**MATHEMATICS**

(Three hours and a quarter)

*Answer **Question 1** from Section **A** and **14** questions from Section **B**.*
*All working, including rough work, should be done on the same sheet as, and adjacent to, the rest of the answer.*
*The intended marks for questions or parts of questions are given in brackets [].*

***Mathematical formulae are given at the end of this question paper.***
***The use of calculator (fx-82/ fx-100) is allowed.***

## Section A (30 marks)

Answer **ALL** the questions

**Direction: Read the following questions carefully. For each question there are four alternatives A, B, C and D. Choose the correct alternative and write it in your answer sheet.**

**Question 1** **[2x15 = 30 Marks]**

i) The value of $\begin{vmatrix} 2 & 3 & 7 \\ 13 & 17 & 5 \\ 15 & 20 & 12 \end{vmatrix}$ is

**A** 486
**B** 0
**C** 416
**D** 70

ii) The Sigma Notation of $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\frac{6}{7}$ is

**A** $\sum_{i=1}^{6}\frac{i}{i+1}$

**B** $\sum_{i=1}^{6}\frac{i}{i-1}$

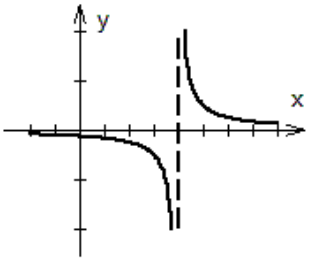
**C** $\sum_{i=0}^{6}\frac{i}{i+1}$

**D** $\sum_{i=0}^{6}\frac{i}{i-1}$



**This booklet contains 12pages**

iii) The rational equation for the given graph is




**A** $f(x)=\dfrac{1}{(x+4)^2}$

**B** $f(x)=\dfrac{1}{x+4}$

**C** $f(x)=\dfrac{1}{x^2-4}$

**D** $f(x)=\dfrac{1}{x-4}$

iv) One of the roots of the equation $x^3+2x^2-9x-18=0$ is

**A** +2

**B** -1

**C** -2

**D** +1

v) The value of '*x*', so that the distance between (1 , *x*, 8 ) and ( -2 , 2, 8) is 5 units

**A** $x=+6 \text{ or } +2$

**B** $x=-6 \text{ or } -2$

**C** $x=-6 \text{ or } +2$

**D** $x=+6 \text{ or } -2$

vi) The value of $\int_0^1 \left(x^2+1\right)^2$ is

**A** $\dfrac{28}{15}$

**B** $\dfrac{14}{15}$

**C** $\dfrac{22}{15}$

**D** $\dfrac{13}{15}$

vii) If $y = x^x$ then $\frac{dy}{dx}$ at $x = 1$ is

**A** 1

**B** $\frac{1}{e}$

***C*** $e$

**D** -1

viii) If $\sin A = -\frac{1}{2}$, $A$ lies in fourth quadrant, the value of $\sin 2A$ is

**A** $\frac{\sqrt{3}}{2}$

**B** $\frac{1}{2}$

**C** $-\frac{\sqrt{3}}{2}$

**D** $-\frac{1}{2}$

ix) The slope of the tangent to the curve $y = x^3 - x$ at $x = 3$ is

**A** 28

**B** 26

**C** 24

**D** 27

x) The standard deviation of the numbers 4 , 1, 3, 5, 12 is

**A** 2.74

**B** 3.47

**C** 2.47

**D** 3.74

xi) Which one of the following pair of surds is similar?

