Oxford Cambridge and RSA

Friday 24 June 2022 – Afternoon

Level 3 Free Standing Mathematics Qualification: Additional Mathematics

6993/01 Paper 1

Time allowed: 2 hours

You must have:
- the Formulae Sheet for Level 3 Free Standing Mathematics Qualification: Additional Mathematics (inside this document)

You can use:
- a scientific or graphical calculator

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number

Candidate number

First name(s)

Last name

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **24** pages.

ADVICE
- Read each question carefully before you start your answer.

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Formulae
FSMQ Additional Mathematics (6993)

Binomial series

$(a+b)^n = a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \ldots + {}^nC_r a^{n-r} b^r + \ldots + b^n$, for positive integers, n,

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$, $r \leqslant n$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

Kinematics

Variable acceleration formulae

$v = \dfrac{ds}{dt}$

$a = \dfrac{dv}{dt} = \dfrac{d^2s}{dt^2}$

$s = \int v \, dt$ and $v = \int a \, dt$

Constant acceleration formulae

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

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Answer **all** the questions.

1 Solve the inequality $-3 < 2(x+1) < 7$. **[3]**

1	

2 A passenger train is 175 m long and is stationary in a station. As it leaves the station it accelerates with uniform acceleration. When the front of the train reaches the end of the platform it is travelling at a speed of $3\,\mathrm{m\,s^{-1}}$ and when the rear of the train reaches the end of the platform it is travelling at a speed of $18\,\mathrm{m\,s^{-1}}$.

(a) Determine the uniform acceleration of the train. **[2]**

(b) Determine the time taken from when the train starts to move until it reaches a speed of $18\,\mathrm{m\,s^{-1}}$. **[2]**

2(a)	
2(b)	

3 A photograph is to be taken of 9 students who have represented their college in sailing this year. The students are to be arranged in a line. The captain is to stand in the middle with the vice-captain standing beside the captain.

How many ways are there of arranging the students? **[3]**

3	

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4 **In this question you must show detailed reasoning.**

You are given the cubic polynomial $f(x) = x^3 + 6x^2 + 5x - 12$.

(a) Show that $f(1) = 0$. **[1]**

(b) Hence solve the equation $f(x) = 0$. **[4]**

4(a)	
4(b)	

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5 **In this question you must show detailed reasoning.**

Find all the values of θ in the range $0° < \theta < 360°$ that satisfy the following equations, giving your answers correct to **1** decimal place.

(a) $\cos 2\theta = 0.6$ **[3]**

(b) $12\cos^2\theta + \sin\theta = 11$ **[5]**

5(a)	
5(b)	

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6 Layla drives to work along a road which has three sets of traffic lights. The lights work independently of each other. Experience indicates that the probability that Layla has to stop at each set of lights is 0.5, 0.6 and 0.7 respectively.

(a) Draw a tree diagram to illustrate the probabilities of Layla having to stop at each set of lights on a particular journey to work. **[3]**

(b) Calculate the probability that Layla has to stop exactly once on a particular journey to work. **[3]**

6(a)	
6(b)	

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7 A local council investigated the pattern of travel to a nearby town in one month.
Residents were asked whether they drove, cycled or walked into the town. They had an opportunity to state if they used more than one mode of transport during the course of the month.

Some of the results are summarised in the table below.

Mode of travel	**Frequency**
Only walked	15
Only cycled	9
Only drove	12
Used all three modes of transport	4
Walked and cycled but did not drive	6

The total number of residents surveyed was 60.

(a) Draw a Venn diagram to illustrate this incomplete set of data. **[3]**

The number that said they drove and walked but did not cycle was the same as the number that said they drove and cycled but did not walk. All those surveyed said that they had travelled to the nearby town by one of these three modes of transport at least once.

(b) Determine how many of these residents drove and walked but did not cycle. **[3]**

7(a)	
7(b)	

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8 The gradient function of a curve is given by $\frac{dy}{dx} = 2 + 4x - 3x^2$ and the curve passes through the point (1, 2).

