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# LEVEL 3 CERTIFICATE

## Examiners' report

# FREE STANDING MATHEMATICS QUALIFICATION: ADDITIONAL MATHS

**6993**

For first teaching in 2018

**6993/01 Summer 2023 series**

# Contents

# Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates.

The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. A selection of candidate answers is also provided. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report.

A full copy of the question paper and the mark scheme can be downloaded from OCR.

# Paper 1 series overview

The paper assessed a wide range of topics within the specification and some high marks attained by candidates indicated a wide understanding of the specification. Some topics overlap and extend the work done at GCSE and it is expected that these topics will be tackled well; others enrich the key stage 4 curriculum with some acceleration into AS Level and it was pleasing to see so many successful responses.

| Candidates who did well on this paper generally: | Candidates who did less well on this paper generally: |
|---|---|
| • were able to set their work out well and understand the requirements of the question<br>• understood command words, such as 'find' or 'determine' and 'hence' were understood<br>• had working that was clear for assessors to see and mark. | • were not secure in the topics being assessed<br>• did not successfully tackle questions that, in particular, could be asked in GCSE<br>• did not set out their work carefully and clearly, which could cause confusion for the candidate as they progressed through their solution. |

## Question 1

**1** Find the term independent of $x$ in the expansion of $\left(x+\frac{2}{x}\right)^6$. **[3]**

Successful responses were split between those finding the full expansion first, and those who recognised that they only needed to find the middle term.

The responses spanned from those who could confidently write down the independent term to those who did not seem to know what to do at all. Some wrote down Pascal's triangle but did not know how to proceed. The formula for the binomial expansion was given on the formula sheet which could have been used. Common mistakes were putting a plus between the two parts of a term and not including powers of 2 in the second term.

A number of candidates only gave the first and last term of the expansion. There were also candidates who attempted to multiply out the six brackets, but they were rarely successful in obtaining correct terms of the expansion.

It was also clear that a number of candidates did not understand the word 'independent'. Candidates that obtained a fully correct expansion, including the 160 term, could only score full marks if they clearly identified the term requested in the question.

## Question 2

**2** Jack throws 4 ordinary six-sided dice numbered 1 to 6.

Find the probability that he throws at least one 3. **[3]**

The most successful responses were where the candidate recognised the need to find the probability of throwing no 3s and then subtracting from 1. There were many responses with the wrong fraction and/or wrong powers. Few who found the four events, where a 3 or more were thrown, were successful in obtaining the correct powers of the probabilities, or the coefficients.

It was pleasing to see a significant number of candidates who were confident in giving their answer as an exact fraction. There were, however, a number of candidates who only gave their answer to 2 significant figures.

## Question 3

**3** Use long division to find the quotient and the remainder when $x^3 + 3x^2 + 5x - 3$ is divided by $x + 1$. **[3]**

Candidates who attempted the traditional long division technique were generally successful although a significant proportion of them then did not identify the quotient, while others gave the remainder as a fraction.

## Question 4 (a)

**4** Simplify the following.

**(a)** $\dfrac{1}{x-2} - \dfrac{2}{x+1}$ **[2]**

This question was generally successfully answered, though some candidates found this question challenging. Although many did correctly multiply and simplify, there were some candidates who did not multiply the numerators or attempted to simplify their fraction by cancelling $x$'s in the final steps. There were errors dealing with the negative sign resulting in $-x-3$ in the numerator.

## Question 4 (b)

**(b)** **In this question you must show detailed reasoning.**

$\dfrac{2}{5-\sqrt{2}} + \dfrac{1}{5+\sqrt{2}}$ **[3]**

This part was not so successfully answered. Sign errors occurred in the numerator and 27 was a popular alternative to the correct denominator.

## Question 5 (a)

**5** You are given that $\sin\theta = -0.6$ for $270° \leqslant \theta \leqslant 360°$.

**(a)** Find the value of $\theta$. **[2]**

There were good responses to this question. Some candidates ignored the **–** sign, thinking 36.9 was the principal angle and so failing to find the required angle.

## Question 5 (b)

**(b)** Using Pythagoras' theorem, determine the **exact** value of $\tan\theta$. **[4]**

This was a more discriminating question, although generally successfully answered. Some candidates did not understand the requirements of the question and simply used calculator methods to find -0.75. Those candidates who did use Pythagoras used a variety of side lengths. Many did not deal with the negative sign correctly so gave 0.75 as their answer. Candidates who attempted to use identities generally had success.

## Question 6 (a)

**6** A car accelerates from rest in a straight line. At time $t$ seconds its velocity, $v\ \mathrm{m\,s^{-1}}$, is given by the equation $v = 20\left(1 - 2^{-\frac{t}{2}}\right)$.

