MATHEMATICS

Three hours and a quarter

(The first 15 minutes of the examination are for reading the paper only. Candidates must NOT start writing during this time).

Total marks: 100

Answer Question 1 from Section A and 10 questions from Section B.
All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answers.
The intended marks for questions or parts of questions are given in brackets [ ].
Mathematical formulae are given at the end of this question paper.
The use of calculator (Fx-82)/(Fx-100) is allowed.

SECTION A

(Answer **ALL** questions)

Directions: *Read the following questions carefully. For each question there are four alternatives, A, B, C and D. Choose the correct alternative and write it in your answer sheet.*

Question 1 **[30 marks]**

(i) The Minor and Co-factor of the element 4 of the determinants $\begin{vmatrix} 2 & 2 & 3 \\ 1 & 4 & 3 \\ 2 & 1 & -4 \end{vmatrix}$ are

- **A** (–14, 14)
- **B** (14, –14)
- **C** (–14, –14)
- **D** (14, 14)

(ii) Name the conic whose eccentricity is $\frac{2}{\sqrt{3}}$

- **A** a circle.
- **B** a parabola.
- **C** an ellipse.
- **D** a hyperbola.

(iii) What is the principal value of $\cos(\sin^{-1}\frac{3}{5})$?

A $\frac{4}{5}$

B $\frac{3}{5}$

C $\frac{5}{4}$

D $\frac{5}{3}$

(iv) The value of $\int_{1}^{3}\left(3x^2+2x-5\right)dx$ is

A 34

B 24

C 42

D 14

(v) If the direction ratios of a line are 2, 3, 6, then the direction cosines of the same line will be

A $\frac{2}{11}, \frac{3}{11}, \frac{6}{11}$

B $\frac{7}{2}, \frac{7}{3}, \frac{7}{6}$

C $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$

D $\frac{11}{2}, \frac{11}{3}, \frac{11}{6}$

(vi) What is the value of 'n', if ${}^{25}C_{2n+12} = {}^{25}C_{3n+2}$?

A 8

B 14

C 12

D 10

(vii) The minimum value of $x^2 + \frac{432}{x}$ is

A 108

B 4

C 0

D 72

(viii) The angle between the pair of straight lines $2x^2 + 7xy - 2y^2 = 0$ is

A 0

B $\frac{\pi}{2}$

C $\frac{\pi}{4}$

D $\frac{\pi}{3}$

(ix) The modulus and amplitude of $\frac{1}{1-i}$ are

A $\left(\sqrt{2}, 30^o\right)$

B $\left(\frac{\sqrt{5}}{2}, 30^o\right)$

C $\left(\frac{2}{\sqrt{5}}, 45^o\right)$

D $\left(\frac{1}{\sqrt{2}}, 45^o\right)$

(x) Find the number of permutations of the letters of the word ‘KHORLO’?

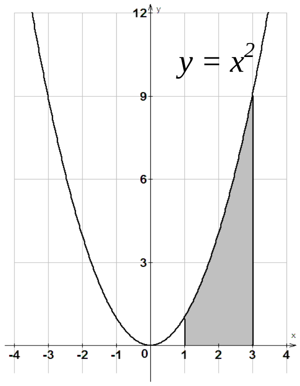
A 360

B 720

C 180

D 90

(xi) The volume of the solid generated when the shaded portion is rotated about the



A $\frac{342\pi}{5}$ *cubic units*

B $\frac{242\pi}{5}$ *cubic units*

C $\frac{404\pi}{5}$ *cubic units*

D $\frac{148\pi}{5}$ *cubic units*

(xii) A die is rolled. If the outcome is an even number, the probability that it is a prime number is

A $\frac{2}{3}$

B 1

C 0

D $\frac{1}{3}$

(xiii) What is the second derivative of $\log \sec x$?

