. Most candidates were able to quote the formula $V = \pi \int x^2\, dy$ for the volume after a rotation about

the y axis. However, a significant number had difficulty in showing the steps involved in making x^2 the subject of the formula $y = e^{x^2}$. Most attempted to take logarithms of both sides, but with varying degrees of success. Some candidates tried unsuccessfully to integrate $\log y$ while others thought they needed to find the integral of $(e^{x^2})^2$.

The mark was awarded to those candidates that were able to successfully show that $x^2 = \log y$ was equivalent to the original equation $y = e^{x^2}$.

(ii) (2 marks)

This proved to be an easy opportunity for nearly all candidates to gain 2 marks. The most common errors came from candidates who had actually calculated $\log_{10} y$ or e^y and not $\log_e y$.

The marking scheme awarded one mark if two of the three values in the table were correct. While the question asked for answers to be given correct to 3 decimal places, considerable leniency was shown and the marks were awarded provided the answer given was correct to at least 1 decimal place. Candidates who simply wrote $\ln 1$, $\ln 4$ and $\ln 7$ in their table and then gave evidence in part (iii) of the use of correct decimal approximations to these values, also received these two marks.

(iii) (2 marks)

This part involved the use of the trapezoidal rule. While many candidates substituted successfully into a variety of learned formulae, others either could not place the given numbers in the correct positions or calculated their $h = \dfrac{b - a}{n}$ incorrectly. Some used the values for y in places where $\log y$ was required in their formula.

The tabular approach was employed by some candidates with considerable success. Using this method, the computation is set out as follows.

y	$f(y)$	w	$w \cdot f(y)$
1	0	1	0
4	1.386	2	2.772
7	1.946	1	1.946
$\sum$		4	4.718

The approximation to the integral can be calculated simply by using the formula $\dfrac{b - a}{\sum w} \sum w f(y) = \dfrac{7 - 1}{4} \times 4.718$ to obtain the solution. The method is easily adapted to Simpson's rule by changing the weightings in the w column to 1, 4 and 1.

Many candidates struggled with the concept of finding a volume using the trapezoidal rule, not realising that the trapezoidal rule is used to evaluate the integral and all that was required was to multiply this value by π. Some multiplied by 6 or 7, explaining that this was the height of the solid. A number of others thought they had to square their answer before multiplying by π. Many confused Simpson's rule and the trapezoidal rule.

One mark was awarded to candidates for a correct substitution into a correct trapezoidal rule using their values from their table in part (ii), while the second mark was awarded for multiplying their answer from their trapezoidal rule by π.

(b) (i) (1 mark)
Candidates were asked to find the probability of drawing a '5' then a '3'. This was understandably the best answered of the three probability parts of this question, with most candidates able to obtain the correct answer, 1/5. This was often expressed in unsimplified form as 4/20.

A significant number thought that there were two separate questions contained in this part. They found the probability of drawing a '5' and the probability of drawing a '3' and gave these as two stand alone answers without taking the product. Examiners took the view that this was not a valid interpretation of the question. Adding the respective probabilities for a '5' and a '3' was also common. The marking scheme awarded the mark for the product $\frac{2}{5} \times \frac{2}{4}$ and ignored subsequent errors.

(ii) (2 marks)
The requirement was that the sum of the two numbers be at least 8. Approximately half of the candidature drew a tree diagram showing the three possible outcomes with regard to the value of each card drawn and wrote the respective probabilities on each branch. The other half drew a tree diagram or grid showing twenty possible outcomes for drawing the two cards without replacement. The latter group seemed to have greater success in obtaining the correct answer to both this part and part (iii) because the question was reduced to simply counting the number of displayed outcomes that satisfied the requirements. Examiners noted that candidates who drew a large tree diagram taking up plenty of space made less errors in counting and were less likely to omit branches.

The key phrases 'without replacement' and 'at least 8' were sources of much confusuion. Answers using replacement were very common while many intentionally or unintentionally ignored the possible oc-

currence of a sum of 10, simply writing $P(\text{sum of } 8) = \dfrac{2}{5}$. Attempts using the complement, $P(\text{sum of at least } 8) = 1 - P(\text{sum less than } 8)$ were not well done. Multiplying probabilities instead of adding was not unusual and fraction manipulation was often inaccurate. A single mark was awarded for correctly obtaining the values in product form of the three required terms, $P(3,5)$, $P(5,3)$ and $P(5,5)$, or for correctly giving two of these three terms as a sum.

(iii) (2 marks)
Candidates were asked to find the probability that the second card drawn was a '3'. Success on this part of the question correlated highly with success in part (ii). Most problems were related to the determination of the sample space, illustrated by answers such as 'it depends which card was chosen first'. A minority answered the question successfully by adding $P(\bar{3}, 3)$ and $P(3, 3)$. Very few showed that they recognised that the probability of drawing a '3' first was exactly the same as the probability of drawing a '3' as the second card.

The marking scheme in part (iii) was very similar to that employed in part (ii). One mark was awarded for correctly obtaining the values of the three required terms, $P(0,3)$, $P(3,3)$ and $P(5,3)$, in product form, or for correctly expressing two of these three terms as a sum. The scheme allowed candidates who used replacement to gain up to 4 marks in part (b) and candidates who would have otherwise scored no marks for part (b) were awarded 1 mark here if a correct tree diagram or grid could be found in any part of their answer to part (b).

(c) (2 marks)
This proved to be the most difficult part, and many candidates simply did not understand the question. Many appeared to read the question as if it said that the diagram showed the graph of the function $y = f(x)$, while others noticed the difference but stated that the question must be in error and told the examiners how it should have been written. An overwhelming majority of the candidature wrote that $x = 3$ was the local minimum because it was the value of x where the lowest part of the given curve occurred.

Another common answer was 'the minimum is $1 < x < 5$ as $\dfrac{dy}{dx} < 0$'. Those who identified $x = 1$ and $x = 5$ as stationary points usually either stopped there or gave incorrect justifications for their answer. The term 'concavity' was often misused.

The marking scheme awarded one mark for correctly identifying $x = 5$ as the x coordinate of the local minimum and one mark for correctly justifying this claim. Inaccurate or vague language used in the justification often made it difficult to award marks.

Question 9

This question consisted of two parts. Part (a) required candidates to find the area of the shaded region between a curve and a straight line. Part (b) was a problem concerning parallel railings which were a fixed distance apart. A set of crossbars, attached to the lower railing, intersected to form two triangles. Candidates were required to show that a proportional relationship existed between corresponding sides and corresponding altitudes of a pair of similar triangles. They were then required to find the altitude of the lower triangle for which the total area of the two triangles was a minimum.

A significant number of candidates provided very good answers to this question.

(a) (3 marks)

This part was handled reasonably well, even in quite a few instances by candidates who did not score any marks in part (b). Most candidates approached the question by finding the difference between the areas under the two curves.

The integral, $\int \sec^2 x \, dx$, associated with the area under the upper curve, could be obtained from the table of standard integrals and those who found the area of the triangle by computing $\int_0^{\frac{\pi}{4}} x \, dx$ generally did so fairly well. The correct substitution of limits in the definite integrals was pleasing. Those who found the area of the triangle using 'half the base times the height' usually managed to write down $\frac{1}{2} \cdot \frac{\pi^2}{4^2}$, but this was often 'simplified' to $\frac{\pi^2}{8}$ or $\frac{\pi^2}{16}$. It was quite disturbing to notice how frequently this error occurred on scripts that were otherwise well done.

(b) In this part, some candidates found the calculus straightforward but could not prove the triangles similar. Others provided an elegant similar triangles proof but had inadequate responses to the parts involving calculus.

(i) (3 marks)

Many candidates proved or attempted to prove that there were some similar triangles. However, the majority of these candidates incorrectly assumed they had, at that point, proved the required relationship. Errors included the absence of any reference to parallel lines to justify the equality of alternate angles, non-standard symbols such as ... and = used to (presumably) denote similarity, and the claim that the line VU was a perpendicular bisector of SR and PQ.

Most candidates labelled the angles in their similarity proofs correctly. The decision to prove two pairs of small triangles similar then required more effort to reach the conclusion than the proof where one pair of small triangles and the pair of large triangles were chosen. Some of the successful candidates presented a proof which used trigonometry.

Few candidates succeeded in establishing the link required to show that $\frac{SR}{PQ} = \frac{VT}{UT}$.

(ii) (1 mark)

This was an opportunity for candidates to demonstrate their ability to express one variable in terms of another. It was particularly well done.

(iii) (2 marks)

Here candidates were required to show sufficient working to indicate that they had actually expanded the expression obtained for the area of $\triangle RTS$. Some candidates left out parentheses and were given the benefit of the doubt if the next line was consistent with the implied parentheses. This part was also particularly well done.

(iv) (3 marks)

Candidates seemed to appreciate the opportunity to demonstrate their prowess with the first and second derivative. Justification of the nature of the stationary point by noting the sign of the second derivative was well done. Some candidates chose to use a first derivative test.

