## BUSINESS MATHEMATICS

(Three hours and a quarter)

Answer Question 1 from Section A and $\mathbf{1 0}$ questions from Section $\boldsymbol{B}$.
All working, including rough work, should be done on the same sheet adjacent to the rest of the answer.
The intended marks for questions or parts of questions are given in brackets [].

Mathematical formulae are given at the end of this question paper. The use of calculator ( $f x-82 / f x-100$ ) is allowed.

## Section A

Answer ALL questions.
Directions: Read the following questions carefully. For each question there are four alternatives $A, B, C$ and $D$. Choose the correct alternative and write it in your answer sheet.
[ $2 \times 15=30]$
i) The value of $\left|\begin{array}{cc}2 & 3 \\ -4 & 7\end{array}\right|$, is

A 2
B 26
C $\quad-13$
D 34
ii) $\quad \int \cos 2 x d x$ is equal to

A $2 \sin 2 x+C$
B $\frac{\sin 2 x}{2}+C$
C $\sin x+C$
D $\sin ^{2} x+C$
iii) The box contains 4 red, 5 black and 7 green balls. If one ball is drawn at randon the probability of getting a red or green ball is

A $\frac{11}{16}$
B $\frac{9}{16}$
C $\frac{4}{11}$
D $\frac{7}{11}$
iv) The derivative of $\log (5 x-6)$ is

A $5 x-6$
B $\frac{1}{5 x-6}$
C $\frac{5 x-6}{5}$
D $\frac{5}{5 x-6}$
v) A company starts producing badminton rackets and finds that the production cost of each racket is Nu 450 and the fixed expenditure of production is $\mathrm{Nu} 25,000$.
The cost function is
A $25000-450 x$
B $25000(450+x)$
C $25000+450 x$
D $25000(450-x)$
vi) The inverse of $\left(\begin{array}{ll}3 & 2 \\ 7 & 5\end{array}\right)$ is

A $\quad\left(\begin{array}{cc}5 & -2 \\ -7 & 3\end{array}\right)$
B $\quad\left(\begin{array}{cc}5 & 2 \\ 7 & -3\end{array}\right)$
C $\quad\left(\begin{array}{ll}-3 & 2 \\ -7 & 5\end{array}\right)$
D $\quad\left(\begin{array}{cc}-3 & 7 \\ 2 & 5\end{array}\right)$
vii) $\quad \int(3 x+7)^{4} d x$ is equal to

A $\quad 12(3 x+7)^{3}+C$
B $\quad \frac{(3 x+7)^{5}}{5}+c$
C $\quad \frac{(3 x+7)^{5}}{15}+c$
D $\quad 3(3 x+7)^{5}+c$
viii) The arithmetic mean of the baskets scored by four basketball players Karma, Sangay, Yeshey and Chencho in a tournament are $50,48,30$ and 12 respectively. The deviations of their baskets are respectively $15,12,10$ and 2 . Who scores the most of the four?

A Karma
B Chencho
C Sangay
D Yeshi
ix) The total cost function for x units of commodity is $C(x)=\frac{2}{3} x^{3}+5 x^{2}+20 x+0$ Then the marginal cost function is

A $\quad 2 x^{2}+10 x+20$
B $\quad 2 x^{3}+10 x-14$
C $\frac{2}{9} x^{4}+\frac{5}{3} x^{3}-10 x^{2}+6 x$
D $x^{2}+10 x+20$
x) There are 13 questions in a question paper. The number of ways a candidate can select 10 questions from the question paper is

A 715
B 66
C 120
D 286
xi) Derivative of $e^{\sin x}$ with respect to x is

A $\sin x$
B $\quad \cos x e^{\sin x}$
C $\quad \sin x e^{\sin x}$
D $e^{\sin x}$
xii) The equation $5 x^{2}-2 y^{2}=10$ represents a

A Parabola
B Circle
C Hyperbola
D Ellipse
xiii) Pasang wishes to create a fund to provide a cash prize of Nu 3000 for the fir rank holder in BHSEC every year. If the fund is invested at $6 \%$ p.a compound interest the amount to be invested is

