BOARD OF STUDIES

NEW SOUTH WALES

HIGHER SCHOOL CERTIFICATE EXAMINATION

1996

MATHEMATICS IN SOCIETY

2 UNIT

Time allowed—Two hours and a half
(Plus 5 minutes' reading time)

DIRECTIONS TO CANDIDATES

- Board-approved calculators may be used.
- Show all necessary working for Section II and Section III.
- Marks may be deducted for careless or badly arranged work.
- You may ask for extra Writing Booklets if you need them.
- The mark out of 80 will be converted to a mark out of 100.

Section I (20 marks)

- This Section contains 20 multiple-choice questions.
- Attempt ALL questions.
- All questions are of equal value.
- Mark your answers in pencil on the Answer Sheet provided.
- Allow about 45 minutes for this Section.

Section II (36 marks)

- Attempt ALL questions.
- Each question is worth 12 marks.
- Answer each question in a *separate* Writing Booklet.
- Allow about one hour for this Section.

Section III (24 marks)

- Attempt TWO questions.
- Each question is worth 12 marks.
- Answer each question in a *separate* Writing Booklet.
- Allow about 45 minutes for this Section.

SECTION I

Attempt ALL questions.

All questions are of equal value.

Select the alternative A, B, C, or D that best answers the question.

Mark your answers in pencil on the *separate* Answer Sheet provided.

1. The Aswan Dam in Egypt has a volume of 43 000 megalitres of water.
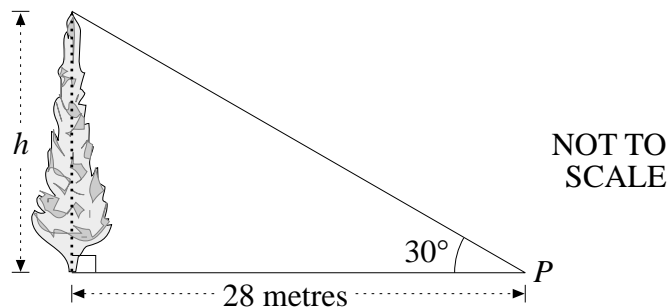
A megalitre is one million litres.

43 000 megalitres is equal to

(A)   4 300 000 litres. (B)   43 000 000 litres.

(C)   4 300 000 000 litres. (D)   43 000 000 000 litres.

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2. Simplify $5 - 2(x + 1)$.

(A) $3 - 2x$ (B) $4 - 2x$ (C) $3 + 3x$ (D) $7 - 2x$

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3. The angle of elevation of the top of a tree from a point P on the ground is 30°. The point P is 28 metres from the base of the tree.

The correct expression for h, the height of the tree, is

(A) $h = 28 \tan 30°$ (B) $h = \dfrac{28}{\tan 30°}$

(C) $h = \dfrac{\tan 30°}{28}$ (D) $h = 30 \tan 28°$

4. Which one of the following groups of scores has a mean of 60 and a median of 50?

(A)   10, 50, 60, 70, 80, 90

(B)   40, 40, 45, 55, 70, 90

(C)   40, 45, 45, 55, 85, 90

(D)   30, 40, 50, 50, 70, 80

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5. Barry's Paint and Wallpaper shop is having a sale.   Barry puts the following advertisement into the local newspaper.

Barry's Paint and Wallpaper

MONSTER SALE

20% off all ten-litre cans of paint!

10% off all wallpaper!

Before the sale started, Jodi purchased two ten-litre cans of paint at $50·00 per can and five rolls of wallpaper at $12·00 per roll.

How much would Jodi have paid altogether if she had purchased the same items during the sale?

(A) $112·00   (B) $134·00   (C) $138·00   (D) $160·00

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6. A microchip is square, with side length 0·000 000 3 mm.

What is its area, expressed in scientific notation?

(A) 3×10^{-7} mm^2

(B) 3×10^{7} mm^2

(C) 9×10^{-14} mm^2

(D) 9×10^{14} mm^2

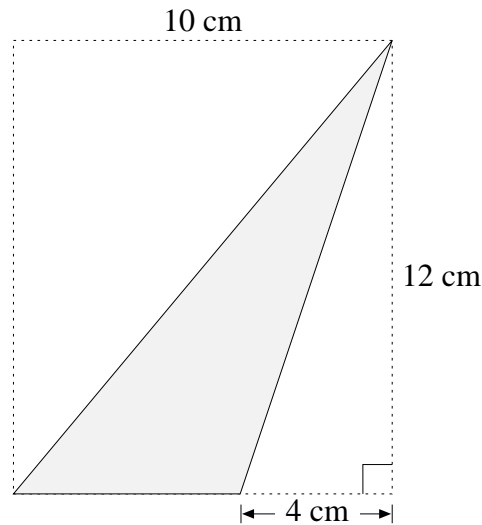**7.** $4^3 \div 4^{-2} = \ ?$

(A)   1   (B)   4   (C) 2^5 (D) 2^{10}

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8. Calculate the area of the shaded part of the rectangle shown below.

(A)   24 cm^2 (B)   36 cm^2 (C)   60 cm^2 (D)   72 cm^2

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9. The following table lists the purchase prices of cars bought in Australia and in the United States of America (U.S.). All prices are in Australian dollars.

Model	Purchase price in Australia	Purchase price in the U.S.
Audi A4 V6	52 690	36 936
BMW Compact	52 450	28 306
Ford Mondeo LX sedan	28 690	20 581
Honda Civic CXi 3dr	24 650	16 001
Honda Accord	38 800	21 478
Jeep Cherokee	41 968	31 258
Toyota Camry CSX	36 340	28 687
Volkswagen Vento GL	35 590	20 269
All prices in Australian dollars		

A Honda Accord is purchased in the U.S.