Working was often well set out, but sloppy notation cost some candidates marks. Unfortunately, some candidates could not use any notation for a first or second derivative other than $\frac{dy}{dx}$ or $\frac{d^2y}{dx^2}$, which proved to be a problem in a question requiring the location of the minimum value of $A(y)$. Other candidates who used the first derivative test chose values for y which were clearly impossible in the context of this question. Values such as 0, 4 or negative values of y were outside the domain of the function, which was restricted by the physical problem to $0 < y < 3$. Candidates who used values outside this domain could not gain full marks.

Question 10

This question dealt with trigonometric functions and plane geometry. Most candidates lacked the level of understanding required to successfully respond to this question, with more than 50% of candidates scoring less than 4 marks (out of 12) and less than 1% obtaining full marks.

(a) (i) (1 mark)

The question required a demonstration that $x = \pi/3$ is a solution of $\sin x = \frac{1}{2}\tan x$, preferably by substitution into each side of the equation. Other methods, such as an algebraic solution of the

equation and identification of $x = \pi/3$ required considerably more work.

While the majority of the candidature gained this mark, it should be noted that very few were aware of the need to follow a 'LHS $=$ ... ' and 'RHS $=$ ... ' development when verifying a solution by substitution. Those who proceeded by algebraic solution often used their calculator to find $\cos^{-1}(1/2)$ and compared this value with $\pi/3$. Examiners accepted this, but not in situations where agreement was not found because of an incorrect setting of the mode of the calculator.

(ii) (2 marks)

This part required sketches of $y = \sin x$ and $y = \frac{1}{2}\tan x$ for the given domain. It was encouraging to see that most candidates used a ruler to draw the axes and measure scale units, and that there were many neat and accurate sketches.

The sine curve was generally well done. The incorrect use of a drawing template was common, with many candidates drawing the shape appropriate for $0 \leq x \leq 2\pi$.

On the other hand, a large proportion of the candidature could not sketch the tangent curve. Many candidates only drew the portion of the graph corresponding to the domain $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$, failing to show the three branches required. Others misinterpreted the $1/2$ as an amplitude, so their graph stopped at $y = \pm 1/2$. There were many curves with the wrong concavities or shapes and multiple inflexions were common.

Other errors included sketching multiple cycles of one or both curves in an attempt to cater for the $1/2$ in the $\frac{1}{2}\tan x$, and drawing correct sketches which were then altered to 'satisfy' the domain in parts (iii) and (iv).

Some of the difficulties encountered by candidates may be attributable to the fact that past examination papers have rarely involved sketching the tangent function.

(iii) (1 mark)

This required either a formal algebraic solution of the equation, or a deduction based on the symmetry of the curve and the value given in part (i).

Most candidates ignored or overlooked the solution $x = 0$. A significant number attempted to indicate or mark the solutions on their graph. The question required an exact solution and such answers did not score the mark. Some candidates misunderstood the question and simply stated the number of solutions which exist in the domain.

(iv) (2 marks)

This required stating the two intervals, $-\dfrac{\pi}{3} \le x \le 0$ and $\dfrac{\pi}{3} \le x < \dfrac{\pi}{2}$, in the domain for which the sine curve is below the tangent curve.

More than half of the candidature failed to attempt this part of the question while most of those who did gave extremely poor answers.

The most common errors were answers which gave only one interval, and those which failed to specify one of the end-points of the interval, such as the answer $x \ge \dfrac{\pi}{3}$ in place of $\dfrac{\pi}{3} \le x < \dfrac{\pi}{2}$. A significant number of candidates responded to this question by shading regions between the curves.

(b) (i) (1 mark)

It should go without saying that some reason is required in explanation for the equality of the angles. Correct reasons referred to alternate angles between the parallel lines AF and BC, similar triangles or the angle sum of a triangle. This part was generally well answered with a majority of candidates being awarded the mark.

A surprising number of candidates did not know the correct term for alternate angles. Some used 'corresponding', others 'adjacent' and even 'alternative angles', while still more wrote ' ... on opposite sides of the line BF'. Some of the claimed proofs for similarity left much to be desired, with vague references to 'sides in ratio', equal sides or congruent triangles.

(ii) (2 marks)

A formal proof was required. Perhaps the most obvious method involved the use of the equal intercept theorem to show $AG = GF$.

The majority of the candidature provided justification for two of the three statements forming the basis of their congruence test, but could not correctly justify their third statement. It was pleasing to note that the stated congruence test was usually consistent with their three statements.

There were two common errors. The first involved the assumption of facts that had not been established, typically claiming that some fact followed because $\triangle AEF$ is isosceles. The other involved an appeal to the SAS test based on the data $AE = EF$, $\angle AGE = \angle FGE = 90°$ and $EG = EG$, which actually requires the RHS test. A significant number of candidates took time out to prove that $AG = GF$ by similarity in $\triangle DAF$ and $\triangle EGF$. An interesting method was to construct a rectangle $AGEX$ or $AFXD$ and subsequently use the opposite sides equality or the fact that diagonals bisect each other.

(iii) (3 marks)

This part required a sequence of deductive steps with full justification, but the task was beyond the capacity of all but the very best candidates. Quite unexpectedly, a few candidates used the sine rule in their proof!

Many missed out on full marks through lack of support for their statements. For example, a number claimed that $\triangle ABE$ is isosceles without any justification indicating that $AB = AE$.

A common error arose when candidates mistakenly believed $AB = BE$, leading either to $\triangle ABE$ being equilateral or being isosceles with $\angle BAE = \angle BEA$. Other errors involved attempts which purported to show that $\triangle ABC$ was similar or congruent to one of the other triangles.

There were many long and very wordy attempts. Correct solutions usually occupied no more than ten lines.

Mathematics 3 Unit (Additional) and 3/4 Unit (Common)

Question 1

Whilst this question was generally well done, with 12 being the most common mark, candidates need to be reminded to show all working, to take care with even the simplest working and not to make the solution unnecessarily difficult.

(a) (2 marks)
Candidates were required to evaluate a definite integral resulting in an inverse trigonometric function. The majority of the candidature easily gained the two marks. Candidates who did not consult the table of standard integrals to find that the integral was $\sin^{-1}(x/2)$ were still able to gain the second mark by correct substitution provided their claimed integral contained an inverse trigonometric function and the solution was given in radians. Common mistakes included incorrect integration, evaluation in degrees and providing multiple solutions.

(b) (2 marks)
Most candidates could easily differentiate $\sin^3 x$ to give $3\sin^2 x \cos x$. Some candidates omitted the $\cos x$ and only gained one of the marks.

Those who wrote $\sin^3 x$ as the product of $\sin^2 x$ and $\sin x$ made the question more difficult. These candidates generally wasted time and often made mistakes, particularly those who used the double angle results for $\sin^2 x$. Common errors included incorrect substitution for $\sin^2 x$, mistakes in the use of the product rule, or badly set out work resulting in silly mistakes. Candidates who attempted to express $\sin^3 x$ in terms of $\sin 3x$ often made mistakes in this process and hence could not gain both marks.

(c) (2 marks)
This part required the candidates to find the coordinates of the point which internally divided an interval in the ratio $2 : 3$. Candidates gained the first mark for correct substitution into the correct formula. The second mark was awarded for correct evaluation to give the coordinates of the point. Whilst this part was generally well done, common errors included the use of the incorrect formula or the wrong ratio, confusing the x and y coordinates of each point, and simple arithmetic mistakes. Candidates using the formula are encouraged to learn it correctly, show all of their working and to take care in substituting values.

Some candidates chose to draw a number plane and use the ratio method for finding the point. Whilst this was generally well done, those making an error with this approach were often unable to gain either mark.

(d) (1 mark)

Most candidates were able to write down the equation of the vertical asymptote, $x = 3$, although $x \neq 3$ was common. Those who were not awarded the mark for this part often showed that they did not understand the word 'vertical', finding instead an equation in terms of y or giving more than one equation. Some of these candidates wasted time finding complex, incorrect equations. Many had written $x \neq 3$ at the start of their working, but clearly did not understand that this gave the equation of the vertical asymptote.

(e) (2 marks)

This part required candidates to find the remainder when a polynomial was divided by $x + 3$. Candidates who correctly used the remainder theorem generally gained the two marks quickly and easily.

Those who attempted to find the remainder by long division often made mistakes during the division. These candidates were awarded one mark provided the remainder was a constant. Some candidates used both methods, with a few choosing the incorrect answer when their two results were different.

(f) (3 marks)

Candidates answered this part, involving the integration of a trigonometric function by substitution, either very well or very badly. Whilst three was the most common mark, candidates who could not differentiate $\tan x$ found it difficult to gain any marks.

Candidates are reminded that, when making a substitution, every occurrence of the variable must be replaced. Candidates who did not understand this often wrote integrals of the form $\int f(u)\,dx$ or $\int f(u)g(x)\,dx$ and then attempted to integrate.

One mark was awarded to candidates for the correct substitution, one for a correct evaluation of the indefinite integral obtained in terms of one variable and the final mark for the evaluation of their expression to give a numerical value. Thus, candidates who made a mistake in the substitution could still earn two of the three marks.

