NEW CURRICULUM

BUSINESS MATHEMATICS
(Three hours)

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*Answer **Question 1** from Section **A** and **10** questions from Section **B**.*
All working, including rough work, should be done on the same sheet as, and adjacent to, the rest of the answer.
The intended marks for questions or parts of questions are given in brackets [].

Mathematical formulae are given at the end of this question paper.
The use of calculator (fx-82/ fx-100) is allowed.

Section A
Answer **ALL** questions.

Directions: Read the following questions carefully. For each question there are four alternatives A, B, C and D. Choose the correct alternative and write it in your answer sheet.

Question 1 (2x15=30 Marks)

(i) $(x-1)$ is a factor of $x^3 - mx^2 + 3x + 1$. The value of m is

A +5.
B –6.
C 8.
D 10.

(ii) $f(x) = 3x^2 + 5x - 8$. $f'(x) = 0$ when x is

A 1.
B $\frac{5}{6}$.
C $\frac{-5}{6}$.
D –8.

(iii) If $\sin A = \frac{4}{5}$ and $\frac{\pi}{2} < A < \pi$, the value of $\cos A$ is

A $-\frac{3}{4}$.

B $-\frac{3}{5}$.

C $\frac{3}{5}$.

D $\frac{4}{3}$.

(iv) $\int \left(4x^3 - 2x\right) dx$ =

A $\frac{x^3}{3} - \frac{x^2}{2} + c$.

B $2x^2 - 2 + c$.

C $12x^2 - 2 + c$.

D $x^4 - x^2 + c$.

(v) For $A = \begin{bmatrix} -1 & 0 & -1 \\ 1 & 2 & 3 \\ -1 & -1 & 2 \end{bmatrix}$, the determinant, $|A|$ is

A –8.

B –6.

C 8.

D 10.

(vi) The solution of $|2x - 1| < 7$ is

A $-4 < x < 3$.

B $-4 < x \leq 3$.

C $-4 < x < 4$.

D $-3 < x < 4$.

(vii) What is the 6^{th} term of the series $\sum_{i=1}^{n} 3^{5-i}(-1)^{i-1}$?

A –3

B $-\frac{1}{3}$

C $\frac{1}{3}$

D 3

(viii) $f(x)=\sqrt{4x+1}$. Find $f'(x)=$

A $4\sqrt{4x+1}$.

B $\dfrac{\sqrt{4x+1}}{2}$.

C $\dfrac{2}{\sqrt{4x+1}}$.

D $\dfrac{1}{2\sqrt{4x+1}}$.

(ix) The derivative of $\log_e\left(x^2\right)$ is

A $2x$.

B $\dfrac{2}{x}$.

C $\dfrac{1}{x^2}$.

D $\dfrac{2}{x^2}$.

(x) $\sum_{i=1}^{10} t_i = 120$ and $\sum_{i=1}^{10} t_i^{\,2} = 1770$. Use this to find $\sum_{i=1}^{10}\left(t_i+1\right)^2$.

A 1780
B 1891
C 2011
D 2020

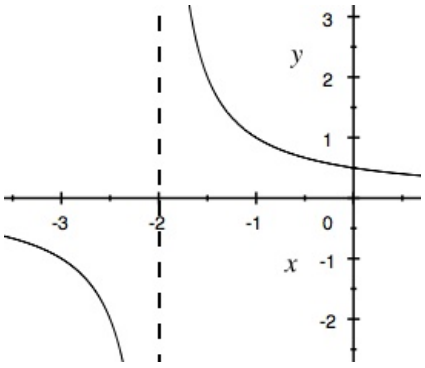
(xi) Determine the equation of the following graph.

A $y=\dfrac{1}{x-2}$

B $y=\dfrac{1}{x+2}$

C $y=\dfrac{1}{(x+2)^2}$

D $y=\dfrac{x+2}{x}$



(xii) Find the derivative of $y = 5^x$.

A 5^x
B 5^{x-1}
C $5^x \log_e 5$
D $\dfrac{5x}{\log_e 5}$

(xiii) $\dfrac{2+\sqrt{2}}{1-\sqrt{2}}$ is equal to

A $-\left(4+3\sqrt{2}\right)$.
B $-4+3\sqrt{2}$.
C $4-3\sqrt{2}$.
D $4+3\sqrt{2}$.

(xiv) Calculate the price index for the following by simple aggregate to the nearest whole number.

