MATHEMATICS

(Three hours and a quarter)

(The first fifteen minutes of the examination are for reading the paper only. Candidates must NOT start writing during this time.)

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*Answer **Question 1** from Section **A** and **14** questions from Section **B**.*
All working, including rough work, should be done on the same sheet as, and adjacent to, the rest of the answer.
The intended marks for questions or parts of questions are given in brackets [].

Mathematical formulae are given at the end of this question paper.
The use of calculator (fx-82/ fx-100) is allowed.

Section A (30 marks)

Answer **ALL** the questions

Direction: Read the following questions carefully. For each question there are four alternatives A, B, C and D. Choose the correct alternative and write it in your answer sheet.

Question 1 **[2x15=30 Marks]**

(i) The sigma notation of the series $5-7-19-31-43$ is

A $\sum_{i=1}^{5}(17+12i)$

B $\sum_{i=1}^{5}(12-17i)$

C $\sum_{i=1}^{5}(17-12i)$

D $\sum_{i=1}^{5}(12+17i)$

 This booklet contains 12 pages.

(ii) The inverse of the matrix $\begin{pmatrix} -3 & 5 \\ 4 & -7 \end{pmatrix}$ is

A $\begin{pmatrix} -7 & -5 \\ -4 & -3 \end{pmatrix}$

B $\begin{pmatrix} 7 & -5 \\ 4 & -3 \end{pmatrix}$

C $\begin{pmatrix} -5 & 7 \\ -3 & 4 \end{pmatrix}$

D $\begin{pmatrix} 5 & -7 \\ 3 & -4 \end{pmatrix}$

(iii) If $(x+3)$ is a factor of the polynomial $x^3 - 6x + m$, then the value of 'm' is

A 3

B -3

C -9

D 9

(iv) The horizontal asymptote of the rational function $f(x) = \dfrac{4x^3 - 2x + 3}{2x^3 + 5x + 2}$ is

A -3

B 2

C -1

D 4

(v) The side of an equilateral triangle is 7cm and is increasing at the rate of $\sqrt{3}$ cm/s. At what rate is the area increasing?

A $10.5\text{cm}^2/\text{s}$

B $15.6\ \text{cm}^2/\text{s}$

C $12.5\ \text{cm}^2/\text{s}$

D $11.2\ \text{cm}^2/\text{s}$

(vi) The point of the curve $y = \sin x + \cos x$ for $x \in [-\pi, \pi]$ where the tangent is parallel to X- axis is

A $\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$

B $\left(\frac{\pi}{2}, \frac{1}{2}\right)$

C $\left(\frac{\pi}{4}, \frac{2}{\sqrt{2}}\right)$

D $\left(\frac{\pi}{3}, \frac{1}{2}\right)$

(vii) If $y = 3\ln x^2 + \ln\sqrt{x}$, then the value of y' will be

A $\frac{-13}{2x}$

B $\frac{2x}{13}$

C $\frac{-2x}{13}$

D $\frac{13}{2x}$

(viii) The integral value of $\int_1^e \frac{\ln x}{x} dx$ is

A 0.5

B 0.4

C 0.3

D 0.2

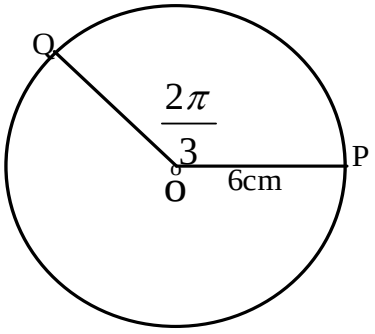
(ix) Area of the sector POQ in the given figure is

A 38.3 cm^2

B 32.2cm^2

C 35.6 cm^2

D 37.7cm^2



(x) Which of the following is the amplitude of $\frac{4+3i}{3-2i}$?

