MATHEMATICS

(Three hours and a quarter)

*Answer **Question 1** from Section **A** and **14** questions from Section **B**.*
All working, including rough work, should be done on the same sheet as, and adjacent to, the rest of the answer.
The intended marks for questions or parts of questions are given in brackets [].

Mathematical formulae are given at the end of this question paper.
The use of calculator (fx-82/ fx-100) is allowed.

Section A (30 marks)

Answer **ALL** the questions

Direction: Read the following questions carefully. For each question there are four alternatives A, B, C and D. Choose the correct alternative and write it in your answer sheet.

Question 1 **[2x15 = 30 Marks]**

i) The value of $\begin{vmatrix} 2 & 3 & 7 \\ 13 & 17 & 5 \\ 15 & 20 & 12 \end{vmatrix}$ is

A 486
B 0
C 416
D 70

ii) The Sigma Notation of $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\frac{6}{7}$ is

A $\sum_{i=1}^{6}\frac{i}{i+1}$

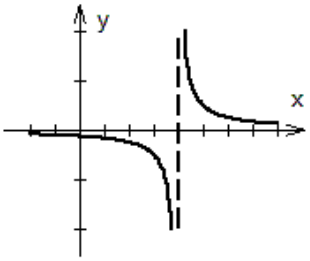
B $\sum_{i=1}^{6}\frac{i}{i-1}$

C $\sum_{i=0}^{6}\frac{i}{i+1}$

D $\sum_{i=0}^{6}\frac{i}{i-1}$

This booklet contains 12pages

iii) The rational equation for the given graph is



A $f(x)=\dfrac{1}{(x+4)^2}$

B $f(x)=\dfrac{1}{x+4}$

C $f(x)=\dfrac{1}{x^2-4}$

D $f(x)=\dfrac{1}{x-4}$

iv) One of the roots of the equation $x^3+2x^2-9x-18=0$ is

A +2

B -1

C -2

D +1

v) The value of '*x*', so that the distance between (1 , *x*, 8) and (-2 , 2, 8) is 5 units

A $x=+6$ or $+2$

B $x=-6$ or -2

C $x=-6$ or $+2$

D $x=+6$ or -2

vi) The value of $\int_0^1\left(x^2+1\right)^2$ is

A $\dfrac{28}{15}$

B $\dfrac{14}{15}$

C $\dfrac{22}{15}$

D $\dfrac{13}{15}$

vii) If $y = x^x$ then $\frac{dy}{dx}$ at $x = 1$ is

A 1

B $\frac{1}{e}$

C e

D -1

viii) If $\sin A = -\frac{1}{2}$, A lies in fourth quadrant, the value of $\sin 2A$ is

A $\frac{\sqrt{3}}{2}$

B $\frac{1}{2}$

C $-\frac{\sqrt{3}}{2}$

D $-\frac{1}{2}$

ix) The slope of the tangent to the curve $y = x^3 - x$ at $x = 3$ is

A 28

B 26

C 24

D 27

x) The standard deviation of the numbers 4 , 1, 3, 5, 12 is

A 2.74

B 3.47

C 2.47

D 3.74

xi) Which one of the following pair of surds is similar?