What is the percentage saving compared to its purchase price in Australia?

(A)   35%   (B)   45%   (C)   55%   (D)   81%

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10. One boy and two girls are about to sit in a row.

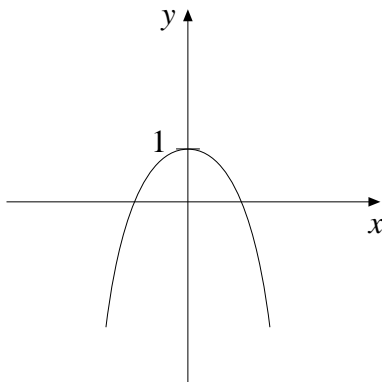Calculate the probability that the two girls will sit together.

(A) $\dfrac{1}{4}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{3}{8}$ (D) $\dfrac{2}{3}$

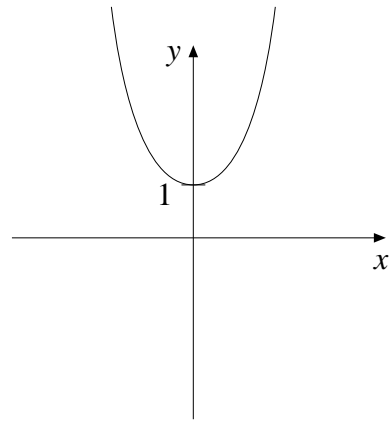**11.** The following table shows ordered pairs for the equation $y = 1 - x^2$.

x	-1	0	1
y	0	1	0

Which of the following graphs could represent the equation $y = 1 - x^2$?
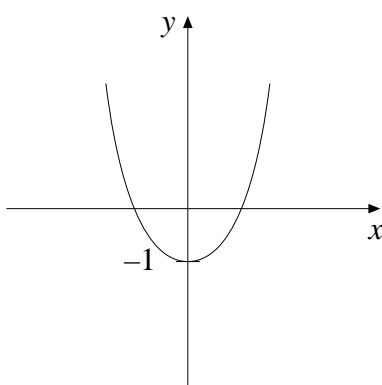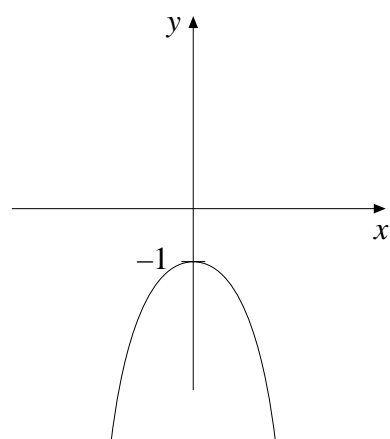
(A)

(B)

(C)

(D)

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12. The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$.

If $V = 100 \text{ cm}^3$, find r to the nearest millimetre.

(A) 29 mm   (B) 35 mm   (C) 62 mm   (D) 75 mm

13. At the beginning of the football season, the Ramsay Street Raiders were given a 70% chance of reaching the semi-final round, and the Summer Bay Sizzlers were given a 20% chance.

What is the probability that neither the Ramsay Street Raiders nor the Summer Bay Sizzlers reach the semi-final round?
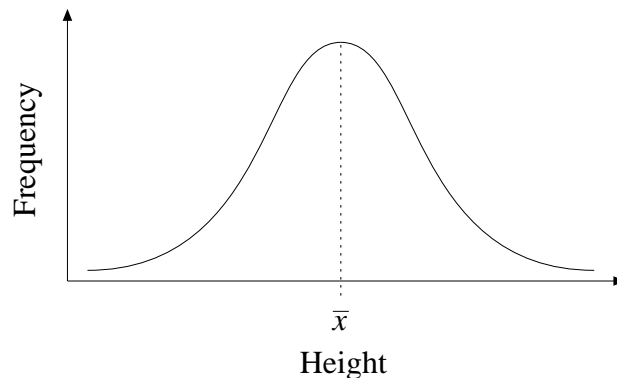
(A) 0·14   (B) 0·24   (C) 0·86   (D) 0·90

14. Three mathematics classes did the same assessment task. The mean marks for the three classes were 60, 75, and 76·5. The numbers of students in the three classes were 22, 18, and 20 respectively.

What was the mean mark for all students on this assessment task?

(A) 45·25   (B) 70   (C) 70·5   (D) 75

15. The frequency graph for the heights of a large group of people is shown.

The mean $(\bar{x})$ of the heights is 155 cm and the standard deviation is 11·2 cm. A person is chosen at random from this group.

Between which two values will the height of this person *almost certainly* lie?

(A) 132·6 cm and 177·4 cm   (B) 155 cm and 177·4 cm

(C) 121·4 cm and 188·6 cm   (D) 155 cm and 188·6 cm

16. The perimeter of a rectangle is 84 cm. The length of the rectangle is twice the breadth.

What is the length of the rectangle?

(A) 7 cm   (B) 14 cm   (C) 21 cm   (D) 28 cm

17. The rate of vibration of a string varies inversely as its length. A string 15 cm long vibrates at 5000 hertz.

What length of string will vibrate at 4000 hertz?

(A) 8·33 cm   (B) 12 cm   (C) 18 cm   (D) 18·75 cm

18. Solve for p:

$$\frac{p-3}{3} - \frac{p-2}{4} = 1$$

(A) $p = 7$ (B) $p = 13$ (C) $p = 18$ (D) $p = 30$

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19. Vietnam has an area of approximately $330\,000$ km^2 and a population of about 74 million. Australia has an area of approximately $7\,680\,000$ km^2 and a population of about 18 million.

The population density (number of people per square kilometre) of Vietnam is greater than that of Australia by approximately

(A) 4 times.   (B) 25 times.   (C) 100 times.   (D) 200 times.

