MATHEMATICS

Three hours and a quarter

(The first 15 minutes of the examination are for reading the paper only. Candidates must NOT start writing during this time).

Total marks: 100

Answer **Question 1** from Section **A** and **10 questions** from Section **B**.
All working, including rough work, should be done on the same sheet adjacent to the rest of the answers.
The intended marks for questions or parts of questions are given in brackets [ ].
Mathematical formulae are given at the end of this question paper.
The use of calculator (Fx-82)/(Fx-100) is allowed.

SECTION A

(Answer **ALL** questions)

Directions: *Read the following questions carefully. For each question there are four alternatives, A, B, C and D. Choose the correct alternative and write it in your answer sheet.*

Question 1 **[2x15 = 30]**

(i) If $\begin{bmatrix} 3 & x \end{bmatrix}\begin{bmatrix} 4 \\ x \end{bmatrix} = \begin{bmatrix} 13 \end{bmatrix}$ then 'x' will be equal to

A ± 5

B $\frac{13}{7}$

C ± 1

D -13

(ii) The distance of the point $(2,-2,1)$ from the origin is

A 3

B $\sqrt{5}$

C $\sqrt{3}$

D 9

(iii) The value of $\int \sin^2 x dx$

A $\cos^2 x + c$

B $\frac{x}{2} - \frac{\sin 2x}{4} + c$

C $-\frac{\sin 2x}{4} + c$

D $\sin 2x + c$

(iv) If $f(x) = e^{\cos x}$, then $f'\left(\frac{\pi}{2}\right)$ is

A -1

B e

C 0

D 1

(v) The direction cosines of the normal to the plane $2x - 9y + 6z + 55 = 0$ are

A $\frac{2}{121}, \frac{-9}{121}, \frac{6}{121}$

B $\frac{2}{11}, \frac{-9}{11}, \frac{6}{11}$

C $2, -9, 6$

D $2, -9, 55$

(vi) The solution of the differential equation $\frac{dy}{dx} = \sec y$ for $y = \frac{\pi}{2}$ when x =1 is

A $y = \sin x$

B $y = \tan x$

C $y = \sec x \tan x$

D $\sin y = x$

(vii) At what value of 'x' is the function $y = \sin x$ maximum?

A $\frac{\pi}{2}$

B 0

C π

D 2π

(viii) The centre of the ellipse $\frac{(x-2)^2}{16} + \frac{(y+5)^2}{9} = 1$ is at

A $(-2, 5)$

B $(2, -5)$

C $(5, -2)$

D $(-5, 2)$

(ix) If 'ω' is a cube root of unity, then $(\omega + \omega^2)^3$ is equal to

A ω

B 0

C 1

D -1

(x) A bag contains 4 red, 5 green and 6 blue balls. A ball is drawn at random, the probability of getting a green or red ball is

A $\frac{1}{15}$

B $\frac{3}{5}$

C $\frac{4}{5}$

D $\frac{5}{13}$

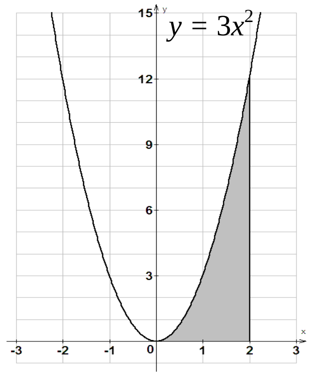
(xi) In the diagram given below, what is the area of the shaded portion?

A 8 sq. units

B $\frac{8}{3}$ sq. units

C 12 sq. units

D 6 sq. units



(xii) The standard deviation of the first five even numbers is.

A $\sqrt{8}$

B 4

C 2

D $\sqrt{2}$

(xiii) If $y = \begin{cases} \cos x, & 0 \le x \le \frac{\pi}{2} \\ 1, & \frac{\pi}{2} \le x \le 5 \end{cases}$, then $\int_0^5 y\,dx$ is equal to

A 5

B $6 - \frac{\pi}{2}$

C $\frac{\pi}{2}$

D $5 - \frac{\pi}{2}$

(xiv) The point of intersection of the pair of lines given by $12x^2 - 10xy + 2y^2 = 0$ is

A $(2, -10)$

B $(12, -10)$

C $(12, 2)$

D $(0, 0)$

(xv) The table below represents the ranks of five students in two different subjects.