A $\quad \mathrm{Nu} 50000$
B $\quad \mathrm{Nu} 30000$
C $\quad \mathrm{Nu} 18000$
D $\quad \mathrm{Nu} 60000$
xiv) The following are the ranks obtained by 5 students in Economics and English

|  | Pema | Dorji | Dechen | Sonam | Tshering |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Economics | 5 | 3 | 1 | 2 | 4 |
| English | 4 | 2 | 1 | 3 | 5 |

The Economics and English marks have
A High positive correlation
B High negative correlation
C Low positive correlation
D Low Negative correlation
xv) The distance between the points $\mathrm{A}(2,-1,3)$ and $\mathrm{B}(1,-3,1)$ is

A 4
B 2
C 9
D 3

## Section B [70 marks]

Answer any 10 questions. All questions in this section have equal marks. Unless otherwise stated, you may round of your answers to two decimal places.
[10 $\times 7=70$ ]

## Question 2

a) Find the vertices and foci of the hyperbola $9 x^{2}-16 y^{2}=144$
b) Find $\frac{d y}{d x}$ for
i) $y^{2}=4 x^{2}$
ii) $y=x^{x}$

## Question 3

a) Find the equation of the parabola whose directrix is $x=0$ and the focus at (2,-3)[3]
b) Solve the following system of equations by matrix method

$$
\begin{aligned}
& 3 x+5 y=-4 \\
& 5 x+3 y=4
\end{aligned}
$$

## Question 4

a) If $A=\left(\begin{array}{cc}2 & 3 \\ 0 & -4\end{array}\right)$ and $B=\left(\begin{array}{cc}1 & 2 \\ 3 & -1\end{array}\right) \quad$ find $(A B)^{T}$
b) Hyundai Company planned to produce a new type of car. During the first year the costs for setting up the new production line is $\mathrm{Nu} 7,500,000$. The additional cost of producing the car is $\mathrm{Nu} 125,000$ each. The company expects the selling price should be $\mathrm{Nu} 250,000$ each. Find the number of cars to be sold for which the company will breakeven. Interpret the result.

## Question 5

a) From the given diagram, determine
i) Equation of ellipse
ii) Equation of major and minor axes
iii) Length of latus rectum

b) Dema decides to construct a chorten after 5 years. The expected budget is Nu 300,000. How much should Dema deposit every year in order to accumulate the amount if the bank calculates the interest at the rate of $5 \%$ p.a compounded annually?

## Question 6

a) There were 128 students studying for commerce stream in a school and their combined mean marks in mathematics was 78 . The mean marks of 54 of them was 68 . Find the mean marks of the remaining students
b) Evaluate $\int \frac{2 x+3}{x^{2}+5 x+6} d x$

## Question 7

a) The total cost $C(x)$ associated with the production and making of x units of an item is given by $C(x)=\frac{1}{3} x^{4}+\frac{3}{5} x^{2}+6 x-30$. Find
i) The average cost function
ii) The average cost of output when 15 units are produced
b) A class consists of 60 students, 35 of them are girls and 25 boys. 15 of them opting computer studies and the remaining opt economics. Find the probability of selecting a girl who is opting computer studies

## Question 8

a) Evaluate $\int e^{3 x^{2}+5 x-6} \times(6 x+5) d x$
b) In how many ways can the letters of the word HAPPINESS be arranged? In how many of them, vowels are together?

## Question 9

a) Prove that the line joining $\mathrm{P}(5,-1,1)$ to $\mathrm{Q}(7,-4,7)$ and $R(1,-6,10)$ to $S(-1,-3,4)$ are parallel.
b) The cost function $C(x)$ is given by $C(x)=2 x^{3}-24 x^{2}+420 x+50$. At what output level the marginal cost is minimum. Also find the marginal cost at this level.

## Question 10

a) For the ellipse $\frac{x^{2}}{81}+\frac{y^{2}}{49}=1$, determine
i) Length of major and minor axes
ii) Eccentricity
iii) Distance between the foci
b) If $y=x^{4}+3 x^{2}+7$, find $\frac{d^{2} y}{d x^{2}} /_{x=2}$

## Question 11

Find the standard deviation of the following data

$$
\begin{equation*}
2,3,5,6,8,10 \tag{4}
\end{equation*}
$$

a) There are 8 black and 4 white balls. How many selections of 7 balls can be made if each selection is to include at least 3 white balls?