**(a)** Calculate the velocity of the car when $t = 4$, 6 and 8 seconds. **[2]**

Almost all candidates scored full marks on this question.

## Question 6 (b)

**(b)** Hence calculate an estimate of the acceleration of the car at $t = 6$ seconds. Give your answer correct to **2** significant figures. **[2]**

By contrast, only a minority of candidates scored any marks here. Those who recognised the fact that the acceleration was the value of the gradient at the point also recognised that a numeric estimate required numerical methods, such as the central estimate. This usually resulted in a correct answer. Those who did not had to search for other ways, ignoring the demand 'hence'. Most candidates proceeding this way decided that the use of the *suvat* formulae was appropriate.

One or two tried using calculus even though the function being used was outside the bounds of this specification.

**Command words**

Candidates need to be aware of the words that are used to ask the question. 'Hence', is a word that should lead a candidate in a particular area. 'Determine' demands that working or justification is required for the response. Disregarding these command words sometimes loses candidates marks, or means that candidates spend longer on a question than necessary.

## Question 6 (c)

**(c)** Explain how this estimate could be improved. **[1]**

Here there were many incorrect responses. Some candidates may have been deceived by being previously asked to give an answer to 2 significant figures and so they suggested more significant figures. Some suggested calculus. It is a standard procedure in numerical estimates of this type to take a smaller interval and this was given by the majority of candidates.

## Question 7 (a)

**7** You are given that the equation $3^x - 4x^2 = 0$ has three roots, $\alpha$, $\beta$ and $\gamma$ where $\alpha < 0$ and $\gamma > 3$.

**(a)** By considering the value of $3^x - 4x^2$ when $x = 0$ and $x = 1$, show that $\beta$ lies between 0 and 1. **[2]**

In almost all cases the first mark was gained by calculating the value of the function when $x = 0$ and 1 The 'sign change' phrase was very frequently used to gain the second mark. Inequality symbols were acceptable as well, as were some more wordy explanations. Rarely was it written well as a formal statement that therefore $\beta$ was in the specific range.

## Question 7 (b)

**(b)** By considering appropriate values of $x$, determine the value of $\beta$ correct to **1** decimal place. **[3]**

The expected method was to see the range for $\beta$ of [0,1] established in part (a) reduced, either by finding a value $x>0$ for which $f(x)>0$ or $x<1$ for which $f(x)<0$, and sight of this established a correct method for the first mark. Reducing the range to [0.7, 0.85] gained the second mark. Candidates who then stated that the root was 0.7 or 0.8 with no justification did not earn the last mark, nor did those who stated that $|f(0.8)| < |f(0.7)|$ meant that the root was 0.8 to 1 decimal place. The justification that the root was 0.8 required the range to be [0.75, 0.85]. The candidates who were the most successful in this part laid out their work carefully, describing the consequence of their calculations. A typical response is shown below.

## Exemplar 1

$3^{0.5} - 4(0.5)^2 = 0.73205$ ... +ve

$3^{0.75} - 4(0.75)^2 = 0.29307$ ... +ve

$3^{0.76} - 4(0.76)^2 = -0.0051193$ ... −ve

$\beta = 0.8$ (1 d.p)

This response gives a clear solution in the minimum of calculations.

## Question 8 (a)

**8** The triangle ABC is such that AB = 12 cm and angle A = 50°.

**(a)** Given that BC = 10 cm, determine the **two** possible values of angle C. **[4]**

The correct use of the sine rule to find the principal angle was managed by the majority of candidates . Many ignored the instruction in the question about the second value. Others found an angle by subtracting their principal angle from 360º, a result of course that gave an angle greater than 180º, apparently with no concern. A clear diagram helped to obtain a correct 2nd value. Some mislabelled the triangle and so caused confusion over angles B and C, while others assumed that the triangle was right-angled. On vary rare occasions a candidate would use the cosine rule but in most cases this led to incorrect answers after rather more working than the question was worth.

## Question 8 (b)

**(b)** State **two** conditions for BC such that if **either** of them is satisfied there will be only **one** value for the angle C. **[2]**

This question was generally poorly done and often not attempted at all. A large number of candidates chose to ignore the instruction to give conditions for the side BC but rather talked about angles B and C. Many hedged their bets saying it needed to be longer than and shorter than AB or AC. There was a lot of mentions of isosceles, right-angled and equilateral triangles. The wording of the question threw many candidates as they thought BC=10cm was given in the first part of the question, so were unsure what to do with this in the second part. Having not given the second angle in part (a) added to the insecurity in the question.

