Level 3

Additional Mathematics

6993/01: Qualification: Additional Mathematics

Free Standing Mathematics Qualification

Mark Scheme for June 2019

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in RM	**Meaning**
BP	Blank Page – this annotation **must** be used on each page of an additional object where there is no candidate response.
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
Dep M1*	The method mark is dependent on the award of the previous M mark.
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Other abbreviations in MS	**Meaning**
AG	Answer given
DM1	M mark dependent on previous M mark
DB1	B mark dependent on previous B mark(s)
Cao	Correct answer only
Oe	Or equivalent
Soi	Seen or implied
www	Without wrong working

Marking Instructions

a Annotations should be used whenever appropriate during your marking. Put a cross on each window which is a NR

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures usually being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g **Rules for replaced work**

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests. This can be by commenting, ringing, or crossing out the response that he/she does not want

to be marked.

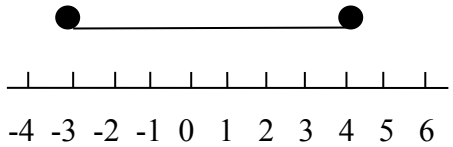
Where a candidate has crossed out a response and provided a clear alternative then the crossed out response is not marked. Where no alternative response has been provided, examiners may give candidates the benefit of the doubt and mark the crossed out response where legible.

If there are two or more attempts at a question which have not been crossed out, examiners should mark the better attempt and ignore the others.

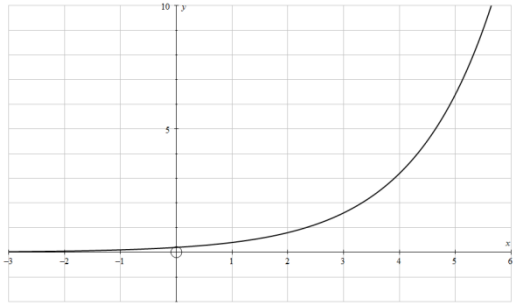
h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			${}_5P_3$ or $5\times4\times3$ or $\frac{5!}{2!}$ **oe** $=60$	**M1** **A1**	Evidence of permutation **soi** Mark final answer	
				[2]		

Question			Answer	Marks	Guidance	
2	(a)		$x^2-x-12=(x\pm3)(x\pm4)$ $\Rightarrow x=-3,4$ $\Rightarrow -3\leqslant x\leqslant 4$	**M1** **A1** **A1**	For factorisation into one of these forms **soi** Correct end values **soi** Correct inequality	Accept $-3\leqslant x$ **and** $x\leqslant 4$ only (i.e. not comma or "or" or spaces or different lines)
				[3]		
	(b)			**B1** **B1**	One line between two end points correct or **FT** from (a) Filled circles at end points of given (single) line	B marks independent Bod "filled in"- be convinced that the circle is not empty.
				[2]		

Question			Answer	Marks	Guidance	
3			$\left(\frac{dy}{dx}\right) = 3x^2 - 4x + 2$	**M1** **A1**	Diffn	At least two powers reduced by 1. Beware division by x
			At (2,8) $\frac{dy}{dx} = 6$	**A1**		
			$\Rightarrow$ gradient of normal $= -\frac{1}{6}$	**M1**	Use of $m_1 m_2 = -1$ on *their* stated (tangent) gradient soi	e.g. $m = k$ gives gradient of normal = $-\frac{1}{k}$
			$\Rightarrow y - 8 = -\frac{1}{6}(x - 2)$ $\Rightarrow 6y = 50 - x$ **oe**	**M1*** **A1**	**Dep** Using *their* **normal** gradient and correct point www	Must be 3 terms only
				[6]		

Question			Answer	Marks	Guidance	
4	(a)			**B1** **B1**	Exponential curve, negative x-axis asymptotic. Sketch must go to at least $x = -1$ Intercept on y-axis seen to be between 0 and 0.5	
				[2]		
	(b)		$\frac{1}{5}2^x = 3 \Rightarrow 2^x = 15$ $\Rightarrow x\log 2 = \log 15 \Rightarrow x = \frac{\log 15}{\log 2} = 3.91$ or $x = \log_2 15 = 3.91$	**M1** **M1** **A1**	Manipulating equation Correct use of logs (to any base) **soi** Ans must be to 3sf. Mark final answer	SC 2 3.91 with no working
				[3]		

