NEW CURRICULUM

MATHEMATICS
(Three hours)

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*Answer **Question 1** from Section **A** and **14** questions from Section **B**.*
All working, including rough work, should be done on the same sheet as, and adjacent to, the rest of the answer.
The intended marks for questions or parts of questions are given in brackets [].

Mathematical formulae are given at the end of this question paper.
The use of calculator (fx-82/ fx-100) is allowed.

Section A
Answer **ALL** questions.

Directions: Read the following questions carefully. For each question there are four alternatives A, B, C and D. Choose the correct alternative and write it in your answer sheet.

Question 1 (2x15=30 Marks)

(i) The value of $\sum_{i=1}^{7}(7-4i)$ is

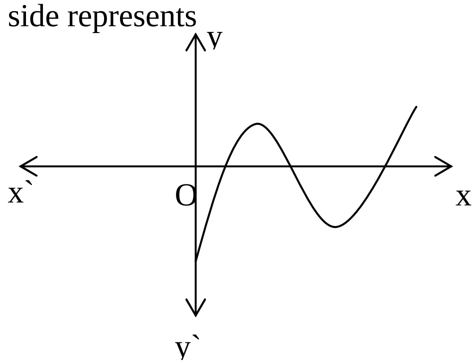
A -63.
B -21.
C 3.
D 169.

(ii) The cofactor of -2 in the matrix $X = \begin{bmatrix} -1 & -2 & 3 \\ 5 & 0 & 8 \\ 7 & 4 & 6 \end{bmatrix}$ is

A -26.
B -32.
C 20.
D 26.

(iii) The graph of the algebraic function along side represents

A cubic function.
B linear function.
C quartic function.
D quadratic function.



(iv) The domain of the function $f(x) = \dfrac{1}{x^2 - 4x + 4}$ are

A $\{x | x \neq -2,\ x \in R\}$

B $\{x | x \neq -2,\ x \in I\}$

C $\{x | x \neq 2,\ x \in R\}$

D $\{x | x \neq 2,\ x \in I\}$

(v) A car moves such that the distance d in meters covered in t seconds is given by $d = 5 + 12t - t^3$. The car will stop after

A 0 seconds.
B 2 seconds.
C 4 seconds.
D 5 seconds.

(vi) The tangent to the curve $y = \sin x,\ x \in [0, \pi]$ is horizontal at the point

A (0, 0).

B $(\pi, 0)$.

C $\left(\dfrac{\pi}{2}, 1\right)$.

D $\left(\dfrac{\pi}{4}, \dfrac{1}{\sqrt{2}}\right)$.

(vii) The value of $\lim_{x \to 0} \left(1 + \dfrac{x}{2}\right)^{\frac{1}{x}}$ is

A $\sqrt{e}$.

B $\dfrac{1}{\sqrt{e}}$.

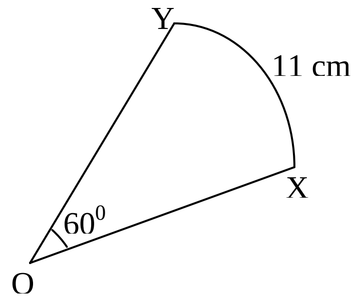
C e^2.

D e^{-2}.

(viii) If $f'(x) = 6x + 4$ and $f(-1) = 1$, then $f(x)$ is

A $3x^2 + 4x$.
B $6x^2 + 4x$.
C $3x^2 + 4x - 6$.
D $3x^2 + 4x + 2$.

(ix) From the diagram of the sector along side, the length of OX is



A $11\frac{11}{21}$ cm .

B $10\frac{1}{2}$ cm .

C $\frac{2}{21}$ cm .

D $\frac{11}{60}$ cm .

(x) The value of $(\sin 3\theta + i\cos 3\theta)^4$ is

A $\cos 12\theta + i\sin 12\theta$.
B $\sin 12\theta + i\cos 12\theta$.
C $\sin 12\theta - i\cos 12\theta$.
D $\cos 12\theta - i\sin 12\theta$.

(xi) The value of $\sqrt{192} + \sqrt{108} - \sqrt{48}$ is

A $18\sqrt{3}$.
B $\sqrt{252}$.
C $10\sqrt{3}$.
D $\sqrt{348}$.