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20. In a race, a boat is sailing a set course. From the first marker, A, it sails on a bearing of $125°$ for $2 \cdot 1$ km. Turning at the second marker, B, it sails due north for $3 \cdot 5$ km. At the third marker, C, it turns and sails directly back to the first marker.

Which of the following diagrams best shows this course?

(A)

(B)

(C)

(D)
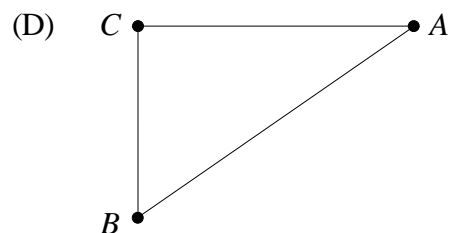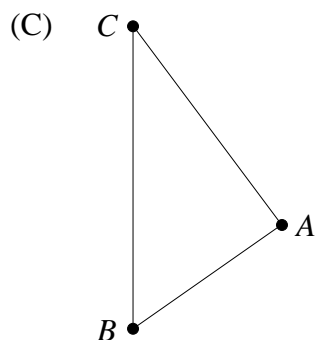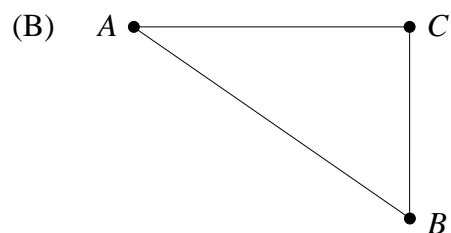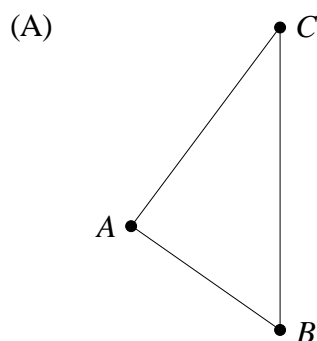
SECTION II

Attempt ALL questions.

Each question is worth 12 marks.

QUESTION 21. Use a *separate* Writing Booklet.

(a)   The diagram shows an in-ground swimming-pool 12 m long and 5 m wide. The depth at the shallow end is 0·8 m and at the deep end is 2·5 m. The depth begins to increase halfway along the pool.

12 m

5 m

2·5 m

0·8 m

6 m

NOT TO SCALE
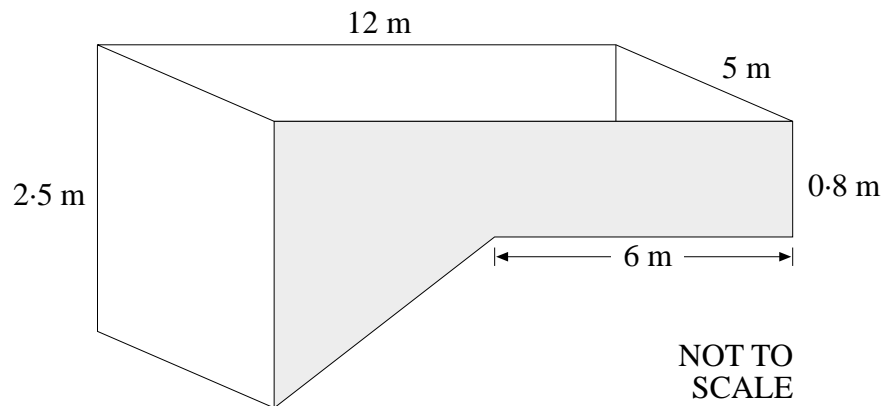
   (i)   Find the area of the shaded wall.

   (ii)   Find the volume of water that the pool can hold when full to the top.

(b)   Assume that there is an equally likely chance of being born on any particular day of the week.

   (i)   What is the probability that a person selected at random was not born on a Tuesday?

   (ii)   What is the probability that, of any two people selected at random, exactly one was born on a Tuesday?

QUESTION 21. (Continued)

(c)

Jo's Clothes is having a sale. To find the percentage discount on an item, a customer rolls three dice and adds the uppermost numbers.

   (i)   What is the greatest percentage discount that could be received?

  (ii)   For an item marked $100, how much would be paid if the smallest possible discount is rolled?

 (iii)   In how many ways can three dice be rolled to obtain a total of 5?

 (iv)   Calculate the probability of rolling a 5% discount.

  (v)   What is the probability of paying $95 or more for an item marked $100?

QUESTION 22. Use a *separate* Writing Booklet.

(a)   Zoe and her two friends, Xavier and Yvonne, are standing on a field. Xavier (X) is 10 m away from Zoe (Z) on a bearing of 040°. Yvonne (Y) is 15 m away from Zoe, on a bearing of 300°.

Draw a diagram in your Writing Booklet to represent the three positions X, Y, and Z. Mark all given information on your diagram.

(b)   The location of three towns, Ubiri (U), Vanati (V), and Wallarah (W) is shown in the diagram. Vanati is due east of Ubiri.
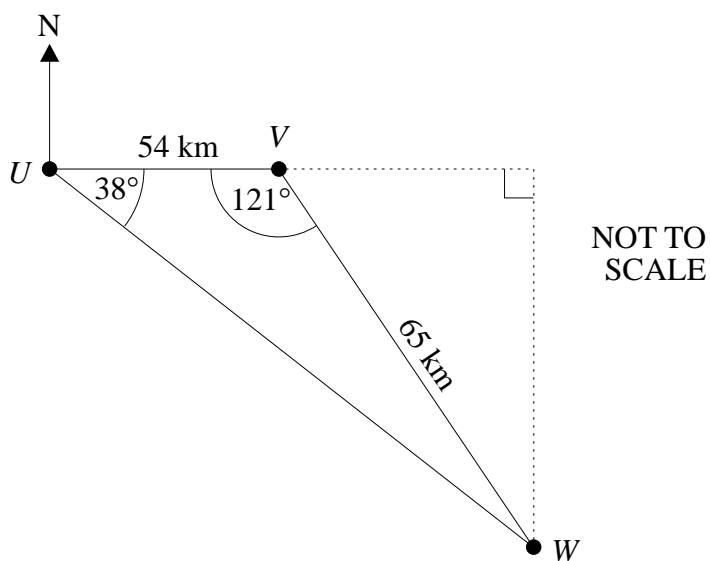
(i)   What is the bearing of W from V?

