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MATHEMATICS
(Three hours and a quarter)
NEW CURRICULUM

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*Answer **Question 1** from Section **A** and **10** questions from Section **B**.*
All working including rough work, should be done on the same sheet adjacent to the rest of the answer.
The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical formulae are given at the end of this question paper.
The use of calculator ($fx-82$/ $fx-100$) ***is allowed.***

Section A (30 Marks)
Answer **ALL** questions

Direction: *Read the following questions carefully. For each question there are four alternatives A, B, C and D. Choose the correct alternative and write it in your answer sheet.*

Question 1 **[2 × 15 = 30]**

i) How many triangles can be formed by joining any three of 9 points when no three points are collinear?

A 74

B 80

C 84

D 90

ii) A bag contains 4 red, 5 green and 6 blue balls. A ball is drawn at random, the probability of getting a green or red ball is

A $\frac{1}{15}$

B $\frac{3}{5}$

C $\frac{4}{5}$

D $\frac{5}{13}$

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iii) The value of $\sin^{-1}\left(\sin\frac{3\pi}{4}\right)$

A $\frac{3\pi}{2}$

B $\frac{\pi}{3}$

C $\frac{5\pi}{3}$

D $\frac{\pi}{4}$

iv) The $a+ib$ form of $(5-3i)(6+2i)$ is

A $11-i$

B $1+i$

C $-11+i$

D $36-8i$

v) The derivative of 2^{2^x}

A $2^{x+1}\log_e 2$

B $2^{x-1}\log_e 2$

C $2^x \log_e 2$

D $2\log_e 2$

vi) The value of $\int \sin^2 x\,dx$

A $\cos^2 x+c$

B $\frac{x}{2}-\frac{\sin 2x}{4}+c$

C $-\frac{\sin 2x}{4}+c$

D $\sin 2x+c$

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vii) The single line which represents the lines $x = 2y$ and $x = y - 3$ is

A $x^2 + 3xy + 2y^2 + 3x - 6y = 0$

B $x^2 - 3xy - 2y^2 + 3x - 6y = 0$

C $x^2 - 3xy + 2y^2 + 3x - 6y = 0$

D $x^2 + 3xy + 2y^2 + 3x + 6y = 0$

viii) The length of the major axis of the ellipse $4x^2 + 36y^2 = 144$

A 6 units

B 4 units

C 2 units

D 3 units

ix) If $f(x) = e^{\cos x}$, then $f'\left(\frac{\pi}{2}\right)$ is

A -1

B e

C 0

D 1

x) The value of 'n' for $np_2 = 56$ is

A 7

B 6

C 8

D 5

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xi) If $\int_0^n 3x^2 dx = 8$, then 'n' is

A -2

B $\frac{4}{3}$

C $\frac{3}{4}$

D 2

xii) The angle between the planes $x - y + z = 1$ and $3x + 2y - z + 2 = 0$ is

A 180°

B 90°

C 0°

D 45°

xiii) The solution of the differential equation $\frac{dy}{dx} = \sec y$ for $y = \frac{\pi}{2}$ when x =1 is

A $y = \sin x$

B $y = \tan x$

C $y = \sec x \tan x$

D $\sin y = x$

xiv) The standard deviation of the first five even numbers is.

A $\sqrt{8}$

B 4

C 2

D $\sqrt{2}$

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xv) The inverse of the matrix $A = \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$ is

A $\begin{bmatrix} -\cos\theta & \cos\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

B $\begin{bmatrix} \cos\theta & -\cos\theta \\ \sin\theta & -\sin\theta \end{bmatrix}$

C $\begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}$

D $\begin{bmatrix} \sin^2\theta & 0 \\ 0 & \cos^2\theta \end{bmatrix}$

Section B (70 marks)

Answer any 10 questions.

All questions in this section have equal marks. Unless otherwise stated, you may round your answers to two decimal places.