Determine the equation of the curve. **[4]**

8	

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9 **In this question you must show detailed reasoning.**

(a) You are given that $y = 4\log_3 x$.

Rewrite this equation with x as the subject. **[2]**

(b) Write $2\log_{10}5 + \frac{1}{2}\log_{10}16$ as a single number. **[3]**

(c) The equation $a^x = 17$ has the solution $x = 2.58$, correct to 3 significant figures.
Given that a is an integer, determine the value of a. **[2]**

9(a)	
9(b)	
9(c)	

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10 In this question you must show detailed reasoning.

Two curves have the following equations.

C_1: $y = x^2 - 4x + 4$
C_2: $y = -x^2 + 8x - 6$

(a) Find the coordinates of the **two** points of intersection of these curves. **[4]**

(b) Find the area of the region enclosed by these two curves. **[5]**

10(a)	
10(b)	

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11 Nina records the speed, $v\,\mathrm{m\,s^{-1}}$, at which she is travelling in her car t seconds after accelerating from rest. The results are shown in the table below.

Time (t seconds)	0	2	4	6	8	10
Speed ($v\,\mathrm{m\,s^{-1}}$)	0	6.6	9.6	11.7	13.2	14.4

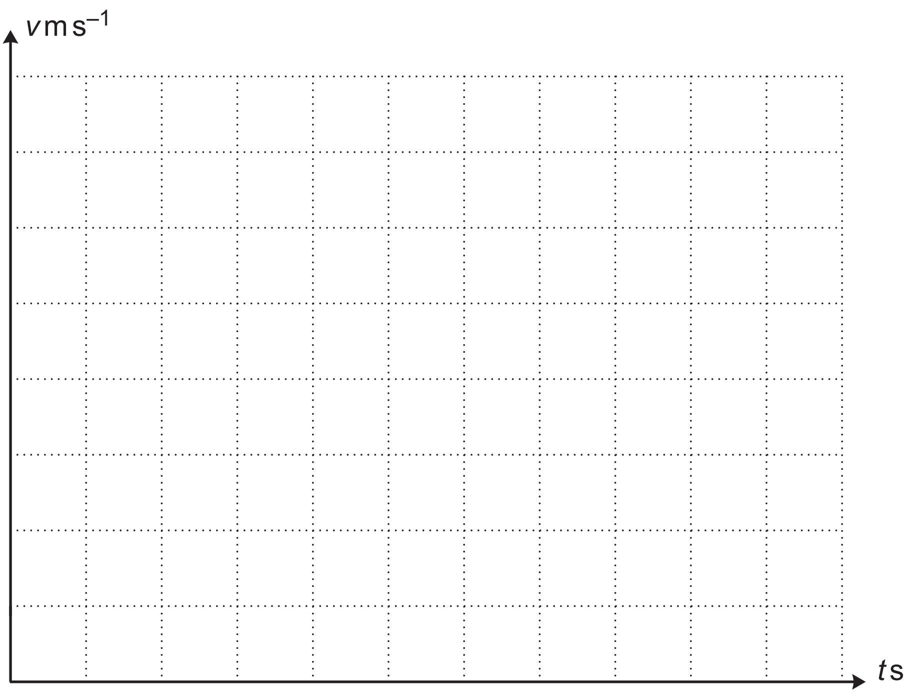
(a) Use these results and the axes below to draw a curve to show how her speed varies with time during these 10 seconds. **[2]**

(b) By constructing 5 rectangles of equal width **above** your curve, estimate the distance she has travelled during the first 10 seconds. **[4]**

(c) Without doing any further calculations, explain how she could obtain a better estimate of the distance she has travelled. **[1]**

11(a)

11(b)