## Question 12

a) Show that $\left|\begin{array}{lll}1 & m & n+p \\ 1 & n & p+m \\ 1 & p & m+n\end{array}\right|=0$
b) The regression equation of two variables x and y are $5 y-2 x+7=0$ and $7 x-4 y-8=0$. Find which equation is meant for Y on X and which one is X on Y and find correlation coefficient of the two series.

## Question 13

a) Evaluate $\int x e^{x} d x$
b) The following are the marks scored by 50 students. Find the mean deviation about mean.

| Marks | $0-10$ | $10-20$ | $20-30$ | $30-40$ | $40-50$ |
| :--- | :---: | :--- | :---: | :--- | :---: |
| No.fo <br> Students | 3 | 8 | 15 | 19 | 5 |

## Question 14

a) Kinley buys a computer paying Nu 7000 in cash and promising to pay Nu 1000 at the end of every month for the next three years. If the rate of interest is $12 \%$ p.a. compounded monthly, what is the cash price of the computer?
b) Find the two positive numbers whose sum is 16 and the product is maximum and find the maximum product?

## MATHFEMATICS FORMULAE

Algebra
$a^{2}-b^{2}=(a+b)(a-b)$
$a^{3}-b^{3}=(a-b)\left(a^{2}+a b+b^{2}\right)$
$a^{3}+b^{3}=(a+b)\left(a^{2}-a b+b^{2}\right)$
$x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$
${ }^{n} P_{r}=\frac{n!}{(n-r)!}$
${ }^{n} C_{r}=\frac{n!}{r!(n-r)!}$
$C_{i j}=(-1)^{i+j} M_{i j}$

$$
A A^{-1}=A^{-1} A=I
$$

$A^{-1}=\frac{1}{\operatorname{det} A} \cdot \operatorname{adj} A$
$x=\frac{\left|D_{x}\right|}{|D|}, y=\frac{\left|D_{y}\right|}{|D|}, z=\frac{\left|D_{z}\right|}{|D|}$

## Calculus

$$
\begin{aligned}
& f^{\prime}(x)=\lim _{n \rightarrow 0} \frac{f(x+h)-f(x)}{h} \\
& y=x^{n}, y^{\prime}=n x^{n-1} \\
& y=c f(x), y^{\prime}=c f^{\prime}(x), \\
& y=f(x) \pm g(x), \\
& y^{\prime}=f^{\prime}(x) \pm g^{\prime}(x) \\
& F(x)=f(x) g(x)
\end{aligned}
$$

$$
F^{\prime}(x)=f(x) g^{\prime}(x)+f^{\prime}(x) g(x)
$$

$F(x)=\frac{f(x)}{g(x)}, F(x)=\frac{g(x) f^{\prime}(x)-f(x) g^{\prime}(x)}{[g(x)]^{2}}$
$\frac{d y}{d x}=\frac{d y}{d u} \times \frac{d u}{d x}$
$\int f(x) g(x) d x=f(x) \int g x d x-\int\left[\left(\frac{d}{d x} f(x)\right) \int g(x) d x\right] d x$

## Co-ordinate Geometry

$$
\begin{aligned}
& D=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}+\left(z_{2}-z_{1}\right)^{2}} \\
& (x, y, z)=\left(\frac{m_{1} x_{2}+m_{2} x_{1}}{m_{1}+m_{2}}, \frac{m_{1} y_{2}+m_{2} y_{1}}{m_{1}+m_{2}}, \frac{m_{1} z_{2}+m_{2} z_{1}}{m_{1}+m_{2}}\right)
\end{aligned}
$$

$$
\frac{x}{b_{1} c_{2}-b_{2} c_{1}}=\frac{y}{c_{1} a_{2}-c_{2} a_{1}}=\frac{z}{a_{1} b_{2}-a_{2} b_{1}}
$$

$$
\cos \theta= \pm \frac{a_{1} a_{2}+b_{1} b_{2}+c_{1} c_{2}}{\sqrt{a_{1}^{2}+b_{1}^{2}+c_{1}^{2}} \sqrt{a_{2}^{2}+b_{2}^{2}+c_{2}^{2}}}
$$

