

BOARD OF STUDIES
NEW SOUTH WALES

HIGHER SCHOOL CERTIFICATE EXAMINATION

1999
**MATHEMATICS IN
SOCIETY**

2 UNIT

*Time allowed—Two hours and a half
(Plus 5 minutes reading time)*

DIRECTIONS TO CANDIDATES

- Only Board-approved calculators are to be used.
- Show all necessary working for Section II and Section III.
- Marks may be deducted for careless or badly arranged work.
- You may ask for extra Writing Booklets if you need them.
- The mark out of 80 will be converted to a mark out of 100.

Section I (20 marks)

- This Section contains 20 multiple-choice questions.
- Attempt ALL questions.
- All questions are of equal value.
- Complete your answers in either blue or black pen on the Answer Sheet provided.
- Allow about 45 minutes for this Section.

Section II (36 marks)

- Attempt ALL questions.
- Each question is worth 12 marks.
- Answer each question in a SEPARATE Writing Booklet.
- Allow about one hour for this Section.

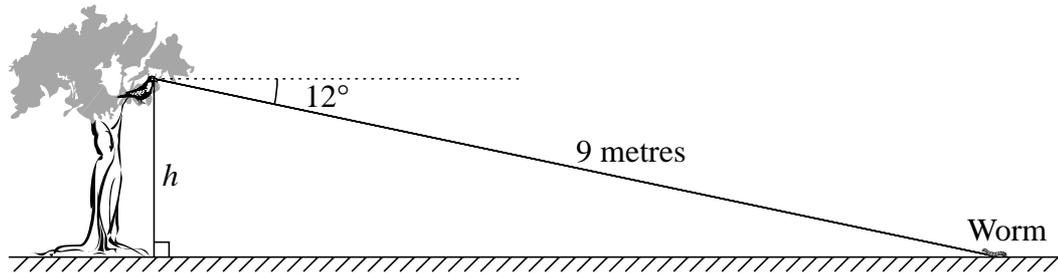
Section III (24 marks)

- Attempt TWO questions.
- Each question is worth 12 marks.
- Answer each question in a SEPARATE Writing Booklet.
- Allow about 45 minutes for this Section.

- 1 Use the formula $R = \sqrt[3]{\frac{3V}{4\pi}}$ to find R (correct to 2 decimal places) if $V = 18.76$.
- (A) 1.65 (B) 2.12 (C) 3.54 (D) 4.49
-

- 2 An unbiased coin is tossed three times. On the first two tosses the result is heads. What is the probability that the result of the third toss will be a head?
- (A) $\frac{1}{8}$ (B) $\frac{1}{6}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$
-

3



A bird nesting in a tree observes a worm on the ground. The angle of depression is 12° and the distance between the bird and the worm is 9 metres.

Which expression gives the height (h) of the bird above the ground?

- (A) $h = \frac{9}{\sin 12^\circ}$ (B) $h = \frac{9}{\sin 78^\circ}$
- (C) $h = 9 \sin 12^\circ$ (D) $h = 9 \sin 78^\circ$
-
- 4 Simplify $10(x + 3) - 2(4x + 2)$.
- (A) $2x + 5$ (B) $2x + 26$ (C) $2x + 28$ (D) $2x + 34$
-

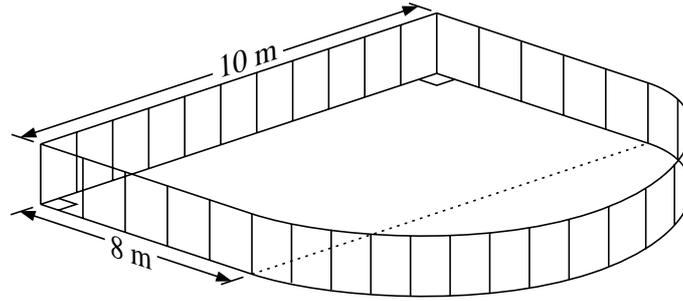
- 5 A set of scores has mean 10 and standard deviation 1.6.

The lowest score is 7 and the range is 5.

What percentage of the scores lie within two standard deviations of the mean?

- (A) 68% (B) 95% (C) 99% (D) 100%
-

6



The paddock enclosed by the fence shown in the diagram consists of a rectangle and a semicircle.

What is the area of the paddock to the nearest square metre?

- (A) 119 (B) 159 (C) 237 (D) 394
-

7 Solve $\frac{3x+5}{x-2} = 7$.

- (A) $x = \frac{7}{4}$ (B) $x = \frac{9}{4}$ (C) $x = \frac{19}{4}$ (D) $x = \frac{7}{10}$
-

8 A new colour of paint is made using a mixture of red, yellow and white paint in the ratio 2 : 3 : 6.

If the new paint comes in two-litre tins, how much yellow paint (to the nearest mL) is needed in each two-litre tin?

