BUSINESS MATHEMATICS
Three hours and a quarter

(The first 15 minutes of the examination are for reading the paper only. Candidates must NOT start writing during this time).

Total marks: 100

Answer Question 1 from Section A and 10 questions from Section B.
All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answers.
The intended marks for questions or parts of questions are given in brackets [ ].
Mathematical formulae are given at the end of this question paper.
The use of calculator (Fx-82)/(Fx-100) is allowed.

SECTION A

(Answer **ALL** questions)

Directions: *Read the following questions carefully. For each question there are four alternatives, A, B, C and D. Choose the correct alternative and write it in your answer sheet.*

Question 1. **[2×15 = 30 marks]**

(i) Find the number of ways in which 3 persons can be selected out of 7

A 7

B 21

C 35

D 210

(ii) The maximum value of the function $y = 12x - 2x^2$ is

A 3

B 4

C 6

D 18

(iii) If the total cost function for n units of a commodity is $c(n) = 15 + 7n + \frac{1}{2}n^2$, what will be the marginal average cost function?

A $-\frac{15}{n^2} + \frac{1}{2}$

B $\frac{15}{n} + 7 + \frac{1}{2}n$

C $15n + 7n^2 + \frac{1}{2}n^3$

D $7 + n$

(iv) What is the correlation coefficient, if $b_{xy} = -0.3$ and $b_{yx} = -1.2$?

A 0.6

B –0.6

C 0.36

D –0.36

(v) Find the value of x in the determinant $\begin{vmatrix} 0 & 2 & x \\ -1 & 8 & 3 \\ 0 & 5 & 1 \end{vmatrix} = 7$

A $-\frac{9}{5}$

B –1

C $\frac{9}{5}$

D 1

(vi) The derivative of the function $y = \frac{1}{x}$ is

A $-\frac{1}{x^2}$

B $\log x$

C $\frac{1}{x^2}$

D $\frac{1}{x}$

(vii) If the total revenue received from the sale of x units of a product is given by $R(x) = 20x^2 + 5x + 3$. What is the average revenue, when $x = 3$?

A 198

B 125

C 66

D 594

(viii) A die is rolled. If the outcome is an odd number, then the probability of getting a prime number is

A $\frac{3}{2}$

B $\frac{2}{6}$

C $\frac{3}{6}$

D $\frac{2}{3}$

(ix) The coordinates of the point, which divides the points $(1,3,7)$ and $(6,3,2)$ internally in the ratio $2:3$ is

A $(-9,3,17)$

B $(4,3,4)$

C $(3,3,5)$

D $(16,-9,-8)$

(x) Find $\int \frac{2x-5}{x^2-5x+2}\ dx$

A $2x^2 - 5x + c$

B $\log(x^2 - 5x + 2) + c$

C $\frac{1}{x^2-5x+2} + c$

D $x^3 - 5x^2 + 2x + c$

(xi) What is the adjoint of $\begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$?

A $\begin{bmatrix} -1 & -1 \\ -4 & 2 \end{bmatrix}$

B $\begin{bmatrix} -1 & 1 \\ 4 & 2 \end{bmatrix}$

C $\begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$

D $\begin{bmatrix} 1 & -1 \\ -4 & -2 \end{bmatrix}$

(xii) A bank pays interest at the rate of 6% per annum compounded quarterly. What amount should be deposited in the bank at the beginning of each half year in order to accumulate Nu.10,000 in 2 years?

A Nu 4580

B Nu 1200

C Nu 1538.5

D Nu 1168.2

(xiii) Find $\int \cot x \ dx$

A $\text{cosec}^2 x + c$

B $\log \sin x + c$

C $\log \cos x + c$

D $-\cot x \, \text{cosec} x + c$

(xiv) What is the eccentricity of the ellipse $3x^2 + 4y^2 = 12$?

A $\frac{1}{2}$

B $\frac{1}{4}$

C $\frac{1}{\sqrt{2}}$

D $\frac{\sqrt{7}}{2}$

(xv) The mean deviation from the median of the data $48, 56, 54, 64, 52$ and 44 is

A 53

B 5

C 0

D 7.67

SECTION B

Answer any 10 questions. All questions in this section have equal marks.

Question 2. **[70 marks]**

(a) Find the value of n, if ${}^{n}p_{2} = 30$. **[3]**

(b) Calculate the co-efficient of variation in respect of the following frequency distribution. **[4]**

Marks	$10-20$	$20-30$	$30-40$	$40-50$	$50-60$	$60-70$
No. of students	5	12	15	20	10	6

Question 3.

(a) What is the amount of annuity due for Nu 200 for 12 years at 8% compound interest per annum payable half yearly? **[3]**

(b) Solve the following system of equations using Cramer's Rule.
$x+4y+z=12$, $x-2y+3z=6$, $x-3y+2z=1$ **[4]**

Question 4.