(ii)   How far south of V is W?

(iii)   Use the cosine rule to find the distance between U and W.

$$\left(\text{Cosine rule: } c^2 = a^2 + b^2 - 2ab \cos C\right)$$

QUESTION 22. (Continued)

(c) An observation tower, OT, is 34 m high. From the top of the tower (T), the angle of depression to the base (Q) of a tree is $17°$, while the angle of depression to the top (R) of the tree is $12°$.
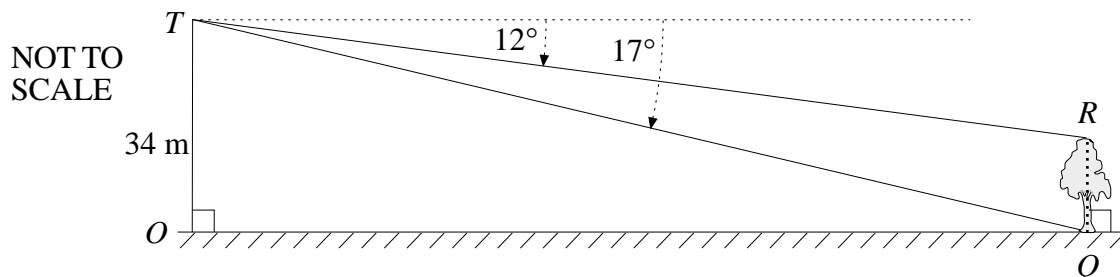
(i) Explain why $\angle OQT = 17°$.

(ii) Show that $QT = 116{\cdot}29$ m (to 2 decimal places).

(iii) Find the size of each of the angles in triangle QRT.

(iv) Use the sine rule in triangle QRT to find the height of the tree, RQ.

$$\left(\text{Sine rule:} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \right)$$

QUESTION 23.   Use a *separate* Writing Booklet.

(a)   A cone has dimensions as shown in the diagram.

NOT TO
SCALE

12 cm

ℓ cm

5 cm
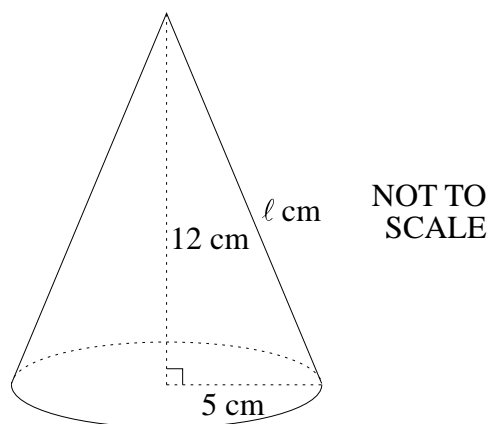
    (i)   Find ℓ, the slant height of the cone.

    (ii)   Find the curved surface area of the cone. $\left(\text{Curved surface area} = \pi r \ell\right)$

(b)   Gary surveyed 100 university students and recorded their heights. He drew the following cumulative frequency graph to represent the information.

HEIGHTS OF UNIVERSITY STUDENTS
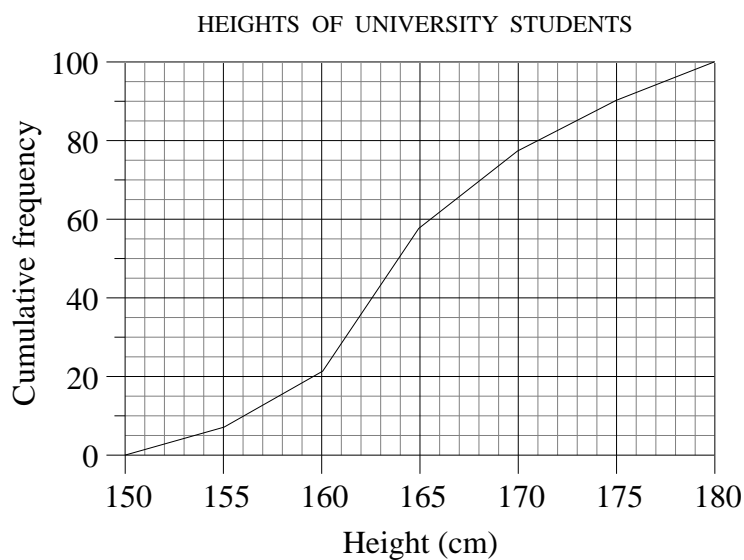
    (i)   Use the graph to estimate the median height.

    (ii)   Use the graph to estimate the interquartile range.

QUESTION 23. (Continued)

(c)   Solve the equation $\dfrac{3}{x} - 2 = \dfrac{4}{x}$.

(d)   The graph shows the monthly maximum temperatures in Tokyo.