(xii) The mean deviation about the mean of the data 12, 14, 18, 20, is

A 0.
B 3.
C 10.
D 16.

(xiii) The equation of the tangent to the curve $y = x^2$ *at* $x = 1$ is

A $y = 2x + 1$.
B $y = 1 - 2x$.
C $y = 2x - 1$.
D $y = -2x + 3$.

(xiv) If $Z_1 = 5 + 2i$ and $Z_2 = 3i - 7$, the value of $\frac{Z_1}{Z_2}$ is

A $\frac{1}{2}(1 - i)$.

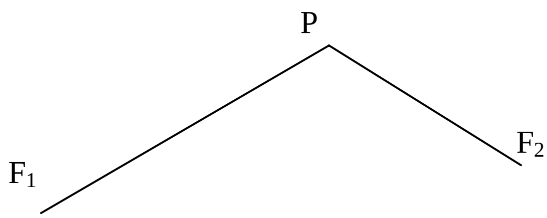
B $\frac{1}{2}(1 + i)$.

C $-\frac{1}{2}(1 - i)$.

D $-\frac{1}{2}(1 + i)$.

(xv) In the diagram to the right, P is a moving point and F_1 and F_2 are two fixed points. If $PF_1 + PF_2$ is constant, then the locus of P is

A circle.
B ellipse.
C hyperbola.
D straight line.



Section B (70 Marks)

Answer any 14 questions. All questions in this section have equal marks.
Unless otherwise stated, you may round answers to 2 decimal places.

Question 2

a) Evaluate $\int_0^{\frac{\pi}{2}} \sin 2x \, dx$. [2]

b) The radius of a circle increases at the rate of 3 cm per second. Find the rate at which the area of the circle increases when the radius is 17 cm. [3]

Question 3

a) Find the inverse of the logarithmic function $y = \log_e x^2$. [2]

b) If $y = e^{\sin \log_e x} \times \log_e \sin e^x$, find $\frac{dy}{dx}$. [3]

Question 4

a) Evaluate $\int x \log_e x \, dx$. [2]

b) Convert the complex number Z = $-1 + \sqrt{3}\, i$ in the polar form. Write the polar co-ordinates and represent them on the complex plane. [3]

Question 5

a) Find the equation of the circle whose center is the point (2, 3) and which passes through the point (5, 7). [2]

b) The foci of hyperbola are (6, 0) and (- 6, 0) and eccentricity is 2. Find the equation of the hyperbola. [3]

Question 6

a) Find the equation of the plane passing through the points (0, - 1, 0), (2, 1, -1) and (1, 1, 1). [3]

b) Prove that the function $y = \sin x$ has a minimum value at $x = \frac{\pi}{2}$. [2]

Question 7

Prove by mathematical induction that $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$, for all positive integers n. [5]

Question 8

A right circular cylinder is to be made so that the sum of its radius and its height is 6 m. Find the maximum volume of the cylinder. [5]

Question 9

Show that the continued product of all the values of $\left(1 + i\sqrt{3}\right)^{\frac{1}{5}}$ is $2\,\mathrm{cis}\left(\frac{13\pi}{3}\right)$. [5]

Question 10

For the ellipse $9x^2 + 16y^2 - 54x + 64y + 1 = 0$, find the: [5]

(i) co-ordinates of center;
(ii) eccentricity;
(iii) vertices;
(iv) foci;

Question 11

If $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 3 \\ 3 & 2 & -1 \end{bmatrix}$, prove that $AA^{-1} = I$,where I is the unit matrix. [5]

Question 12

Draw the graph of $y = 2x^2$ and $y = 2x + 4$ on the same graph paper taking same axes. For what values of x is the graph of $y = 2x^2$ below the graph of $y = 2x + 4$? [5]

