BOARD OF STUDIES
NEW SOUTH WALES

HIGHER SCHOOL CERTIFICATE EXAMINATION

1995
MATHEMATICS
2/3 UNIT (COMMON)

Time allowed—Three hours
(Plus 5 minutes' reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- ALL questions are of equal value.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are printed on page 12.
- Board-approved calculators may be used.
- Each question attempted is to be returned in a *separate* Writing Booklet clearly marked Question 1, Question 2, etc. on the cover. Each booklet must show your Student Number and the Centre Number.
- You may ask for extra Writing Booklets if you need them.

QUESTION 1. Use a *separate* Writing Booklet. **Marks**

(a) Factorize $9x^2 - 16$. **1**

(b) Find the value of $19^{-0\cdot 5}$ to two decimal places. **2**

(c) Convert $\frac{3\pi}{5}$ radians to degrees. **1**

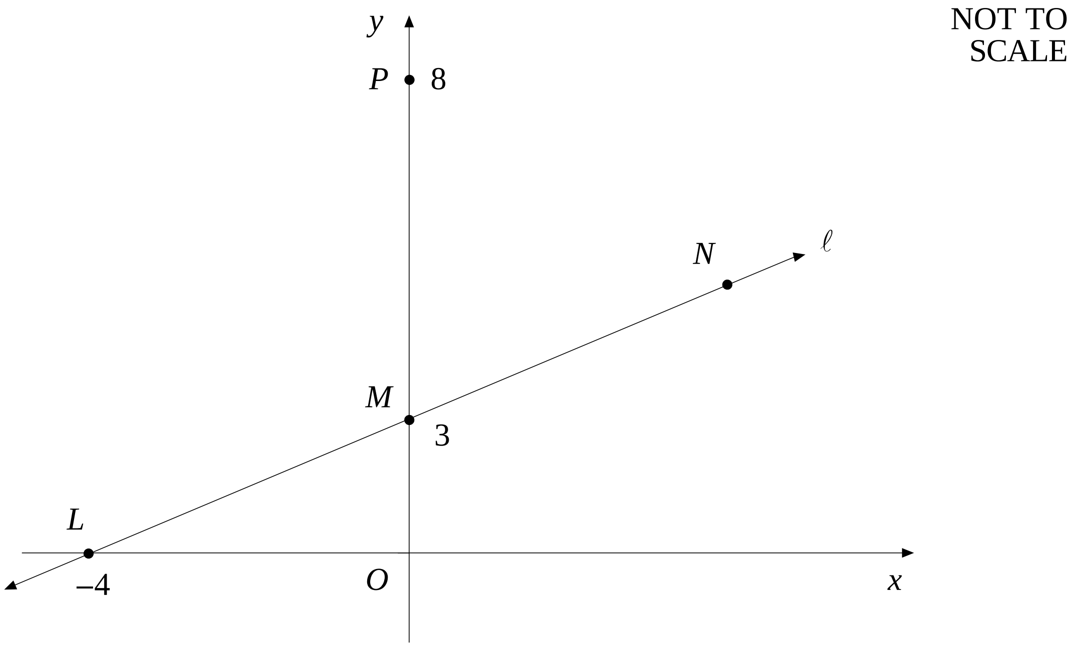
(d) Simplify $\frac{x}{3} + \frac{3x-1}{2}$. **2**

(e) Find a primitive function of $2x + 11$. **2**

(f) Express $0\cdot\dot{2}\dot{3}$ as a fraction. **2**

(g) Solve $5 - 3x < 7$. **2**

QUESTION 2. Use a *separate* Writing Booklet. **Marks**



The line l cuts the x axis at $L(-4, 0)$ and the y axis at $M(0, 3)$ as shown. N is a point on the line l, and P is the point $(0, 8)$.

Copy the diagram into your Writing Booklet.

