BUSINESS MATHEMATICS

(Three hours and quarter)

---

*Answer **Question 1** from Section **A** and **10** questions from Section **B**.*
All working, including rough work, should be done on the same sheet as, and adjacent to, the rest of the answer.
The intended marks for questions or parts of questions are given in brackets [].

Mathematical formulae are given at the end of this question paper.
The use of calculator (fx-82/ fx-100) is allowed.

Section A

Answer **ALL** questions.

Directions: Read the following questions carefully. For each question there are four alternatives A, B, C and D. Choose the correct alternative and write it in your answer sheet.

Question 1 (2x15=30 Marks)

(i) The restrictions on $\frac{x^2-9}{x^2-3x}$ are

A $x \neq 0, 9$.
B $x \neq 0, 3$.
C $x \neq 3, -3$.
D $x \neq 0, -3$.

(ii) If $f(x) = \frac{x^3}{3}$, then $f'(-1)$ is

A 1.
B -1.
C $\frac{1}{3}$.
D $-\frac{1}{3}$.

(iii) A is an angle between 0° and 90°. If sinA = $\frac{3}{5}$, then the value of tanA is

A $\frac{3}{4}$.
B $\frac{4}{5}$.
C $\frac{5}{4}$.
D $\frac{4}{3}$.

(iv) Value of $\int_0^1 \left(4x^3 + 2\right)\, dx$ is.

A 0
B $\frac{1}{4}$
C 1
D 3

(v) M is the matrix $\begin{bmatrix} 1 & 2 & 0 \\ 4 & 5 & 6 \\ -1 & 2 & -3 \end{bmatrix}$. The cofactor of 6 is

A –4.
B –3.
C 3.
D 4.

(vi) A factor of $x^3 - 3x^2 - 4x + 6$ is

A $x - 2$.
B $x + 1$.
C $x - 1$.
D $x - 3$.

(vii) The value of $\sum_{i=1}^{20} i^2$ is.

A 8236900
B 2870
C 210
D 70

(viii) The second derivative of $4(x+1)^{1/2}$ is

A $\dfrac{1}{(x+1)^{-3/2}}$.

B $(x+1)^{-3/2}$.

C $2(x+1)^{-1/2}$.

D $-(x+1)^{-3/2}$.

(ix) If $e^{x+2} = 4$, the value of x correct to 2 decimal places is

A –1.40
B –0.61
C 0.61
D 3.39

(x) The series $5^2 + 7^2 + 9^2 + ... + 21^2$ can be written as

A $\sum_{i=1}^{9} (2i+3)^2$.

B $\sum_{i=1}^{21} (2i+3)^2$.

C $\sum_{i=5}^{21} (2i+3)^2$.

D $\sum_{i=5}^{21} i^2$.

(xi) For the function $y = \dfrac{1}{x+4}$,

A domain $= \{x : x \neq -4\}$ and range $= \left\{y : y \neq \dfrac{1}{4}\right\}$.

B domain $= \{x : x \neq 0\}$ and range $= \{y : y \neq -4\}$.

C domain $= \{x : x \neq -4\}$ and range $= \{y : y \neq 0\}$.

D domain $= \{x : x \neq 4\}$ and range $= \{y : y \neq 0\}$.

(xii) If $y = x \log_e x$, then $\dfrac{dy}{dx}$ is equal to

A $\dfrac{1}{x}$.

B $\dfrac{2}{x}$.

C $1 - \log_e x$.

D $1 + \log_e x$.

(xiii) Simplify $\sqrt{8}+\sqrt{50}$.

A $8\sqrt{2}$
B $2\sqrt{8}$
C $\sqrt{58}$
D 8

(xiv) The regression line of y on x is $3x-5y=10$ and the regression line of x on y is $2x-y=4$. Estimate the value of x when $y=1$.

A 5
B 3
C 2.5
D 1.5

(xv) $\int \frac{x}{x^2+4}dx$ is

A $\frac{1}{2}\log_e\left(x^2+4\right)+c$.