Question 13

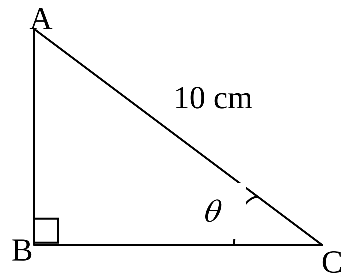
a) Simplify and write the restrictions of $\dfrac{x^2-9}{x^2-x-2} \div \dfrac{x+3}{x-2}$. [2]

b) Solve $\sqrt{x}+\sqrt{x-9}=\dfrac{36}{\sqrt{x-9}}$. [3]

Question 14

a) A pot of water is boiled and then removed from heat. The temperature of water T^0 C after t minutes is given by $T = 80e^{-0.06t}+20$. Find the rate of change of temperature after 10 minutes. [2]

b) In the given diagram, Δ ABC is a right angled triangle with hypotenuse 10 cm. Find the value of θ so that the perimeter of the triangle is maximum. [3]



Question 15

The area enclosed by the curves $x^2 = 4y$ and $y^2 = 4x$ is rotated through four right angles about the x – axis. Calculate the volume of the solid of revolution so formed. [5]

Question 16

a) Evaluate $\int \dfrac{\cos(\log x)}{x}dx$. [2]

b) If the two regression lines are $5x - 6y + 90 = 0$ and $15x - 8y - 130 = 0$, determine which one of these lines is the regression line of y on x and which one is that of x on y. [3]

Question 17

For the function $f(x) = x^4 - 6x^3 + 9x^2$, find: [5

i) intercepts;
ii) critical values;
iii) intervals of increase and decrease;
iv) concavity.

Also Sketch the function.

Question 18

The following are the marks obtained by 10 students in Mathematics and Physics.

Mathematics	50	90	85	60	75	85	60	70	55	60
Physics	60	85	90	65	70	75	70	80	50	55

Calculate Spearman's rank correlation coefficient and interpret the result. [5]

MATHEMATICS FORMULAE

Functions and Equations

(1) $(a \pm b)^2 = a^2 + b^2 \pm 2ab$

(2) $(a \pm b)^3 = a^3 \pm 3a^2 b + 3ab^2 \pm b^3$

(3) $a^2 - b^2 = (a+b)(a-b)$

(4) $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

(5) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(6) $v(t) = h'(t)$

Sequence and series

(1) $\sum_{i=1}^{n} i = \dfrac{n(n+1)}{2}$

(2) $\sum_{i=1}^{n} i^2 = \dfrac{n(n+1)(2n+1)}{6}$

(3) $\sum_{i=1}^{n} i^3 = \left[\dfrac{n(n+1)}{2}\right]^2$

(4) $t_n = ar^{n-1}$

(5) $S_n = \dfrac{a(1-r^n)}{1-r} = \dfrac{a(r^n - 1)}{r-1}$, where $r>1$

(6) $t_n = a + (n-1)d$

(7) $S_n = \dfrac{n}{2}[2a + (n-1)d]$

Differentiation

(1) $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

(2) $y = x^n$, $y' = nx^{n-1}$

(3) $y = cf(x)$, $y' = cf'(x)$

(4) $y = f(x) \pm g(x)$, $y' = f'(x) \pm g'(x)$

(5) $F(x) = f(x)g(x)$,
$F'(x) = f(x)g'(x) + f'(x)g(x)$.

(6) $F(x) = \dfrac{f(x)}{g(x)}$,

$F'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

(7) $(f \circ g)'(x) = f'g(x) \times (g'x)$

(8) $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

Coordinate Geometry

(1) $(y - y_1) = m(x - x_1)$

(2) $d = \sqrt{(x-a)^2 + (y-b)^2}$

Trigonometry

(1) $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

(2) $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

(3) $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

(4) $\sin^2\theta + \cos^2\theta = 1$

Logarithmic Exponentials

(1) $y = y_0(1+r)^x$

(2) $y = y_0 e^{kx}$

(3) $A = P(1+r)^n$

Integration

(1) $\int f(x)g(x)dx = f(x)\int gxdx - \int\left[\left(\dfrac{d}{dx} f(x)\right)\int g(x)dx\right]dx$