(a) Evaluate $\int x\sec^2 x \ dx$ **[3]**

(b) Find the inverse of $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ **[4]**

Question 5.

(a) The means of two samples 500 and 600 were 186 and 175 respectively. The corresponding standard deviations were 9 and 10 respectively. Find the standard deviation of the combined sample. **[3]**

(b) Calculate Karl Pearson's correlation coefficient between the marks in Mathematics and Economics obtained by 6 students in the table below. **[4]**

Marks in Mathematics (Out of 10)	8	6	10	4	9	5
Marks in Economics (Out of 10)	9	8	9	6	3	7

Question 6.

(a) Find the present value of an annuity of Nu 1200 payable at the end of every 6 months for 3 years, when the interest is earned at 8% per year compounded semi-annually. **[3]**

(b) Find the axes, coordinates of vertices, foci, eccentricity, equations of directrices, and length of latus rectum of the hyperbola $9x^2 - 16y^2 = 144$ **[4]**

Question 7.

(a) Prove that the lines joining $A(5, 2, -3)$ to $B(6, 1, 4)$ and $C(-3, -2, -1)$ to $D(-1, -4, 13)$ are parallel. **[3]**

(b) A company sells its products at Nu 10 per unit. The fixed cost for the company is Nu 35,000 and variable costs is estimated to run 30% of total revenue. Determine the break-even point. **[4]**

Question 8.

(a) A bag contains 8 black and 10 red balls. Two balls are drawn at random. What is the probability that one ball is black and the other is red? **[3]**

(b) Find the derivative of $y = (\sin x)^{\log x}$ **[4]**

Question 9.

(a) Find the number of permutations of the letters of the word "CONSEQUENCE" in which all the 2 N's are together. **[3]**

(b) The details of 5 observations for the variables x and y are given as $\sum x = 15, \sum y = 25, \sum x^2 = 55, \sum y^2 = 135$, and $\sum xy = 83$. Find the equation of lines of regressions and also estimate the value of x when $y = 12$. **[4]**

Question 10.

(a) If $y = ae^{mx} + be^{-mx}$, prove that $\frac{d^2y}{dx^2} = m^2 y$ **[3]**

(b) Using the properties of determinants, prove that **[4]**

$$\begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = xyz(x-y)(y-z)(z-x)$$

Question 11.

(a) Find the equation of the parabola whose focus is at $(-3, 0)$ and the directrix $x + 5 = 0$ **[3]**

(b) Evaluate $\int \frac{3x+1}{(x-1)^2(x+3)}\ dx$ **[4]**

Question 12.

(a) Find $\frac{dy}{dx}$, when $y^2 = x^2 + 2xy$ **[3]**

(b) Find the least number of years for an annuity of Nu 1500 per annum such that the amount just exceeds Nu 30,000 @ 9% compounded annually. **[4]**

Question 13.

(a) Show that the points $A(0,1,2)$, $B(2,-1,3)$ and $C(1,-3,1)$ are the vertices of an isosceles right-angled triangle. **[3]**

(b) Differentiate *w.r.t.* x **[4]**

i) $y = \cos^2(x^2)$

ii) $y = (x^2 + 3x - 1)^4$

Question 14.

(a) Evaluate $\int (3x^2 + 4x + 5)^4 (3x + 2)\ dx$ **[3]**

(b) Find the maximum profit that a company can make, if the profit function is given by $P(x) = 2x^3 + 21x^2 + 36x - 20$ **[4]**

FORMULAE

Co-ordinate Geometry

$$D=\sqrt{\left(x_2-x_2\right)^2+\left(y_2-y_1\right)^2+\left(z_2-z_1\right)^2}$$

$$(x,y,z)=\left(\frac{m_1x_2+m_2x_1}{m_1+m_2},\frac{m_1y_2+m_2y_1}{m_1+m_2},\frac{m_1z_2+m_2z_1}{m_1+m_2}\right)$$

$$a_1x+b_1y+c_1z=0 \text{ and } a_2x+b_2y+c_2z=0$$

$$\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{z}{a_1b_2-a_2b_1}$$

$$\cos\theta=\pm\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^{\,2}+b_1^{\,2}+c_1^{\,2}}\sqrt{a_2^{\,2}+b_2^{\,2}+c_2^{\,2}}}$$