- (A) 364 mL (B) 545 mL (C) 667 mL (D) 1091 mL
-

9 A car is travelling at a speed of v kilometres per hour. This speed can be converted to metres per second by using the expression:

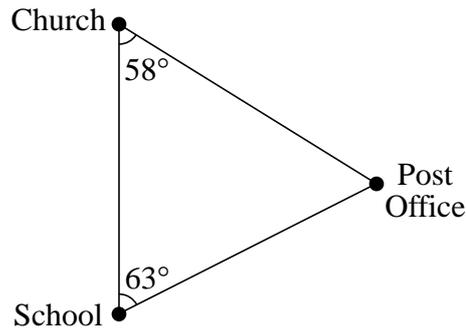
- (A) $\frac{1000 \times v}{60 \times 60}$ (B) $\frac{60 \times 60 \times v}{1000}$
 (C) $1000 \times 60 \times 60 \times v$ (D) $\frac{v}{1000 \times 60 \times 60}$
-

10 The population of Australia in December 1998 was estimated to be 18 615 837.

If the population increases by 0.96% per annum, what will the estimated population be in December 2000?

- (A) 17 871 204 (B) 18 794 549 (C) 18 973 261 (D) 18 974 977
-

- 11 The diagram shows the positions of three buildings in a town.



The church is due north of the school. What is the bearing of the post office from the church?

- (A) 058° (B) 059° (C) 122° (D) 302°
-

- 12 After six mathematics tests, Nadine's mean score was 71. She hopes to raise her mean score to 75 after the next test.

What mark will Nadine need to achieve on her seventh test if her mean mark is to be 75?

- (A) 75 (B) 79 (C) 95 (D) 99
-

- 13 The height of a container is inversely proportional to the square of the radius of its base.

When the height is 5 cm, the radius of the base is 3 cm.

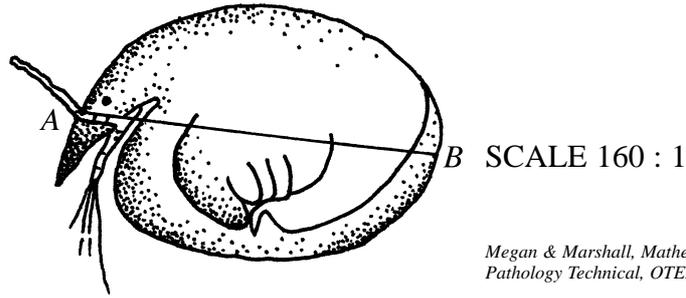
What is the height if the radius of the base is 5 cm?

- (A) $1\frac{4}{5}$ cm (B) 3 cm (C) $8\frac{1}{3}$ cm (D) $13\frac{8}{9}$ cm
-

- 14 Which of the following expressions is equivalent to $(6t^3)^2$?

- (A) $12t^6$ (B) $36t^6$ (C) $12t^5$ (D) $36t^5$
-

- 15 The tiny animal illustrated below, called a *chydorus*, lives in fresh-water ponds.

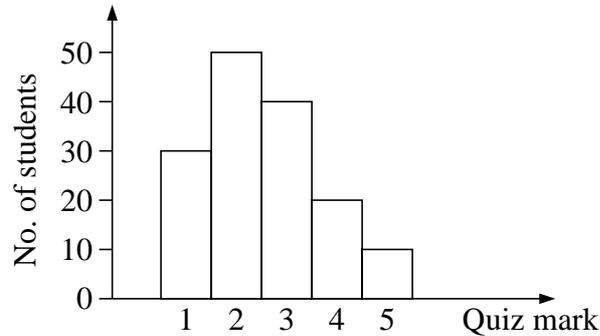


*Megan & Marshall, Mathematics for Biology/
Pathology Technical, OTEN-DET, Sydney.*

The length of the *chydorus* is measured from *A* to *B*. In the diagram, *AB* is 48 mm long. What is the length of the *chydorus*?

- (A) 3.0×10^{-1} mm (B) 3.3×10^{-1} mm
(C) 3.0 mm (D) 3.3 mm

16



The graph shows the results of 150 students on a maths quiz.

Which statement is correct?

- (A) The median is 3 and the mode is 2.
(B) The median is 2 and the mode is 2.
(C) The median is 2 and the mode is 3.
(D) The median is 3 and the mode is 3.

- 17 One litre of water was used to make exactly 64 ice cubes. These ice cubes melted too quickly. It was decided to make larger ice cubes by doubling all the dimensions.

How many of these larger ice cubes can be made using one litre of water?

- (A) 4 (B) 8 (C) 16 (D) 32

- 18 In a primary school class there are 13 boys and 14 girls.

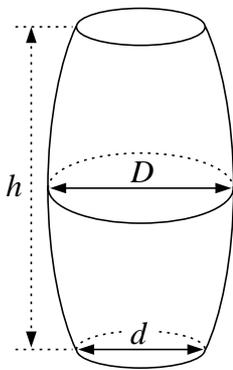
In order to choose two class representatives, the teacher places the names of all the students in a box. Two names are selected at random.

What is the probability that two girls' names will be selected?

- (A) $\frac{182}{729}$ (B) $\frac{196}{729}$ (C) $\frac{7}{27}$ (D) $\frac{14}{27}$
-

- 19 The formula below gives the approximate volume of a barrel:

$$V = \frac{\pi}{12}h(2D^2 + d^2),$$



where h is the height of the barrel,
 D is the diameter at the centre of the barrel, and
 d is the diameter at the base of the barrel.

Find the approximate volume, in litres, of a barrel with dimensions $h = 1.1$ metres, $D = 0.85$ metres and $d = 0.70$ metres. ($1 \text{ m}^3 = 1000 \text{ L}$)

- (A) 0.490 (B) 0.557 (C) 490 (D) 557
-

- 20 In five spelling tests, Jenny made the following numbers of mistakes:

7 1 5 4 3

The mean number of mistakes is 4 and the standard deviation is 2.