Question 2

Examiners were very disappointed by the poor standard of the arithmetic in answers to this question, and noted that algebraic manipulation was also poorly done. Nevertheless there were many candidates who scored 12 marks and very few who did not earn any marks.

(a) (2 marks)

This asked for the number of committees of 5 including exactly 3 females

that can be formed from 4 men and 7 women. It was very well answered, but candidates should be advised not to spend a lot of time showing detailed calculations when a numerical expression can be readily evaluated using a calculator.

Most common errors involved an attempt to use the binomial expansion, or were the result of adding rather than multiplying $\binom{4}{2}$ and $\binom{7}{3}$. Non-attempts were rare.

(b) (4 marks)
Candidates were required to find the solutions of $\cos\theta + \sqrt{3}\sin\theta = 1$ in the range $0 \le \theta \le 2\pi$. As expected, attempts to answer this used a variety of methods.

The auxiliary angle method seemed to be most prone to errors, for several reasons. Too many candidates tried to quote results such as $a\cos\theta + b\sin\theta = c$ implies $\alpha = \tan^{-1}\dfrac{b}{a}$, without realising that the result they quoted did not apply to the form in which they had written the trigonometric function using an auxiliary angle. Incorrect expansion of expressions such as $\sin(\theta+\alpha)$ occurred frequently, particularly when the expansion being used was not stated at the outset. Attempts to 'recognise' $2(\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta)$ as $2\cos(\theta - \pi/3)$ or an equivalent were prone to error, and examiners do not recommend this approach.

The specified range was often disregarded, usually leading to the omission of solutions rather than the inclusion of solutions outside this range. Examiners were surprised that so many gave one of 0 and 2π as a solution but did not include the other.

The technique of squaring both sides was generally done badly, and very few of those who were successful in applying this technique remembered to check the validity of their solutions.

The t method was used well, apart from the expected loss of $t = 0$ from the solution set. Many successful answers put t back into the expression for $\sin\theta$, $\cos\theta$ or $\tan\theta$.

Graphical solutions were less common, and good graphical solutions were even rarer. Many candidates merely sketched the two functions $y = \cos\theta$ and $y = \sqrt{3}\sin\theta$, but were then unable to use them. Scales and labels were rarely features of the sketches.

(c) (i) (1 mark)
This part, which asked for the natural domain of $f(x) = x + \log_e x$, was well answered. However, many thought that they needed to distinguish the natural domain from 'the unnatural domain', and many who answered this part correctly then ignored this answer in their work for subsequent parts.

(ii) (2 marks)
Most showed that $f(x)$ was increasing by showing that the derivative was positive over the domain, but some graphed the function, while others argued that since both x and $\log x$ were increasing then so was their sum. Candidates should be reminded that it is not sufficient to merely show, for example, that $f(1) < f(2) < f(3)$.

There were many confused explanations such as 'the curve is concave down and therefore increasing', 'the gradient is increasing' and 'since the turning point occurs at $x = -1$ which is outside the domain, the function must be increasing'. Confusion between positive and increasing was rife. One candidate had the rather quaint explanation, 'it's a log function, so it is increasing exponentially', while others wrote that the functions were increasing at a negative rate, or that the natural domain was increasing. A very common incorrect argument was 'since $x \to \infty$ as $y \to \infty$, the function is increasing'.

(iii) (1 mark)
The fact that a change of sign of the function was required in order to show that a root occurred in the required interval was very well understood. A very high proportion of the candiature earned this mark. However, it was not unusual for computational errors, such as using $\log_{10}$ instead of $\log_e$, to lead a candidate to conclude that the claim of the question was incorrect.

(iv) (2 marks)
This part, involving the use of Newton's method, was very well answered. It was not unusual for this to be the only part that a candidate answered correctly.

As expected, many candidates misquoted the formula, and there were too many instances of poor arithmetic. Examiners were astonished to find that hundreds of candidates substituted 0.5 for x in $(1 + \frac{1}{x})$ as $0.5 + \frac{1}{0.5}$. Despite having already shown that the root lies between 0.5 and 1, many appeared unconcerned when arithmetic mistakes resulted in a second approximation to the root which was outside that interval. Others thought that a second approximation required the formula to be applied at least twice.

Question 3

(a) (4 marks)
This part involved the calculation of the volume of revolution of part of the curve $y = 3 \sin x$ about the x axis. It was generally answered fairly well, with most candidates recognising the need to make use of the double angle trigonometric identities. There were the predictable errors, in which

candidates replaced $\sin^2 x$ with expressions such as $\frac{1-\cos x}{2}$, $\frac{1+\cos 2x}{2}$, $1-\cos 2x$ or $\frac{1-\sin 2x}{2}$.

The safest approach was to start with the double angle formula for $\cos 2x$ and to carefully derive the correct expression for $\sin^2 x$. Those who quoted the memorised formula $\int \sin^2 x\, dx = \frac{1}{2}(x - \frac{1}{2}\sin 2x)$ denied themselves the chance to be rewarded for intermediate steps if their memory was faulty, and so many missed out on 2 marks.

(b) (2 marks)
This required the use of binomial probability to determine the likelihood of obtaining exactly two sixes in seven throws of a fair die. Full marks were awarded for the expression $^7C_2(\frac{1}{6})^2(\frac{5}{6})^5$. A common error was to omit the binomial coefficient, $^7C_2$. Candidates were awarded 1 mark if their answer involved multiplication of the probabilities $(\frac{1}{6})^2$ and $(\frac{5}{6})^5$.

(c) (2 marks)
This was a fairly straightforward proof involving circle geometry. Essentially, it involved two steps of reasoning which were each worth 1 mark. While full marks could be obtained in only a couple of lines, it was not uncommon for solutions to go on for a page or more.

There were several successful approaches, with the most obvious one making direct use of the angle properties of a cyclic quadrilateral. A popular method involved showing that the two triangles were similar either by using the ratio properties of intersecting chords or by using the cyclic quadrilateral to show the triangles were equiangular. Another technique was to show that AQ was a diameter of the larger circle since it subtended a right angle at the circumference. Expression was generally poor and reasons were often incomplete or imprecise. There was some confusion in candidates' answers as to whether opposite angles in a cyclic quadrilateral are equal or supplementary, although the fact that the angle in question was a right angle may have contributed to this.

(d) (i) (2 marks)
A large number of candidates were baffled by this part, which required the expression $\sin x + 8\cos x$ to be rewritten in the form $A(2\sin x + \cos x) + B(2\cos x - \sin x)$. The first mark was awarded for correctly setting up a pair of simultaneous equations and the second mark was given for correctly solving them. Although there were many correct solutions, there were also many of the usual algebraic errors. Many chose to ignore or did not understand the hint 'by equating coefficients'. While candidates may not have encountered this before in a trigonometric context, it was not particularly difficult. Many candidates were able to write down the two simultaneous equations, $2A - B = 1$ and $A + 2B = 8$, and solve them by inspection.

Candidates who were daunted by the unfamiliar often chose to use

their trigonometric heavy machinery such as subsidiary angles or the t-method. This proved to be a waste of effort.

(ii) (2 marks)
This part proved to be more difficult, with relatively few candidates recognising that part (i) enabled them to rewrite the integrand as $2 + 3\dfrac{2\cos x - \sin x}{2\sin x + \cos x}$.

Even fewer candidates recognised that the second term was of the form $f'(x)/f(x)$ and so contributed $\ln(2\sin x + \cos x)$ to the integral. Algebraic manipulation was poor, with cancelling errors such as

$$\frac{2(2\sin x + \cos x) + 3(2\cos x - \sin x)}{2\sin x + \cos x} = 2 + 3(2\cos x - \sin x)$$

quite common.

Question 4

The question was generally well done. Candidates found parts (b) (iii) and (c) (iii) more difficult than other parts of the question.

(a) (1 mark)
The question used sigma notation to ask candidates to find the sum of four terms. Although this was generally well done, many candidates did not attempt this part at all. Too many responses showed a lack of understanding of this reasonably basic concept. Common errors included listing the four terms 2, -3, 4 and -5 without summing to obtain the correct answer of -2, using arithmetic or geometric series formulae, and attempts to relate this to a binomial expansion.

(b) (i) (2 marks)
Deriving the tangent, $y = px - p^2$, was a simple application of bookwork using the parametric locus $x^2 = 4y$. This part was very well done, although some candidates made it more complicated by attempting to reproduce remembered bookwork for the locus $x^2 = 4ay$, without letting $a = 1$.

As only two marks were assigned to this part, examiners did not insist that candidates use calculus to determine that the slope of the tangent at P was p. Candidates who simply quoted this fact were able to obtain full marks.

(ii) (2 marks)
Candidates needed to find the equation of the normal in terms of the parameter p and substitute $x = 0$ to obtain $y = p^2 + 2$. Although this was well done, many responses showed attempts to 'fudge' the final

step from an incorrect equation. Candidates making genuine attempts in questions where the answer is given should ensure that each step is clearly presented to avoid any doubt about its authenticity. For instance, candidates should explicitly state 'let $x = 0$' and show the corresponding substitution clearly.