**A** $\sqrt{16}, \sqrt[3]{54}$

**B** $\sqrt{54}, \sqrt[3]{48}$

**C** $\sqrt{32}, \sqrt{72}$

**D** $\sqrt{48}, \sqrt{80}$

xii) The $x+iy$ form of $(2+3i)(1-i)$ is

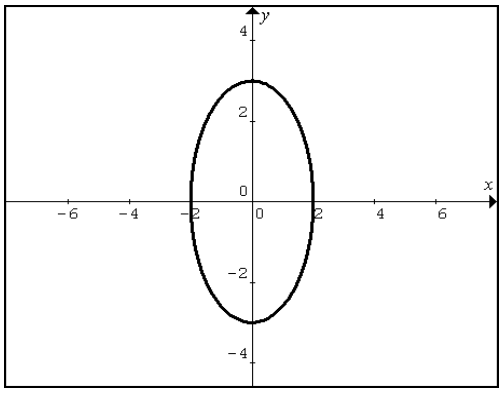
**A** $5-i$

**B** $5+i$

**C** $1+5i$

**D** $1-5i$

xiii) The equation of the conic in the given diagram is




**A** $\frac{x^2}{4}+\frac{y^2}{9}=1$

**B** $\frac{x^2}{9}-\frac{y^2}{4}=1$

**C** $\frac{x^2}{4}-\frac{y^2}{9}=1$

**D** $\frac{x^2}{9}+\frac{y^2}{4}=1$

xiv) The side of an equilateral triangle is 3m and is increasing at the rate of $\sqrt{3}\,m/\sec$. At what rate is area increasing?

**A** $3.5\,m^2/\sec$

**B** $4.5\,m^2/\sec$

**C** $4.8\,m^2/\sec$

**D** $3.9\,m^2/\sec$

xv) The polar form of $(1+i)^5$ is

**A** $5.93cis225°$

**B** $5.41cis225°$

**C** $5.66cis225°$

**D** $5.56cis225°$

## Section B (70 marks)

Answer any **14** questions. All questions in this section have equal marks.
Unless otherwise stated, you may round answers to 2 decimal places.

**Question 2**

a) Determine $f''(2)$ for $f(x) = (3x-2)^4$ **[3]**

b) Solve for '$x$' and find the intervals of inequation $|2x-3| < 7$ **[2]**

**Question 3**

The roots of the polynomial equation are $-2, +1$ and $+2$. Determine the equation.
Sketch the graph of this equation **[5]**

**Question 4**

Using the Principle of Mathematical Induction, prove

$$\sum_{i=1}^{n} \frac{1}{(2i+1)(2i-1)} = \frac{n}{2n+1}, \qquad n \in N$$ **[5]**

**Question 5**

A ladder 3.4m long rests against a vertical wall with the lower end on the horizontal ground. The lower end slips away from the wall at the rate of 1 m/min. Find the rate at which its upper end is descending at the instant when the lower end is 1.6 m from the wall. **[5]**

**Question 6**

For the function $f(x) = 2x^3 + 4x^2 - 10x - 12$, determine

i) $x$ and $y$ intercepts
ii) critical values
iii) inflection points **[5]**

**Question 7**

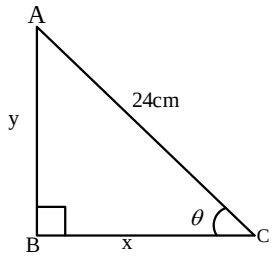
a) Determine $\frac{dy}{dx}$ for $y = \frac{\cos x}{\sin^2 x}$ **[2]**

b) If $x^y = e^{x-y}$, determine $\frac{dy}{dx}$ **[3]**

**Question 8**

a) Ms. Kinzang's uncle invested Nu.4000 in BOB when she was born with the interest rate at 4% per year compounded semi-annually. How long will it take the amount to grow to Nu.16,000 **[2]**

b) Determine the maximum perimeter of a right triangle with hypotenuse 24cm **[3]**




**Question 9**

a) Evaluate $\int x \sin x dx$ **[2]**

b) Determine the modulus and argument of $\frac{3-4i}{4-2i}$ **[3]**

**Question 10**

a) Determine the anti-derivative of $f(x) = -3e^{-x} + 6e^{2x} - \sin x$ **[2]**

b) A circular tent is constructed to have only a cone-shaped top or roof. It is constructed from a circular piece of canvas with a radius of 14 m with an angle $30^0$ sector removed. The cut lines are then sewn together. Find the area of the resulting canopy. **[3]**

**Question 11**

a) Show that the four points $(0,4,3),(-1,-5,-3),(-2,-2,1)$ and $(1,1,-1)$ are coplanar **[3]**

b) Evaluate $\int \frac{\log x}{x} dx$ **[2]**

**Question 12**

Determine the area bounded by the curve $y = x(2-x)$ and the lines $x = 0, y = 0$ and $x = 2$. This area is rotated through four right angles about the x-axis, determine the volume of the solid so formed. **[5]**