B $\log_e\left(x^2+4\right)+c$.

C $\frac{1}{2}\log_e(x)+c$.

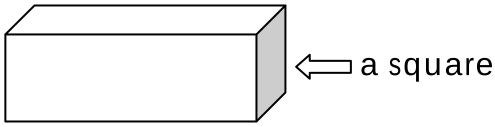
D $\log_e(x)+c$.

Section B (70 Marks)

Answer any 10 questions. All questions in this section have equal marks.
Unless otherwise stated, you may round answers to 2 decimal places.

Question 2

a) A box has square ends. The length is 2 cm greater than the width. The volume of the box is 45 cm^3. Find the dimensions of the box.



[4]

b) Find the value of $\sum_{i=1}^{35}(i+3)^2$. [3]

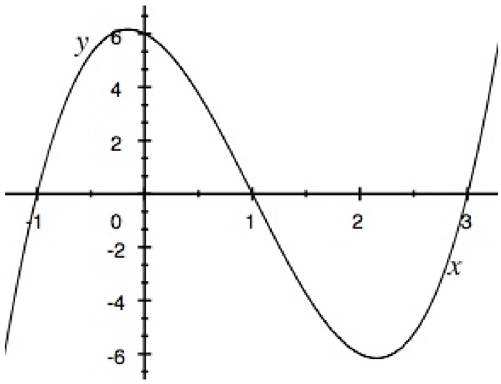
Question 3

a) Use proof by induction to prove that $\sum_{i=1}^{n}(2i+9)=n(n+10)$. [4]

b) Simplify $\frac{13-3x}{(x+1)(x-3)}+\frac{4}{x+1}$, stating restrictions. [3]

Question 4

Find the equation of the following cubic function. [3]



a) Find the derivative of $y=\frac{x}{x+1}$. [2]

b) If sin A = $\frac{1}{2}$ and $0<A<\frac{\pi}{2}$, find sin (2A). [2]

Question 5

a) A rectangle has a perimeter of 36 m. Find the size that maximizes the area. [4]

b) Differentiate $y = x\cos(2x)$. [3]

Question 6

a) Find the equation of the tangent to the curve $y = -x^4 + 2x^3 - 5x + 2$ at the point where $x = 1$. [3]

b) $f(x) = \dfrac{x+1}{x^2 - 1}$.

(i) Where is f not continuous, and what kinds of discontinuity? [2]

(ii) Find all the asymptotes. [2]

Question 7

a) $f(x) = x^3 - 12x + 4$. Find all maximum and minimum points. [4]

b) Differentiate $y = \log_e (1 + 2x)^3$. [3]

Question 8

a) The number of people, P, in a town x years from now is given by $P(x) = 1000 - 0.25x^2 + 200x$.

(i) Find the rate of increase of P five years from now.

(ii) When will the rate of increase be 190 people per year? [4]

b) Find $\int \sec^2 x \tan x dx$. [3]

Question 9

a) Prove $\sin 3x = 3\sin x - 4\sin^3 x$. [4]

b) Evaluate $\int_1^9 \dfrac{1}{\sqrt{x^3}} dx$. [3]

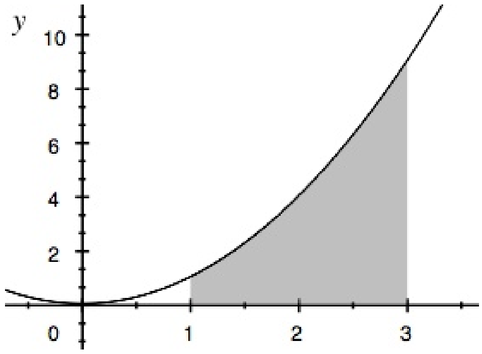
Question 10

a) Deki invests some money. The amount, Nu A, after t years is given by the formula $A = 2500e^{0.12t}$.