(a) Find the equation of the line l. **2**

(b) Show that the point $(16, 15)$ lies on the line l. **1**

(c) By considering the lengths of ML and MP, show that ΔLMP is isosceles. **2**

(d) Calculate the gradient of the line PL. **1**

(e) M is the midpoint of the interval LN. Find the coordinates of the point N. **2**

(f) Show that $\angle NPL$ is a right angle. **2**

(g) Find the equation of the circle that passes through the points N, P, and L. **2**

QUESTION 3. Use a *separate* Writing Booklet. **Marks**

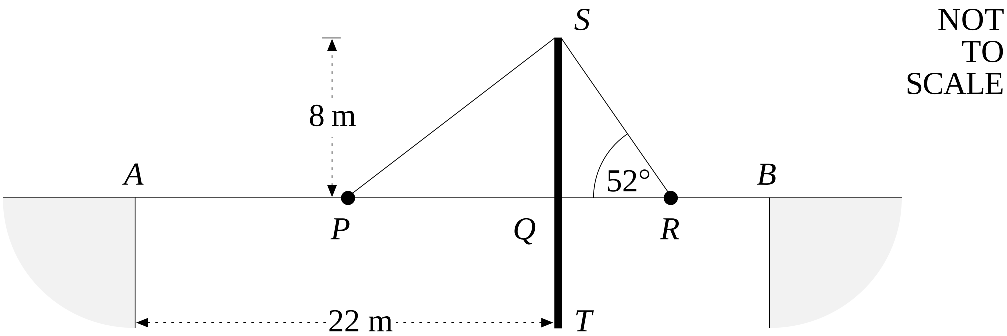
(a) Differentiate: **4**

(i) $2x^{-3}$

(ii) $4e^{2x}$

(iii) $x \log_e x$.

(b) **4**



A horizontal bridge is built between points A and B. The bridge is supported by cables SP and SR, which are attached to the top of a vertical pylon ST.

The section of the pylon, SQ, above the bridge is 8 metres long and $\angle SRQ = 52°$.

The distance AQ is 22 metres, and P is the midpoint of AQ.

(i) Find the length of the cable SR.

(ii) Find the size of $\angle SPQ$ to the nearest degree.

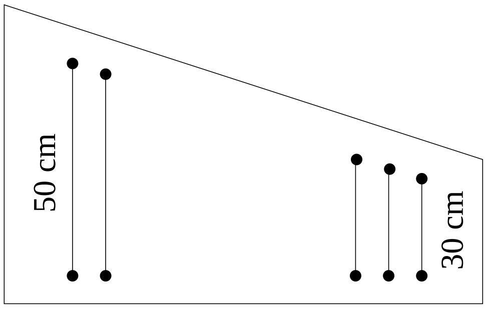
(c) Find: **4**

(i) $\int e^{3x}\,dx$

(ii) $\int_0^{\frac{\pi}{2}} \sin 2x\,dx$.

QUESTION 4. Use a *separate* Writing Booklet. **Marks**

(a) **4**



NOT TO SCALE

A simple instrument has many strings, attached as shown in the diagram. The difference between the lengths of adjacent strings is a constant, so that the lengths of the strings are the terms of an arithmetic series.

The shortest string is 30 cm long and the longest string is 50 cm. The sum of the lengths of all the strings is 1240 cm.

(i) Find the number of strings.

(ii) Find the difference in length between adjacent strings.

(b) (i) Draw the graphs of $y = |x|$ and $y = x + 4$ on the same set of axes. **4**

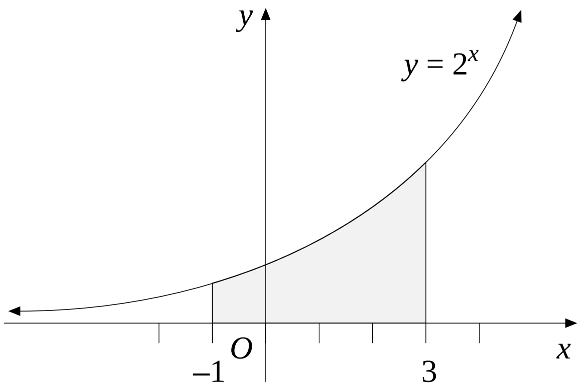
(ii) Find the coordinates of the point of intersection of these two graphs.