A 69.8°
B 70.6°
C 65.7°
D 75.3°

(xi) If $\sqrt{x}+\sqrt{x+1}=2$, then the value of 'x' is

A $\frac{9}{16}$
B $\frac{3}{13}$
C $\frac{2}{14}$
D $\frac{4}{15}$

(xii) The trend value of the following data using a three yearly moving average is

Year	1	2	3	4	5	6
values	4	5	3	2	8	7

A 2.00, 4.22, 5.06, 3.56
B 3.30, 6.23, 7.22, 0.22
C 4.00, 3.33, 4.33, 5.66
D 1.21, 7.15, 8.15, 9.15

(xiii) If $y=\sqrt{x^2-3x}$, then of the value of $\frac{dy}{dx}$ at x = -1 is

A $\frac{-5}{4}$
B $\frac{4}{3}$
C $\frac{2}{5}$
D $\frac{1}{4}$

(xiv) The rectangular form of $5cis126.9°$ is

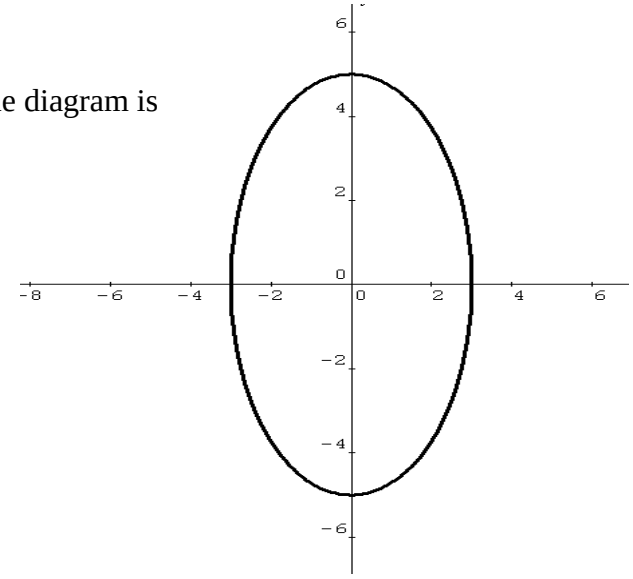
A $3-4i$

B $-3-4i$

C $3+4i$

D $-3+4i$

(xv) The equation of the conic given in the diagram is



A $\frac{x^2}{9}-\frac{y^2}{25}=1$

B $\frac{x^2}{25}+\frac{y^2}{9}=1$

C $\frac{x^2}{9}+\frac{y^2}{25}=1$

D $\frac{x^2}{25}-\frac{y^2}{9}=1$

Section B (70 marks)

Answer any 14 questions. All questions in this section have equal marks.

Unless otherwise stated, you may round answers to 2 decimal places.

Question 2

(a) Evaluate $\int \frac{\sec^2 x}{\sqrt[3]{1+\tan x}}dx$ **[2]**

(b) Determine a quadratic function $f(x)$ such that

$f(1)=5, f'(3)=7$ *and* $f''(4)=10$ **[3]**

Question 3

(a) A bacteria culture starts with 2000 bacteria and after 10 hours estimated count is 18000 bacteria. Find the number of bacteria after 3 hrs. **[2]**

(b) Find $\frac{dy}{dx}$ if $y=\frac{e^x \ln x}{x^2}$ **[3]**

Question 4

(a) Determine $\int 2xe^{3x}dx$ **[2]**

(b) If $z_1 = 3-2i$ and $z_2 = 1+i$, find the value of $\frac{z_1}{z_2} - z_1 z_2$ **[3]**

Question 5

(a) Find the equation of the circle whose centre is $(-3,4)$ and passes through the point $(1,1)$ **[2]**

(b) If the eccentricity is $\sqrt{10}$ and the sum of those semi-axes is 12, then find the equation of the hyperbola. **[3]**

Question 6

(a) Determine the equation of the plane passing through the points $A(0,4,3), B(-1,-5,-3)$ and $C(-2,-2,1)$ **[3]**

(b) Find the derivative of $y = \cos x$ by using first principle method **[2]**

Question 7

Prove $\sum_{i=1}^{n} \frac{1}{4i^2-1} = \frac{n}{2n+1}, n \in N$ by mathematical induction method **[5]**

Question 8

The length of slant height of a right circular cone is 6cm. Find the radius of the base so that the volume may be maximum. **[5]**

Question 9

Show that the continued product of the four values of $\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{\frac{3}{4}}$ is equal to 1. **[5]**