A $\sqrt{16}, \sqrt[3]{54}$

B $\sqrt{54}, \sqrt[3]{48}$

C $\sqrt{32}, \sqrt{72}$

D $\sqrt{48}, \sqrt{80}$

xii) The $x+iy$ form of $(2+3i)(1-i)$ is

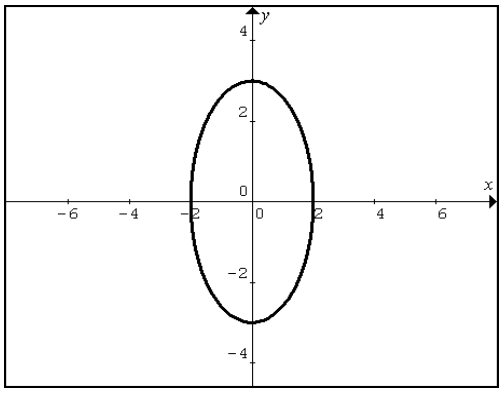
A $5-i$

B $5+i$

C $1+5i$

D $1-5i$

xiii) The equation of the conic in the given diagram is



A $\frac{x^2}{4}+\frac{y^2}{9}=1$

B $\frac{x^2}{9}-\frac{y^2}{4}=1$

C $\frac{x^2}{4}-\frac{y^2}{9}=1$

D $\frac{x^2}{9}+\frac{y^2}{4}=1$

xiv) The side of an equilateral triangle is 3m and is increasing at the rate of $\sqrt{3}\,m/\sec$. At what rate is area increasing?

A $3.5\,m^2/\sec$

B $4.5\,m^2/\sec$

C $4.8\,m^2/\sec$

D $3.9\,m^2/\sec$

xv) The polar form of $(1+i)^5$ is

A $5.93cis225°$

B $5.41cis225°$

C $5.66cis225°$

D $5.56cis225°$

Section B (70 marks)

Answer any **14** questions. All questions in this section have equal marks.
Unless otherwise stated, you may round answers to 2 decimal places.

Question 2

a) Determine $f''(2)$ for $f(x) = (3x-2)^4$ **[3]**

b) Solve for 'x' and find the intervals of inequation $|2x-3| < 7$ **[2]**

Question 3

The roots of the polynomial equation are $-2, +1$ and $+2$. Determine the equation.
Sketch the graph of this equation **[5]**

Question 4

Using the Principle of Mathematical Induction, prove

$$\sum_{i=1}^{n} \frac{1}{(2i+1)(2i-1)} = \frac{n}{2n+1}, \qquad n \in N$$ **[5]**

Question 5

A ladder 3.4m long rests against a vertical wall with the lower end on the horizontal ground. The lower end slips away from the wall at the rate of 1 m/min. Find the rate at which its upper end is descending at the instant when the lower end is 1.6 m from the wall. **[5]**

Question 6

For the function $f(x) = 2x^3 + 4x^2 - 10x - 12$, determine

i) x and y intercepts
ii) critical values
iii) inflection points **[5]**

Question 7

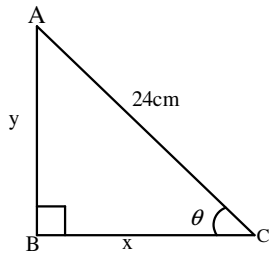
a) Determine $\frac{dy}{dx}$ for $y = \frac{\cos x}{\sin^2 x}$ **[2]**

b) If $x^y = e^{x-y}$, determine $\frac{dy}{dx}$ **[3]**

Question 8

a) Ms. Kinzang's uncle invested Nu.4000 in BOB when she was born with the interest rate at 4% per year compounded semi-annually. How long will it take the amount to grow to Nu.16,000 **[2]**

b) Determine the maximum perimeter of a right triangle with hypotenuse 24cm **[3]**



Question 9

a) Evaluate $\int x \sin x dx$ **[2]**

b) Determine the modulus and argument of $\frac{3-4i}{4-2i}$ **[3]**

Question 10

a) Determine the anti-derivative of $f(x) = -3e^{-x} + 6e^{2x} - \sin x$ **[2]**

b) A circular tent is constructed to have only a cone-shaped top or roof. It is constructed from a circular piece of canvas with a radius of 14 m with an angle 30^0 sector removed. The cut lines are then sewn together. Find the area of the resulting canopy. **[3]**

Question 11

a) Show that the four points $(0,4,3),(-1,-5,-3),(-2,-2,1)$ and $(1,1,-1)$ are coplanar **[3]**

b) Evaluate $\int \frac{\log x}{x} dx$ **[2]**

Question 12

Determine the area bounded by the curve $y = x(2-x)$ and the lines $x = 0, y = 0$ and $x = 2$. This area is rotated through four right angles about the x-axis, determine the volume of the solid so formed. **[5]**

Question 13

Using Demoivre's theorem, determine all the values of $(2-2i)^{\frac{1}{3}}$ **[5]**