(c) Consider the function $y = 2^x$. **4**

x	–1	0	1	2	3
2^x					

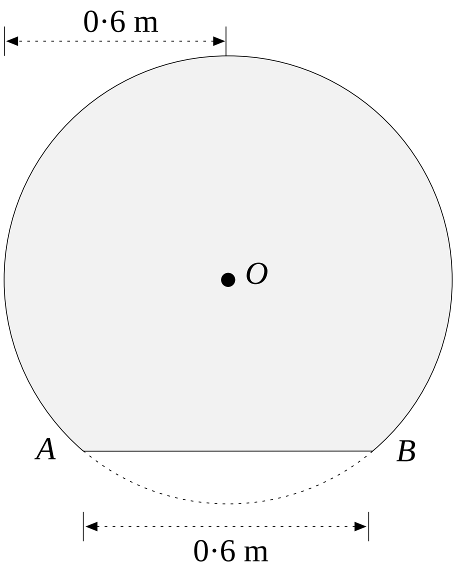
(i) Copy and complete the above table in your Writing Booklet.

(ii) Using Simpson's rule with these five function values, find an estimate for the area shaded in the diagram below.



QUESTION 5. Use a *separate* Writing Booklet. **Marks**

(a) **4**



NOT TO SCALE

A table-top is in the shape of a circle with a small segment removed as shown. The circle has centre O and radius 0·6 metres. The length of the straight edge AB is also 0·6 metres.

(i) Explain why $\angle AOB = \dfrac{\pi}{3}$.

(ii) Find the area of the table-top.

(b) Consider the curve given by $y = 7 + 4x^3 - 3x^4$. **8**

(i) Find the coordinates of the two stationary points.

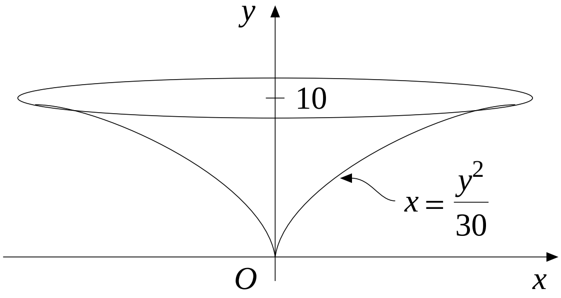
(ii) Find all values of x for which $\dfrac{d^2y}{dx^2} = 0$.

(iii) Determine the nature of the stationary points.

(iv) Sketch the curve for the domain $-1 \le x \le 2$.

QUESTION 6. Use a *separate* Writing Booklet. **Marks**

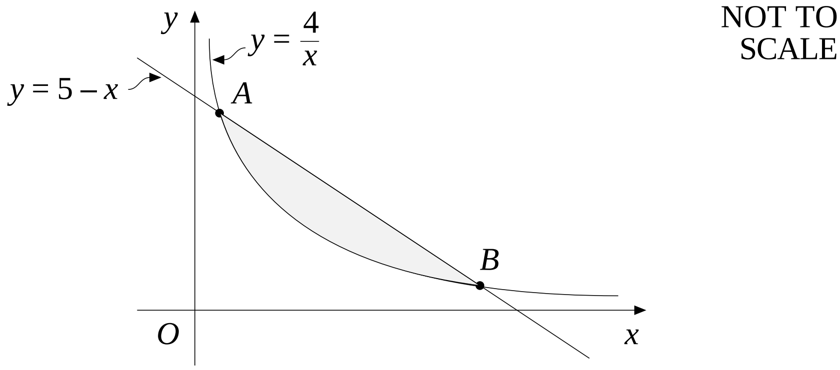
(a) **3**



A glass has a shape obtained by rotating part of the parabola $x = \frac{y^2}{30}$ about the y axis as shown. The glass is 10 cm deep.

Find the volume of liquid which the glass will hold.

(b) **5**



The diagram shows the graphs of $y = \frac{4}{x}$ and $y = 5 - x$.

The graphs intersect at the points A and B as shown.

(i) Find the x coordinates of the points A and B.

(ii) Find the area of the shaded region between $y = \frac{4}{x}$ and $y = 5 - x$.

(c) Coal is extracted from a mine at a rate that is proportional to the amount of coal remaining in the mine. Hence the amount R remaining after t years is given by **4**

$$R = R_0 e^{-kt},$$

where k is a constant and R_0 is the initial amount of coal.