Question			Answer	Marks	Guidance	
5			$\Rightarrow \log_{10} x(x+2)$ $= \log_{10} 2^3$	**B1** **B1**	lhs use of log multiplication rule rhs use of log power rule	Beware incorrect method – see below
			$\Rightarrow x^2+2x-8(=0)$	**M1**	Remove logs and get 3 term quadratic	
			$\Rightarrow x=-4, 2$	**A1**	For both	
			$x \neq -4$ $\Rightarrow x=2$	**A1**	www And reject -4	NB "reject negative root" oe is OK. N.B. B1 B1 M1 A0 A1 is possible
				[5]		

Q5 beware

$\log x + \log(x+2) = 3\log 2 \Rightarrow \log x + \log x + \log 2 = 3\log 2$

$\Rightarrow 2\log x = 2\log 2 \Rightarrow \log x = \log 2$

$\Rightarrow x = 2$

This gets 0

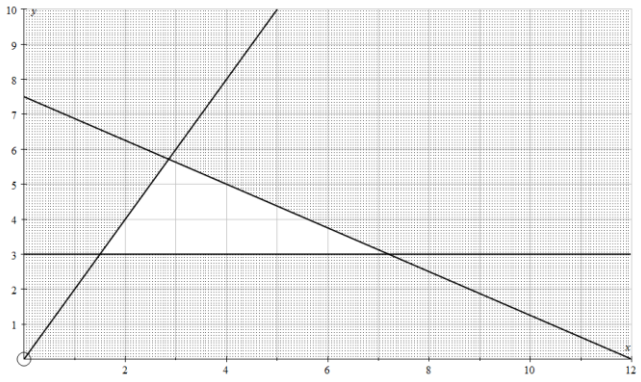
Question			Answer	Marks	Guidance	
6	(a)		56.3º 236.3º	B1 B1	**FT** 1st angle + 180º	Accept 56° Accept 236° Withhold 2nd B mark if there are other angles within the range – ignore any outside range
				[2]		
	(b)		hypotenuse $= \sqrt{3^2+2^2} = \sqrt{13}$ $\Rightarrow \sin\theta = \frac{3}{13}\sqrt{13}$ and $-\frac{3}{13}\sqrt{13}$ **isw**	M1 A1 A1	Using Pythagoras First value 2nd value –ve of the first**. FT**	 i.e. Accept $\sin\theta = \frac{3}{\sqrt{13}}$ and $-\frac{3}{\sqrt{13}}$
			Alternative method: $\tan\theta = 1.5 \Rightarrow \frac{\sin\theta}{\cos\theta} = 1.5$ $\Rightarrow \sin^2\theta = 2.25\left(1-\sin^2\theta\right)$ $\Rightarrow 3.25\sin^2\theta = 2.25$ $\Rightarrow \sin^2\theta = \frac{2.25}{3.25} \quad \left(=\frac{9}{13}\right)$ $\Rightarrow \sin\theta = \frac{3}{\sqrt{13}}$ or $-\frac{3}{\sqrt{13}}$ **isw**	M1 A1 A1	Square and use Pythagoras	 Accept $\sin\theta = \frac{3}{\sqrt{13}}$ and $-\frac{3}{\sqrt{13}}$
				[3]		

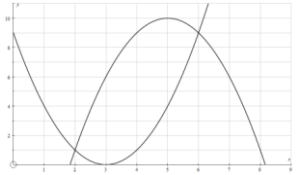
Question			Answer	Marks	Guidance	
7			$f(2)=0 \Rightarrow 8-6+k=0$ $\Rightarrow k=-2$	**M1** **A1**	Substitute $x = 2$	
			$x^3-3x-2=0$ $\Rightarrow (x-2)(x^2+2x+1)=0$ $\Rightarrow x=-1,-1$	**M1** **A1** **A1**	Attempt to solve by division For quadratic	Sight of x^2 and x^3-2x^2 subtracted Accept –1 written only once in answer providing three factors of cubic seen
			Alternative method: for last 3 marks $x^3-3x-2=0$ $\Rightarrow (x-2)(x^2+...+1)=0$ $=(x-2)(x^2+2x+1)$ $\Rightarrow x=-1,-1$	 **M1** **A1** **A1**	 Solve by factorisation by inspection	 Accept –1 written only once in answer providing three factors of cubic seen
			Alternative method: for last 3 marks Trial of any f(a) $x = -1$ found $x = -1$ is a repeated root	 **M1** **A1** **A1**	 on correct function	
			Alternative method: for all marks Sight of x^2 and x^3-2x^2 subtracted Remainder = 0 $k = -2$ **soi** Quadratic factor x^2+2x+1 $x=-1,-1$	 **M1** **M1** **A1** **A1** **A1**	 Cubic divided by $(x-2)$ k chosen to make it a factor	 Accept –1 written only once in answer providing three factors of cubic seen
				[5]		