Question 14

Determine the length of the axes, coordinates of the foci, the eccentricity and length of latus rectum of the conic $16x^2 - 9y^2 - 144 = 0$ **[5]**

Question 15

a) Determine the equation of the tangent to the curve 4^x at $(1,0)$ **[2]**

b) Determine the square root of $18 + 6\sqrt{5}$ **[3]**

Question 16

a) Determine the centre and radius of the circle $x^2 + y^2 + 4x - 4y - 1 = 0$ **[2]**

b) The daily attendance of class XII Science students in a school is given below

Day	1	2	3	4	5	6	7
Attendance	53	48	64	68	54	70	72

Calculate three day moving averages and plot this on a graph paper. **[3]**

Question 17

If $A = \begin{bmatrix} 1 & 1 & 3 \\ 2 & -4 & 0 \\ 1 & 1 & -3 \end{bmatrix}$, determine A^{-1}.

Hence solve the following system of equations

$$x + y + 3z = 5$$
$$2x - 4y = -2$$
$$x + y - 3z = -1$$

[5]

Question 18

Determine Karl Pearson's coefficient of correlation between the marks in Dzongkha and Mathematics obtained by 10 students of a class in the monthly test. Interpret the result. **[5]**

Marks in Dzongkha	20	13	18	21	11	12	17	14	19	15
Marks in Mathematics	17	12	23	25	14	7	19	21	22	19

Functions and Equations

(1) $(a \pm b)^2 = a^2 + b^2 \pm 2ab$

(2) $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

(3) $a^2 - b^2 = (a+b)(a-b)$

(4) $a^3 \pm b^3 = \left(a \pm b\right)\left(a^2 \mp ab + b^2\right)$

(5) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sequence and series

(1) $\sum_{i=1}^{n} i = \dfrac{n(n+1)}{2}$

(2) $\sum_{i=1}^{n} i^2 = \dfrac{n(n+1)(2n+1)}{6}$

(3) $\sum_{i=1}^{n} i^3 = \left[\dfrac{n(n+1)}{2}\right]^2$

(4) $t_n = ar^{n-1}$

(5) $t_n = a + (n-1)d.$

(6) $S_n = \dfrac{a\left(1-r^n\right)}{1-r}$ where $r<1$

$= \dfrac{a\left(r^n - 1\right)}{r-1}$, Where $r > 1$

(7) $S_n = \dfrac{n}{2}\left[2a + \left(n-1\right)d\right]$

Differentiation

(1) $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

(2) $y = \boldsymbol{x^n}$, $y' = nx^{n-1}$,

(3) $y = cf(x)$, $y' = cf'(x)$

(4) $y = f(x) \pm g(x)$, $y' = f'(x) \pm g'(x)$

(5) $F(x) = f(x)g(x)$,

$F'(x) = f(x)g'(x) + f'(x)g(x)$

(6) $F(x) = \dfrac{f(x)}{g(x)}$,

$F'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{\left[g(x)\right]^2}$

(7) $f \circ g(x)' = f'g(x) \times (g'x)$

(8) $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

(9) $v(t) = h'(t)$

Coordinate Geometry

(1) $\left(y - y_1\right) = m(x - x_1)$

(2) $\sqrt{\left(x-a\right)^2 + \left(y-b\right)^2}$

Trigonometry

(1) $Sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

(2) $Cos(A \pm B) = CosA \cos B \mp \sin A \sin B$

(3) $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

(4) $\sin^2\theta + \cos^2\theta = 1$

Logarithmic Exponentials

(1) $y = y_0(1+r)^x$

(2) $y = y_0 e^{kx}$

(3) $A = P(1+r)^n$

Integration

(1) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[\left(\dfrac{d}{dx}f(x)\right)\int g(x)dx\right]dx$

(2) $\int_a^b f(x)dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\Delta x$