## Question 9

**9** The point A has the coordinate (3, 7) and the point B has the coordinate (7, 1).

Find the equation of the perpendicular bisector of AB. **[5]**

Most candidates managed this question with great success. A large number of candidates also found the equation of the line AB as if this was something that one must always do when finding the perpendicular bisector of AB. There were a few who did not find the midpoint, and a few who got the original gradient of AB wrong, hence made errors from there. A few candidates managed the first 4 marks, used $y = mx + c$ rather than $y - y_1 = m(x - x_1)$, and got the wrong value of $c$ because of numerical slips.

## Question 10 (a)

**10 (a)** On the grid below, indicate the region for which the following inequalities hold. You should shade the region that is **not** satisfied by the inequalities.

$y \geqslant x + 1$

$x \geqslant 1$

$x + 2y \leqslant 11$ **[4]**

There were few problems with this question. The vast majority of candidates understood what was required, drew the 3 lines and found the correct region.

## Question 10 (b)

**(b)** Find the maximum value of $x + y$ subject to these conditions. **[2]**

Most candidates were able to pick the correct point in the region to give the maximum value of $x + y$. A number checked the other two vertices of the triangular region.

## Question 11 (a)

**11** Amir asked 80 people about their preferences for the drinks tea, coffee or hot chocolate.
The results of his investigation were as follows.

25 liked all three drinks.
3 liked tea and coffee but not hot chocolate.
4 liked hot chocolate and coffee but not tea.
5 liked tea but neither of the other two drinks.
35 liked tea and hot chocolate.
48 liked hot chocolate.
47 liked coffee.

**(a)** Draw a Venn diagram to represent these data. **[3]**

Most candidates successfully achieved full marks. Not drawing the universal set was accepted, because it was thought possible that candidates took the rectangular space for the response to be the universal set.

## Question 11 (b)

**(b)** Hence determine how many people said that they did **not** like any of the drinks. **[2]**

Even those that did not get all the regions of the diagram in part (a) correct were able to earn the marks for this part.

## Question 12 (a)

**12** The point A (1, 3) lies on a circle with centre (4, 5).

**(a)** Determine the equation of the circle. **[2]**

Most candidates managed to find the radius of the circle ( $\sqrt{13}$ ) but then a few took this to be the square of the radius when writing the equation of the circle. Some simply wrote $x^2 + y^2 = 13$ as their equation.

## Question 12 (b)

B is a point on the circle such that AB is a diameter of the circle.

**(b)** Find the coordinates of B. **[2]**

A few wrote the coordinates of B as $(x, y)$ and solved the equations $4=\left(\frac{x+1}{2}\right), 5=\left(\frac{y+3}{2}\right)$.

Most candidates, however, were able to use a vector method given that the vector from A to the midpoint was the same as the vector from the midpoint to B. Most were able to write the coordinates without showing any working.

## Question 12 (c)

D is the point (2, 8).

**(c)** Show that AD and DB are perpendicular. **[3]**

As in Question 9, a number of candidates thought that they had to find the equation of each line. That caused problems as a few did not specifically state what the gradients of their lines were or demonstrated the criterion for perpendicular lines. A few also lost a mark for not completing the 'show that' process by stating that because of what they had shown, the lines were perpendicular.

## Question 12 (d)

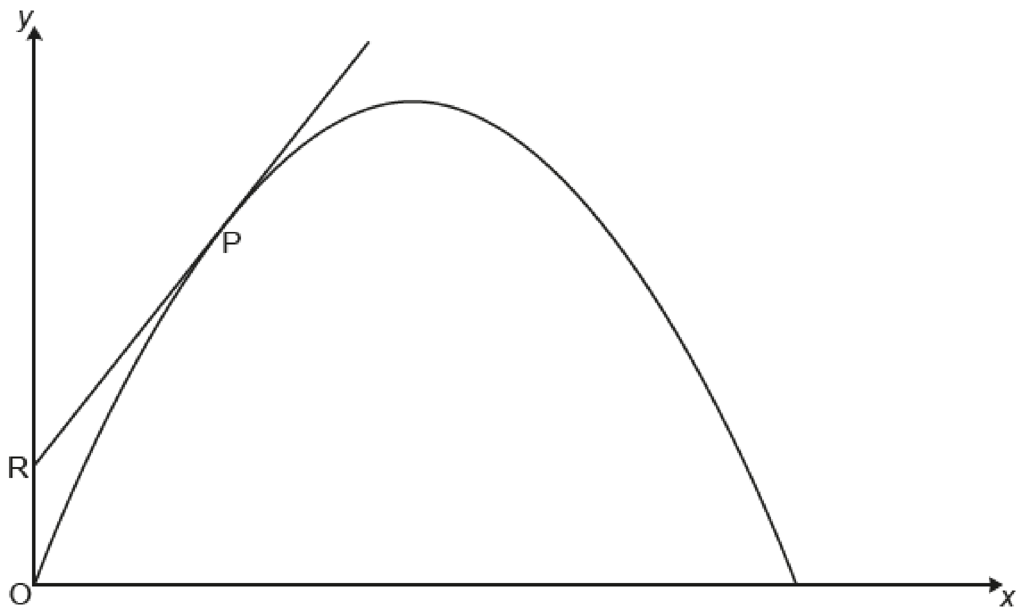
**(d)** Explain what this tells you about the point D. **[1]**