Question			Answer	Marks	Guidance	
8	(a)		$\left(\frac{1}{2}\right)^5 = \frac{1}{32}\left(=0.03125\right)$	**B1** **B1**	For prob and power soi Must be correct fraction or correct decimal or 0.0313	Allow missing brackets if correct answer seen
				[2]		
	(b)		$10\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2$ $3\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)$ $= \frac{15}{128}\left(=0.117...\right)$	**B1** **B1** **M1** **A1**	1st term soi 2nd term soi *Their* probs multiplied together	0.3125 0.375 Terms must be less than 1
				[4]		

Question			Answer	Marks	Guidance	
9	(a)		$f(0) = -1,\ f(1) = 1$	**M1** **A1**	Substitution of both 0 and 1 Both correct	
				[2]		
	(b)		$x^3 + 2x^2 - x - 1 = 0 \Rightarrow x^2(x+2) = (x+1)$ $\Rightarrow x^2 = \frac{(x+1)}{(x+2)} \Rightarrow x = \sqrt{\frac{(x+1)}{(x+2)}}$ **AG**	**M1** **A1**	Factorising the first two terms; x^2 must be seen as a factor Ignore ±	Alternatively: start from given eqn Square both sides and multiply up M1 Correct algebra seen to give original equation A1
				[2]		
	(c)		0.8 0.801784 0.801926 0.801937 x_3 $(\Rightarrow \gamma) = 0.802$	**M1** **A1** **A1**	Use of formula x_1 correct to at least 4 dp or exact At least x_3 correct to at least 4 dp seen and answer correct to 3sf	Accept $\frac{3\sqrt{14}}{14}$ **oe** for x_1 Accept $x_3 = 0.802$ as final answer
				[3]		

Question			Answer	Marks	Guidance	
10	(a)		$x^2+(2x+k)^2=5$ $\Rightarrow x^2+4x^2+4kx+k^2-5=0$ $\Rightarrow 5x^2+4kx+(k^2-5)=0$ **AG**	**M1** **A1**	Substitute and attempt to simplify	
				[2]		
	(b)		For coincident roots, the terms under the square root sign sum to 0 $\Rightarrow 16k^2-20(k^2-5)=0$ $\Rightarrow -4k^2+100=0$ $\Rightarrow k=\pm 5$	**M1** **A1**	Evidence of understanding that tangent implies coincident roots	"Evidence" means some indication that roots are coincident
				[2]		

Question			Answer	Marks	Guidance	
11	(a)		$5x+8y \leqslant 60$ **oe**	**B1**		
				[1]		
	(b)		$y \geqslant 3$ $y \leqslant 2x$	**B1** **B1**		
				[2]		
	(c)			**B3,2,1** **B1**	B1 for each line. Ignore errors in (a) and (b) Ignore labels for lines on graph Correct shading	Check for time line goes through (4,5) Do not accept the shaded region being the triangle N.B. You might see lines on the graph that relate to (d) and (e) - ignore
				[4]		
	(d)		$(x + y =)$ 10 Time = 59 (hours)	**B1** **B1**	$x = 7$, $y = 3$ only is B0	
				[2]		
	(e)		 4 small and 5 large Profit = £412	**M1** **A1** **A1**	Sight of $(P =)28x+60y$ on grid Or listing at least two correct feasible points and calculating P for each point **soi** by £412	 SC no working seen but answer given, B1 B1 Allow B2 by implication from £412
				[3]		