(2) $\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$

(3) $V = \pi \int_a^b y^2 dx$

(4) $A = \int_a^b y dx$

Measurement

(1) Cone: $V = \dfrac{\pi}{3} r^2 h$

(2) Cone: $SA = \pi r l + \pi r^2$

(3) Sphere: $V = \dfrac{4\pi}{3} r^3$

(4) Sphere: $SA = 4\pi r^2$

(5) Cylinder: $SA = 2\pi r^2 + 2\pi rh$

(6) Cylinder: $V = \pi r^2 h$

(7) Circle: $A = \pi r^2$

(8) Circle: $C = 2\pi r$

(9) Triangle: $A = \dfrac{bh}{2}$, $A = \dfrac{\sqrt{3}}{4} x^2$,

$A = \sqrt{s(s-a)(s-b)(s-c)}$

(10) Rectangle: $A = lw$,

(11) Rectangle: $P = 2l + 2w$

(12) Square: $A = s^2$,

(13) Square: $P = 4S$

(14) Rectangular Prism: $V = lwh$

Complex numbers

(1) $r = \sqrt{a^2 + b^2}$

(2) $\tan\theta = \frac{b}{a} \Rightarrow \theta = \tan^{-1}\left(\frac{b}{a}\right)$

(3) If $z = r\text{cis}\theta$ then $z^n = r^n \text{cis} n\theta$

(4) $z^{\frac{1}{n}} = r^{\frac{1}{n}} cis(\frac{\theta}{n} + k \cdot \frac{360°}{n})\ f\ or\ k = 0,1,2,3,...n-1$

Second Degree Relations

(1) *Ellipse:* $\frac{X^2}{a^2} + \frac{y^2}{b^2} = 1$

(2) *Hyperbola:* $\frac{X^2}{a^2} - \frac{y^2}{b^2} = 1$

(3) $e = \frac{c}{a}$

Geometry

(1) $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

(2) $(x, y, z) = \left(\frac{lx_2 + mx_1}{l+m}, \frac{ly_2 + my_1}{l+m}, \frac{lz_2 + mz_1}{l+m}\right)$

(3) For $a_1x + b_1y + c_1z = 0$ and $a_2x + b_2y + c_2z = 0$,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{z}{a_1b_2 - a_2b_1}$$

(4) $l = \frac{\theta}{360^0} 2\pi r$

(5) $A = \frac{\theta}{360^0} \pi r^2$

Matrices

(1) $C_{ij} = (-1)^{i+j} M_{ij}$

(2) $AA^{-1} = A^{-1}A = I$

(3) Inverse of $A = A^{-1} = \frac{1}{\det A} \cdot adjA$

Data & Probability

(1) $\bar{x} = \frac{\sum fx}{n}$

(2) $\text{Median} = l_1 + \frac{l_2 - l_1}{f_1}(m - c)$

(3) $\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$

(4) $\sigma_{12} = \sqrt{\frac{n_1\sigma_1^2 + n_2\sigma_2^2 + n_1d_1^2 + n_2d_2^2}{n_1 + n_2}}$

(5) $\bar{x}_{12} = \frac{m\bar{x}_1 + n\bar{x}_2}{m+n}$

(6) $I = \frac{\sum \frac{P_1}{P_0} \times 100}{n}$

(7) $I = \frac{\sum p_1 w}{\sum p_o w} \times 100$

(8) $\text{Cov}(X, Y) = \frac{1}{n}\sum (X - \bar{X})(Y - \bar{Y})$

(9) $r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2 \sum (y - \bar{y})^2}}$

(10) $r = \frac{\sum (x - \bar{x})(y - \bar{y})}{n\sigma_x \sigma_y}$

(11) $b_{YX} = \frac{\text{cov}(X, Y)}{\sigma_X^2} = r\frac{\sigma_Y}{\sigma_X}$

(12) $Y - \bar{Y} = \frac{\text{cov}(X, Y)}{\sigma_x^2}(X - \bar{X})$

$= r\frac{\sigma_x}{\sigma_y}(X - \bar{X})$

(13) $b_{xy} \times b_{yx} = r\frac{\sigma_y}{\sigma_x} \times r\frac{\sigma_x}{\sigma_y}$

(14) $\tau = \frac{2S}{n(n-1)}$

(15) $r = 1 - \frac{6\sum d^2}{n(n^2 - 1)}$