**Question 13**

Using Demoivre's theorem, determine all the values of $(2-2i)^{\frac{1}{3}}$ **[5]**

**Question 14**

Determine the length of the axes, coordinates of the foci, the eccentricity and length of latus rectum of the conic $16x^2 - 9y^2 - 144 = 0$ **[5]**

**Question 15**

a) Determine the equation of the tangent to the curve $4^x$ at $(1,0)$ **[2]**

b) Determine the square root of $18 + 6\sqrt{5}$ **[3]**

**Question 16**

a) Determine the centre and radius of the circle $x^2 + y^2 + 4x - 4y - 1 = 0$ **[2]**

b) The daily attendance of class XII Science students in a school is given below

| Day | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Attendance | 53 | 48 | 64 | 68 | 54 | 70 | 72 |

Calculate three day moving averages and plot this on a graph paper. **[3]**

**Question 17**

If $A = \begin{bmatrix} 1 & 1 & 3 \\ 2 & -4 & 0 \\ 1 & 1 & -3 \end{bmatrix}$, determine $A^{-1}$.

Hence solve the following system of equations

$$x + y + 3z = 5$$
$$2x - 4y = -2$$
$$x + y - 3z = -1$$

**[5]**

**Question 18**

Determine Karl Pearson's coefficient of correlation between the marks in Dzongkha and Mathematics obtained by 10 students of a class in the monthly test. Interpret the result. **[5]**

| Marks in Dzongkha | 20 | 13 | 18 | 21 | 11 | 12 | 17 | 14 | 19 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|
| Marks in Mathematics | 17 | 12 | 23 | 25 | 14 | 7 | 19 | 21 | 22 | 19 |

**Functions and Equations**

(1) $(a \pm b)^2 = a^2 + b^2 \pm 2ab$

(2) $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

(3) $a^2 - b^2 = (a+b)(a-b)$

(4) $a^3 \pm b^3 = \left(a \pm b\right)\left(a^2 \mp ab + b^2\right)$

(5) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

**Sequence and series**

(1) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

(2) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

(3) $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$

(4) $t_n = ar^{n-1}$

(5) $t_n = a + (n-1)d.$

*(6)* $S_n = \frac{a\left(1-r^n\right)}{1-r}$ where $r<1$

$= \frac{a\left(r^n - 1\right)}{r-1}$, Where $r > 1$

(7) $S_n = \frac{n}{2}\left[2a + \left(n-1\right)d\right]$

**Differentiation**

(1) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

(2) $y = x^n,\ y' = nx^{n-1},$

(3) $y = cf(x),\ y' = cf'(x)$

(4) $y = f(x) \pm g(x),\ y' = f'(x) \pm g'(x)$

(5) $F(x) = f(x)g(x),$

$F'(x) = f(x)g'(x) + f'(x)g(x)$

(6) $F(x) = \frac{f(x)}{g(x)}$ ,

$F'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{\left[g(x)\right]^2}$

(7) $f \circ g(x)' = f'g(x) \times (g'x)$

(8) $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

(9) $v(t) = h'(t)$

**Coordinate Geometry**

(1) $\left(y - y_1\right) = m(x - x_1)$

(2) $\sqrt{\left(x-a\right)^2 + \left(y-b\right)^2}$

**Trigonometry**

(1) $Sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

(2) $Cos(A \pm B) = CosA\cos B \mp \sin A\sin B$

(3) $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

(4) $\sin^2\theta + \cos^2\theta = 1$

**Logarithmic Exponentials**

(1) $y = y_0(1+r)^x$

(2) $y = y_0e^{kx}$

(3) $A = P(1+r)^n$

**Integration**

(1) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[\left(\frac{d}{dx}f(x)\right)\int g(x)dx\right]dx$