(iii) (2 marks)

The majority of the candidature found the midpoint $(\frac{p}{2}, \frac{p^2+2}{2})$, but then found it very difficult to get the second mark by solving simultaneously using $p = 2x$, to obtain the Cartesian form of locus, $y = 2x^2 + 1$. Many candidates obtained the equation $y = \frac{4x^2+2}{2}$ and then made errors in attempts to simplify this expression. Such errors were ignored, and these candidates were awarded both marks.

A number of candidates tried to used the distance formula with $AC = BC$. This approach had no chance of success, as all points on the perpendicular bisector of AB are equidistant from A and B.

(c) (i) (1 mark)

This question, requiring the evaluation of a definite integral to get $\ln 2$, was very well answered. The most common errors involved not knowing the value of $\ln 1$ or using $\log_{10}$, instead of $\log_e$, on the calculator. Candidates might avoid this mistake if they were encouraged to write ln, instead of log, although it was frustrating to see how many candidates wrote 'In' instead of ln.

(ii) (2 marks)

Simpson's rule was usually applied correctly to the given integral, but many candidates were unsure whether the multiplier had a denominator of 2, 3 or 6. The correct multiplier received a mark, as did the use of the 3 function values in the correct weighted sum.

Many responses involved the use of the trapezoidal rule, and received no marks. Common errors arose from attempts to use the Simpson's formula remembered using the words 'evens' and 'odds'. Candidates did not know whether 2 was an 'odd' or an 'even' in this context. The use of five function values was more time consuming, and more susceptible to error, but full marks were awarded if it was done correctly.

An alarming number of attempts used $f(x) = \ln x$ rather than $f(x) = 1/x$, with the resulting answer being a very poor 'approximation' to $\ln 2$.

(iii) (2 marks)

This part, which was poorly done, involved equating the answer from part (i) with the answer to part (ii) and applying either index or logarithmic laws to obtain an approximation for e. Ideally, the

statement $\ln 2 = 25/36$ should lead to $e \approx 2.713$. Bizarre values for e were often obtained by candidates who had incorrect answers in the earlier parts. These values were accepted if they were correctly developed.

Candidates were often at a loss to obtain a reasonable approximation, or simply did not understand the question. Many simply wrote down $e = 2.718$ from their calculator.

The first mark was awarded to candidates who wrote $2 = e^{25/36}$ or $\frac{\log 2}{\log e} = 25/36$. Candidates rarely progressed from $2 = e^{25/36}$ to $e = 2^{36/25}$ which was necessary to find the desired approximation, and so the second mark was awarded to very few candidates. Candidates who thought that an 'approximation' meant they should apply Simpson's rule again, or use Newton's method for roots seldom gained marks.

Question 5

Few candidates scored zero, since most were able to differentiate to earn some marks in part (b). However, not many scored 12. The induction in part (a) proved to be far too hard for the majority of candidates, and parts (b) (i) and (b) (vi) were not well done. Written expression, grammar and spelling were poor.

(a) (3 marks)
Most candidates had some idea of the process involved in proof by induction, although there was not much evidence to suggest that they understood why the process proved something. Most knew that the statement should be established for $n = 1$, and that something should then be assumed. Statements such as 'assume $n = k$', followed by something like 'let $n = k+1$' were all too common, suggesting that induction has been learnt as a mechanical process with virtually no understanding.

Many candidates clearly did not understand the expression $(n+1)(n+2)\cdots(2n-1)2n$. The most common attempt at substituting $n = 1$ into this expression was $(2 \times 1 - 1) \times 2$, which gives the correct value while indicating a complete misconception.

Candidates were thrown by the fact that the statement to be proved involved products, rather than a sum. It seemed that many candidates had never experienced a proof by induction not involving a sum, and so many decided to treat it as a sum anyway, writing such things as $S_{k+1} = S_k + T_{k+1}$. Substituting $n = k + 1$ into the expression also caused problems, with many candidates obtaining $(k+1)(k+2)\cdots(2k+1)(2k+2)$, rather than $(k+2)(k+3)\cdots(2k+1)(2k+2)$. Even those who managed to obtain the correct expression were sometimes unable to rearrange it into a useful form.

(b) (i) (3 marks)

The vast majority of the candidature failed to realise that they were being asked to show that the given function has an absolute minimum at $x = 0$. Most were able to gain two marks for showing that there was a local minimum at $x = 0$, and some scored the third mark in the course of their answer to part (ii).

Almost all candidates were able to correctly differentiate $e^x - 1 - x$, which was pleasing. The second derivative test for a minimum was not always well done, with some candidates claiming that $f''(0) \geq 0$ gave the result. Others presented the diagram

0^-	0	0^+
$-$	0	$+$

as their proof, without any further explanation. Such minimal working is not sufficient to earn the marks, as examiners cannot supply the missing details for the candidate.

(ii) (1 mark)

This very simple part was not handled at all well. Many candidates failed to evaluate f at zero, claiming that $f(x) \geq 0$ because there is a minimum at $x = 0$. Others claimed that f is an increasing function since $f''(x) > 0$. Some seemed to think that the question asked them to deduce that $f(x) \geq 0$ for $x \geq 0$, or to prove that $x \geq 0$.

Some candidates deduced that $f(x) \geq 0$ because $e^x \geq 1 + x$, totally missing connection with part (i). The examiners were somewhat perplexed by a number of candidates who attempted to treat f as a quadratic, and show that it is 'positive definite' by considering $\sqrt{b^2 - 4ac}$.

(iii) (2 marks)

Candidates generally seemed to know the basic shape of the curve $y = e^x - 1$ and the line $y = x$, although some needed to plot several points in order to sketch the line. The examiners were looking for a clear indication that the candidate was aware of the asymptotic behaviour of $y = e^x - 1$. Candidates would be well advised to include, and label, all the important features of a function on its graph.

It was evident that the connection between this part and part (i) was overlooked by many candidates, with a significant number drawing the line cutting the curve in two distinct places. Candidates who used a plastic template to draw the curve often drew diagrams which were very clear, and for which it was easy to award the two marks. In some cases, however, the examiners thought that the use of a template contributed to the error of drawing the line cutting the curve. It also appears that the exponential curve on some templates stops before it flattens out.

Only a very small number of candidates made the mistake of attempting to sketch $y = e^x - 1 - x$.

(iv) (1 mark)

This part was done very well, with most candidates demonstrating that they had learnt the mechanical process for finding an inverse function. Most were able to write $x = e^y - 1$, and solve for y. The examiners took the view that it was necessary to write the inverse function explicitly as a function of x, as the question was trivial if this was not required. While mistakes such as $\ln(e^y - 1) = y - 0$ or $\ln(x + 1) = \ln x + \ln 1$ appeared, they were not common.

(v) (1 mark)

Unfortunately, the responses to this part indicated that some candidates did not know that $g^{-1}(x)$ was the inverse function they had found in part (iv). Predictably, a number of candidates wrote $g^{-1}(x) = 1/(e^x - 1)$. Those who knew $g^{-1}(x) = \ln(x + 1)$ were generally able to write down the domain immediately. Some candidates who had failed to find the correct inverse function were still able to gain the mark here by correctly arguing that the domain was the range of g.

(vi) (1 mark)

The most successful candidates in this part were those who drew $y = g^{-1}(x)$ by reflecting a correct sketch of $y = g(x)$ drawn in part (ii) in the line $y = x$. They were then able to see that the graph of the inverse function was below the line $y = x$. Those who attempted to apply this procedure to a graph in part (ii) in which the line cut the curve in two places, were obviously in trouble. Many candidates seem aware of the fact that a function and its inverse have graphs which are reflections of each other in the line $y = x$. However, they often used words such as flipped, rotated, reflexed, inflexed and spun to describe reflection. A small number of candidates, including some with correct graphs, believed that the inequality did not hold at $x = 0$.

Many candidates clearly saw no connection between this and previous parts. New graphs were drawn of $y = \ln(x+1)$ and $y = x$, without any reference to any previous part. Candidates who argued algebraically that '$\ln(x + 1) \leq x$ for all x because $e^x - 1 \geq x$ for all x', by taking logarithms of both sides of the inequality $e^x < x+1$, were not awarded the mark.

Question 6

Most candidates managed to gain some marks for this question, but very few managed to obtain high marks. It was common for candidates to perform well on one part of the question but not the other.

(a) This part of the question was testing the candidate's knowledge of simple harmonic motion, and many candidates failed to gain any marks. Candidates who simplified at the outset by noting

$$\cos^2 3t = \frac{1 + \cos 6t}{2} = \frac{1}{2} + \frac{\cos 6t}{2}$$

had considerably less difficulty than those who did not.

(i) (1 mark)
This was generally well done. The most common mistake was to find the answer in degrees, thus making the subsequent parts harder. A significant number of candidates failed to complete this part, leaving their answer in the form $3t = \pi/6$.