Most candidates were able to state that D lay on the circle, or the circumference of the circle. Justification was not required but many did state the angle in a semicircle theorem.

## Question 13 (a)

**13 In this question you must show detailed reasoning.**

The point P with coordinates (2, 12) lies on the curve $y = 8x - x^2$. The tangent to this curve at P meets the $y$-axis at the point R as shown in the diagram. The origin is O.




**(a)** Determine the coordinates of the point R. **[5]**

The majority of candidates answered this question well, attaining full marks. A small proportion of candidates offered R as '4' rather than giving coordinates and so lost the last A mark. Occasionally some candidates did not find $m$ and then tried to use $8–2x$ in an incorrect way, such as setting this = 0 and taking the result (fortuitously, $x$ = 4). Others found the gradient function correctly to be $8 – 2x$ and then taking $–2$ to be the gradient of the tangent.

## Question 13 (b)

**(b)** Determine the exact area of the region OPR that is bounded by the curve from O to P, the tangent PR and the $y$-axis. **[6]**

A large number of candidates either answered the question successfully or attained high marks. A few lost the last mark by giving an approximate, decimal, answer even though the question demanded an exact answer. Methods used were to subtract the functions before integrating or subtracting the integrated functions before applying the limits as well as the standard way of finding the area of the trapezium and subtracting the area under the curve. An interesting method that some candidates used was creating a rectangle, taking the area under the curve from this and then subtracting the triangle. Common errors were using the wrong limits, either 8 or 12. A small number of candidates did not know how to answer this question.

## Question 14 (a) (i)

**14** Sarah brings a saucepan of water to the boil. She leaves the water to cool, measuring its temperature every 10 minutes for 30 minutes.
The results are shown in the table below.

| Time ($t$ minutes) | 0 | 10 | 20 | 30 |
|---|---|---|---|---|
| Temperature ($T$ °C) | 100 | 60 | 40 | 30 |

Sarah believes that the temperature of the water as it cools can be modelled by the equation $T-20=A\times 2^{-\frac{t}{b}}$ where $A$ and $b$ are constants.

**(a)** **(i)** Explain the significance of the number 20 in this equation. **[1]**

A significant proportion of candidates did not contextually understand that the number represented room temperature. Consideration of the long term temperature from the model was very rarely seen.

## Question 14 (a) (ii)

**(ii)** Use the fact that the initial temperature of the water is 100 °C to determine the value of $A$. **[2]**

There was no difficulty with this calculation for those who knew that $2^0=1$.

## Question 14 (b)

**(b)** By taking logs of both sides of Sarah's equation, show that plotting $\log_{10}(T-20)$ against $t$ will give a straight line. **[3]**

This was a question that really exposed the understanding of log laws and relating the equation to a straight line graph. Many candidates could not combine correctly applying the two laws, either one or the other, or both. Additionally, some candidates took the log of the LHS incorrectly. Of those who successfully used the log laws only a proportion could the correctly identify the form $y=mx+c$. Most common issues were multiplying the two RHS logs and taking the '2' out of the log.

## Question 14 (c)

**(c)** Complete the table below. **[2]**

| Time ($t$ minutes) | 0 | 10 | 20 | 30 |
|---|---|---|---|---|
| Temperature ($T$ °C) | 100 | 60 | 40 | 30 |
| $T - 20$ | | | | |
| $\log_{10}(T - 20)$ | | | | |

There were no problems with the filling in of the table. A number 'read ahead' and gave the values only to an accuracy that they knew they could manage on the graph – i.e. one decimal place – and that was perfectly acceptable. Others wrote down a large number of significant figures.

## Question 14 (d)

**(d)** Plot the values of $\log_{10}(T-20)$ against $t$ on the grid below. **[1]**

There were only a handful of rather odd graphs. Given that the question inferred that a straight line should result, failure to obtain one from their values should have alerted them to the fact that an error had been made somewhere. Although the question did not specifically require the straight line to be drawn, most did so.