MONTHLY MAXIMUM TEMPERATURE
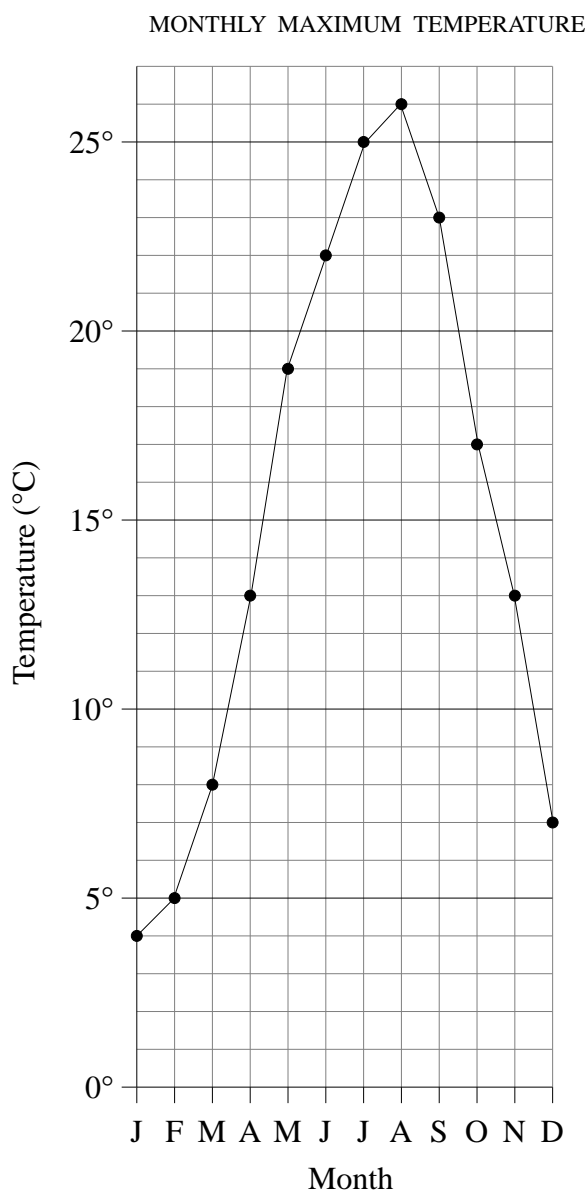
(i)   What is the range of the monthly maximum temperatures?

(ii)   Find the mean and standard deviation of the monthly maximum temperatures.

(iii) It is predicted that global warming will result in an increase in Tokyo's monthly maximum temperature of about 5°C in December, January, February, and March. It will also result in a small increase (about 1°C) in the other months.

Describe the effect that these predicted increases in temperature would have on the mean and standard deviation as found in part (ii).

SECTION III

Attempt TWO questions.

Each question is worth 12 marks.

QUESTION 24. Space Mathematics. Use a *separate* Writing Booklet.

(a)
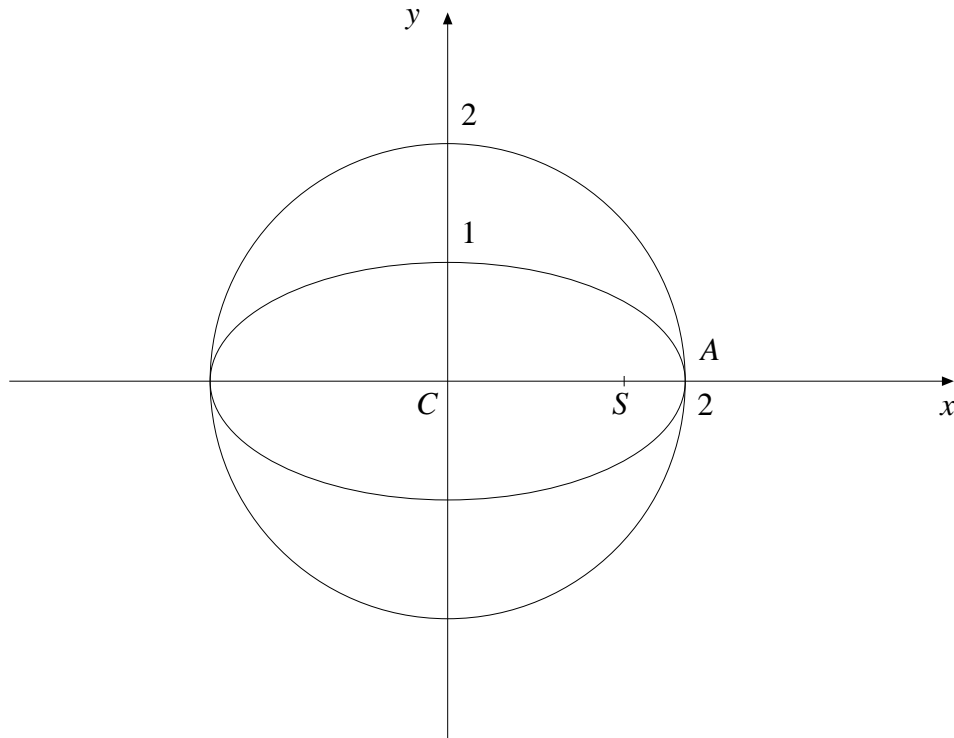
An ellipse is drawn within an auxiliary circle of radius 2 units as shown.

  (i)   What is the value of $\dfrac{b}{a}$ for this ellipse?

  (ii) Calculate the value of the eccentricity of this ellipse. $\left(e^2 = 1 - \dfrac{b^2}{a^2} \right)$

  (iii) Give the coordinates of the focus S. $(CS = eCA)$

  (iv) Another ellipse has eccentricity 0·6. What is the value of $\dfrac{b}{a}$ for this new ellipse?

  (v) Describe what happens to the shape of an ellipse and the positions of the foci as the eccentricity decreases.

QUESTION 24. (Continued)

(b)   The table below shows information about the planet Jupiter.

Jupiter	
Position	786 million km from Sun
Diameter	142 984 km
Atmosphere	89% hydrogen, 10% helium
Jupiter year	4333 days
Volume	1330 times that of Earth
Mass	318 times that of Earth
Jupiter has 20 moons.	