(ii) (1 mark)
As in Question 7 of the 2/3 Unit (Common) Mathematics paper, the question created a difficulty for candidates by not providing an orientation for the line along which the particle was moving. The most common acceptable answer was to say the particle was travelling in a negative direction, with 'left' being almost as popular. Some said 'down' and a small number claimed the particle was travelling west. Other acceptable answers included 'toward the origin' or 'toward the centre of its motion'. One unfortunate answer was to claim that the particle was travelling in the opposite direction to its original motion. The particle was stationary at the outset, and subsequently travelled left or west or down.

Apart from this, mistakes often resulted from poor differentiation skills or from evaluating $\dot{x}|_{t=3/4}$. Another common mistake was to claim that as the particle is at $x = 3/4$, which is to the right of the origin, the particle must be travelling to the right.

(iii) (2 marks)
About one third of the candidature managed to complete this part. Most had either written x in terms of $\cos 6t$ or $\dot{x}$ in terms of $\sin 6t$. Many reached the conclusion that $\ddot{x} = -18(\cos^2 3t - \sin^2 3t) = -18\cos 6t$, but did not write this expression in terms of x and so only gained one of the two marks available. Those making an attempt to rewrite this in terms of x often left their answer as $\ddot{x} = -18(x - \sin^2 3t)$.

The most common alternative solution was to write t as a function of x and then proceed to find $\frac{dt}{dx}$, usually incorrectly, and hence $\frac{dx}{dt}$. This could be completed differentiating $\frac{1}{2}v^2$ with respect to x to find $\ddot{x}$. Only one or two candidates managed to use this approach correctly. Several candidates used their answer to part (ii) and the $\frac{1}{2}v^2$ approach to correctly evaluate $\ddot{x}$ in terms of x.

The most common incorrect answer involved either very poor differentiation or a 'fudge'. Noticing that the period was required in part (v) and believing that the period was clearly $2\pi/3$, many candidates wrote

$$x = \cos^2 3t$$
$$\dot{x} = -3\sin^2 3t$$
$$\ddot{x} = -(3)^2 \cos^2 3t = -3^2 x.$$

It was impossible to distinguish between those who genuinely thought they had differentiated and those trying to organise a period of $2\pi/3$.

(iv) (1 mark)
A few candidates chose the 'otherwise' option, having noted that since $x = \frac{1}{2} + \frac{1}{2}\cos 6t$, the motion is in the form $a + b\cos(nt + \alpha)$ and hence is clearly simple harmonic motion.

The rest relied upon their answer to part (iii). It was not unusual for candidates who had left some terms involving t in their expression for $\ddot{x}$ to realise that this was a problem here and to complete part (iii) as the first step in their answer for part (iv). Those who did so had this taken into account in determining their mark for part (iii).

Amongst those who had an expression for $\ddot{x}$ in terms of x, a common incorrect answer resulted from not expanding completely, leaving the answer in the form $\ddot{x} = -18(2x - 1)$. From this point it was difficult to claim that $\ddot{x}$ was in the form $\ddot{x} = -n^2 x$, although most made just this claim, obtaining $n = \sqrt{18}$ instead of the correct value which is 6.

(v) (1 mark)
Most candidates knew how to find the period from their answer to part (iv). This meant that $2\pi/\sqrt{18}$ was almost as common as the obvious, but incorrect answer, $2\pi/3$. Once again, those who had written x in terms of $\cos 6t$ had the fewest problems.

(b) In contrast to part (a), candidates who attempted this part usually gained some marks.

(i) (2 marks)
Most candidates attempting this tried unsuccessfully to use similarity. Those who noted that $OD = 10\,\text{cm}$ almost always completed this part. Noticing OD was apparently quite difficult. Quite a few candidates returned to part (i) after completing part (iii) which gave them a hint by drawing their attention to $\triangle DOE$.

A common error was to find FE instead of FD, but this was also awarded the two marks if it was done correctly.

(ii) (1 mark)

It was very unusual for candidates to score marks elsewhere in Question 6 without earning this mark. The only common error was to include a factor of π, giving the answer as $\angle DFE = 0.4\,\pi$ radians.

(iii) (3 marks)

Roughly half of those attempting this part used a correct method. This generally involved using the cosine rule to find DE in $\triangle DFE$ and then using this in $\triangle DOE$ to find the required angle. However, some used $\frac{1}{2}DE = 8\sin 0.2$ and repeated this for $\triangle DOE$, while others used the sine rule in an isosceles triangle.

The most common incorrect method involved using part (ii) to deduce that the length of the arc DE on the circle centred at F was 3.2 cm. Candidates then incorrectly used this as a great circle arc length on the sphere centred at O to deduce that $\angle DOE = 0.32$. Many candidates used the arc length $DE = 3.2$ cm as the length of the side DE in applying the cosine rule to $\triangle ODE$ and so found $\angle DOE = 0.321$.

There did not seem to be any incorrect method which resulted in the correct answer of 0.319. Candidates who rounded too early often found the answer 0.321, but were awarded full marks if their method was correct.

A mark was awarded to any candidate who used the line or arc DE in the two triangles or sectors DFE and DOE to find a value for $\angle DOE$. More marks could only be obtained by calculating the length of the line DE correctly or using $\triangle DOE$ to calculate the required angle. This meant that candidates using the two arcs method could obtain at most one mark, while those using $DE = 3.2$ cm in the $\triangle DOE$ could obtain two marks.

Question 7

This question was rather well done given its position on the paper, with far fewer non-attempts than most would have expected.

(a)   (i) (4 marks)

Many candidates were well versed in deriving the equations of motion for a projectile and generally did this very well. Amongst those who had some idea of what was required, the most common mistake was not to notice that the initial vertical velocity was negative. Many went back later to alter their equation for vertical displacement after noticing that it was different to the one given in the question.

Others were much more obvious in their attempts to fudge the correct equations. Several candidates wrote down the correct 6 equations but

failed to show constants of integration and state initial conditions. Examiners were not convinced that these equations were not the result of differentiating the expressions for the coordinates of the ball given in the question.

Candidates who used $v = 30$ and $\theta = -5°$ from the outset fared better than those that worked with equations in general terms and tried to substitute later.

Some candidates used formulae learnt in physics instead of calculus. These candidates were generally awarded zero, although a few gained a mark for stating the correct initial conditions.

There were some instances in which candidates integrated with respect to θ instead of t. In order to end up with the correct equations, such candidates often started with equations for $\ddot{x}$ and $\ddot{y}$ which bore no resemblance to those given in the question.

(ii) (2 marks)
Most candidates correctly identified that $y = -2$ when the ball strikes the ground, but many did not recognise that this gave them a quadratic equation in t which could be solved using the formula. The algebraic manipulation presented in this part was often below the standard which would be expected of 3 Unit candidates.

Some candidates found the Cartesian equation for the trajectory to find the horizontal distance which had been travelled at the time the ball hit the ground. Even if they succeeded in doing this, few went on to determine the time at which the impact occurred.

(iii) (2 marks)
The fact that this was projectile motion and the path of flight was not a straight line seemed to escape many candidates. A popular answer, requiring no justification, was that the angle was $5°$.

Another common mistake was to use $\tan\theta = y/x$, rather than $\tan\theta = \dot{y}/\dot{x}$.

Some candidates tried to use the Cartesian equation and solve for θ not realising that the θ in their expression was a constant, namely $5°$.

(b) (4 marks)
Hardly any candidates scored more than two marks on this part. It is hard to ascertain whether this was due to fatigue or lack of the required knowledge. Most of those who attempted the question were able to write down at least one binomial expansion correctly. Quite a few recognised the connection with the coefficient of x^2 or x^{-2}, but rarely proceeded to compute this coefficient in the expansion of $(\frac{1}{x} - x)^n$.

73

Only a handful of candidates distinguished between odd and even n, which was necessary to earn full marks.

Mathematics 4 Unit (Additional)

Question 1

This question was generally well done. There were very few non-attempts, approximately one in three candidates gained full marks, and the mean was about 13. On this evidence, the methods of integration are well-taught and well-practised, but students still need to be encouraged to check their answers by differentiating.

(a) (2 marks)

Attempts to evaluate $\int_0^1 xe^{-x^2}\, dx$ gained one mark for obtaining $-\frac{1}{2}e^{-x^2}$ and a further mark for arriving at $(e-1)/2e$, or its equivalent. Many candidates lost a mark in evaluating the limits by writing $e^0 = 0$. Many errors were made in 'simplifying' the answer to one with no negative indices.

(b) (2 marks)

The first mark for finding $\int \dfrac{e^x\, dx}{\sqrt{1 - e^{2x}}}$ was obtained by substituting correctly to arrive at $\int \dfrac{du}{\sqrt{1 - u^2}}$, and a further mark by arriving at $\sin^{-1} e^x$. Many candidates left their answer as $\sin^{-1} u$ and so failed to earn the second mark. No mark was lost for the omission of the constant.

(c) (3 marks)

Candidates were awarded one mark for the division of $4x^3 - 2x^2 + 1$ by $2x - 1$, and a further two marks for integrating the quotient and remainder. It was apparent that while many knew that the first step was to perform a division, they were not sure what to do with the result of this operation.