(2) $\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$

(3) $V = \pi\int_a^b y^2dx$

(4) $A = \int_a^b ydx$

**Measurement**

(1) Cone: $V = \frac{\pi}{3}r^2h$

(2) Cone: $SA = \pi rl + \pi r^2$

(3) Sphere: $V = \frac{4\pi}{3} r^3$

(4) Sphere: $SA = 4\pi r^2$

(5) Cylinder: $SA = 2\pi rh + 2\pi r^2$

(6) Cylinder: $V = \pi r^2 h$

(7) Circle: $A = \pi r^2$

(8) Circle: $C = 2\pi r$

(9) Triangle: $A = \frac{bh}{2}$, $A = \frac{\sqrt{3}}{4} x^2$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

(10) Rectangle: $A = lw$,

(11) Rectangle $P = 2l + 2w$

(12) Square: $A = s^2$,

(13) Square $P = 4s$

(14) Rectangular Prism: $V = lwh$

**Complex Numbers**

(1) $r = \sqrt{a^2 + b^2}$

(2) $\tan\theta = \frac{b}{a} \quad \Rightarrow \theta = \tan^{-1}\left(\frac{b}{a}\right)$

(3) If $z = r\text{cis}\theta$ then $z^n = r^n \text{cisn}\theta$

(4) $z^{\frac{1}{n}} = r^{\frac{1}{n}} cis(\frac{\theta}{n} + k \cdot \frac{360°}{n}) f \text{ or } k = 0,1,2,3,...n-1$

Second Degree Relations

(1) $\text{Ellipse}: \frac{X^2}{a^2} + \frac{y^2}{b^2} = 1$

(2) $\text{Hyperbola}: \frac{X^2}{a^2} + \frac{y^2}{b^2} = 1$

(3) $e = \frac{c}{a}$

Geometry

(1) $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

(2) $(x, y, z) = \left(\frac{lx_2 + mx_1}{l+m}, \frac{ly_2 + my_1}{l+m}, \frac{lz_2 + mz_1}{l+m}\right)$

(3) For

$a_1x + b_1y + c_1z = 0$ *and* $a_2x + b_2y + c_2z = 0$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{z}{a_1b_2 - a_2b_1}$$

(4) $l = \frac{\theta}{360} 2\pi r$

(5) $A = \frac{\theta}{360} \pi r^2$

Matrices

(1) $C_{ij} = (-1)^{i+j} M_{ij}$

(2) $AA^{-1} = A^{-1}A = I$

(3) *Inverse of* $A = A^{-1} = \frac{1}{\det A} \cdot adjA$

## Data & Probability

(1) $\bar{x} = \frac{\sum fx}{n}$

(2) $\text{Median} = l_1 + \frac{l_2 - l_1}{f1}(m - c)$

(3) $\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$

(4) $\sigma_{12} = \sqrt{\frac{n_1\sigma_1^2 + n_1\sigma_2^2 + n_1d_1^2 + n_2d_2^2}{n_1 + n_2}}$

(5) $\sigma = \sqrt{\frac{\sum f(x_i - \bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{N} - \left(\frac{\sum fx}{N}\right)^2}$

(6) $\bar{x}_{12} = \frac{m\bar{x}_1 + n\bar{x}_2}{m+n}$

(7) $I = \frac{\sum \frac{P_1}{P_0} \times 100}{n}$

(8) $I = \frac{\sum p_1 w}{\sum p_0 w} \times 100$

(9) $\text{Cov(X, Y)} = \frac{1}{n} \sum (X - \bar{X})(Y - \bar{Y})$

(10) $r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2 \sum (y - \bar{y})^2}}$

(11) $r = \frac{\sum (x - \bar{x})(y - \bar{y})}{n\sigma_x \sigma_y}$

(12) $b_{YX} = \frac{\text{cov(X, Y)}}{\sigma_x^2} = r\frac{\sigma_y}{\sigma_x}$

(13) $Y - \bar{Y} = \frac{\text{cov}(X, Y)}{\sigma_x^2}(X - \bar{X})$

$$= r\frac{\sigma_x}{\sigma_y}\left(X - \bar{X}\right)$$

(14) $$b_{xy} \times b_{yx} = r\frac{\sigma_y}{\sigma_x} \times r\frac{\sigma_x}{\sigma_y}$$

(15) $$\tau = \frac{2S}{n(n-1)}$$

(16) $$r = 1 - \frac{6\sum d^2}{n(n^2-1)}$$