A $\sec x \tan x$

B $-\cos ec^2 x$

C $\sec^2 x$

D $\tan x$

(xiv) Find the distance of the point $(2, 5, -7)$ from the plane $2x + 3y - 6z = 5$

A 8

B 6

C 5

D 7

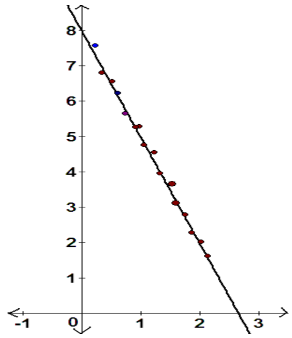
(xv) What type of correlation is shown in the diagram given below?

A High positive correlation

B Low positive correlation

C High negative correlation

D Low negative correlation



SECTION B [70 marks]

Answer any 10 questions. All questions in this section have equal marks.

Question 2

(a) Using the properties of determinants prove that **[3]**

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = abc\,(a-b)(b-c)(c-a)$$

(b) Evaluate: $\int \frac{2x-3}{x^2-x-2}\,dx$ **[4]**

Question 3

(a) Prove that $2\tan^{-1}\left(\frac{1}{3}\right)+\sin^{-1}\left(\frac{4}{5}\right) = \frac{\pi}{2}$ **[3]**

b) Show that the equation $2x^2-2y^2-3xy+21x+18y-36=0$ represents a pair of straight lines. **[4]**

Question 4

(a) Differentiate $e^{\sin x}$ with respect to $\tan x$. **[3]**

(b) A bag contains 4 white, 7 red and 9 black balls. If 4 balls are drawn one by one without replacement, find the probability of getting all white balls. **[4]**

Question 5

(a) Reduce the equation $3x-2y+6z-35=0$ to the normal form and then determine the direction cosines and the length of the normal from the origin. [

(b) If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$ **[4]**

Question 6

(a) Calculate the mean deviation from the mean for the following frequency distribution. **[3]**

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Number of students	2	3	5	8	4

(b) How many different words, each containing 2 vowels and 3 consonants, can be formed with 4 vowels and 15 consonants? **[4]**

Question 7

(a) Differentiate with respect to x: $y = \sin^{-1}(\cos x)$ **[3]**

(b) Solve for x, if $\tan(\cos^{-1} x) = \sin(\cot^{-1} \frac{1}{2})$ **[4]**

Question 8

(a) Find the dimensions of the rectangle of area 64 sq.cm whose perimeter is the least. Also find its perimeter. **[3]**

(b) Find the general solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = \cos x$. **[4]**

Question 9

(a) Find the eccentricity, co-ordinates of foci and length of latus rectum of the hyperbola $9x^2 - 27y^2 = 243$. **[3]**

(b) Find the equation of the ellipse whose focus is $(-1, 1)$, eccentricity is $\frac{1}{\sqrt{2}}$ and the directrix is $x+y=2$ **[4]**

Question 10

(a) The correlation coefficient between the variables ' x ' and ' y ' is $r = 0.63$.
If $\sigma_x = 7$, $\sigma_y = 9$, $\bar{x} = 25$ and $\bar{y} = 60$, find the two regression equations. [

(b) Find the equation of the plane passing through the points $(2, 3, -4)$, $(3, -2, 1)$ and $(4, 3, 2)$. **[4]**

Question 11

(a) Find the square root of $7 + 24i$ **[3]**

(c) Calculate Karl Pearson's correlation coefficient between the marks in Mathematics and English obtained by 10 students and interpret the result. **[4]**

Marks in Mathematics	15	18	21	24	27	30	36	39	42	48
Marks in English	25	25	27	27	31	33	35	41	41	45

Question 12

(a) Evaluate $\int \cos^5 \theta \sin \theta \, d\theta$ **[2]**

(b) Using De Moivre's theorem find all the values of $\left(1 + i\sqrt{3}\right)^{1/4}$. Also find the continued product of the four values. **[5]**

Question 13

(a) Show that the lines PQ and QR are perpendicular to each other, if the points are $P(2,6,2)$, $Q(0,4,1)$ and $R(2,3,-1)$. **[2]**

(b) Solve the following system of equations using determinants:

$$x - 2y + 3z = 4$$
$$2x + y - 3z = 5$$
$$-x + y + 2z = 3$$

[5]

Question 14

(a) For the parabola $x^2 = 16y$, find the co-ordinates of the focus and equation of the directrix. **[2]**