If she makes no mistakes in either of the next two tests, then

- (A) the mean increases and the standard deviation increases.
 (B) the mean increases and the standard deviation decreases.
 (C) the mean decreases and the standard deviation increases.
 (D) the mean decreases and the standard deviation decreases.
-

SECTION II

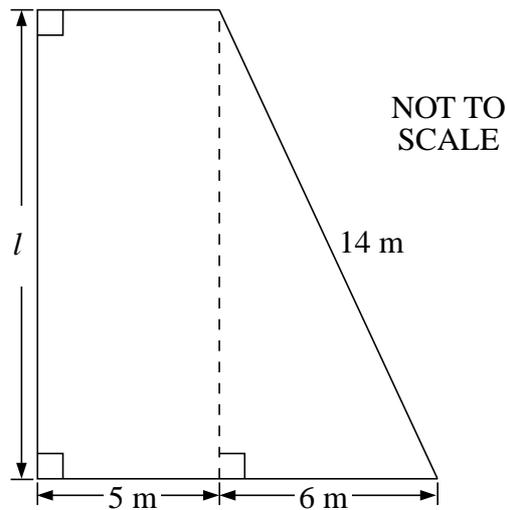
Attempt ALL questions.

Each question is worth 12 marks.

Show all necessary working.

QUESTION 21 Use a SEPARATE Writing Booklet.

(a) The following diagram represents the shape of a garden bed.

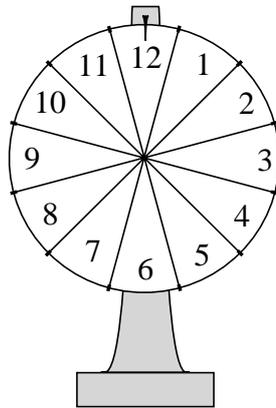


- (i) Find the length l , to the nearest tenth of a metre.
- (ii) Calculate the area of the garden bed, to the nearest square metre.
- (iii) The fertiliser used for this garden bed comes in 4 kg bags. The fertiliser should be used at a rate of 0.3 kg per square metre.

How many bags of fertiliser need to be purchased to cover this garden bed?

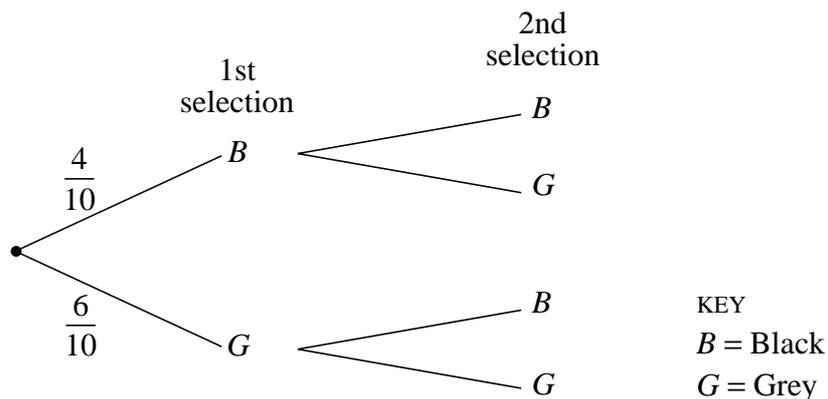
QUESTION 21 (Continued)

- (b) On the 'Wheel of Luck', Toni spins the wheel shown below.



Toni's spin must land on 1 in order to win a car. If her first spin does not land on 1, she may have one more spin.

- (i) Find the probability that Toni wins a car after one spin.
 - (ii) Find the probability that Toni wins a car after two spins.
 - (iii) Find the probability that Toni does not win a car.
- (c) Sebastian has 4 black socks and 6 grey socks in a drawer. He selects 2 socks at random.
- (i) Copy the tree diagram shown below into your Writing Booklet, and write the probability on each branch.



- (ii) What is the probability that Sebastian selects a pair of socks matching in colour?
- (d) A bucket holds 4 litres of water when full. The bucket is placed under a tap, which leaks at a constant rate of 50 drops/minute. Each drop of water is 0.1 mL.
- How long does it take for the drops of water to fill the bucket if it was empty when placed under the tap? Give your answer in hours and minutes.

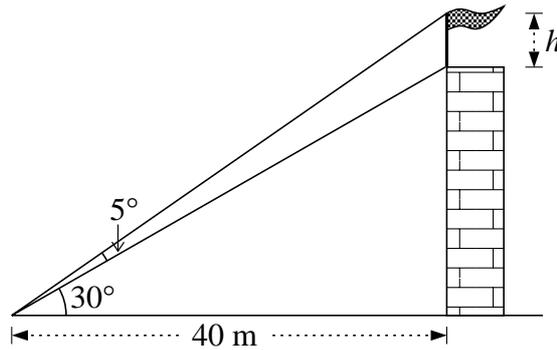
QUESTION 22 Use a SEPARATE Writing Booklet.

- (a) A molecule of hydrogen has a mass of 3.32×10^{-24} g.

What is the mass of 40 billion hydrogen molecules? (1 billion = 1 000 000 000)

- (b) Solve the equation $\sqrt{5x+1} = 36$.