Question 10

For the equation of the ellipse $9x^2+16y^2-54x+64y+1=0$, find the

i) coordinates of the centre

ii) vertices

iii) eccentricity

iv) length of major and minor axes **[5]**

Question 11

Solve the equation by matrix method $x+y+z=4, 2x-y+3z=1, 3x+2y-z=1$ **[5]**

Question 12

A rectangular shopping complex is to be built on a 80 m by 60 m rectangular plot in such a way that there is a path *x* meter width surrounding the building. The building can occupy up to 70% of the plot's area. What is the range of possible integral values for the width of the path? **[5]**

Question 13

(a) Simplify and state the restrictions $\frac{x^2-4}{x^2+x-6}\times\frac{x^2+7x+12}{x^2+4x}$ **[2]**

(b) Express in the simplest surd form $\sqrt{112}-\sqrt{63}-\frac{224}{\sqrt{28}}$ **[3]**

Question 14

(a) Wangchuk invested Nu.20,000 for 5 years at the rate of 10% compounded annually. How much money will he get back after completion of the time period? **[2]**

(b) A ladder 12 m long rests against a vertical wall. If the bottom of the ladder slides away from the wall at the speed of 3 m/sec, how fast is the angle between the top of the ladder and the wall changing the angle when the angle is $\frac{\pi}{4}$? **[3]**

Question 15

Find the area of the region bounded by the curves $y^2 = 6x$ and $x^2 = 6y$ **[5]**

Question16

(a) If $f'(x) = 3x^2 + 2x + 4$ and $f(x) = 2$ then find the value of $f(2)$ **[2]**

(b) Find the standard deviation of the following distribution **[3]**

Class	1-3	3-5	5-7	7-9	9-11	11-13	13-15
Frequency	3	9	25	35	17	10	1

Question 17

For the function $y = x^4 - 2x^2 + 1$, find

i) intercepts

ii) critical values

iii) inflection points, and

iv) sketch the graph **[5]**

Question 18

Find the rank correlation coefficient for the following data of marks obtained by 10 students in Physics and Chemistry **[5]**

Roll No	1	2	3	4	5	6	7	8	9	10
Physics	80	38	95	30	74	84	91	60	66	40
Chemistry	85	50	92	58	70	65	88	56	52	46

Functions and Equations

(1) $(a \pm b)^2 = a^2 + b^2 \pm 2ab$

(2) $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

(3) $a^2 - b^2 = (a+b)(a-b)$

(4) $a^3 \pm b^3 = \left(a \pm b\right)\left(a^2 \mp ab + b^2\right)$

(5) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sequence and series

(1) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

(2) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

(3) $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$

(4) $t_n = ar^{n-1}$

(5) $t_n = a + (n-1)d.$

(6) $S_n = \frac{a\left(1-r^n\right)}{1-r}$ where $r<1$

$= \frac{a\left(r^n - 1\right)}{r-1}$, Where $r > 1$

(7) $S_n = \frac{n}{2}\left[2a + \left(n-1\right)d\right]$

Differentiation

(1) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

(2) $y = x^n,\ y' = nx^{n-1},$

(3) $y = cf(x),\ y' = cf'(x)$

(4) $y = f(x) \pm g(x),\ y' = f'(x) \pm g'(x)$

(5) $F(x) = f(x)g(x),$

$F'(x) = f(x)g'(x) + f'(x)g(x)$

(6) $F(x) = \frac{f(x)}{g(x)}$,

$F'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{\left[g(x)\right]^2}$

(7) $f \circ g(x)' = f'g(x) \times (g'x)$

(8) $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

(9) $v(t) = h'(t)$

Coordinate Geometry

(1) $\left(y - y_1\right) = m(x - x_1)$

(2) $\sqrt{\left(x-a\right)^2 + \left(y-b\right)^2} = r$

Trigonometry

(1) $Sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

(2) $Cos(A \pm B) = CosA\cos B \mp \sin A\sin B$

(3) $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

(4) $\sin^2\theta + \cos^2\theta = 1$

Logarithmic Exponentials

(1) $y = y_0(1+r)^x$

(2) $y = y_0e^{kx}$

(3) $A = P(1+r)^n$

Integration

(1) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[\left(\frac{d}{dx}f(x)\right)\int g(x)dx\right]dx$