(i)   Taking Jupiter's orbit about the Sun as a circle, find the length of its orbit in metres. $(C = 2\pi r)$

(ii)   Find the average speed in metres per second at which Jupiter orbits the Sun.

(iii) Calculate the time in minutes taken for light to travel from the Sun to Jupiter. $\left(\text{Speed of light} = 3 \times 10^8 \text{ m/s}\right)$

(iv)   The Earth is 150 million kilometres from the Sun. This distance is represented in the scale diagram below.

⊢─────────────────────────┼────────────────────────────

Sun   Earth
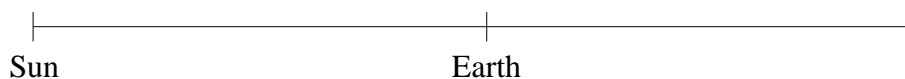
If Jupiter were to be marked on the scale diagram above, calculate the distance, in centimetres, between the position of Jupiter and the position of the Sun.

QUESTION 25. Mathematics of Chance and Gambling
Use a *separate* Writing Booklet.

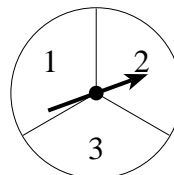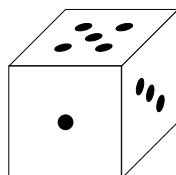(a)   In a game of chance, a die is rolled and a spinner (shown below) is spun. For the die, the uppermost number is noted. For the spinner, the number to which the arrow points is noted. The score is obtained by subtracting the smaller number from the larger number.

For example, if Mark rolls a 5 and spins a 2, the score is 3.

The table below shows some of the scores.

Spinner

Die	1	2	3
1			2
2		0	
3		1	
4			
5		3	
6	5		

(i)   Copy the table into your Writing Booklet. Complete the table to show all the possible scores.

(ii)   What is the probability of getting a score of 1?

(iii)   In 90 rounds of the game, how many times would Mark expect a score of 1?

(iv)   Mark and Tim play a game where Mark receives a point every time the score is 0, 1, or 2, and Tim receives a point every time the score is 3, 4, or 5.

Explain why these rules are not fair.

(v)   Describe how you could change the rules of the game described in part (iv) to make the game fair.

QUESTION 25. (Continued)

(b)   The following odds are set for a dog race.

Dog	Odds
Lucky Boy	8/1
Star Gazer	4/1 on

(i)   Jamo bets $20 on Lucky Boy to win.

How much will he collect if Lucky Boy wins?

(ii)   Rosie bets $20 on Star Gazer.

How much will she win if Star Gazer wins?

(c)   Eight horses will run in a race. To win a trifecta, a punter must select the horses that will come 1st, 2nd, and 3rd in the race, in that order.

(i)   What is the probability of winning the trifecta?

(ii)   In a box trifecta, a punter selects the horses that will come 1st, 2nd, and 3rd in the race in *any* order.

What is the probability of winning the box trifecta?

(d)   The 5th and 6th rows of Pascal's triangle are shown below.

$$1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1$$
$$1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1$$

(i)   Write down the 7th row of Pascal's triangle.

(ii)   In how many ways can a basketball team of 5 players be chosen from a group of 7 players?

(iii)   The captain will lead the team onto the court.

In how many ways can the remaining 4 players be lined up behind the captain?

QUESTION 26. Land and Time Measurement. Use a *separate* Writing Booklet.

(a) A surveyor's notebook contained the following field entries for a traverse survey of a field. All entries are in metres.

$$
\begin{array}{c}
B \\
\hline
52 \\
38 \quad 20 \\
18 \quad 25 \\
14 \quad 17 \\
0 \\
\hline
A
\end{array}
$$

Use these notebook entries to draw an accurate scale diagram of the field. Use a scale of 1 mm = 0·5 m.

(b) A radial survey of a tract of land is shown.

J (351°)

NOT TO
SCALE

63 m

52 m   K (090°)

(255°) M   O   40 m

61 m

L (172°)
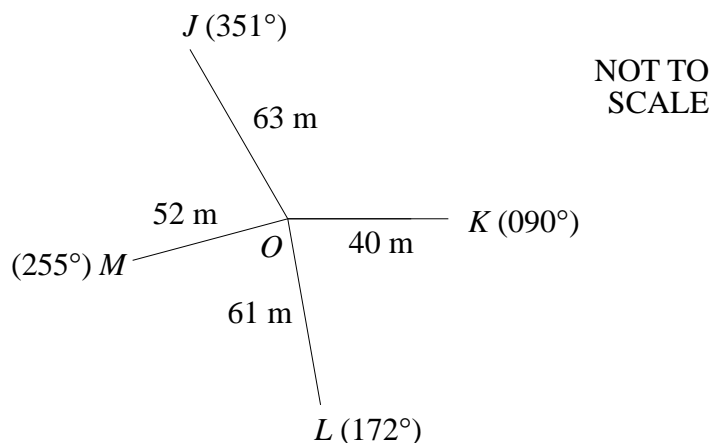
(i) Find the size of ∠JOK .

(ii) Find the area of triangle *JOK* to the nearest square metre.

$$\left(\text{Area } = \frac{1}{2} ab \sin C\right)$$

(iii) Use the cosine rule to find the length of the boundary *JK*. Give your answer to the nearest metre.

$$\left(\text{Cosine rule: } c^2 = a^2 + b^2 - 2ab \cos C\right)$$

QUESTION 26. (Continued)

(c)   A yacht leaves the Pacific island of Nauru (1°S, 167°E) and sails due south to Vanuatu (16°S, 167°E). The yacht leaves Nauru at 10:00 a.m. on Tuesday, 12 December.

  (i)   Given that the radius of the Earth is 6400 kilometres, find the distance the yacht sails between Nauru and Vanuatu. (Give your answer to the nearest kilometre.)