- (c)



Calculate the height (h) of the flagpole, to the nearest metre.

- (d) Bryce walks 2 kilometres east, then 3 kilometres north-east.
- In your Writing Booklet, draw a neat sketch showing all of the above information.
 - Calculate the distance, in a straight line, from Bryce's starting position to his present position.

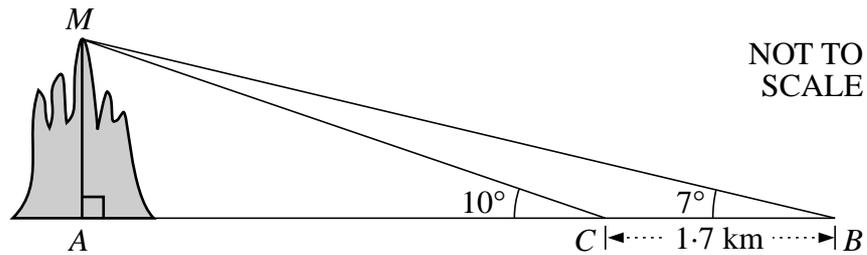
$$\left(\text{Cosine rule : } a^2 = b^2 + c^2 - 2bc \cos A \right)$$

QUESTION 22 (Continued)

- (e) During a trek in Nepal, Anne recorded the angle of elevation from various points to the top of a mountain.

From a point B , the angle was 7° . From another point C on the same level as B , the angle was 10° . The point C was 1.7 km closer to the base of the mountain than was B .

The following diagram represents this information. In the diagram, AM represents the height of the mountain and ACB is a straight line.



- (i) Find the size of $\angle BMC$.
- (ii) Calculate the length CM to two decimal places.
- (iii) Find the height of the mountain.

$$\left(\text{Sine rule: } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \right)$$

Please turn over

QUESTION 23 Use a SEPARATE Writing Booklet.

- (a) The maximum temperature for each day of September was recorded. The information is listed in the table below.

<i>Temperature (°C)</i> x	<i>Frequency</i> f	fx	<i>Cumulative frequency</i>
19	4	76	4
20	6	120	10
21	9	A	19
22	0	0	19
23	7	161	B
24	4	96	30

- (i) What is the mode of this distribution?
 - (ii) Find the value of A in the fx column.
 - (iii) Calculate the mean maximum temperature for the month of September.
 - (iv) Find the standard deviation of this distribution.
 - (v) Find the value of B in the *Cumulative frequency* column.
 - (vi) Complete the cumulative frequency histogram on the graph paper provided on page 29.
 - (vii) Draw a cumulative frequency polygon on the same graph.
 - (viii) Use the graph to find the interquartile range of this distribution.
- (b) An advertisement for Happy Teeth toothpaste makes the following claim:

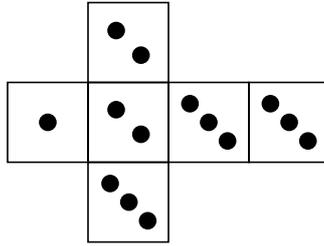
‘Two out of every three dentists surveyed recommend Happy Teeth toothpaste for sparkling healthy teeth.’

Georgio concludes from this advertisement that $\frac{2}{3}$ of all dentists recommend Happy Teeth toothpaste.

Is Georgio’s conclusion correct? Briefly justify your answer.

QUESTION 23 (Continued)

(c) A die is made from the net illustrated below.



The die is rolled twice, and the sum of the results is recorded.

		<i>First roll</i>					
		1	2	2	3	3	3
<i>Second roll</i>	1	2	3	3	4	4	4
	2	3	4	4	5	5	5
	2	3	4	4	5	5	5
	3	4	5	5	6	6	6
	3	4	5	5	6	6	6
	3	4	5	5	6	6	6

- (i) What is the probability that the sum is bigger than 2?
- (ii) What is the probability that the sum is an odd number?
- (iii) What is the probability that the number obtained on the second roll is greater than the number obtained on the first roll?

Please turn over

SECTION III

Attempt TWO questions.

Each question is worth 12 marks.

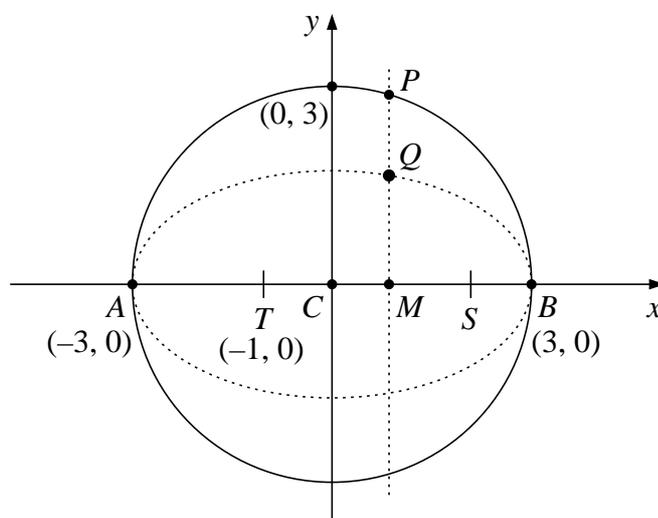
Show all necessary working.

QUESTION 24 Space Mathematics Use a SEPARATE Writing Booklet.