## Question 14 (e)

**(e)** Hence estimate the value of $b$. **[2]**

The word 'hence' here should have led candidates to find the gradient of their line and then equate that to their '$m$' of part (b) and many did so successfully. Others ignored their work to produce the line with its gradient and, having found $A$ by substitution in part (a) (ii) proceeded to do the same using another pair of values to find $b$. This was perfectly acceptable providing the substitution was using another pair of values. Using (0, 100) would not find $b$ and a small number were confused by their inability to find $b$.

## Question 15 (a) (i)

**15 In this question you must show detailed reasoning.**

You are given that the curve $y = 2x^3 + 3x^2 - 12x + 8$ has two stationary points.

**(a)** **(i)** Show that one of the stationary points has coordinates (1, 1). **[4]**

A large majority of candidates were successful with this question. A significant number gained 3 marks but neglected to show that $y$ = 1 and so lost the last A mark. Those that showed that (1, 1) lay on the curve earned no marks just for this piece of information without doing some calculus to attempt to also show that it was a stationary point. As the $x$ coordinate ($x$ = 1) had been given, it was satisfactory merely to verify that when $x$ = 1, the gradient of the curve was 0. The majority chose to set the gradient function = 0 and to solve the quadratic equation, thus saving themselves work in the last part.

## Question 15 (a) (ii)

**(ii)** Determine the nature of this stationary point. **[2]**

The usual method used was to demonstrate that the sign of the 2nd differential was positive, the criterion for a minimum point. Just finding the value of $\frac{d^2y}{dx^2}$ or just stating that $\frac{d^2y}{dx^2} > 0$ was not sufficient, the significance of both was needed since this was a 'determine' question. Those that used the values of $y$ or $\frac{dy}{dx}$ as you pass through the turning point were often not explained well. Some attempted to argue that the value of $y$ at this turning point was smaller than that at the other turning point (thus pre-empting part (b)) needed to add in other necessary criteria, such as the shape of the curve and the continuity of the curve and this was not done.

## Question 15 (b)

**(b)** Find the coordinates of the other stationary point. **[2]**

Some candidates had already worked out the $x$ coordinate of the other turning point (at $x$ = -2) in part (a) (i) and used what they had calculated from there. This was perfectly acceptable; others had to solve the quadratic equation, albeit easily done given one of the roots, but this was no difficulty for most candidates.

## Question 16 (a)

**16** I can drive my motor boat at a maximum speed of 4 kilometres per hour in still water.
One day I drive at maximum speed up a river from a point A to a point B, a distance of 9 km.
The constant speed of the current down the river is $r$ kilometres per hour.

**(a)** Show that the time it takes me to drive up the river from A to B is $\frac{9}{4-r}$ hours. **[2]**

Candidates were able to provide a version of distance = speed × time, although some resorted to quoting one or other *suvat* equations. Few were able to explain the denominator satisfactorily.

### Exemplar 2

A → B = 9km

max speed = 4km/hr

total velocity = forward velocity − backwards velocity

↳ 4 − r

$\text{time} = \frac{\text{distance}}{\text{speed}}$

$T = \frac{9}{4-r}$ hours

This response outlines why the speed is 4 – *r*. The explanation was often omitted.

## Question 16 (b)

I also drive at maximum speed on my return journey down the river from B to A.

**(b)** Write down, in terms of $r$, the time it takes me to drive down the river from B to A. **[1]**

There was little difficulty in writing down the correct fraction.

## Question 16 (c)

**(c)** Given that the difference between the time to drive up the river **(a)** and the time to drive down the river **(b)** is 1.2 hours, form an equation in $r$ and show that it simplifies to $r^2 + 15r - 16 = 0$. **[4]**

Most identified the need to subtract the two fractions, although there were many who gave the subtraction the wrong way round. Perhaps taking a moment to think why -18*r* is achieved instead of 18*r* would have encouraged reconsideration of the order of the difference. To complete the question and achieve the given answer the difference in times had to be the correct way round but those who gave the fractions the wrong way round had a problem with a negative sign. Most just deleted it. Once the equation with the fractions was established, the elimination of the fractions was generally well attempted.

## Question 16 (d)

**(d)** Hence find the speed of the current down the river. **[3]**

The equation was generally correctly solved. However, many lost the last mark for failing to give the units for the speed of the current, either correctly or at all. The question asked for the speed of the current. $r = 1$ was therefore not the complete answer. $r = 1$ km/hr was also incorrect as $r$ is a variable with no units. The complete answer of 'speed = 1 km/hr' was not always seen.

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