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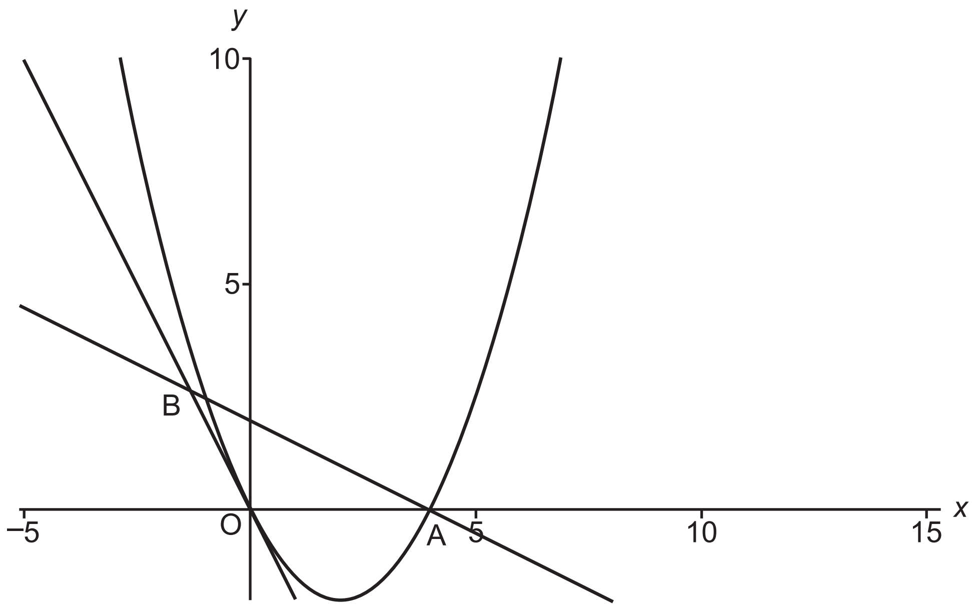
11(c)

12



In the diagram the curve with equation $y = \frac{1}{2}x^2 - 2x$ crosses the x-axis at the origin, O, and the point A. The tangent to this curve at O and the normal to this curve at A intersect at the point B.

(a) Determine the equation of the line OB. **[3]**

(b) Determine the equation of the line AB. **[6]**

(c) Hence determine the coordinates of the point B. **[3]**

12(a)	

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12(b)	
12(c)	

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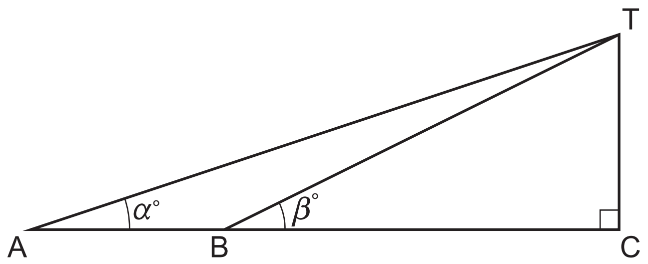
13 A vertical tower CT stands with its base, C, on horizontal ground.
Amir stands at a point A and observes that the angle of elevation of the top of the tower, T, is $\alpha°$.
He then walks directly towards the base of the tower to a point B where he observes that the angle of elevation of the top of the tower is $\beta°$.



Not to scale

(a) Show that $BC = \dfrac{AB\tan\alpha}{\tan\beta - \tan\alpha}$. **[3]**

(b) You are given that $AB = 25\,m$, $\alpha = 15°$ and $\beta = 20°$.

Find

(i) BC, **[2]**

(ii) the height of the tower. **[2]**

13(a)	

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13(b)(i)	
13(b)(ii)	

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14 **(a)** Ben wishes to estimate the gradient of a curve at the point where $x = 1.4$.

He calculates points on the curve which are given in the table below.

x	1.2	1.4	1.8
y	1.0732	1.5358	3.1447

(i) Explain why he should **not** use the coordinates at $x = 1.2$ and $x = 1.8$ to obtain a reasonable estimate for the gradient of the curve at $x = 1.4$. **[1]**

(ii) Calculate an estimate for the gradient of the curve at $x = 1.4$ by using the coordinates at $x = 1.4$ and $x = 1.8$. **[2]**

(iii) Calculate an estimate for the gradient of the curve at $x = 1.4$ by using the coordinates at $x = 1.2$ and $x = 1.4$. **[1]**

(b) Mia wishes to estimate the gradient of another curve at the point where $x = 1.4$.