- (a) A space probe is launched from Earth towards a star which is 10^{11} km away. It will take the space probe 150 years to reach the star.

Calculate the speed, in km/h, at which the probe will travel. Give your answer in scientific notation, correct to three significant figures.

- (b)



An ellipse (with major axis AB) is drawn within the auxiliary circle shown in the diagram, through the point Q .

The point Q is such that $\frac{QM}{PM} = \frac{2}{3}$.

- What is the value of $\frac{b}{a}$ for this ellipse?
- Calculate the eccentricity of this ellipse. $\left(e^2 = 1 - \frac{b^2}{a^2} \right)$
- The point S shown on the diagram is a focus of this ellipse.

Find the coordinates of S . ($CS = e \cdot CB$)

- The point T shown on the diagram is a focus of another ellipse drawn within the same auxiliary circle.
 - Find the eccentricity of this ellipse.
 - Will this ellipse lie inside or outside the ellipse shown in the diagram? Give a reason for your answer.

QUESTION 24 (Continued)

- (c) The average distance from the planet Neptune to the Sun is 30 AU. ($1 \text{ AU} \approx 1.5 \times 10^8 \text{ km}$)
- (i) What is this distance in kilometres? Give your answer in scientific notation.
 - (ii) What is this distance in light years? (Light travels at $3 \times 10^5 \text{ km/s}$.)
Give your answer correct to one significant figure.
 - (iii) Use the formula $R^3 = T^2$ (where R is in AU and T is in years) to find the period of Neptune.
 - (iv) Neptune's mass is approximately 18 times that of Earth. The escape velocity of Neptune (V_P) is 25 km/s. The escape velocity of Earth (V_E) is 11 km/s.

Use the formula $V_P = V_E \sqrt{\frac{m}{r}}$ to find the radius of Neptune in terms of the radius of Earth.

Please turn over

QUESTION 25 Mathematics of Chance and Gambling

Use a SEPARATE Writing Booklet.

- (a) A pack of cards consists of 52 cards in 4 suits. Within each suit there are 13 cards:

2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King, Ace (in ascending order of value).

Susan and Stephen play a game in which they are dealt one card each at random from the pack. The person with the higher value card wins.

Susan shows her card first and it is a 10.

- (i) What is the probability that Stephen has another 10?
- (ii) What is the probability that Stephen has a higher valued card?
- (b) One hundred raffle tickets are sold at \$5 each.
- First prize is \$300.
 - Second prize is a dinner voucher at a local restaurant.
 - There are 10 prizes of \$10 each.

- (i) Hussein buys one ticket in the raffle.

What is the probability that he wins a prize?

- (ii) If the raffle is fair, what is the value of the dinner voucher?
- (c) (i) Write down the next line of Pascal's Triangle, following the lines:

$$\begin{array}{cccc}
 1 & 1 & & \\
 1 & 2 & 1 & \\
 1 & 3 & 3 & 1 \\
 1 & 4 & 6 & 4 & 1
 \end{array}$$

- (ii) Five fair coins are tossed.

Using your answer to part (c) (i), or otherwise, find the probability that the result is 2 heads and 3 tails.

QUESTION 25 (Continued)

- (d) With four teams remaining in a football competition, a bookmaker offers the following odds on which team will win the Grand Final.

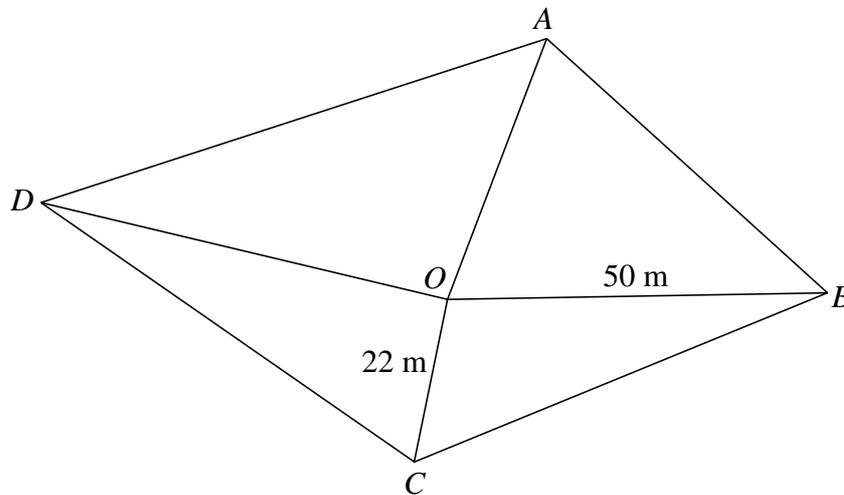
ANTELOPES	3/2 ON
BILLY GOATS	4/1
CAMELS	2/1
LIZARDS	7/2

- (i) Which team does the bookmaker believe is least likely to win the Grand Final?
- (ii) Lawrence bets \$10 on the Camels. If the Camels win, how much will Lawrence collect from the bookmaker?
- (iii) If Tanya bets \$30 on the Antelopes, and the Antelopes win, how much does Tanya win?
- (iv) How much must Adam bet on the Billy Goats in order to win \$100?
- (v) Calculate the bookmaker's percentage margin.

Please turn over

QUESTION 26 Land and Time Measurement Use a SEPARATE Writing Booklet.