## Data and Probability

$$
\begin{aligned}
& \bar{x}=\frac{m \overline{x_{1}}+n \bar{x}_{2}}{m+n} \\
& \bar{x}=\frac{\sum f x}{n}
\end{aligned}
$$

$$
\text { Median }=l_{1}+\frac{l_{2}-l_{1}}{f_{1}}(m-c)
$$

$$
\sigma=\sqrt{\frac{\sum\left(x_{i}-\bar{x}\right)^{2}}{n}}
$$

$$
\sigma_{12}=\sqrt{\frac{n_{1} \sigma_{1}^{2}+n_{2} \sigma_{2}^{2}+n_{1} d_{1}^{2}+n_{2} d_{2}^{2}}{n_{1}+n_{2}}}
$$

$$
\operatorname{Cov}(\mathrm{X}, \mathrm{Y})=\frac{1}{\mathrm{n}} \sum(\mathrm{X}-\overline{\mathrm{X}})(\mathrm{Y}-\overline{\mathrm{Y}})
$$

$$
r=\frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\left.\sum(x-\bar{x})^{2} \sum y-\bar{y}\right)^{2}}}=\frac{n \sum x y-\sum x \sum y}{\sqrt{n \sum x^{2}-\left(\sum x\right)^{2}} \sqrt{n \sum y^{2}-\left(\sum y\right)^{2}}}
$$

$$
r=\frac{\sum(x-\bar{x})(y-\bar{y})}{n \sigma_{x} \sigma_{y}}
$$

$$
r=1-\frac{6 \sum d^{2}}{n\left(n^{2}-1\right)}
$$

$b_{\mathrm{YX}}=r \frac{\sigma_{y}}{\sigma_{x}}=\frac{n \sum x y-\sum x \sum y}{n \sum x^{2}-\left(\sum x\right)^{2}}$
$M C \frac{d}{d x}(C(x))$
$b_{X Y}=r \frac{\sigma_{x}}{\sigma_{y}}=\frac{n \sum x y-\sum x \sum y}{n \sum y^{2}-\left(\sum y\right)^{2}}$
$Y-\bar{Y}=\frac{\operatorname{cov}(X, Y)}{\sigma_{x}{ }^{2}}(X-\bar{X})=r \frac{\sigma_{x}}{\sigma_{y}}(X-\bar{X})$
$X-\bar{X}=\frac{\operatorname{cov}(X, Y)}{\sigma_{x}{ }^{2}}(Y-\bar{Y})=r \frac{\sigma_{y}}{\sigma_{x}}(Y-\bar{Y})$
$\mathrm{b}_{x y} \times \mathrm{b}_{y x}=r \frac{\sigma_{x}}{\sigma_{y}} \times r \frac{\sigma_{y}}{\sigma_{x}}$
$\sum y=n a+b \sum x$
$\sum x y=a \sum x+b \sum x^{2}$
$y-\bar{y}=b_{x y}(x-\bar{x})$
$x-\bar{x}=b_{y x}(y-\bar{y})$
$P(A \cup B)=P(A)+P(B)-P(A \cap B)$
$P(A)+P(\bar{A})=1$
$P(B / A)=\frac{P(A \cap B)}{P(A)}$
$P(A / B)=\frac{P(A \cap B)}{P(B)}$

## Commercial Mathematics

$$
\begin{aligned}
& A=\frac{a}{i}(1+i)\left[(1+i)^{n}-1\right] \\
& P=\frac{a}{i}\left[1-(1+i)^{-n}\right] \\
& A(x)=\frac{C(x)}{x}, M(x)=\frac{d}{d x}(C(x)) \\
& C(x)=F+V(x) \\
& R(x)=x G(x) \\
& P(x)=R(x)-C(x)
\end{aligned}
$$