Subject A	2	5	1	4	3
Subject B	4	1	5	2	3

Which statement below best describes the relation between the ranks in two subjects?

A No correlation

B Perfect positive correlation

C Perfect negative correlation

D Moderate negative correlation

SECTION B

Answer any 10 questions. All questions in this section have equal marks

Question 2

(a) Using the properties of determinants prove that **[3]**

$$\begin{vmatrix} p & 1 & q+r \\ q & 1 & p+r \\ r & 1 & p+q \end{vmatrix} = 0$$

(b) Find $\frac{dy}{dx}$ if $y = x^{\tan x}$ **[4]**

Question 3

(a) If $\sin(\cos^{-1} x) = \frac{2}{3}$, find the values of x. [

(b) Show that the four points $(0,-1,2)$, $(2,-1,-1)$, $(1,1,1)$ and $(0,3,3)$ are coplanar. **[4]**

Question 4

(a) Evaluate: $\int \log x \, dx$ **[3]**

(b) The probability that Dorji will pass in Dzongkha test is $\frac{4}{5}$ and the probability that he will not pass in English test is $\frac{1}{8}$. If the probability of passing in at least one of the tests is $\frac{3}{4}$, what is the probability that he will pass in both the tests? **[4]**

Question 5

(a) Find the equation of the parabola whose focus is $(1,-2)$ and the directrix is $x+3=0$. **[3]**

(b) Solve the differential equation $\frac{d^2y}{dx^2} = 1 + \cos x$, given that $y = 2$ and $\frac{dy}{dx} = 0$, when $x = 0$.

[4]

Question 6

(a) The mean monthly salary paid to all employees of a company was Nu 2500. The mean annual salaries of male and female employees were Nu 2600 and Nu 2100 respectively. Determine the percentage of males and females employed by the company. **[3]**

(b) Ten teachers were invited for a meeting by the Principal. In how many ways can the teachers and the Principal be seated around a circular table? In how many ways will two particular teachers be seated on either side of the Principal? **[4]**

Question 7

(a) Solve: $(x^2 + 2x)\dfrac{dy}{dx} = 2x + 2$ [3]

(b) Evaluate: $\sin\left(\tan^{-1}\dfrac{3}{4} + \tan^{-1}\dfrac{5}{12}\right)$ **[4]**

Question 8

(a) Evaluate: $\displaystyle\int \frac{2x+1}{x^2 - x - 6}\,dx$ **[3]**

(b) Differentiate with respect to x: $\sin^{-1}(3x - 4x^3)$ **[4]**

Question 9

(a) Find the equation of a plane which passes through the point $(4,0,1)$ and is parallel to the plane $4x + 3y - 12z + 6 = 0$ **[3]**

(b) Find the value of 'λ' so that the equation $x^2 + 4xy + 4y^2 + \lambda x + 10y + 4 = 0$ represents a pair of straight lines. **[4]**

Question 10

(a) The arithmetic mean of the points scored by four archers Pema, Dawa, Tshering and Tashi are 70, 90, 60, and 40 respectively. The standard deviations of their points are 12, 16, 9 and 8. Who is the most consistent archer of the four? **[3]**

(b) Find the vertices and foci of the hyperbola $5x^2 - 4y^2 - 10x - 8y - 19 = 0$. **[4]**

Question 11

(a) Find the modulus and amplitude of $\dfrac{1 + i\sqrt{3}}{\sqrt{3} + i}$. **[3]**

(b) The marks of five candidates in Mathematics and Physics tests were as follows.