Item	2005 Price	2006 Price
Sugar	15	20
Tea	120	145
Rice	15	21

A 186
B 136
C 124
D 81

(xv) $\int \dfrac{1}{(x-1)^2}\,dx$ is equal to

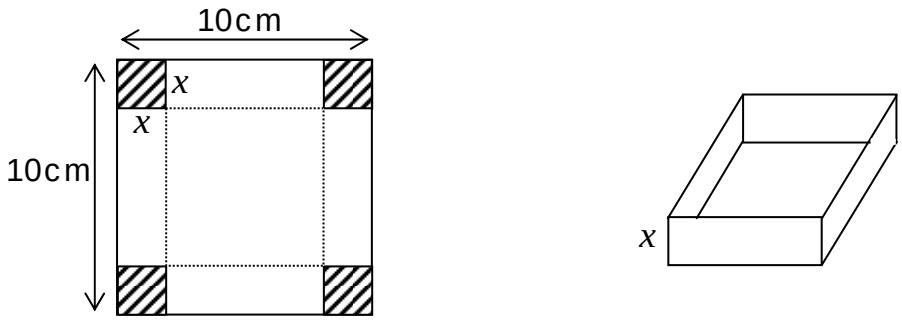
A $\log_e (x-1)^2$.
B $\dfrac{1}{(x-1)^3}$.
C $-\dfrac{1}{(x-1)^3}$.
D $\dfrac{-1}{x-1}$.

Section B (70 Marks)

Answer any 10 questions. All questions in this section have equal marks.
Unless otherwise stated, you may round answers to 2 decimal places.

Question 2

a) Squares of size x cm by x cm are cut from a card 10 cm by 10 cm. The four sides are folded up to make a box with no top.
The volume of the box is 72 cm^3.



Write an equation to show this and solve it to find the possible value(s) of x. [4]

b) Evaluate $\sum_{i=1}^{20}\left(i^3+1\right)$. [3]

Question 3

a) Use proof by induction to prove that $n^2 - n$ is divisible by 2 for all natural numbers n. [4]

b) The graph of $y = x^3 + 4x^2 + x - 6$ cuts the x-axis at (1,0).
Find the other x-intercepts. [3]

Question 4

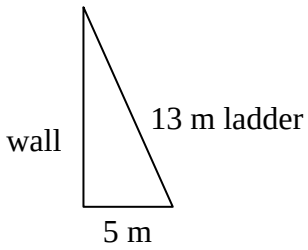
a) Solve the inequality $(x-2)^2 < 8 - x$. [3]

b) $y = x\left(\sqrt{x}+3\right)$. Find y' [2]

c) $\cos A = -\frac{3}{5},\ \pi < A < \frac{3\pi}{2}$. Find $\tan A$. [2]

Question 5

a) A ladder 13 m long is leaning against a wall. The bottom slides away from the wall at 0.24 m/s. How quickly is the top of the ladder sliding down the wall when the bottom is 5 m from the wall?



[4]

b) $y = \sin^2(3x)$. Find and simplify $\frac{dy}{dx}$. [3]

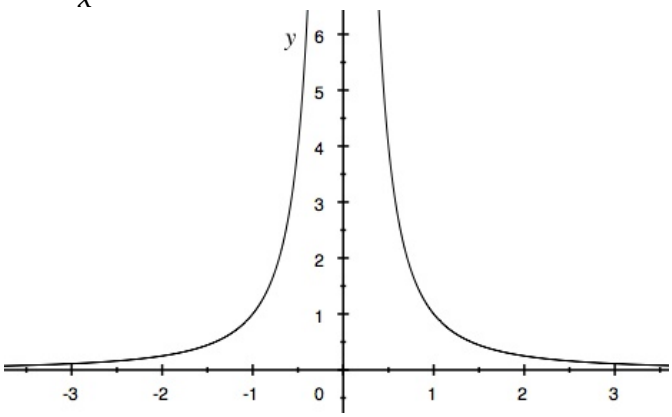
Question 6

a) $x^2 + y^2 = 25$. Find the value of $\frac{dy}{dx}$ at the point $(3,4)$. [3]

b) Find the point(s) of inflection of the graph of $f(x) = 5x^4 - x^5$.
You must prove your answer. [4]

Question 7

a) The graph of $y = \frac{1}{x^2}$ is shown below:



(i) Find $\lim_{x \to \infty} f(x)$. [1]

(ii) Sketch a graph of $y = \frac{1}{(x+1)^2} + 2$. Label the y-intercept. [3]

b) Pema invests Nu 10,000. The bank pays her 5% per year compounded annually. How long (to the nearest year) will it take to grow to Nu 20,000? [3]

Question 8

a) Two positive numbers add to 16. The sum of their squares is a minimum. Find the two numbers. [4]

b) Evaluate $\int_1^e (\log_e x)dx$. [3]

Question 9

a) Use $\cos(A-B) = \cos A\cos B + \sin A\sin B$ to prove the rule $\sin(A+B) = \sin A\cos B + \cos A\sin B$. [4]

b) Find $\int\left(\frac{1}{x^2} - \frac{1}{\sqrt{x}}\right)dx$. [3]

Question 10

a) The value of a car, Nu *V*, after time *t* years can be modeled by $V = 250{,}000e^{-0.24t}$.