(3) $V = \pi\int_a^b y^2 dx$

(4) $A = \int_a^b y dx$

Measurement

(1) Cone: $V = \dfrac{\pi}{3}r^2 h$

(2) Cone: $SA = \pi rl + \pi r^2$

(3) Sphere: $V = \frac{4\pi}{3} r^3$

(4) Sphere: $SA = 4\pi r^2$

(5) Cylinder: $SA = 2\pi rh + 2\pi r^2$

(6) Cylinder: $V = \pi r^2 h$

(7) Circle: $A = \pi r^2$

(8) Circle: $C = 2\pi r$

(9) Triangle: $A = \frac{bh}{2}$, $A = \frac{\sqrt{3}}{4} x^2$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

(10) Rectangle: $A = lw$,

(11) Rectangle $P = 2l + 2w$

(12) Square: $A = s^2$,

(13) Square $P = 4s$

(14) Rectangular Prism: $V = lwh$

Complex Numbers

(1) $r = \sqrt{a^2 + b^2}$

(2) $\tan\theta = \frac{b}{a} \quad \Rightarrow \theta = \tan^{-1}\left(\frac{b}{a}\right)$

(3) If $z = r\text{cis}\theta$ then $z^n = r^n \text{cisn}\theta$

(4) $z^{\frac{1}{n}} = r^{\frac{1}{n}} cis(\frac{\theta}{n} + k \cdot \frac{360°}{n}) f \text{ or } k = 0,1,2,3,...n-1$

Second Degree Relations

(1) $\text{Ellipse}: \frac{X^2}{a^2} + \frac{y^2}{b^2} = 1$

(2) $\text{Hyperbola}: \frac{X^2}{a^2} + \frac{y^2}{b^2} = 1$

(3) $e = \frac{c}{a}$

Geometry

(1) $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

(2) $(x, y, z) = \left(\frac{lx_2 + mx_1}{l+m}, \frac{ly_2 + my_1}{l+m}, \frac{lz_2 + mz_1}{l+m}\right)$

(3) For

$a_1x + b_1y + c_1z = 0$ *and* $a_2x + b_2y + c_2z = 0$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{z}{a_1b_2 - a_2b_1}$$

(4) $l = \frac{\theta}{360} 2\pi r$

(5) $A = \frac{\theta}{360} \pi r^2$

Matrices

(1) $C_{ij} = (-1)^{i+j} M_{ij}$

(2) $A A^{-1} = A^{-1} A = I$

(3) *Inverse of* $A = A^{-1} = \frac{1}{\det A} \cdot adjA$

Data & Probability

(1) $\bar{x} = \frac{\sum fx}{n}$

(2) $\text{Median} = l_1 + \frac{l_2 - l_1}{f1}(m - c)$

(3) $\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$

(4) $\sigma_{12} = \sqrt{\frac{n_1\sigma_1^2 + n_1\sigma_2^2 + n_1d_1^2 + n_2d_2^2}{n_1 + n_2}}$

(5) $\sigma = \sqrt{\frac{\sum f(x_i - \bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{N} - \left(\frac{\sum fx}{N}\right)^2}$

(6) $\bar{x}_{12} = \frac{m\bar{x}_1 + n\bar{x}_2}{m+n}$

(7) $I = \frac{\sum \frac{P_1}{P_0} \times 100}{n}$

(8) $I = \frac{\sum p_1 w}{\sum p_0 w} \times 100$

(9) $\text{Cov}(X, Y) = \frac{1}{n} \sum (X - \bar{X})(Y - \bar{Y})$

(10) $r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2 \sum (y - \bar{y})^2}}$

(11) $r = \frac{\sum (x - \bar{x})(y - \bar{y})}{n\sigma_x \sigma_y}$

(12) $b_{YX} = \frac{\text{cov}(X, Y)}{\sigma_x^2} = r\frac{\sigma_y}{\sigma_x}$

(13) $Y - \bar{Y} = \frac{\text{cov}(X, Y)}{\sigma_x^2}(X - \bar{X})$

$$= r\frac{\sigma_x}{\sigma_y}\left(X - \bar{X}\right)$$

(14) $$b_{xy} \times b_{yx} = r\frac{\sigma_y}{\sigma_x} \times r\frac{\sigma_x}{\sigma_y}$$

(15) $$\tau = \frac{2S}{n(n-1)}$$

(16) $$r = 1 - \frac{6\sum d^2}{n(n^2-1)}$$