  (ii)   The yacht sails at an average speed of 8 knots.

      How many hours will it take to sail from Nauru to Vanuatu?
      (1 knot = 1·852 km/h)

  (iii)   Give the date and time of arrival of the yacht in Vanuatu.

QUESTION 27. Personal Finance. Use a *separate* Writing Booklet.

(a) A new refrigerator costs $1699. Mahal buys the refrigerator on terms of $500 deposit and weekly instalments of $26·50 over one year.

    (i) How much does Mahal pay in total for the refrigerator?

    (ii) What rate of interest is he charged?

(b) Sarah is employed, on a casual basis, by a fast food outlet. Her rate of pay is shown in the table below.

Rate of pay	
Weekdays	$9 per hour
Saturday	Time-and-a-quarter
Sunday	Time-and-a-half

Last week Sarah worked from 12:30 p.m. till 3:30 p.m. on Thursday, from 8:30 a.m. till 1:00 p.m. on Saturday, and from 12 noon till 5 p.m. on Sunday.

How much did Sarah earn last week?

(c) A bank has three different types of savings accounts, as described in the table.

	Type A	Type B	Type C
Account service fees per month:			
• if minimum monthly balance stays at or above $350	Nil	$5·00	Nil
• if balance drops below $350	$3·00	$5·00	Nil
Number of fee-free transactions per month	10	Unlimited	6
Fee per transaction over the free limit	80 cents	Nil	80 cents

    (i) Cameron has a type *A* account. In September, his minimum balance was $362, and he made thirteen transactions.

        Calculate the fee he was charged for September.

    (ii) In any month, Jesse normally has between $200 and $300 in the bank. He usually makes about eight transactions each month.

        Explain why Jesse should choose a type *C* account at this bank in order to minimise fees.

QUESTION 27. (Continued)

(d)   For the 1994–95 financial year, Claudia's taxable income was $49 540.

    (i)   Calculate the tax on her income using the table below.

Taxable income range	Tax payable
$20 701 – $38 000	$3060 plus 34c for each $1 over $20 700
$38 001 – $50 000	$8942 plus 43c for each $1 over $38 000
$50 000 and over	$14 102 plus 47c for each $1 over $50 000

    (ii)   The total amount of tax that Claudia had to pay is the amount calculated in part (i) plus the Medicare levy. The Medicare levy was 1·5% of taxable income.

Calculate the total amount of tax that Claudia paid.

    (iii)   During the 1994–95 financial year, tax instalments of $573·30 per fortnight were deducted from Claudia's salary.

Calculate the refund Claudia received for the 1994–95 financial year.

    (iv)   For the 1995–96 financial year, Claudia's taxable income increased by $3148.

Using the above table, calculate how much *more* tax (including the Medicare levy) Claudia had to pay for 1995–96.

QUESTION 28. Mathematics in Construction. Use a *separate* Writing Booklet.

(a)   The drawings on page 23 show the first floor and ground floor plans of a new home.

   (i)   What scale is used on the plans on page 23?

  (ii)   Draw a sketch of the plan of the verandah and write its external dimensions on your sketch.

  (iii)   Calculate the actual area of the verandah.

  (iv)   Which elevation of the house is shown below?