(d)   (i) (2 marks)

One mark was awarded for a correct method, with the second mark awarded only if all three values were correct. The variety of values obtained for a, b and c was remarkable. Many candidates displayed poor skills at solving three simultaneous equations. A few wisely checked and, if necessary, corrected their answers, but most simply moved on to part (ii) without any further thought.

   (ii) (2 marks)

This part was very well done, with most candidates recognising that the integrals involved $\tan^{-1}$ and $\ln$. Since the correct value of a was 0, one mark was available for $\int \dfrac{b\, dx}{x^2 + 4}$ and the other for $\int \dfrac{c\, dx}{x - 2}$. However, those who had made the question more difficult by obtaining a non-zero value for a in part (i) had to integrate all three functions correctly to obtain the two marks.

(e) (4 marks)

This was a standard question on integration by parts. Apart from the 5% of the candidature making an inappropriate selection of u and dv, almost all candidates recognised that two applications of the method would yield a solution. Despite the familiarity with this method, there were a distressing number of common errors including $\int \sin x \, dx = \cos x$, omitting the factor of 2 in the second application, $\cos 0 = 0$ and the inexplicably frequent occurrence of $2(\frac{\pi}{2} - 1) = \pi - \frac{1}{2}$. Candidates did not lose marks for simplification errors made after they had arrived at a correct numerical expression.

Question 2

This question was generally well done, with hardly any non-attempts and many candidates scoring full marks.

(a) (i) (1 mark)

Almost every candidate correctly evaluated the product of the two complex numbers $z = 3 + 2i$ and $w = -1 + i$.

(ii) (2 marks)

Candidates were asked to express $2/iw$ in Cartesian form. One mark was awarded for knowing how to realise the denominator and the remaining mark was for correctly finding the answer, which was $-1+i$. Most candidates did well on this question, but a number made silly arithmetic mistakes which cost them a mark. A few candidates tried to write w in polar form but were usually unable to complete the problem with this approach.

(b) (i) (2 marks)

Candidates were awarded one mark each for the modulus and argument of the complex number $\alpha = 1 + i\sqrt{3}$. The fact that the number was in the first quadrant made this relatively easy, and almost every candidate gained two marks.

(ii) (2 marks)

In this part, candidates were asked to find α^{11} in Cartesian form. This was also generally well done. One mark was given for expressing α in polar form, $2\operatorname{cis}(\pi/3)$ was sufficient, and one mark awarded for correctly applying de Moivre's theorem.

The examiners took the view that $2^{11} \operatorname{cis}(11\pi/3)$ was sufficient for full marks since $2^{11} \cos(11\pi/3) + 2^{11} \sin(11\pi/3)i$ is technically in the form $a + bi$. This was fortunate for the many candidates who made subsequent errors in evaluating the trigonometric functions.

Only a few tried to use the binomial theorem, and only one candidate succeeded by this method.

(c) (2 marks)

In this part, candidates were required to sketch the region in the plane where two inequalities held. One inequality corresponded to the disc of radius 2, centre $(0, 1)$ and the other to a wedge with its vertex $(-1, 0)$.

The fact that the centre of the circle and the vertex of the wedge were not the same was not apparent in many answers. Responses also predictably included circles with centre $(0, -1)$ and wedges with vertex $(1, 0)$. If appropriate shading was present in such answers, one mark was awarded.

A number of candidates drew a seemingly correct picture, but failed to put any scale on their axes or give any indication or description of the location of the centre of the circle and the vertex. These candidates were also awarded only one mark.

(d)   (i) (1 mark)

A polynomial with real coefficients was given and candidates were told that $1 - 3i$ was a root. They were asked to explain why $1 + 3i$ was also a root. Many candidates correctly commented on the fact that the coefficients were real and that, for such polynomials, complex roots occur as conjugates. However, candidates who referred to a 'conjugate root theorem', without any mention of the coefficients, did not receive the mark.

An alarming number of candidates seemed to believe that complex conjugates occur because the 'constant' of the polynomial was real, or because the sum of the roots was real. The example, $x^3 + 9x^2 + 16x - 84ix - 170$, which has three complex roots $1 + 3i, 1 + 4i, -11 - 7i$ whose sum and product are both real, shows that this is false. A small, but significant, number substituted the complex number $1 - 3i$ into the polynomial and tried to show the result was zero.

   (ii) (1 mark)

Candidates were asked to find all the roots of the polynomial in part (i). The third root was $-1/2$, and candidates did not need to explicitly mention the other two roots in order for the mark to be awarded.

The easiest way to find this root was to use the fact that the sum of the roots was $3/2$ and many used this method. A large number spent quite some time finding the quadratic factor arising from the two given complex roots and then performing the long division required to find the third factor. Although correct, this was time consuming and a number made numerous attempts at the process over several pages.

There was also a worrying confusion between roots and factors, with a significant proportion of the candidature claiming that $2z + 1$ was one of the roots.

(e)   (i) (2 marks)
This question asked for an explanation as to why the point P on the given diagram corresponded to the complex number $(1 + i)z_1$. There were three equally popular basic approaches to the problem.

The first used some basic geometry and Pythagoras' theorem to deduce that OP was obtained by rotating OA about the origin through $\pi/4$ radians and stretching it by a factor of $\sqrt{2}$ so that $OP = \sqrt{2}z_1 \operatorname{cis}(\pi/4) = (1 + i)z_1$. One mark was awarded for each of these two steps provided there was some link with $(1 + i)$.

The second method involved letting OA' be the vector OA rotated anticlockwise about the origin through $90°$, which corresponds to iz_1, and observing that $OP = OA + OA'$, giving $z_1 + iz_1$ as required. Again, one mark was awarded for each step. A properly marked diagram illustrating these facts was sufficient to obtain full marks.

The third method was to say that the vector AP was parallel to the vector OA' defined above and so, as vectors, $AP = iz_1$. Then $OP = OA + AP = z_1 + iz_1$. There were some difficulties with this method, as candidates often claimed explicitly that $AP = iz_1$ because clockwise rotation through $90°$ was effected by multiplication by i, without any reference to the centre of rotation. There was also general confusion between complex numbers, points and vectors. However, the marking scheme was fairly liberal on such matters.

(ii) (2 marks)
Candidates were asked to find the complex number corresponding to M, the midpoint of P and another point Q in the diagram. One mark was awarded for writing down the complex number represented by Q and the other mark for writing down the average of this number and $(1 + i)z_1$.

Many candidates gained full marks in this part without have succeeded in obtaining both marks in part (i). Many unnecessarily wasted time in long attempts to express their answer in Cartesian form. Of more concern was the large number of candidates who wrote answers such as $(\frac{1}{2}(1 + i)z_1, \frac{1}{2}(1 - i)z_2)$.

Question 3

This question was well done with the majority of candidates gaining full marks or close to full marks. Responses frequently contained some beautiful mathematics that was clearly argued and presented.

(a)   (i) (2 marks)
     This graph of $y = |f(x)|$ based on a sketch of $y = f(x)$ given in the
     question was well done, with most candidates gaining full marks.

(ii) (2 marks)
     The sketch of $y = 1/f(x)$ was also well done, with most candidates
     clearly marking asymptotes and the y-intercept. However, the lower
     branch, between $x = 1$ and $x = 3$ often had a local maximum which
     was greater than -1. Candidates were awarded one mark if two of
     the three branches were correct.

(iii) (2 marks)
     The sketch of $y = \ln f(x)$ was not so well done. Candidates were
     awarded one mark for indicating that the function was undefined
     between 1 and 3. A few candidates circled or put crosses at points
     of interest such as 0, 1 and 3, which made it difficult to determine
     clearly whether or not the points were part of their graph.

     Some candidates also created the potential for confusion by drawing
     the original graph and the new graph on the same diagram, without
     providing labels to distinguish between them.

(b)   (i) (2 marks)
     This part was very well done. Candidates were awarded one mark for
     identifying the major and minor axis and the formula for eccentricity,
     and one mark for the answer. Candidates who gave the answer $e =
     \pm 4/5$ were awarded one mark.

(ii) (2 marks)
     This part was also well done. The main error was in identifying the
     two directrices as $x = \pm 25/4$. Many candidates simply wrote $\pm 25/4$,
     while many others answered $y = \pm 25/4$ or $d = \pm 25/4$.

(iii) (2 marks)
     One mark was awarded to those candidates who established that
     $y' = -9x_0/25y_0$ at P. The second mark was awarded for a correct
     derivation from here of the equation of the tangent, making use of
     the fact that P lies on E.

     Many candidates successfully answered this question using polar
     representations.

(iv) (3 marks)
     Several methods were used to answer this question. The most
     successful was to show that the product of the gradients of PF and
     FL was -1. One mark was awarded for finding the y coordinate of
     the point L, one mark was awarded for the gradient of PF or FL, and
     one for the conclusion. A variation on this method involved showing

that the tangent of the angle between the two lines PF and FL is undefined.