(i) What is the rate of growth after 2 years? [2]

(ii) Find how long it will take to get $A = 4000$? [2]

b) Find the area enclosed by the curve $y = x^2$, the x-axis, and the ordinates $x = 1$ and $x = 3$. [3]



Question 11

a) Sketch a graph of $y = e^x$ and describe three features. [3]

b) Use integration by parts to find $\int x \log_e x dx$. [4]

Question 12

a) Find the exact square root of $5 + \sqrt{24}$. [3]

b) Compute the Karl Pearson's coefficient of correlation between X and Y for the following data.

X	5	7	1	3	4
Y	3	2	4	5	6

[4]

Question 13

a) Find the inverse of $M = \begin{bmatrix} 1 & -2 & -1 \\ -3 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$. [4]

b) The table shows the prices of milk and coffee in 2006 and 2007 and the weights used to calculate the cost of a cup of coffee. Calculate (to one decimal place) the index number for the cost of a cup of coffee in 2007 using weighted price-relatives.

	2006 Cost (per kg) Nu	2007 Cost (per kg) Nu	Weights
milk	24	26	4
coffee	120	125	2

[3]

Question 14

a) Use the matrix method to solve the following system of equations: [4]

$$3x - 2y = 10$$

$$2x + y = 7$$

b) Two samples of size 40 and 50 respectively have the same mean 53, but different standard deviations, 19 and 8 respectively. Find the standard deviation of the combined sample of size 90. [3]

MATHEMATICS FORMULAE FOR BHSEC

Functions and Equations

(1) $(a \pm b)^2 = a^2 + b^2 \pm 2ab$

(2) $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

(3) $a^2 - b^2 = (a+b)(a-b)$

(4) $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

(5) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(6) $v(t) = h'(t)$

Sequence and series

(1) $\sum_{i=1}^{n} i = \dfrac{n(n+1)}{2}$

(2) $\sum_{i=1}^{n} i^2 = \dfrac{n(n+1)(2n+1)}{6}$

(3) $\sum_{i=1}^{n} i^3 = \left[\dfrac{n(n+1)}{2}\right]^2$

(4) $t_n = ar^{n-1}$

(5) $S_n = \dfrac{a(1-r^n)}{1-r} = \dfrac{a(r^n - 1)}{r-1}$, where $r>1$

(6) $t_n = a + (n-1)d$

(7) $S_n = \dfrac{n}{2}[a + (n-1)d]$

Differentiation

(1) $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

(2) $y = x^n$, $y' = nx^{n-1}$

(3) $y = cf(x)$, $y' = cf'(x)$

(4) $y = f(x) \pm g(x)$, $y' = f'(x) \pm g'(x)$

(5) $F(x) = f(x)g(x)$,
$F'(x) = f(x)g'(x) + f'(x)g(x)$.

(6) $F(x) = \dfrac{f(x)}{g(x)}$,

$F'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

(7) $(f \circ g)'(x) = f'g(x) \times (g'x)$

(8) $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

Coordinate Geometry

(1) $(y - y_1) = m(x - x_1)$

(2) $d = \sqrt{(x-a)^2 + (y-b)^2}$

Trigonometry

(1) $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

(2) $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

(3) $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

(4) $\sin^2\theta + \cos^2\theta = 1$

Logarithmic Exponentials

(1) $y = y_0(1+r)^x$

(2) $y = y_0 e^{kx}$

(3) $A = P(1+r)^n$

Integration

(1) $\int f(x)g(x)dx = f(x)\int gxdx - \int\left[\left(\dfrac{d}{dx}f(x)\right)\int g(x)dx\right]dx$

(2) $\int_a^b f(x)dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\Delta x$