(i) Calculate the gradient of the curve when $t = 6$, stating the units. [3]

(ii) What does the gradient represent? [1]

b) Calculate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin 2x dx$. [3]

Question 11

a) $y = xe^x$. Find $\frac{d^2y}{dx^2}$. [3]

b) Sketch the graph of $y = \sqrt{x}$, stating its domain. Then find the volume when the region between the curve and the x-axis from $x = 0$ to $x = 4$ is rotated about the *x*-axis. [4]

Question 12

a) Solve the equation $\sqrt{3x-2} = 1 + \sqrt{2x-3}$. [4]

b) The ranking of 10 people at the start and finish of a course of training are as follows.

Name	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
Rank before	1	6	3	9	5	2	7	10	8	4
Rank after	6	8	3	7	2	1	5	9	4	10

Calculate the Spearman's coefficient of correlation. [4]

Question 13

a) Use the matrix method to solve the simultaneous equations:

$$6x = 2 - 7y \text{ and}$$
$$4x = 3 - 5y.$$

[4]

b) Calculate the 5-yearly moving averages of the number of students who have studied in the school given below.

Year	**98**	**99**	**00**	**01**	**02**	**03**	**04**	**05**	**06**	**07**
No of students	332	317	357	392	402	105	410	417	405	431

[3]

Question 14

a) Find the inverse of $M = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 0 \\ 4 & 1 & 2 \end{bmatrix}$. [4]

b) The annual rainfall (cm) at a place over the period from 2002 to 2007 was as follows:

Year	**2002**	**2003**	**2004**	**2005**	**2006**	**2007**
Rainfall (nearest cm)	73	79	111	79	55	59

Calculate the standard deviation. [3]

Functions and Equations

(1) $(a \pm b)^2 = a^2 + b^2 \pm 2ab$

(2) $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

(3) $a^2 - b^2 = (a + b)(a - b)$

(4) $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

(5) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(6) $v(t) = h'(t)$

Sequence and series

(1) $\sum_{i=1}^{n} i = \dfrac{n(n+1)}{2}$

(2) $\sum_{i=1}^{n} i^2 = \dfrac{n(n+1)(2n+1)}{6}$

(3) $\sum_{i=1}^{n} i^3 = \left[\dfrac{n(n+1)}{2}\right]^2$

(4) $t_n = ar^{n-1}$

(5) $S_n = \dfrac{a(1-r^n)}{1-r} = \dfrac{a(r^n-1)}{r-1}$, where $r>1$

(6) $t_n = a + (n-1)d$

(7) $S_n = \dfrac{n}{2}[2a + (n-1)d]$

Differentiation

(1) $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

(2) $y = x^n,\ y' = nx^{n-1}$

(3) $y = cf(x),\ y' = cf'(x)$

(4) $y = f(x) \pm g(x),\ y' = f'(x) \pm g'(x)$

(5) $F(x) = f(x)g(x)$,
$F'(x) = f(x)g'(x) + f'(x)g(x)$.

(6) $F(x) = \dfrac{f(x)}{g(x)}$,
$F'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

(7) $(f \circ g)'(x) = f'g(x) \times (g'x)$

(8) $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

Coordinate Geometry

(1) $(y - y_1) = m(x - x_1)$

(2) $d = \sqrt{(x-a)^2 + (y-b)^2}$

Trigonometry

(1) $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

(2) $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

(3) $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

(4) $\sin^2\theta + \cos^2\theta = 1$

Logarithmic Exponentials

(1) $y = y_0(1+r)^x$

(2) $y = y_0 e^{kx}$

(3) $A = P(1+r)^n$

Integration

(1) $\int f(x)g(x)dx = f(x)\int gxdx - \int\left[\left(\dfrac{d}{dx}f(x)\right)\int g(x)dx\right]dx$