After 20 years, 50% of the initial amount of coal remains.

(i) Find the value of k.

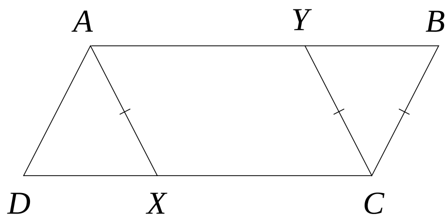
(ii) How many *more* years will elapse before only 30% of the original amount remains?

QUESTION 7. Use a *separate* Writing Booklet. **Marks**

(a) A factory assembles torches. Each torch requires one battery and one bulb. It is known that 6% of all batteries and 4% of all bulbs are defective. **2**

Find the probability that, in a torch selected at random, both the battery and the bulb are NOT defective. Give your answer in exact form.

(b) **5**



$ABCD$ is a parallelogram. The point X lies on CD, the point Y lies on AB, and $AX = CY = BC$, as shown in the diagram.

(i) Copy the diagram into your Writing Booklet.

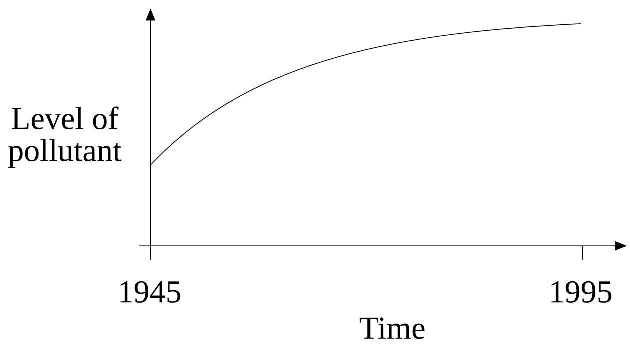
(ii) Explain why $\angle ADX = \angle CBY$.

(iii) Show that $AD = AX$.

(iv) Show that triangles ADX and CBY are congruent.

(v) Hence prove that $AYCX$ is a parallelogram.

(c) **2**



The graph shows the levels of a pollutant in the atmosphere over the past 50 years.

Describe briefly how the level of this pollutant has changed over this period of time. Include mention of the rate of change.

QUESTION 7. (Continued) **Marks**

(d) Given that $\log_a b = 2 \cdot 75$ and $\log_a c = 0 \cdot 25$, find the value of: **3**

(i) $\log_a \left(\dfrac{b}{c} \right)$

(ii) $\log_a (bc)^2$.

QUESTION 8. Use a *separate* Writing Booklet.

(a) Greg and Jack are playing in a golf tournament. They will play two rounds and each has an equal chance of winning the first round. **4**

If Greg wins the first round, his probability of winning the second round is increased to 0·6.

If Greg loses the first round, his probability of winning the second round is reduced to 0·3.

(i) Draw a tree diagram for the two-round sequence. Label each branch of the diagram with the appropriate probability.

(ii) Find the probability that Greg wins exactly one round.

(b) On 1 July 1985, Anna invested \$10 000 in a bank account that paid interest at a fixed rate of 8% per annum, compounded annually. **8**

(i) How much would be in the account after the payment of interest on 1 July 1995 if no additional deposits were made?

(ii) In fact, Anna added \$1000 to her account on 1 July each year, beginning on 1 July 1986.

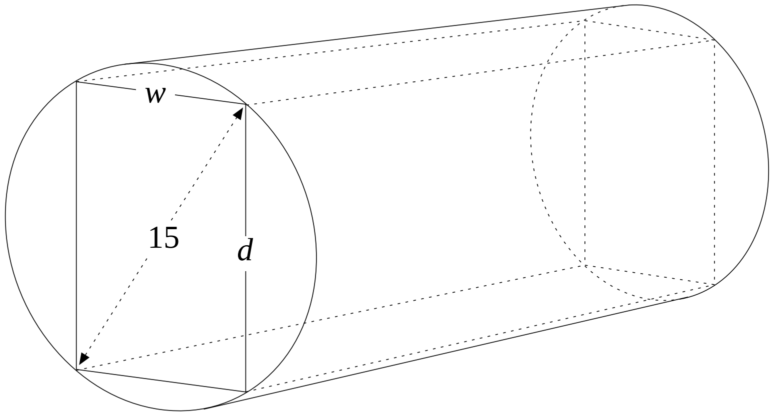
How much was in her account on 1 July 1995 after the payment of interest and her deposit?