Question 2

a) Show that $\begin{vmatrix} 1 & -x & y \\ x & 1 & z \\ -y & -z & 1 \end{vmatrix} = 1 + x^2 + y^2 + z^2$ **[4]**

b) Calculate $\tan^{-1}\frac{1}{2} + \cot^{-1} 3$ **[3]**

Question 3

a) How many different words each containing 3 vowels and 4 consonants can be formed with 5 vowels and 11 consonants? **[4]**

b) Find $\frac{dy}{dx}$ for $y = \cos^{-1}\sqrt{\frac{1+\cos x}{2}}$ **[3]**

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Question 4

a) Determine the value of *x, y* and *z* for the following system of equations

$$x - y + z = 2$$
$$2x - y = 0$$
$$2y - z = 1$$

[4]

b) The events A and B are such that $P(A) = \frac{2}{5}, P(B) = \frac{1}{4}$ and $P\left(\frac{A}{B}\right) = \frac{3}{5}$.

Determine the probability that neither A nor B occurs. **[3]**

Question 5

a) Examine the equality of the statement

$$\sin\left[\tan^{-1}\left\{\cos\left(\sec^{-1} x\right)\right\}\right] = \frac{1}{\sqrt{1+x^2}}$$

[4]

b) Show that the points $(0,4,3),(-1,-5,-3)$ and $(-2,-2,1)$ are coplanar **[3]**

Question 6

An open tank with a square base and vertical side is to be constructed to hold 500 litres of water. Find the height of the tank that will minimize the cost of the material to construct the tank. **[7]**

Question 7

Find the area of the region bounded by the curve $y = 8x$ and its latus rectum. Also find the volume of the solid generated when the region is rotated about x-axis. **[7]**

Question 8

a) Find the vertices, foci, eccentricity and the equation of the directrices of the hyperbola $9x^2 - 25y^2 = 225$ **[5]**

b) Explain whether $y = ax^2 + bx + c$ is the solution of the differential equation

$$x\frac{d^2y}{dx^2} - \frac{dy}{dx} + b = 0$$

[2]

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Question 9

Find the two lines represented by the equation $4x^2 + 4xy + y^2 - 4x - 2y - 24 = 0$. Are the two lines are parallel? Explain. If parallel, find the perpendicular distance between them? **[7]**