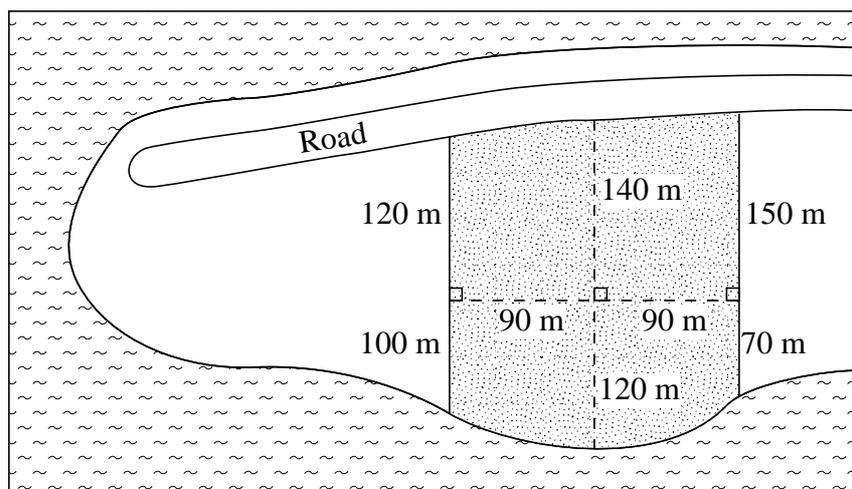
- (a) A field was surveyed using a plane table survey. The scale drawing below is the result of the survey.



- What scale has been used?
- Use the drawing to find the perimeter of the field (to the nearest metre).
- Find the size of $\angle BOC$.
- Calculate the area of that part of the field represented by triangle BOC .

$$\left(\text{Area of triangle} = \frac{1}{2} ab \sin C \right)$$

- (b)



Use Simpson's rule to find the approximate area of the shaded block of waterfront land.

$$\left(A \approx \frac{h}{3} (d_F + 4d_M + d_L) \right)$$

QUESTION 26 (Continued)

- (c) Pam lives in Sydney (34°S , 150°E). Her daughter Cara is an exchange student in Denver, Colorado USA (40°N , 105°W).
- (i) What is the time difference (ignoring time zones) between Sydney and Denver?
 - (ii) Pam wants to telephone Cara at 4 pm Friday (Denver time). At what time and on what day (Sydney time) should Pam phone?
- (d) A cargo plane flies from Capetown, South Africa (34°S , 18°E) to Sydney (34°S , 150°E), along the 34°S parallel of latitude.
- (i) Find the radius (r) of the small circle along which the plane flies.
($r = R \cos \alpha$; $R = 6400$ km)
 - (ii) Find the distance of the flight.

Please turn over

QUESTION 27 Personal Finance Use a SEPARATE Writing Booklet.

- (a) Joyce is a sales representative at WESELLIT Real Estate agency. The agency charges commission on all sales as follows:

WESELLIT COMMISSION RATES	
<i>Selling price</i>	<i>Rate</i>
Up to \$160 000	4%
\$160 000 and over	5%

Joyce is paid \$150 per week plus 6% of the commission received by WESELLIT.

In the week beginning 6 September 1999, Joyce sold one property for \$130 000 and one for \$210 000. She sold no properties over the next two weeks.

- (i) What is the commission paid to WESELLIT on the sale of the property worth \$130 000?
 - (ii) What is the total commission paid to WESELLIT on the sale of both properties?
 - (iii) Calculate Joyce's pay for the week beginning 6 September 1999.
 - (iv) What is Joyce's average weekly income for the three-week period?
- (b) The following table is used by ARMNLEG Insurance to calculate the premium to be charged for life insurance.

Premium Rate Table (rate in \$ per \$1000 of cover)

<i>Age</i>	<i>Male non-smoker</i>	<i>Male smoker</i>	<i>Female non-smoker</i>	<i>Female smoker</i>
33	0.88	1.54	0.66	1.10
34	0.88	1.65	0.66	1.21
35	0.88	1.65	0.77	1.21
36	0.88	1.87	0.77	1.32
37	0.92	1.98	0.88	1.32

- (i) Jason, aged 35, decides to take out insurance for \$90 000. Jason is a smoker.
How much premium will he pay?
- (ii) How much would Jason save on this premium if he gave up smoking?
- (iii) Leanne does not smoke and is aged 36. Her premium is \$84.70.
For how much is Leanne insured?

QUESTION 27 (Continued)

(c) The table below gives personal income tax rates.

<i>Taxable income</i>	<i>Tax payable</i>
\$0 – \$5 500	Nil
\$5 501 – \$21 200	Nil plus 20 cents for each \$1 over \$5 500
\$21 201 – \$39 400	\$3 140 plus \boxed{A} for each \$1 over \$21 200
\$39 401 – \$54 200	\$9 146 plus 44 cents for each \$1 over \$39 400
Over \$54 200	\$15 658 plus 48 cents for each \$1 over \$54 200

- (i) Sonia has a taxable income of \$45 000. Use the table to calculate the tax on Sonia's income.
- (ii) Sonia must also pay the Medicare levy of 1.5% of taxable income.
Calculate the amount Sonia must pay for Medicare.
- (iii) Throughout the year Sonia has \$460 tax per fortnight deducted from her salary.
How much does Sonia owe the tax department at the end of the financial year?
- (iv) This table was designed so that a person with a taxable income of \$35 000 would pay 22% of that income in tax (not including the Medicare levy).