(3) $V = \pi\int_a^b y^2 dx$

(4) $A = \int_a^b y dx$

Measurement

(1) Cone: $V = \dfrac{\pi}{3} r^2 h$

(2) Cone: $SA = \pi rl + \pi r^2$

(3) Sphere: $V = \dfrac{4\pi}{3} r^3$

(4) Sphere: $SA = 4\pi r^2$

(5) Cylinder: $SA = 2\pi r^2 + 2\pi rh$

(6) *Cylinder:* $V = \pi r^2 h$

(7) Circle: $A = \pi r^2$

(8) Circle: C $= 2\pi$r

(9) Triangle: $A = \frac{bh}{2}$, $A = \dfrac{\sqrt{3}}{4}x^2$,

A = $\sqrt{s(s-a)(s-b)(s-c)}$

(10) Rectangle: $A = lw$,

(11) Rectangle: $P = 2l + 2w$

(12) Square: $A = s^2$,

(13) Square: $P = 4S$

(14) Rectangular Prism: $V = lwh$

Complex numbers

(1) $r = \sqrt{a^2 + b^2}$

(2) $\tan\theta = \frac{b}{a} \Rightarrow \theta = \tan^{-1}\left(\frac{b}{a}\right)$

(3) If $z = r\text{cis}\theta$ then $z^n = r^n \text{cis} n\theta$

(4) $z^{\frac{1}{n}} = r^{\frac{1}{n}} cis(\frac{\theta}{n} + k \cdot \frac{360°}{n}) f \text{ or } k = 0,1,2,3,...n-1$

Second Degree Relations

(1) *Ellipse:* $\frac{X^2}{a^2} + \frac{y^2}{b^2} = 1$

(2) *Hyperbola:* $\frac{X^2}{a^2} - \frac{y^2}{b^2} = 1$

(3) $e = \frac{c}{a}$

Geometry

(1) $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

(2) $(x, y, z) = \left(\frac{lx_2 + mx_1}{l+m}, \frac{ly_2 + my_1}{l+m}, \frac{lz_2 + mz_1}{l+m}\right)$

(3) For

$a_1x + b_1y + c_1z = 0$ and $a_2x + b_2y + c_2z = 0$,

$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{z}{a_1b_2 - a_2b_1}$

(4) $l = \frac{\theta}{360^0} 2\pi r$

(5) $A = \frac{\theta}{360^0} \pi r^2$

Matrices

(1) $C_{ij} = (-1)^{i+j} M_{ij}$

(2) $AA^{-1} = A^{-1}A = I$

(3) Inverse of $A = A^{-1} = \frac{1}{\det A} \cdot adjA$

Data & Probability

(1) $\bar{x} = \frac{\sum fx}{n}$

(2) Median $= l_1 + \frac{l_2 - l_1}{f_1}(m - c)$

(3) $\sigma = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n}}$

(4) $\sigma_{12} = \sqrt{\frac{n_1\sigma_1^2 + n_2\sigma_2^2 + n_1d_1^2 + n_2d_2^2}{n_1 + n_2}}$

(5) $\bar{x}_{12} = \frac{m\bar{x}_1 + n\bar{x}_2}{m+n}$

(6) $I = \frac{\sum \frac{P_1}{P_0} \times 100}{n}$

(7) $I = \sum \frac{P_i W}{P_0 W} \times 100$

(8) $\text{Cov}(X, Y) = \frac{1}{n}\sum(X - \bar{X})(Y - \bar{Y})$

(9) $r = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sqrt{\sum(x - \bar{x})^2 \sum(y - \bar{y})^2}}$

(10) $r = \frac{\sum(x - \bar{x})(y - \bar{y})}{n\sigma_x \sigma_y}$

(11) $b_{YX} = \frac{\text{cov}(X, Y)}{\sigma_x^2} = r\frac{\sigma_y}{\sigma_x}$

(12) $Y - \bar{Y} = \frac{\text{cov}(X, Y)}{\sigma_x^2}(X - \bar{X})$

$= r\frac{\sigma_x}{\sigma_y}(X - \bar{X})$

(13) $b_{xy} \times b_{yx} = r\frac{\sigma_y}{\sigma_x} \times r\frac{\sigma_x}{\sigma_y}$

(14) $\tau = \frac{2S}{n(n-1)}$

(15) $r = 1 - \frac{6\sum d^2}{n(n^2 - 1)}$