(2) $\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$

(3) $V = \pi\int_a^b y^2dx$

(4) $A = \int_a^b ydx$

Measurement

(1) Cone: $V = \frac{\pi}{3}r^2h$

(2) Cone: $SA = \pi rl + \pi r^2$

(3) Sphere: $V = \frac{4\pi}{3}r^3$

(4) Sphere: $SA = 4\pi r^2$

(5) Cylinder: $SA = 2\pi rh + 2\pi r^2$

(6) Cylinder: $V = \pi r^2h$

(7) Circle: $A = \pi r^2$

(8) Circle: $C = 2\pi r$

(9) Triangle: $A = \frac{bh}{2}$, $A = \frac{\sqrt{3}}{4}x^2$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

(10) Rectangle: $A = lw$,

(11) Rectangle $P = 2l + 2w$

(12) Square: $A = s^2$,

(13) Square $P = 4s$

(14) Rectangular Prism: $V = lwh$

Complex Numbers

(1) $r = \sqrt{a^2 + b^2}$

(2) $\tan\theta = \frac{b}{a} \quad \Rightarrow \theta = \tan^{-1}\left(\frac{b}{a}\right)$

(3) If $z = r\text{cis}\theta$ then $z^n = r^n \text{cis} n\theta$

(4) $z^{\frac{1}{n}} = r^{\frac{1}{n}} cis(\frac{\theta}{n} + k \cdot \frac{360°}{n})\ f\ or\ k = 0,1,2,3,...n-1$

Second Degree Relations

(1) $\text{Ellipse}: \frac{X^2}{a^2} + \frac{y^2}{b^2} = 1$

(2) $\text{Hyperbola}: \frac{X^2}{a^2} + \frac{y^2}{b^2} = 1$

(3) $e = \frac{c}{a}$

Geometry

(1) $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

(2) $(x, y, z) = \left(\frac{lx_2 + mx_1}{l+m}, \frac{ly_2 + my_1}{l+m}, \frac{lz_2 + mz_1}{l+m}\right)$

(3) For

$a_1x + b_1y + c_1z = 0\ and\ a_2x + b_2y + c_2z = 0$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{z}{a_1b_2 - a_2b_1}$$

(4) $l = \frac{\theta}{360} 2\pi r$

(5) $A = \frac{\theta}{360} \pi r^2$

Matrices

(1) $C_{ij} = (-1)^{i+j} M_{ij}$

(2) $AA^{-1} = A^{-1}A = I$

(3) $Inverse\ of\ A = A^{-1} = \frac{1}{\det A} \cdot adjA$

Data & Probability

(1) $\bar{x} = \frac{\sum fx}{n}$

(2) $\text{Median} = l_1 + \frac{l_2 - l_1}{f1}(m - c)$

(3) $\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$

(4) $\sigma_{12} = \sqrt{\frac{n_1\sigma_1^2 + n_1\sigma_2^2 + n_1d_1^2 + n_2d_2^2}{n_1 + n_2}}$

(5) $\sigma = \sqrt{\frac{\sum f(x_i - \bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{N} - \left(\frac{\sum fx}{N}\right)^2}$

(6) $\bar{x}_{12} = \frac{m\bar{x}_1 + n\bar{x}_2}{m+n}$

(7) $I = \frac{\sum \frac{P_1}{P_0} \times 100}{n}$

(8) $I = \frac{\sum p_1w}{\sum p_0w} \times 100$

(9) $\text{Cov}(X, Y) = \frac{1}{n}\sum (X - \bar{X})(Y - \bar{Y})$

(10) $r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2 \sum (y - \bar{y})^2}}$

(11) $r = \frac{\sum (x - \bar{x})(y - \bar{y})}{n\sigma_x\sigma_y}$

(12) $b_{YX} = \frac{\text{cov}(X, Y)}{\sigma_x^2} = r\frac{\sigma_y}{\sigma_x}$

(13) $Y - \bar{Y} = \frac{\text{cov}(X, Y)}{\sigma_x^2}(X - \bar{X})$

$$= r\frac{\sigma_x}{\sigma_y}(X - \bar{X})$$

(14) $b_{xy} \times b_{yx} = r\frac{\sigma_y}{\sigma_x} \times r\frac{\sigma_x}{\sigma_y}$

(15) $\tau = \frac{2S}{n(n-1)}$

(16) $r = 1 - \frac{6\sum d^2}{n(n^2 - 1)}$

Rough Work

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