What is the value of A in the table?

Please turn over

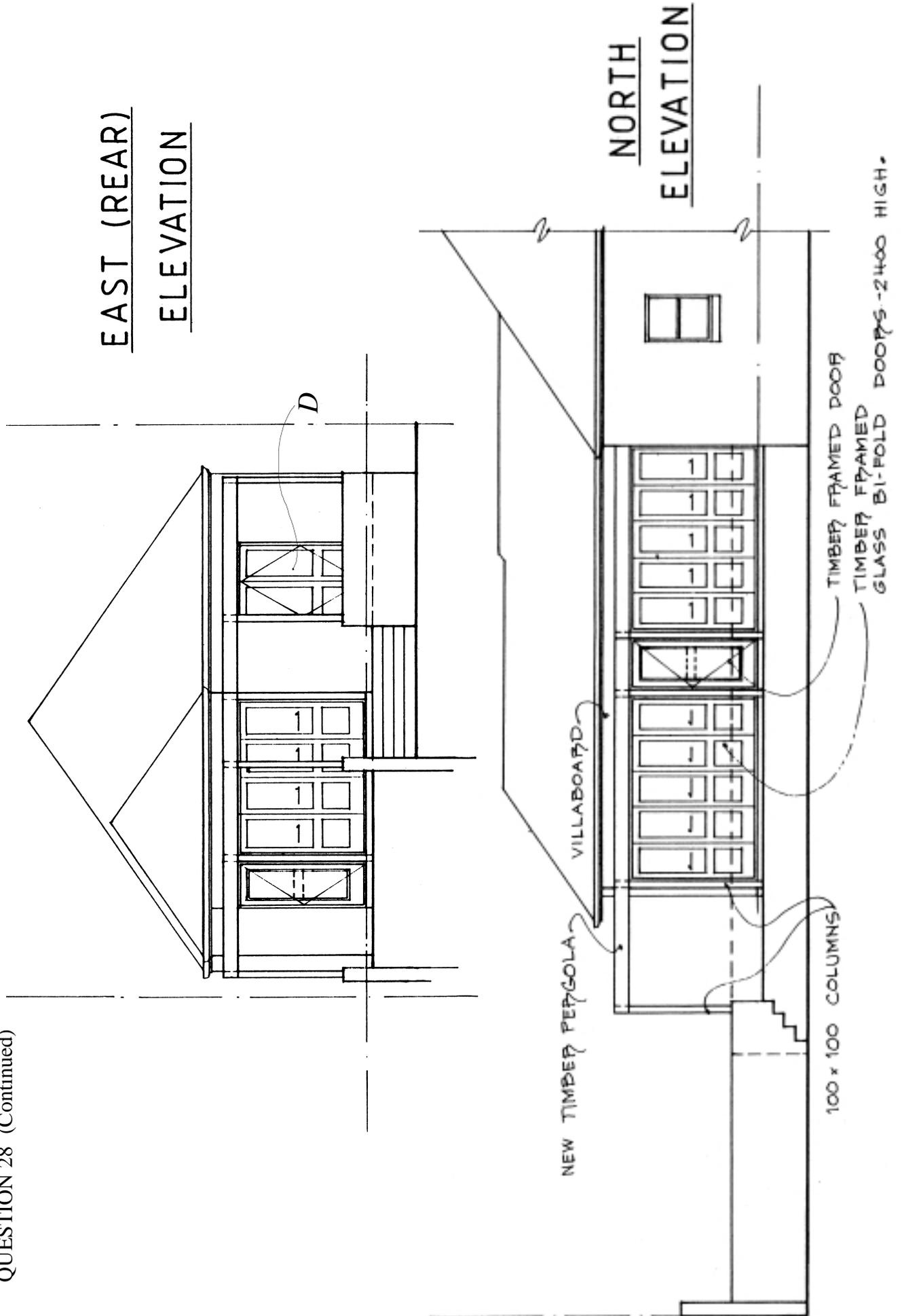
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QUESTION 28 Mathematics in Construction Use a SEPARATE Writing Booklet.

- (a) The plans (on pages 24 and 25) show the rear extension of a house.
- (i) What is the thickness of the wall between the rumpus room and the hall?
 - (ii) What is the scale used on these plans?
 - (iii) What is the height of the pergola above the paved play area?
 - (iv) What is the area of the bedroom in square metres?
 - (v) The carpet to be used in the bedroom costs \$155 per metre. A roll of this carpet is 3.6 m wide. What is the cost of carpeting the bedroom so that there are no joins in the carpet?
 - (vi) On the east elevation a door is labelled *D*. Into which room does this door lead?
 - (vii) How many windows are on the south elevation of this extension?