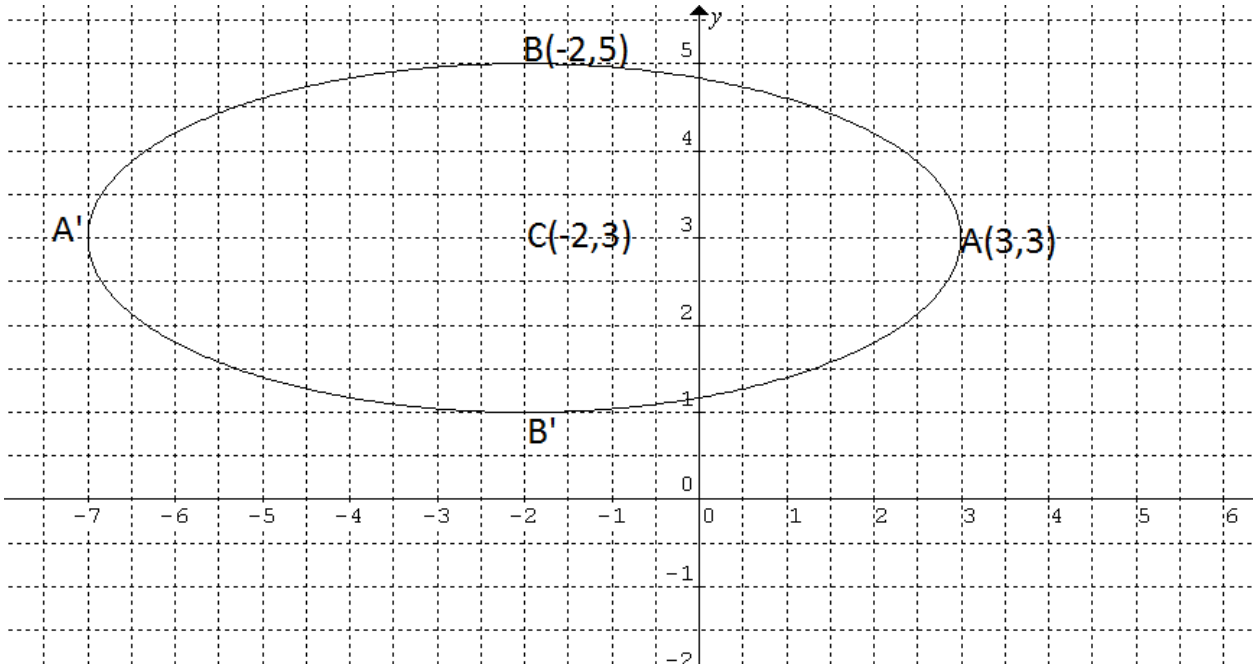
Question 10

a) Find the ratio in which YZ plane divides the join of $(-4,3,7)$ and $(6,3,2)$. Also obtain the point of intersection of the line with the plane. **[5]**

b) Verify if the planes $x + y - z + 4 = 0$ and $2x + 2y - 2z + 10 = 0$ are parallel. **[2]**

Question 11

a) Write the equation of the ellipse for the given figure below **[2]**



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b) Compute the coefficient of correlation between the income and expenditure of six families for a particular month in the data given below **[5]**

Income (in thousands)	16	19	20	27	29	33
Expenditure(in thousands)	14	15	17	21	23	24

Question 12

a) The mean monthly salary paid to the teaching and non-teaching staff of a school is Nu 25000 and Nu 15000 respectively. What is the mean monthly salary paid to all the staff if there are 30 teaching and 10 non-teaching staff? **[3]**

b) Decide the value of $\left(\sqrt{3}+i\right)^6$ **[4]**

Question 13

a) Evaluate $\int x\left(\log_e x\right)^2 dx$ **[4]**

b) Calculate the quartile deviation of the following data **[3]**

Marks	0-10	10-20	20-30	30-40	40-50
No. of students	32	45	54	42	27

Question 14

a) Write the complex number $4\left(\cos 135° + i\sin 135°\right)$ in Cartesian form. **[3]**

b) Solve the differential equation of $2x\dfrac{dy}{dx} = x + 2y$ **[4]**

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MATHEMATICS FORMULAE

Algebra

$${}^{n}p_{r} = \frac{n!}{(n-r)!}$$

$${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$C_{ij} = (-1)^{i+j} M_{ij}$$

$$A\,A^{-1} = A^{-1}A = I$$

$$A^{-1} = \frac{1}{\det A} \cdot adjA$$

$$x = \frac{|D_x|}{|D|}, y = \frac{|D_y|}{|D|}, z = \frac{|D_z|}{|D|}$$

Trigonometry

$$\sin^{-1} x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}}$$

$$\sin^{-1} x \pm \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\right)$$

$$\cos^{-1} x \pm \cos^{-1} y = \cos^{-1}\left(xy \mp \sqrt{1-x^2}\sqrt{1-y^2}\right)$$

$$\tan^{-1} x \pm \tan^{-1} y = \tan^{-1}\left(\frac{x \pm y}{1 \mp xy}\right), xy < 1 \text{ or } xy > -1$$

Calculus

$y = x^n,\ y' = nx^{n-1}$,

$y = cf(x),\ y' = cf'(x)$,

$y = f(x) \pm g(x),\ y' = f'(x) \pm g'(x)$

$F(x) = f(x)g(x)$

$F'(x) = f(x)g'(x) + f'(x)g(x).$

$$F(x) = \frac{f(x)}{g(x)},\ F'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\int f(x)g(x)dx = f(x)\int gxdx - \int\left[\left(\frac{d}{dx}f(x)\right)\int g(x)dx\right]$$

$$\int_a^b f(x)dx = \lim_{h\to 0} h\left[\sum_{r=0}^{n-1} f(a+rh)\right]$$

$$V = \pi\int_a^b y^2 dx$$

$$A = \int_a^b y dx$$

$$\frac{dy}{dx} + py = Q \Rightarrow ye^{\int pdx} = \int Qe^{\int pdx}dx + c$$

Co-ordinate Geometry

$$D = \sqrt{(x_2 - x_2)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$(x, y, z) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}, \frac{m_1z_2 + m_2z_1}{m_1 + m_2}\right)$$