Another method was to use Pythagoras' theorem. Candidates were awarded one mark for finding the y coordinate of the point L, one mark for the lengths of at least two of the sides of the triangle, and one mark for proceeding to the correct conclusion. However, only one candidate managed to work through the algebra required to get all three marks via this method.

Question 4

Candidates handled both parts of this question well, with the average mark being almost 11.5.

(a) (4 marks)
The computation of the volume by the method of cylindrical shells required the evaluation of $\int_1^3 2\pi x(4x - x^2)\,dx$. Many candidates who made small errors in setting up the integral typically thought that the volume was given by $2\pi \int_1^4 x(4x - x^2)\,dx$, $2\pi \int_0^4 x(4x - x^2)\,dx$, $2 \int_1^2 2\pi x(4x - x^2)\,dx$, $2\pi \int_1^3 (1 - x)(4x - x^2)\,dx$ or even $2\pi \int_2^3 x(4x - x^2)\,dx$.

Some candidates produced excellent concise arguments beginning with $\delta V = [\pi(x+\delta x)^2 - \pi x^2]y$, and more than half the candidature obtained the correct answer, which was $88\pi/3$. However, many thought that $29\frac{1}{3}\pi$ could be written as $29\frac{\pi}{3}$.

Some candidates with good work elsewhere in this question wrote the integral as $2\pi \int_1^3 2(4x - x^2)\,dx$ without any explanation. This is correct, as can be seen by noting that the original integral is equal to $2\pi \int_1^3 2(4x - x^2)\,dx + 2\pi \int_1^3 (x-2)(4 - (x-2)^2)\,dx$ and observing that the second integral is zero by symmetry about the line $x = 2$. However, it was not possible to award marks for this in the absence of any indication that the candidate had arrived at their answer by some legitimate means related to cylindrical shells.

(b) (i) (2 marks)
Candidates who displayed an understanding of the meaning of a double root by writing $P(x) = (x - \alpha)^2 Q(x)$ were awarded the first mark.

Most were able to complete the question by showing that $(x - \alpha)$ was a factor of $P'(x)$, although a number of candidates thought that $P'(x) = 2(x - \alpha)Q(x) + (x - \alpha)Q'(x)$.

(ii) (2 marks)
Correct differentiation and two substitutions produced the simultaneous equations $4a + 2b = -52$ and $4a + b = -32$ which could be solved

to yield $a = -3$ and $b = -20$. Although a number made arithmetic mistakes, most candidates who managed to find the simultaneous equations were able to complete this part successfully.

Some candidates divided by $(x - 2)$ twice to obtain a remainder as a function of a and b. Despite the relative difficulty of the algebra, a few candidates arrived at the correct result by this method.

Others effectively answered part (iii) at the same time by using the fact that the remaining two roots, α and β, must satisfy $\alpha + \beta = -4$ and $\alpha\beta = 9$ to deduce that the remaining factor must be $x^2 + 4x + 9$. The answer to this part could then be obtained by computing $(x^2 + 4x + 9)(x^2 - 4x + 4)$. Others reversed this process by multiplying out $(x^2 + kx + l)(x^2 - 4x + 4)$ and equating coefficients.

(iii) (2 marks)
The marks were awarded for the coefficients 4 and 9 of the remaining quadratic factor. Even the 4 was immediately clear once part (ii) had been answered.

(c) (i) (1 mark)
Most knew that x in the domain of $\sin^{-1}(3x + 1)$ must satisfy $-1 \le 3x+1 \le 1$ and to deduce from this that the domain was $-2/3 \le x \le 0$.

(ii) (2 marks)
One mark was awarded for the correct shape, including concavity, of the graph of $y = \sin^{-1}(3x + 1)$ provided the graph was aligned with the domain. The second mark was awarded for the correct range, $-\pi/2 < y < \pi/2$.

(iii) (2 marks)
Candidates who added a correct sketch of $y = \cos^{-1} x$ to the previous sketch and noted that $x = 0$ was the only solution, were awarded two marks. Many claimed that $x = \pi/2$ was another solution.

The best algebraic solution, found by a handful of candidates, was as follows. Let $\sin^{-1}(3x + 1) = \cos^{-1} x = \alpha$. Since $\sin^2 \alpha + \cos^2 \alpha = 1$, it follows that $(3x + 1)^2 + x^2 = 1$ and so the only possibilities for x are 0 and $-3/5$. A check shows that $x = 0$ is the only solution.

Question 5

About 10% of the candidature gained full marks on this question and few scored low marks. Parts (a) and (b) were especially well done.

(a) (3 marks)
Although this part was subdivided in the examination paper, the fact that candidates often solved part (ii) in the course of solving part (i), meant

that from a marking perspective, it was simpler to regard this as a single part worth three marks.

In part (i) candidates were asked to form the polynomial equation with roots α^2, β^2 and γ^2 where α, β and γ are the roots of $x^3 + 5x^2 + 11 = 0$. Part (ii) then asked them to find the value of $\alpha^2 + \beta^2 + \gamma^2$.

The majority of the candidature found an equation with the desired roots by substituting $y = \sqrt{x}$ into the original polynomial. Whilst this is the easiest method, many could not proceed beyond $y^{3/2} + 5y + 11 = 0$ to obtain the polynomial equation $y^3 = (-5y - 11)^2$.

A surprising number of candidates who had used this method to successfully answer part (i) then did not use the coefficient of y^2 in their polynomial as the basis for their answer to part (ii).

Many approached the question of finding the polynomial by first finding $\alpha^2 + \beta^2 + \gamma^2$, $\alpha^2\beta^2\gamma^2$ and $\alpha^2\beta^2 + \beta^2\alpha^2 + \gamma^2\alpha^2$. Since the coefficient of x in the original cubic was zero, this method was only slightly longer.

(b) This part was a fairly routine problem on circular motion involving a conical pendulum. Candidates were asked to use resolution of forces to show that, in the usual notation, $w^2 = g/h$.

  (i) (1 mark)
     The vast majority knew that the force towards the centre was mrw^2, or some equivalent expression. However, there was a significant number who confused the tension, T, with the force towards the centre.

  (ii) (3 marks)
     A large number of candidates gave no evidence of resolution of forces and simply wrote down expressions like $\tan\theta = \dfrac{mg}{mrw^2}$ without any explanation. These candidates could earn at most two marks.

(c) This part was a two-stage exponential modelling problem and was less well done.

  (i) (2 marks)
     Given $\frac{dw}{dt} = k_1 w$ and $w(0) = 1$, candidates were asked to find $w(s)$. It was sufficient to write down $w = Ae^{kt}$ and then find A, but many spent time deriving this equation.

  (ii) (3 marks)
     In this part, most candidates did not realise the essential fact that $\frac{dr}{dt}$ was constant with time.

  (iii) (3 marks)
     Candidates were required to find the value of s which maximises the value of $r(t) = k_2 e^{k_1 s}(t - s)$ when $t = 100$.

Most realised the need to substitute $t = 100$ to obtain the expression $R = k_2 e^{0.04s}(100 - s)$ for the value at this time as a function of s. Of these, about half knew they needed to solve $\frac{dR}{ds} = 0$ and a substantial proportion were able to arrive at the correct answer, $s = 75$.

Question 6

The first part of this question involved proving a well-known inequality relationship by mathematical induction. Surprisingly, this was not well done. The second part was on the mechanics of a downwards motion of a particle encountering resistance proportional to its velocity. Through a series of steps, candidates were led to show that a particle projected upwards takes longer to return without air resistance. Examiners were pleased with the skill displayed in the considerable amount of algebraic manipulation required in the course of attempting this question.

(a) (i) (3 marks)
Candidates did not appear to have had much experience in presenting solutions involving the application of mathematical induction to inequalities. Candidates would benefit from more practice with inequalities.

One mark was awarded for showing that the inequality held for $n = 2$. The second mark was awarded for an appropriate use of the induction hypothesis at $n = k$ in the context of an attempt to prove the required result, and the third mark was awarded if the attempt to prove the result was correct.

Many simply wrote that since $(1 + x)^k > 1 + kx$ and $1 + x > x$, it follows that $(1 + x)^{k+1} = (1 + x)(1 + x)^k = (1 + x)^k + x(1 + x)^k) > 1 + kx + x$ without discussion as to why $x(1 + x)^k > x$. Candidates approaching the question by this method needed to discuss the cases $-1 < x < 0$ and $x > 0$ separately. However, it was much easier to write $(1+x)(1+x)^k > (1+x)(1+kx) = 1+(k+1)x+kx^2 > 1+(k+1)x$.

(ii) (1 mark)
This followed immediately from part (i) by substituting $x = -1/2n$, which clearly satisfies the condition $x > -1$. Many tried to use mathematical induction again and so wasted time.

(b) (i) (2 marks)
This was well done. One mark was awarded for explaining why $v(0) = 0$, which simply required a reference to the top of the motion.

The other mark was for explaining why $\frac{dv}{dt} = g - kv$, usually via a diagram of forces or a short sentence.

(ii) (2 marks)

This was generally well done. The only problems were due to careless errors in the integration, particularly in evaluating the constant.