Question 28 continues on pages 24, 25 and 26

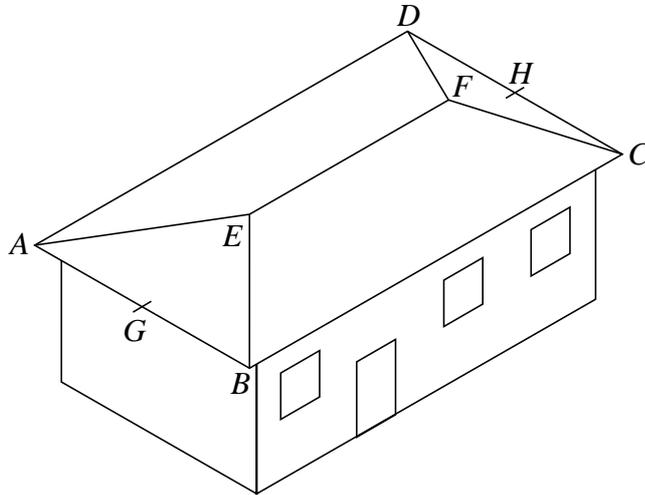
QUESTION 28 (Continued)



QUESTION 28 (Continued)

(b) This diagram shows a house with a hipped roof.

G and H are the midpoints of AB and CD respectively.



- (i) Name a pair of skew lines in the roof structure.
- (ii) Sketch the vertical section of the roof through the line EF .
- (iii) Name the angle between the roof plane ABE and the ceiling plane $ABCD$.
- (iv) Given $AB = 7210$ mm and $GE = 3800$ mm, find the length of the ridge BE to the nearest millimetre.

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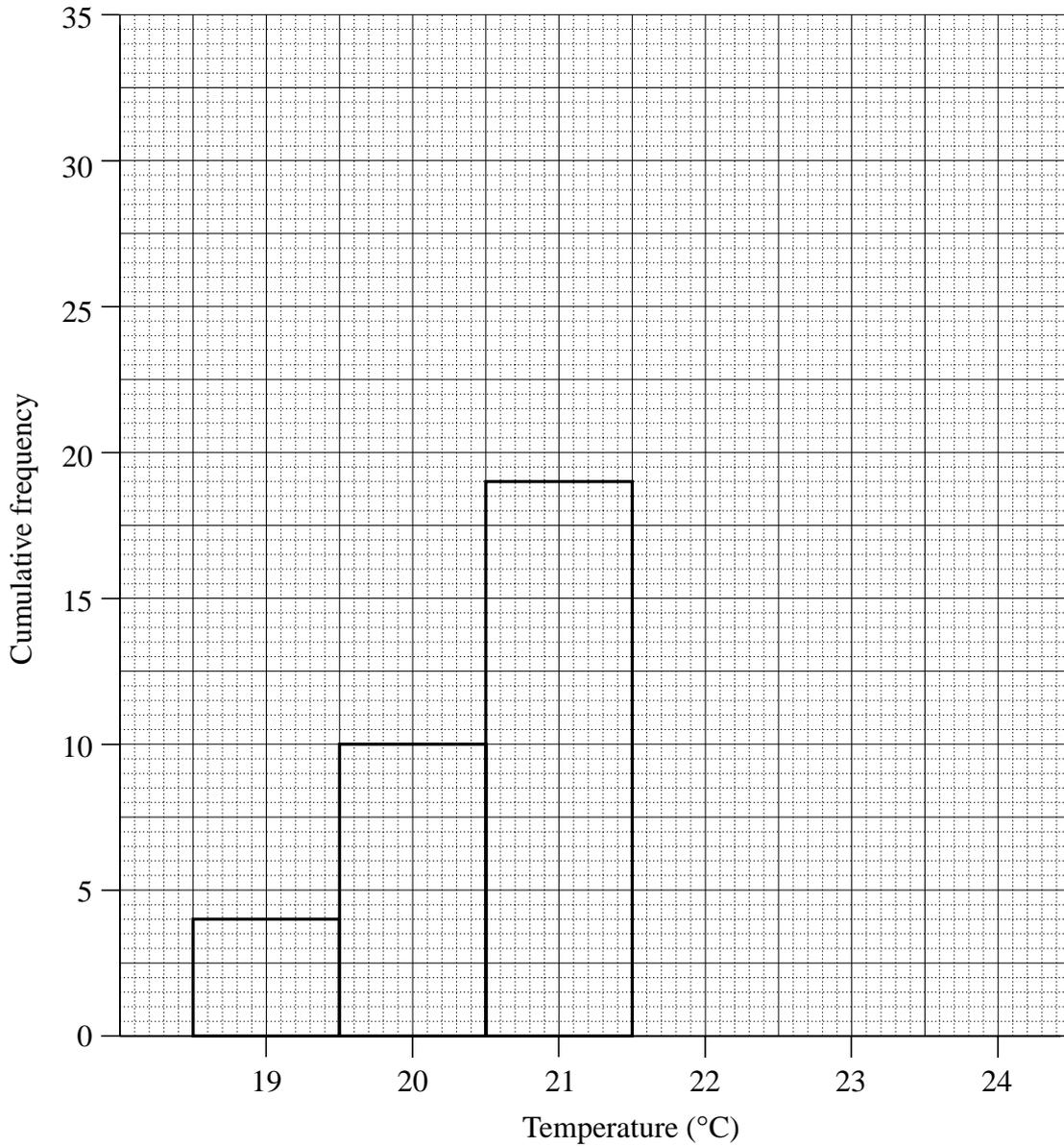
STUDENT NUMBER

**1999
HIGHER SCHOOL CERTIFICATE EXAMINATION
MATHEMATICS IN SOCIETY
2 UNIT**

CENTRE NUMBER

This page is to be detached, completed and attached to the inside front cover of your Writing Booklet for Question 23.

QUESTION 23



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