She calculates points on this curve which are given in the table below.

x	1.2	1.4	1.6
y	0.6899	0.9518	1.3132

Calculate a reasonable estimate of the gradient of this curve when $x = 1.4$. **[3]**

14(a)(i)	
14(a)(ii)	

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14(a)(iii)	
14(b)	

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15 The points A and B have coordinates (3, 3) and (5, 7) respectively.

(a) On the grid below plot

(i) the points A and B, **[1]**

(ii) the line l with equation $x + 2y = 14$. **[2]**

(b) Verify that the line l is the perpendicular bisector of AB. **[4]**

AB is a diameter of the circle C.

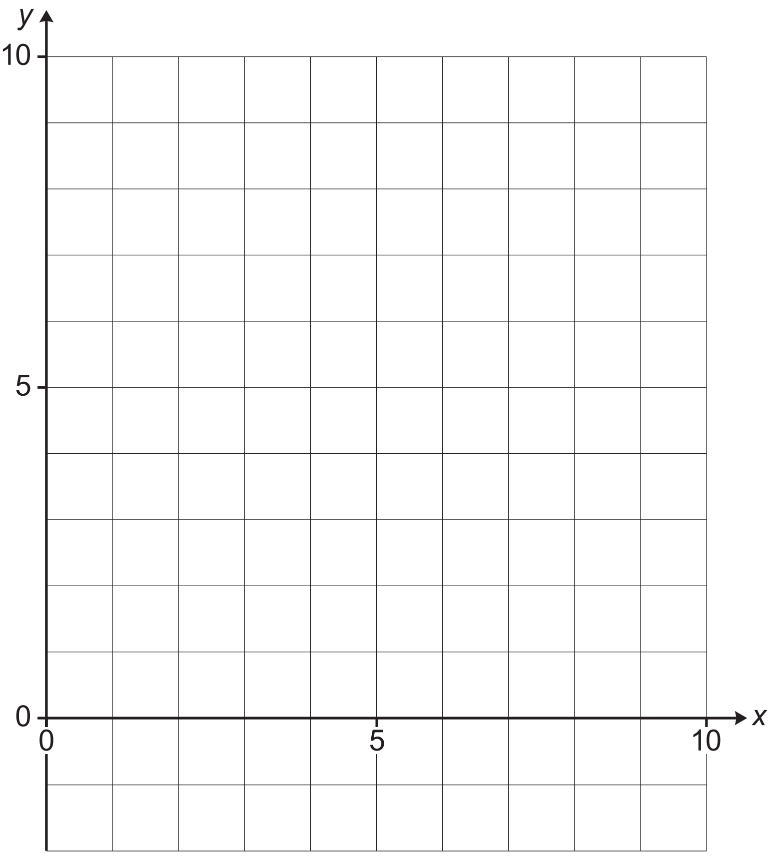
(c) Find the equation of the circle C. **[2]**

The line l cuts the circle C in two points, P and Q.

(d) Determine the coordinates of P and Q. **[3]**

15(a)(i)

(ii)



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15(b)	
15(c)	
15(d)	