(2) $\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$

(3) $V = \pi\int_a^b y^2 dx$

(4) $A = \int_a^b y dx$

Measurement

(1) Cone: $V = \dfrac{\pi}{3}r^2h$

(2) Cone: $SA = \pi rl + \pi r^2$

(3) Sphere: $V = \dfrac{4\pi}{3}r^3$

(4) Sphere: $SA = 4\pi r^2$

(5) Cylinder: $SA = 2\pi r^2 + 2\pi rh$

(6) Cylinder: $V = \pi r^2 h$

(7) Circle: $A = \pi r^2$

(8) Circle: $C = 2\pi r$

(9) Triangle: $A = \dfrac{bh}{2}$, $A = \dfrac{\sqrt{3}}{4}x^2$,
$A = \sqrt{s(s-a)(s-b)(s-c)}$

(10) Rectangle: $A = lw$,

(11) Rectangle: $P = 2l + 2w$

(12) Square: $A = s^2$,

(13) Square: $P = 4S$

(14) Rectangular Prism: $V = lwh$

Complex numbers

(1) $r = \sqrt{a^2 + b^2}$

(2) $\tan\theta = \frac{b}{a} \quad \Rightarrow \theta = \tan^{-1}\left(\frac{b}{a}\right)$

(3) If $z = r\text{cis}\theta$ then $z^n = r^n \text{cis}n\theta$

(4) $z^{\frac{1}{n}} = r^{\frac{1}{n}} cis(\frac{\theta}{n} + k \cdot \frac{360°}{n})\ f\ or\ k = 0,1,2,3,...n-1$

Second Degree Relations

(1) *Ellipse:* $\frac{X^2}{a^2} + \frac{y^2}{b^2} = 1$

(2) *Hyperbola:* $\frac{X^2}{a^2} - \frac{y^2}{b^2} = 1$

(3) $e = \frac{c}{a}$

Geometry

(1) $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

(2) $(x, y, z) = \left(\frac{lx_2 + mx_1}{l+m}, \frac{ly_2 + my_1}{l+m}, \frac{lz_2 + mz_1}{l+m}\right)$

(3) For
$a_1x + b_1y + c_1z = 0$ and $a_2x + b_2y + c_2z = 0$,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{z}{a_1b_2 - a_2b_1}$$

(4) $l = \frac{\theta}{360^0} 2\pi r$

(5) $A = \frac{\theta}{360^0} \pi r^2$

Matrices

(1) $C_{ij} = (-1)^{i+j} M_{ij}$

(2) $AA^{-1} = A^{-1}A = I$

(3) Inverse of $A = A^{-1} = \frac{1}{\det A} \cdot adjA$

Data & Probability

(1) $\overline{x} = \frac{\sum fx}{n}$

(2) $\text{Median} = l_1 + \frac{l_2 - l_1}{f_1}(m - c)$

(3) $\sigma = \sqrt{\frac{\sum (x_i - \overline{x})^2}{n}}$

(4) $\sigma_{12} = \sqrt{\frac{n_1\sigma_1^{\ 2} + n_2\sigma_2^{\ 2} + n_1d_1^{\ 2} + n_2d_2^{\ 2}}{n_1 + n_2}}$

(5) $\overline{x}_{12} = \frac{m\overline{x}_1 + n\overline{x}_2}{m+n}$

(6) $I = \frac{\sum \frac{P_1}{P_0} \times 100}{n}$

(7) $I = \frac{\sum P_i W}{\sum P_o W} \times 100$

(8) $\text{Cov}(\text{X}, \text{Y}) = \frac{1}{\text{n}} \sum (\text{X} - \overline{\text{X}})(\text{Y} - \overline{\text{Y}})$

(9) $r = \frac{\sum (x - \overline{x})(y - \overline{y})}{\sqrt{\sum (x - \overline{x})^2 \sum (y - \overline{y})^2}}$

(10) $r = \frac{\sum (x - \overline{x})(y - \overline{y})}{n\sigma_x \sigma_y}$

(11) $\text{b}_{\text{YX}} = \frac{\text{cov}(\text{X}, \text{Y})}{\sigma_{\text{x}}^{\ 2}} = \text{r}\frac{\sigma_{\text{y}}}{\sigma_{\text{x}}}$

(12) $Y - \overline{Y} = \frac{\text{cov}(X, Y)}{\sigma_x^{\ 2}}(X - \overline{X})$

$= r\frac{\sigma_x}{\sigma_y}(X - \overline{X})$

(13) $b_{xy} \times b_{yx} = r\frac{\sigma_y}{\sigma_x} \times r\frac{\sigma_x}{\sigma_y}$

(14) $\tau = \frac{2S}{n(n-1)}$

(15) $r = 1 - \frac{6\sum d^2}{n(n^2 - 1)}$