Mathematics	90	72	82	70	85
Physics	95	65	70	73	80

A candidate who scored 75 in Mathematics was absent from the Physics test. Estimate his probable score in Physics by finding the appropriate regression equation. **[4]**

Question 12

(a) If $x^2 + xy + y = 5$, find $\frac{dy}{dx}$. [

(b) Using De Moivre's theorem, find all the values of $(1+i)^{\frac{1}{3}}$. Also find the continued product of the values. **[5]**

Question 13

(a) Find the length of latus rectum of the ellipse $\frac{x^2}{9} + \frac{y^2}{36} = 1$ **[2]**

(b) Using matrix method, solve the following system of linear equations. **[5]**

$$2a + 3b = 13$$
$$4b - 2c = 4$$
$$3a + c = 10$$

Question 14

(a) Given that $A(-2,3,5)$, $B(1,2,3)$ and $C(7,0,-1)$ are collinear. Find the ratio in which B divides AC. **[2]**

(b) A right circular cylinder is to be made so that the sum of its radius and its height is 9 metres. Find the maximum volume of the cylinder. **[5]**

MATHEMATICS FORMULAE

Trigonometry

$$\sin^{-1} x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}}$$

$$\sin^{-1} x \pm \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\right)$$

$$\cos^{-1} x \pm \cos^{-1} y = \cos^{-1}\left(xy \mp \sqrt{1-x^2}\sqrt{1-y^2}\right)$$

$$\tan^{-1} x \pm \tan^{-1} y = \tan^{-1}\left(\frac{x \pm y}{1 \mp xy}\right), xy < 1$$

$$2\tan^{-1} x = \tan^{-1}\frac{2x}{1-x^2} = \sin^{-1}\frac{2x}{1+x^2} = \cos^{-1}\frac{1-x^2}{1+x^2}$$

$$\cos ec^{-1} x = \sin^{-1}\frac{1}{x}$$

$$\sec^{-1} x = \cos^{-1}\frac{1}{x}$$

$$\cot^{-1} x = \tan^{-1}\frac{1}{x}$$

Complex Numbers

$$r = \sqrt{a^2 + b^2}$$

$$\tan\theta = \frac{b}{a} \Rightarrow \theta = \tan^{-1}\left(\frac{b}{a}\right)$$

If $z = r(\cos\theta + i\sin\theta)$ then

$$z^n = r^n(\cos n\theta + i\sin n\theta)$$

$$z^{\frac{1}{n}} = r^{\frac{1}{n}}\left(\cos\frac{2k\pi+\theta}{n} + i\sin\frac{2k\pi+\theta}{n}\right),$$

$$k = 0,1,2,3,\ldots n-1$$

Co-ordinate Geometry

$$D = \sqrt{(x_2 - x_2)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$(x, y, z) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}, \frac{m_1z_2 + m_2z_1}{m_1 + m_2}\right)$$

$a_1x + b_1y + c_1z = 0$ and $a_2x + b_2y + c_2z = 0$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{z}{a_1b_2 - a_2b_1}$$

Angle between two planes,

$$\cos\theta = \pm\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

distance of a point from a plane $= \pm\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}$

$$(x, y) = \left(\frac{m_1x_1 + m_2x_2}{m_1 + m_2}, \frac{m_1y_1 + m_2y_2}{m_1 + m_2}\right)$$

Angle between the lines $ax^2 + 2hxy + by^2 = 0$,

$$\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|$$

equation of bisector, $\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$

points of intersection, $\left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2}\right)$

Algebra

$$a^2 - b^2 = (a+b)(a-b)$$

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

In the quadratic equation $ax^2 + bx + c = 0$, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$^n p_r = \frac{n!}{(n-r)!}$$

$$^n C_r = \frac{n!}{r!(n-r)!}$$

$$C_{ij} = (-1)^{i+j} M_{ij}$$

$$AA^{-1} = A^{-1}A = I$$

$$A^{-1} = \frac{1}{\det A} \cdot adjA$$

$$x=\frac{|D_x|}{|D|},\ y=\frac{|D_y|}{|D|},\ z=\frac{|D_z|}{|D|}$$

$$1+2+3+.....+(n-1)=\frac{1}{2}n(n-1)$$

$$1^2+2^2+3^2+......+(n-1)^2=\frac{1}{6}n(n-1)(2n-1)$$

$$1^3+2^3+3^3+......+(n-1)^3=\left\{\frac{n(n-1)}{2}\right\}^2$$

CALCULUS

$$y = x^n,\ y' = nx^{n-1},$$

$$y = cf(x),\ y' = cf'(x),$$

$$If\ \ y = u \pm v, then\ \frac{dy}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$$

$$If\ \ y = uv, then\ \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$If\ \ y = \frac{u}{v},\ then\ \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}$$