(iii) (2 marks)

This part was reasonably well done.

Some had no idea how to find $\int \dfrac{vdv}{g - kv}$, with quite a few trying integration by parts. Successful methods involved rewriting the integral by long division, a rearrangement or substitution for $g - kv$.

There were many errors with signs and the terms involving k and g. However, skills with properties of logarithms were used to good advantage in this part.

(iv) (2 marks)

A great many found it quite easy to combine the information from part (ii) with part (iii) to show $t = (v + ky)/g$.

Others tried to substitute the expression for v in part (ii) into part (iii). If they only substituted for v in the ln term they were usually successful. However, substitution into the other term in part (iii) usually resulted in a mess, with only a few able to proceed to obtain the result.

Some only used part (ii), computing $y = \frac{g}{k} \int 1 - e^{-kt} dt$ to obtain the desired expression.

(v) (2 marks)

Many candidates set this part out well, and showed a good understanding of what was going on. However, those who had had problems in part (iv) usually did not handle this part well.

One mark was awarded if they applied the result of part (iv) at the time when the ball returned to ground level to obtain $t_{\text{down}} = (V + kh)/g$. Many forgot that distance was measured from the top of the motion and substituted $y = 0$ to obtain $t_{\text{down}} = V/g$, while others confused the sign and substituted $y = -h$.

Quite frequently, candidates were able to show that $T = \frac{1}{k} \ln(\dfrac{g + kU}{g - kV})$ but they were not able to see that the process in part (iv) could be used to rewrite this as $\dfrac{U + V}{g}$.

(vi) (1 mark)

This part was reasonably well done. Candidates needed to provide a physical explanation for the fact that $V_0 > V$ and deduce from this that $T_0 > T$. Many provided this explanation, but then concluded that it takes less time, so T is longer.

83

Question 7

Although this was the second last question on the paper, there were many good attempts at all parts and there were very few non-attempts.

(a) (i) (2 marks)
     The graph of $y = \ln x$ and its tangent at $x = 1$ served as an introduction to the question. As expected, it caused very little difficulty.

   (ii) (2 marks)
     At times this was answered in too much detail. The most common methods used were to integrate the equation of the tangent or find the area of the triangle formed by the tangent, the x axis, and the line $x = 3/2$. The most common error in the first method was substitution of the slope of the tangent as $1/x$ rather than 1, echoing the similar error in Question 3 (b) (iii). A common error in the second method was the use of $\ln(3/2)$ as the height of the triangle. Many found the exact area under the curve, ignoring the written instructions in the question.

  (iii) (2 marks)
     This part caused difficulty to most candidates, many of whom again ignored the written instructions in the question. Those who drew another diagram had more chance of success. This provided a better explanation than a long written argument. The key was to realise that $\ln k$ was the area of a rectangle of height $\ln k$ and width one unit. A statement on the concavity of the curve then sealed the result.

     Some candidates found the equation of the tangent to $y = \ln x$ at $x = k$ and either integrated between $x = k - \frac{1}{2}$ and $x = k + \frac{1}{2}$ or computed the area of the trapezium. By either method, this was a time consuming way of obtaining $\ln k$ as an upper bound for the integral. Others attempted to find the area of the trapezium under the curve, which did not lead anywhere.

   (iv) (2 marks)
     Some candidates thought that a proof by mathematical induction was required. However, most realised that a combination of the previous results was necessary. It was not sufficient to merely state that the result followed from parts (ii) and (iii). It was necessary to show that the integral had been effectively broken up into integrals over the intervals $[1, 3/2], [3/2, 5/2], [5/2, 7/2] \ldots [n - 3/2, n - 1/2]$ and $[n - 1/2, n]$.

     Most candidates realized that the last interval required separate treatment, and a diagram clearly showed $\int_{n-\frac{1}{2}}^{n} \ln x \, dx < \frac{1}{2} \ln n$. Another

successful method was to state that $\int_{n}^{n+\frac{1}{2}} \ln x \, dx > \int_{n-\frac{1}{2}}^{n} \ln x \, dx$ due to the nature of the curve, and then apply the result of part (iii).

(v) (2 marks)
Most saw the connection with part (iv) and so algebraic errors were the main source of error. Those who were able to transform $\ln(n-1)! + \frac{1}{2}\ln n$ into $\ln n! - \frac{1}{2}\ln n$ usually gained full marks. The fact that the required expression was stated in the question proved to be of great assistance to many candidates who managed to track down and correct errors in their algebra.

(b) Underlining of key phrases followed by concentration on the given diagram would presumably have been the best approach to this question, especially if accompanied by a mental reminder that 'and' generally implies multiplication of probablities while 'or' usually requires addition. Some candidates forced denominators that were wrong or not necessary, into the probabilities in parts (i) and (ii). Some used $p = 1/2$ in all parts of the question.

(i) (1 mark)
This part was very well done. Errors which did occur were usually conceptual errors which generally prevented candidates from even attempting the remaining parts.

(ii) (2 marks)
Many obtained the required result, $p^2q^3 + p^3q^3$, early in their answer and then spent much time rewriting the answer in terms of p only.

The answer p^2q^3 was common, with many candidates not realising that the path $1 \to 2 \to 4 \to 3 \to 1 \to 2 \to 0$ was also possible.

(iii) (2 marks)
Many candidates were able to see that the probability of winning a prize without returning to 1 was $p^3 + p^3q = \dfrac{3}{16}$. This was sufficient for the first mark.

To gain the second mark the loop $1 \to 2 \to 4 \to 3 \to 1$ and its associated probability $q^2q^2 = \dfrac{1}{16}$ had to be recognized and a geometric series formed. Conversion to numbers early in the process made this recognition more likely.

A number of candidates used the alternative method that $P(\text{win}) = 1 - P(\text{losing})$. This was developed by then writing $P(\text{win}) = 1 - [P(\text{never } 4) + P(\text{once } 4) + P(\text{twice } 4) + \ldots]$, with the first two terms in the brackets actually being the results from parts (i) and (ii).

Question 8

The first part of this question tested aspects of complex numbers and polynomials while the second part was on harder 3 Unit circle geometry. The marks gained were spread evenly across parts (a) and (b). It was clear that many candidates did not have much time available in which to answer this question. Many candidates with good examination technique were able to gain one or two of the easy marks in part (a) (i), part (b) (ii) and part (b) (iii) in the time allowed, but did not attempt the remainder of the question.

(a) This question required candidates to work with complex roots of unity and apply the fact that the zeroes of polynomials with real coefficients occur in conjugate pairs, to establish a trigonometric result.

(i) (1 mark)
This was a standard result on the complex roots of unity and the question was very well done. The two most common approaches were looking at the sum of the roots of $z^7 - 1 = 0$ or by noting that the given expression was a sum of terms in a geometric progression. Candidates who used their calculators to show that the result was approximately true scored no marks.

(ii) (2 marks)
Most of those who attempted this part gained the mark for noting that $\beta = \bar{\alpha}$, but few were awarded the second mark for deducing that $\beta = \rho^3 + \rho^5 + \rho^6$ or $\beta = (\rho^3 + \rho^2 + 1)/\rho^4$.

(iii) (3 marks)
One mark was awarded for noting $\alpha + \beta = -a$ and $\alpha\beta = b$. Most candidates obtained this mark. The remaining marks were awarded for showing that $a = 1$ and $b = 2$. The degree of success in gaining these marks depended on how well candidates could manipulate the conjugates in part (ii).

(iv) (2 marks)
This part required the candidates to equate the imaginary parts of the two expressions for the roots of the quadratic and then manipulate the trigonometric terms to obtain the stated result. Very few attempted this part. Some did gain one mark for correctly identifying $-\sin(\pi/7) + \sin(2\pi/7) + \sin(3\pi/7)$ as the imaginary part of α.

(b) The first three parts were generally well done by those who had time to make a serious attempt at the question.

(i) (2 marks)
There were three common ways of approaching this part. Successful candidates were fairly evenly split between those using the result

$OP.OP'' = OT^2$, where OT is the length of a tangent from O to the circle, and those using the externally intersecting chord result. Unsuccessful candidates generally tried to use similar triangles to obtain the result, without appreciating that the ratios of the sides of the triangles will change as the position of P changes.

(ii) (1 mark)
This part was very well done. Candidates had to use part (i) and the given fact that $OP.OP'$ is constant. Candidates who appeared to be out of time often obtained their only mark on Question 8 here.

(iii) (2 marks)
This part was generally well done. One mark was awarded for using similar triangles or results relating to parallel lines to obtain a statement which was equivalent to

$$\frac{OP''}{OP'} = \frac{OC}{OC'}.$$

The second mark was awarded for using part (ii) and the fact that O and C are fixed to deduce that C' is a fixed point.

(iv) (2 marks)
There were very few serious attempts at this part. Many stated a shape for the locus without any attempt to justify their choice. One mark was awarded for stating that the locus was a circle centre C' or for showing that the length $C'P'$ was constant. The final mark required an explanation of why the locus was the full circle.