   (v)   The living-room and dining-room are on different levels. The actual height of each step between the levels is 18 cm.
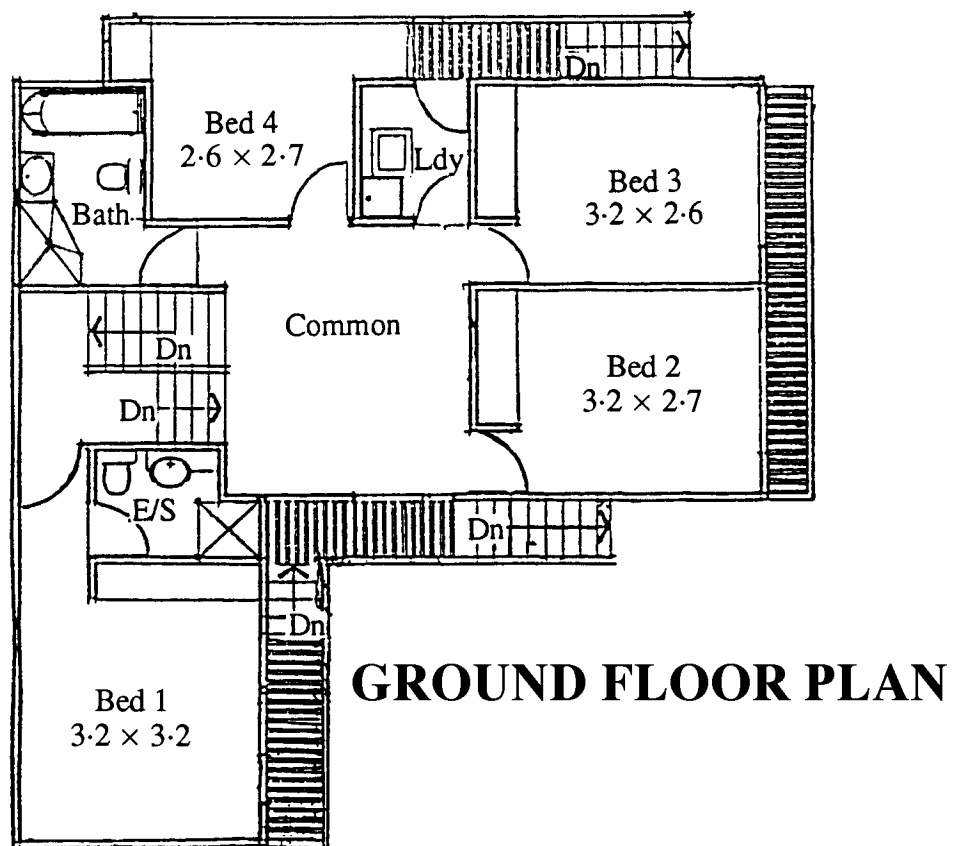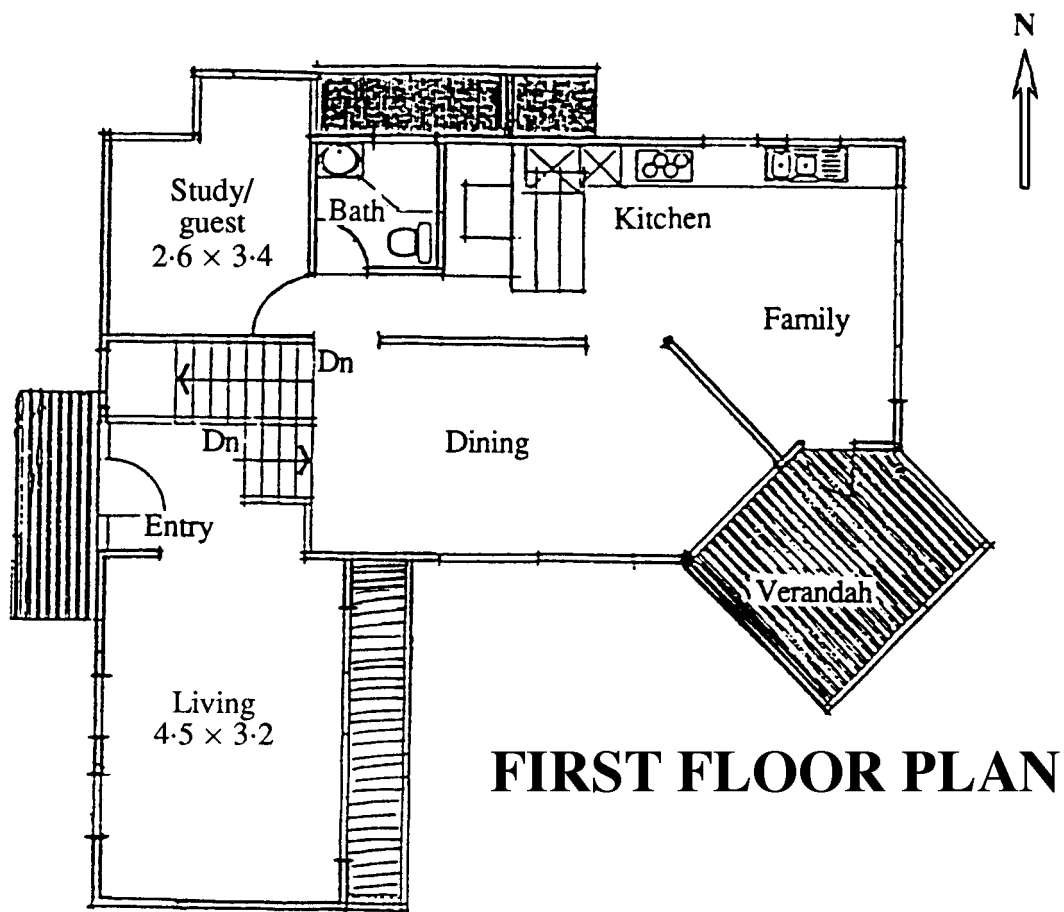
         What is the difference in floor level between the living-room and the dining-room?

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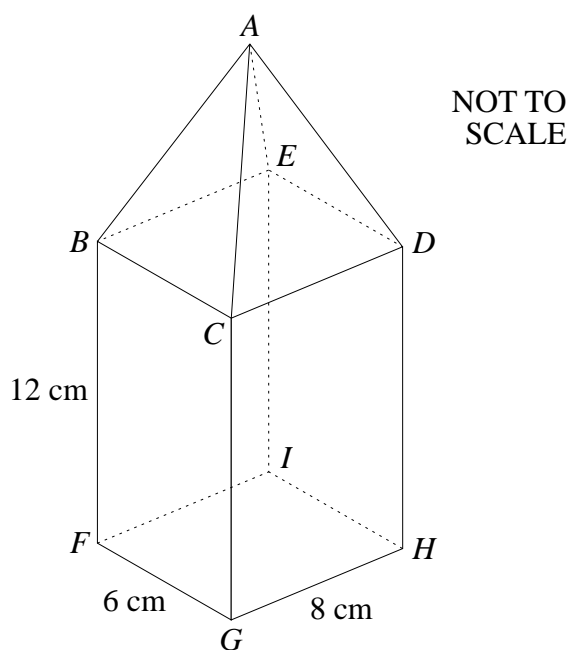
AN ELEVATION OF THE HOUSE

QUESTION 28. (Continued)

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Study/
guest
2·6 × 3·4

Bath

Kitchen

Family

Dn

Dn

Dining

Entry

Verandah

Living
4·5 × 3·2

FIRST FLOOR PLAN

Bed 4
2·6 × 2·7

Dn

Ldy

Bed 3
3·2 × 2·6

Bath

Dn

Common

Bed 2
3·2 × 2·7

Dn

E/S

Dn

Bed 1
3·2 × 3·2

GROUND FLOOR PLAN

QUESTION 28. (Continued)

(b) The solid below is composed of two common solids.

NOT TO
SCALE

12 cm

6 cm   8 cm

(i)   Give the geometric name for the solid that has vertices *ABCDE*.

(ii) Name a line which is skew to *CD*.

(iii) Which line in the plane *FGHI* represents the projection of *BH* onto this plane?

(iv) Calculate the angle between the line *BH* and the plane *FGHI*.