(b) Evaluate the following integral as limit of sums.

$$\int_0^1 (2x+3)\, dx.$$ **[5]**

FORMULAE

Trigonometry

$$\sin^{-1}x=\cos^{-1}\sqrt{1-x^2}=\tan^{-1}\frac{x}{\sqrt{1-x^2}}$$

$$\sin^{-1}x\pm\sin^{-1}y=\sin^{-1}\left(x\sqrt{1-y^2}\pm y\sqrt{1-x^2}\right)$$

$$\cos^{-1}x\pm\cos^{-1}y=\cos^{-1}\left(xy\mp\sqrt{1-x^2}\sqrt{1-y^2}\right)$$

$$\tan^{-1}x\pm\tan^{-1}y=\tan^{-1}\left(\frac{x\pm y}{1\mp xy}\right), xy<1$$

$$2\tan^{-1}x=\tan^{-1}\frac{2x}{1-x^2}=\sin^{-1}\frac{2x}{1+x^2}=\cos^{-1}\frac{1-x^2}{1+x^2}$$

$$\operatorname{cosec}^{-1}x=\sin^{-1}\frac{1}{x}$$

$$\sec^{-1}x=\cos^{-1}\frac{1}{x}$$

$$\cot^{-1}x=\tan^{-1}\frac{1}{x}$$

Complex Numbers

$$r=\sqrt{a^2+b^2}$$

$$\tan\theta=\frac{b}{a}\Rightarrow\theta=\tan^{-1}\left(\frac{b}{a}\right)$$

If $z=r\left(\cos\theta+i\sin\theta\right)$ then

$$z^n=r^n\left(\cos n\theta+i\sin n\theta\right)$$

$$z^{\frac{1}{n}}=r^{\frac{1}{n}}\left(\cos\frac{2k\pi+\theta}{n}+i\sin\frac{2k\pi+\theta}{n}\right),$$

$$k=0,1,2,3,\ldots n-1$$

Co-ordinate Geometry

$$D=\sqrt{\left(x_2-x_2\right)^2+\left(y_2-y_1\right)^2+\left(z_2-z_1\right)^2}$$

$$(x,y,z)=\left(\frac{m_1x_2+m_2x_1}{m_1+m_2},\frac{m_1y_2+m_2y_1}{m_1+m_2},\frac{m_1z_2+m_2z_1}{m_1+m_2}\right)$$

$a_1x+b_1y+c_1z=0$ and $a_2x+b_2y+c_2z=0$

$$\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{z}{a_1b_2-a_2b_1}$$

Angle between two planes,

$$\cos\theta=\pm\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

distance of a point from a plane $=\pm\dfrac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}$

$$(x,y)=\left(\frac{m_1x_1+m_2x_2}{m_1+m_2},\frac{m_1y_1+m_2y_2}{m_1+m_2}\right)$$

Angle between the lines $ax^2+2hxy+by^2=0,$

$$\tan\theta=\left|\frac{2\sqrt{h^2-ab}}{a+b}\right|$$

equation of bisector, $\dfrac{x^2-y^2}{a-b}=\dfrac{xy}{h}$

points of intersection, $\left(\dfrac{hf-bg}{ab-h^2},\dfrac{gh-af}{ab-h^2}\right)$

Algebra

$$a^2-b^2=(a+b)(a-b)$$

$$(a\pm b)^2=a^2\pm 2ab+b^2$$

In the quadratic equation $ax^2+bx+c=0,\ x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$${}^np_r=\frac{n!}{(n-r)!}$$

$${}^nC_r=\frac{n!}{r!(n-r)!}$$

$$C_{ij}=(-1)^{i+j}M_{ij}$$

$$AA^{-1}=A^{-1}A=I$$

$$A^{-1}=\frac{1}{\det A}\cdot adjA$$

$$x = \frac{|D_x|}{|D|}, y = \frac{|D_y|}{|D|}, z = \frac{|D_z|}{|D|}$$

$$1+2+3+.....+(n-1) = \frac{1}{2}n(n-1)$$

$$1^2+2^2+3^2+......+(n-1)^2 = \frac{1}{6}n(n-1)(2n-1)$$

$$1^3+2^3+3^3+......+(n-1)^3 = \left\{\frac{n(n-1)}{2}\right\}^2$$

Calculus

$y = x^n$, $y' = nx^{n-1}$,

$y = cf(x)$, $y' = cf'(x)$,

$$If\ \ y = u \pm v, then \frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$$

$$If\ \ y = uv, then \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$If\ \ y = \frac{u}{v}, then \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\int uv\,dx = u\int vdx - \int\left(\frac{du}{dx}\int vdx\right)dx.$$