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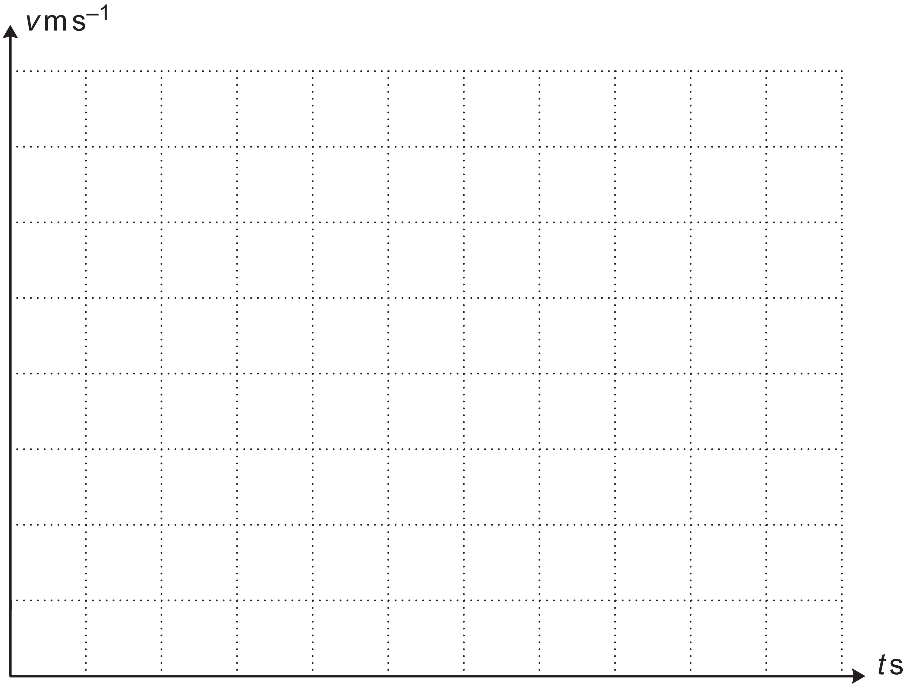
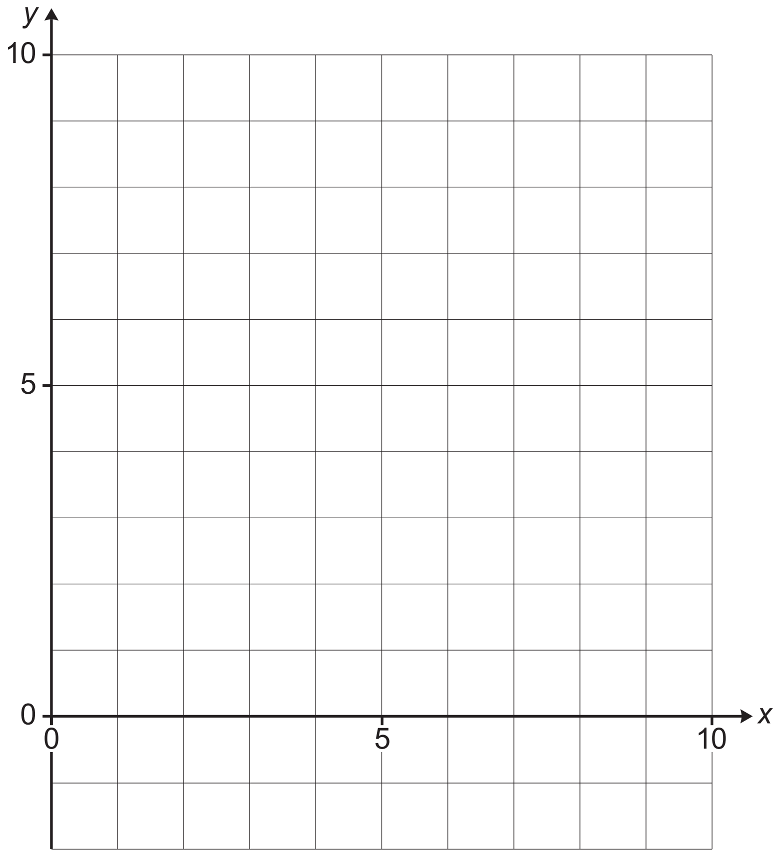
	This is a spare copy of the axes for question 11(a) and (b). Only write on this page if you want to offer a second attempt. If you do so, then you must cross through the first attempt on page 13.
11(a) **11(b)**	

	This is a spare copy of the grid for question 15(a). Only write on this page if you want to offer a second attempt. If you do so, then you must cross through the first attempt on page 20.
15(a)(i) **(ii)**	

END OF QUESTION PAPER

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