Algebra

$$a^2-b^2=(a+b)(a-b)$$

$$(a\pm b)^2=a^2\pm 2ab+b^2$$

$$In\,the\,quadratic\,equation\,ax^2+bx+c=0,\ x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$${}^nP_r=\frac{n!}{(n-r)!}$$

$${}^nC_r=\frac{n!}{r!(n-r)!}$$

$$C_{ij}=(-1)^{i+j}M_{ij}$$

$$AA^{-1}=A^{-1}A=I$$

$$A^{-1}=\frac{1}{\det A}\cdot adjA$$

$$x=\frac{|D_x|}{|D|},\ y=\frac{|D_y|}{|D|},\ z=\frac{|D_z|}{|D|}$$

Commercial Mathematics

$$A=\frac{a}{i}(1+i)\left[(1+i)^n-1\right]$$

$$P=\frac{a}{i}\left[1-(1+i)^{-n}\right]$$

$$A(x)=\frac{C(x)}{x},\ M(x)=\frac{d}{dx}\left(C(x)\right)$$

$$C(x)=F+V(x)$$

$$R(x)=xG(x)$$

$$P(x)=R(x)-C(x)$$

$$MC=\frac{d}{dx}\left(C(x)\right)$$

Calculus

$y = x^n,\ y'=nx^{n-1},$

$y = cf(x),\ y'=cf'(x),$

$$If\ y=u\pm v,\ then\ \frac{dy}{dx}=\frac{du}{dx}\pm\frac{dv}{dx}$$

$$If\ y=uv,\ then\ \frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}$$

$$If\ y=\frac{u}{v},\ then\ \frac{dy}{dx}=\frac{v\dfrac{du}{dx}-u\dfrac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$$

$$\int uv\,dx=u\int v dx-\int\left(\frac{du}{dx}\int v dx\right)dx.$$

Data and Probability

$$\overline{X}=\frac{\sum fx}{\sum f}$$

$$Median=l_1+\frac{l_2-l_1}{f_1}(m-c)$$

$$\sigma = \sqrt{\frac{\sum\left(x_i - \bar{x}\right)^2}{n}} \text{ or } \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2}$$

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

$$\overline{X} = \frac{m\overline{x_1} + n\overline{x_2}}{m+n}$$

$$\sigma_{12} = \sqrt{\frac{n_1\sigma_1^{\,2} + n_2\sigma_2^{\,2} + n_1 d_1^{\,2} + n_2 d_2^{\,2}}{n_1 + n_2}}$$

$$\text{Cov}(X, Y) = \frac{1}{n}\sum\left(X - \overline{X}\right)\left(Y - \overline{Y}\right)$$

$$r = \frac{\sum\left(x-\bar{x}\right)\left(y-\bar{y}\right)}{\sqrt{\sum\left(x-\bar{x}\right)^2\sum\left(y-\bar{y}\right)^2}} = \frac{n\sum xy - \sum x\sum y}{\sqrt{n\sum x^2 - \left(\sum x\right)^2}\sqrt{n\sum y^2 - \left(\sum y\right)^2}}$$

$$r = \frac{\sum\left(x-\bar{x}\right)\left(y-\bar{y}\right)}{n\sigma_x\sigma_y}$$

$$r = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

$$r = \sqrt{b_{xy}.b_{yx}}$$

$$b_{YX} = r\frac{\sigma_y}{\sigma_x} = \frac{n\sum xy - \sum x\sum y}{n\sum x^2 - \left(\sum x\right)^2}$$

$$b_{XY} = r\frac{\sigma_x}{\sigma_y} = \frac{n\sum xy - \sum x\sum y}{n\sum y^2 - \left(\sum y\right)^2}$$

$$Y - \bar{Y} = \frac{\mathbf{cov}(X,Y)}{\sigma_x^{\,2}}\left(X - \bar{X}\right) = r\frac{\sigma_x}{\sigma_y}\left(X - \bar{X}\right)$$

$$X - \bar{X} = \frac{\mathbf{cov}(X,Y)}{\sigma_x^{\,2}}\left(Y - \bar{Y}\right) = r\frac{\sigma_y}{\sigma_x}\left(Y - \bar{Y}\right)$$

$$\text{b}_{xy} \times \text{b}_{yx} = r\frac{\sigma_x}{\sigma_y} \times r\frac{\sigma_y}{\sigma_x}$$

$$\sum y = na + b\sum x$$

$$\sum xy = a\sum x + b\sum x^2$$

$$y - \bar{y} = b_{yx}\left(x - \bar{x}\right)$$

$$x - \bar{x} = b_{xy}\left(y - \bar{y}\right)$$

$$P\left(A \cup B\right) = P\left(A\right) + P\left(B\right) - P\left(A \cap B\right)$$

$$P\left(A\right) + P\left(\overline{A}\right) = 1$$

$$P\left(B/A\right) = \frac{P\left(A \cap B\right)}{P\left(A\right)}$$

$$P\left(A/B\right) = \frac{P\left(A \cap B\right)}{P\left(B\right)}$$