(iii) Anna's friend, Jennifer, invested \$10 000 in an account at another bank on 1 July 1985 and made no further deposits. On 1 July 1995, the balance of Jennifer's account was \$35 478.

What was the annual rate of compound interest paid on Jennifer's account?

QUESTION 9. Use a *separate* Writing Booklet. **Marks**

(a) **7**



A rectangular beam of width w cm and depth d cm is cut from a cylindrical pine log as shown.

The diameter of the cross-section of the log (and hence the diagonal of the cross-section of the beam) is 15 cm.

The strength S of the beam is proportional to the product of its width and the square of its depth, so that

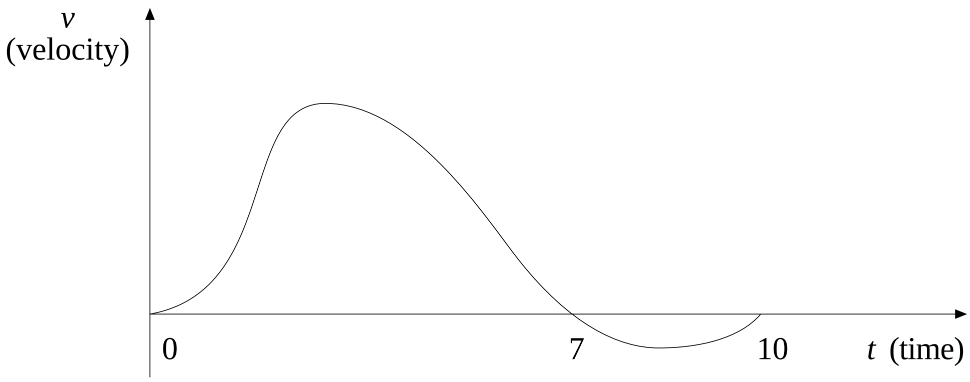
$$S = kd^2w.$$

(i) Show that $S = k\left(225w - w^3\right)$.

(ii) What numerical dimensions will give a beam of maximum strength? Justify your answer.

(iii) A square beam with diagonal 15 cm could have been cut from the log. Show that the rectangular beam of maximum strength is more than 8% stronger than this square beam.

(b) **5**



A particle is observed as it moves in a straight line in the period between $t = 0$ and $t = 10$. Its velocity v at time t is shown on the graph above.

QUESTION 9. (Continued) **Marks**

Copy or trace this graph into your Writing Booklet.

(i) *On the time axis*, mark and clearly label with the letter Z the times when the acceleration of the particle is zero.

(ii) *On the time axis*, mark and clearly label with the letter G the time when the acceleration is greatest.

(iii) There are three occasions when the particle is at rest, i.e. $t = 0$, $t = 7$, and $t = 10$.

The particle is furthest from its initial position on one of these occasions. Indicate which occasion, giving reasons for your answer.

QUESTION 10. Use a *separate* Writing Booklet.

(a) (i) Draw the graphs of $y = 4\cos x$ and $y = 2 - x$ on the same set of axes for $-2\pi \le x \le 2\pi$. **3**

(ii) Explain why all the solutions of the equation $4\cos x = 2 - x$ must lie between $x = -2$ and $x = 6$.

(b) Two particles A and B start moving on the x axis at time $t = 0$. The position of particle A at time t is given by **9**

$$x = -6 + 2t - \tfrac{1}{2}t^2$$

and the position of particle B at time t is given by

$$x = 4\sin t.$$

(i) Find expressions for the velocities of the two particles.

(ii) Use part (a) to show that there are exactly two occasions, t_1 and t_2, when these particles have the same velocity.

(iii) Show that the distance travelled by particle A between these two occasions is

$$4 - 2(t_1 + t_2) + \tfrac{1}{2}(t_1{}^2 + t_2{}^2).$$

(iv) Show that the two particles never meet.

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \, \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$