$a_1x + b_1y + c_1z = 0$ and $a_2x + b_2y + c_2z = 0$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{z}{a_1b_2 - a_2b_1}$$

$$\cos\theta = \pm\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$(x, y) = \left(\frac{m_1x_1 + m_2x_2}{m_1 + m_2}, \frac{m_1y_1 + m_2y_2}{m_1 + m_2}\right)$$

Data and Probability

$$\bar{x} = \frac{m\bar{x_1} + n\bar{x_2}}{m+n}$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$Median = l_1 + \frac{l_2 - l_1}{f_1}(m - c)$$

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$$\sigma = \sqrt{\frac{\sum\left(x_i - \bar{x}\right)^2}{n}}$$

$$\sigma_{12} = \sqrt{\frac{n_1\sigma_1^{\,2} + n_2\sigma_2^{\,2} + n_1 d_1^{\,2} + n_2 d_2^{\,2}}{n_1 + n_2}}$$

$$\mathrm{Cov(X,Y)} = \frac{1}{\mathrm{n}}\sum\left(\mathrm{X}-\overline{\mathrm{X}}\right)\left(\mathrm{Y}-\overline{\mathrm{Y}}\right)$$

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2\sum(y-\bar{y})^2}} = \frac{n\sum xy - \sum x\sum y}{\sqrt{n\sum x^2 - \left(\sum x\right)^2}\sqrt{n\sum y^2 - \left(\sum y\right)^2}}$$

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{n\sigma_x\sigma_y}$$

$$r = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

$$b_{YX} = r\frac{\sigma_y}{\sigma_x} = \frac{n\sum xy - \sum x\sum y}{n\sum x^2 - \left(\sum x\right)^2}$$

$$b_{XY} = r\frac{\sigma_x}{\sigma_y} = \frac{n\sum xy - \sum x\sum y}{n\sum y^2 - \left(\sum y\right)^2}$$

$$Y - \bar{Y} = \frac{\mathbf{cov}(X,Y)}{\sigma_x^{\,2}}\left(X - \bar{X}\right) = r\frac{\sigma_x}{\sigma_y}\left(X - \bar{X}\right)$$

$$X - \bar{X} = \frac{\mathbf{cov}(X,Y)}{\sigma_x^{\,2}}\left(Y - \bar{Y}\right) = r\frac{\sigma_y}{\sigma_x}\left(Y - \bar{Y}\right)$$

$$\mathrm{b}_{xy} \times \mathrm{b}_{yx} = r\frac{\sigma_x}{\sigma_y} \times r\frac{\sigma_y}{\sigma_x}$$

$$\sum y = na + b\sum x$$

$$\sum xy = a\sum x + b\sum x^2$$

$$y - \bar{y} = b_{yx}\left(x - \bar{x}\right)$$

$$x - \bar{x} = b_{xy}\left(y - \bar{y}\right)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A) + P\left(\overline{A}\right) = 1$$

$$P\left(B/A\right) = \frac{P(A \cap B)}{P(A)}$$

$$P\left(A/B\right) = \frac{P(A \cap B)}{P(B)}$$

Complex Numbers

$$r = \sqrt{a^2 + b^2}$$

$$\tan\theta = \frac{b}{a} \Rightarrow \theta = \tan^{-1}\left(\frac{b}{a}\right)$$

If $z = r\left(\cos\theta + i\sin\theta\right)$ then

$$z^n = r^n\left(\mathrm{cosn}\theta + \mathrm{isinn}\theta\right)$$

$$z^{\frac{1}{n}} = r^{\frac{1}{n}}\left(\cos\frac{\theta + 2k\pi}{n} + i\sin\frac{\theta + 2k\pi}{n}\right),$$

$$k = 0, 1, 2, 3, \ldots n-1$$

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