$$\int uv\,dx = u\int vdx - \int\left(\frac{du}{dx}\int vdx\right)dx.$$

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[\sum_{r=0}^{n-1} f(a+rh)\right]$$

$$\frac{dy}{dx}+py=Q, I.F = ye^{\int pdx},$$

$$general\ solution, y.IF = \int (Q.IF)dx + c$$

$$V=\pi\int_a^b y^2dx \qquad A=\int_a^b ydx$$

$$Volume\,of\ Cone = \frac{1}{3}\pi r^2 h$$

$$Volume\,of\ Sphere = \frac{4}{3}\pi r^3 h$$

$$Volume\,of\ Cylinder = \pi r^2 h$$

$$S.Area\,of\ Cone = \pi rl + \pi r^2$$

$$S.Area\,of\ Sphere = 4\pi r^2$$

$$S.Area\,of\ Cylinder = 2\pi rh + 2\pi r^2$$

Data and Probability

$$\overline{X}=\frac{\sum fx}{\sum f}$$

$$Median = l_1 + \frac{l_2 - l_1}{f_1}(m-c)$$

$$\sigma=\sqrt{\frac{\sum(x_i-\overline{x})^2}{n}}\ or\ \sqrt{\frac{\sum x^2}{n}-\left(\frac{\sum x}{n}\right)^2}$$

$$\sigma=\sqrt{\frac{\sum fx^2}{\sum f}-\left(\frac{\sum fx}{\sum f}\right)^2}$$

$$\overline{X}=\frac{m\overline{x_1}+n\overline{x_2}}{m+n}$$

$$\sigma_{12}=\sqrt{\frac{n_1\sigma_1{}^2+n_2\sigma_2{}^2+n_1d_1{}^2+n_2d_2{}^2}{n_1+n_2}}$$

$$\mathrm{Cov(X, Y)}=\frac{1}{\mathrm{n}}\sum\left(\mathrm{X}-\overline{\mathrm{X}}\right)\left(\mathrm{Y}-\overline{\mathrm{Y}}\right)$$

$$r=\frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2\sum(y-\bar{y})^2}}=\frac{n\sum xy-\sum x\sum y}{\sqrt{n\sum x^2-\left(\sum x\right)^2}\sqrt{n\sum y^2-\left(\sum y\right)^2}}$$

$$r=\frac{\sum(x-\bar{x})(y-\bar{y})}{n\sigma_x\sigma_y}$$

$$r=1-\frac{6\sum d^2}{n(n^2-1)}$$

$$r=\sqrt{b_{xy}.b_{yx}}$$

$$b_{YX} = r\frac{\sigma_y}{\sigma_x} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2}$$

$$b_{XY} = r\frac{\sigma_x}{\sigma_y} = \frac{n\sum xy - \sum x \sum y}{n\sum y^2 - \left(\sum y\right)^2}$$

$$Y - \bar{Y} = \frac{\textbf{cov}(X, Y)}{\sigma_x^{\ 2}}\left(X - \bar{X}\right) \ = \ r\frac{\sigma_x}{\sigma_y}\left(X - \bar{X}\right)$$

$$X - \bar{X} = \frac{\textbf{cov}(X, Y)}{\sigma_x^{\ 2}}\left(Y - \bar{Y}\right) \ = \ r\frac{\sigma_y}{\sigma_x}\left(Y - \bar{Y}\right)$$

$$\mathrm{b}_{xy} \times \mathrm{b}_{yx} = r\frac{\sigma_x}{\sigma_y} \times \ r\frac{\sigma_y}{\sigma_x}$$

$$\sum y = na + b\sum x$$

$$\sum xy = a\sum x + b\sum x^2$$

$$y - \bar{y} = b_{yx}\left(x - \bar{x}\right)$$

$$x - \bar{x} = b_{xy}\left(y - \bar{y}\right)$$

$$P\left(A \cup B\right) = P\left(A\right) + P\left(B\right) - P\left(A \cap B\right)$$

$$P\left(A\right) + P\left(\bar{A}\right) = 1$$

$$P\left(B/A\right) = \frac{P\left(A \cap B\right)}{P\left(A\right)}$$

$$P\left(A/B\right) = \frac{P\left(A \cap B\right)}{P\left(B\right)}$$

Rough Work