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[\sum_{r=0}^{n-1} f(a+rh)\right]$$

$$\frac{dy}{dx} + py = Q, I.F = ye^{\int pdx},$$

$$general\ solution, y.IF = \int (Q.IF)dx + c$$

$$V = \pi\int_a^b y^2 dx \qquad A = \int_a^b ydx$$

$$Volume\ of\ Cone = \frac{1}{3}\pi r^2 h$$

$$Volume\ of\ Sphere = \frac{4}{3}\pi r^3 h$$

$$Volume\ of\ Cylinder = \pi r^2 h$$

$$S.Area\ of\ Cone = \pi rl + \pi r^2$$

$$S.Area\ of\ Sphere = 4\pi r^2$$

$$S.Area\ of\ Cylinder = 2\pi rh + 2\pi r^2$$

Data and Probability

$$\overline{X} = \frac{\sum fx}{\sum f}$$

$$Median = l_1 + \frac{l_2 - l_1}{f_1}(m-c)$$

$$\sigma = \sqrt{\frac{\sum(x_i - \overline{x})^2}{n}}\ or \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

$$\overline{X} = \frac{m\overline{x_1} + n\overline{x_2}}{m+n}$$

$$\sigma_{12} = \sqrt{\frac{n_1\sigma_1^{\ 2} + n_2\sigma_2^{\ 2} + n_1 d_1^{\ 2} + n_2 d_2^{\ 2}}{n_1 + n_2}}$$

$$\mathrm{Cov(X, Y)} = \frac{1}{\mathrm{n}}\sum\left(\mathrm{X} - \overline{\mathrm{X}}\right)\left(\mathrm{Y} - \overline{\mathrm{Y}}\right)$$

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2\sum(y-\bar{y})^2}} = \frac{n\sum xy - \sum x\sum y}{\sqrt{n\sum x^2 - \left(\sum x\right)^2}\sqrt{n\sum y^2 - \left(\sum y\right)^2}}$$

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{n\sigma_x\sigma_y}$$

$$r = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

$$r = \sqrt{b_{xy}.b_{yx}}$$

$$b_{YX} = r\frac{\sigma_y}{\sigma_x} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2}$$

$$b_{XY} = r\frac{\sigma_x}{\sigma_y} = \frac{n\sum xy - \sum x \sum y}{n\sum y^2 - \left(\sum y\right)^2}$$

$$Y - \bar{Y} = \frac{\textbf{cov}(X, Y)}{\sigma_x{}^2}\left(X - \bar{X}\right) = r\frac{\sigma_x}{\sigma_y}\left(X - \bar{X}\right)$$

$$X - \bar{X} = \frac{\textbf{cov}(X, Y)}{\sigma_x{}^2}\left(Y - \bar{Y}\right) = r\frac{\sigma_y}{\sigma_x}\left(Y - \bar{Y}\right)$$

$$\mathrm{b}_{xy} \times \mathrm{b}_{yx} = r\frac{\sigma_x}{\sigma_y} \times r\frac{\sigma_y}{\sigma_x}$$

$$\sum y = na + b\sum x$$

$$\sum xy = a\sum x + b\sum x^2$$

$$y - \bar{y} = b_{yx}\left(x - \bar{x}\right)$$

$$x - \bar{x} = b_{xy}\left(y - \bar{y}\right)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A) + P(\bar{A}) = 1$$

$$P\left(B/A\right) = \frac{P(A \cap B)}{P(A)}$$

$$P\left(A/B\right) = \frac{P(A \cap B)}{P(B)}$$