Question			Answer	Marks	Guidance	
12	(a)		$10 = 10\times5 - 5^2 + k$ **oe** $\Rightarrow k = 10 - 25 = -15$ **AG**	**M1** **A1**	Substitute	Either substitute (5, 10) to obtain k or substitute $k = -15$ and show that it is satisfied by (5, 10)
				[2]		
	(b)		$\frac{dy}{dx} = 10 - 2x$ $= 0$ when $x = 5$	**M1** **A1** **A1**	Diffn and set = 0 or find = 0 from x = 5 Any demonstration to show maximum	All powers reduced by 1 – beware of division by x e.g. gradient either side of turning points or values either side or $\frac{d^2y}{dx^2} = -2 < 0$ stated meaning maximum
			Alternative method: $y = a \pm (x \pm b)^2$ $= \left(10 - (x-5)^2\right)$ $\Rightarrow$ Maximum value of 10 when $x = 5$ **Alternative method:** Statement that Quadratic function is symmetric Pair of symmetric coordinates either side of (5, 10) Gives maximum	**M1** **A1** **A1** **M1** **A1** **A1**	Completing the square	$a \neq -15$ and $b \neq 10$ e.g. at x =4, 6, y = 9 and 10 > 9
				[3]		
	(c)			**B1** **B1**	Upside down parabola Maximum (5, 10); Meets other curve at two points	
				[2]		

	(d)		(2, 1) and (6, 9)	**B1** **B1**	**FT** *their* graphs **FT** *their* graphs	Accept an algebraic method that may differ from their intersections in (c)
				[2]		
	(e)		$\text{Area} = \int_2^6 (y_1 - y_2)\,dx$	**M1**	Difference of curves	In either order
			$= \int_2^6 \left(16x - 2x^2 - 24\right)dx$	**A1**	Or – this function	A0 if divided by 2
			$= \left[8x^2 - \frac{2}{3}x^3 - 24x\right]_2^6$	**M1***	**Dep** Integrate; ignore limits	At least two powers increased by 1; beware multiplication by x
			$= \left(288 - 144 - 144\right) - \left(32 - \frac{16}{3} - 48\right)$	**M1***	**Dep** Apply limits by substitution of *their* intersections, subtract in correct order	
			$= 0 - \left(-\frac{64}{3}\right) = \pm\frac{64}{3}$	**A1**	Accept to 3sf or better	SC answer with no working B5
			Alternative method: $\text{Area} = \int_2^6 y_1\,dx - \int_2^6 y_2\,dx$	**M1**	Integrate both; ignore limits	
			$= \int_2^6 \left(10x - x^2 - 15\right)dx - \int_2^6 \left(x^2 - 6x + 9\right)dx$	**M1**	Apply limits in one integral by substitution of *their* intersections, subtract in correct order	
			$= \left[5x^2 - \frac{x^3}{3} - 15x\right]_2^6 - \left[\frac{x^3}{3} - 3x^2 + 9x\right]_2^6$			
			$= \left(\left(180 - 72 - 90\right) - \left(20 - \frac{8}{3} - 30\right)\right) - \left(\left(72 - 108 + 54\right) - \left(\frac{8}{3} - 12 + 18\right)\right)$	**A1**	For one value	
			$= \left(18 + \frac{38}{3}\right) - \left(18 - \frac{26}{3}\right)$	**M1***	**Dep on both** Subtract	
			$= \frac{92}{3} - \frac{28}{3} = \frac{64}{3}$	**A1**		
				[5]		

Question			Answer	Marks	Guidance	
13	(a)		$\frac{h}{300} = \tan 7 \Rightarrow h = 36.8....$	**M1** **A1**	Use tan ratio	Sine rule may be used
				[2]		
	(b)		$CB^2 = 400^2 + 300^2 - 2\times 300\times 400\times \cos 60$ $= 130000$ $\Rightarrow CB = 360.55.....$ elevation $= \tan^{-1}\left(\frac{their\ \ h}{their\ \ \text{CB}}\right)$ $= 5.83...$	**M1** **A1** **A1** **M1** **A1**	Find CB by correct cosine rule Correct substitutions Ignore any rounding Correct angle Correct to 3 sf	
				[5]		
	(c)		e.g $\frac{\sin B}{300} = \frac{\sin 60}{their\ CB}$ $\Rightarrow B = 46.1$ or $C = 73.9$ Bearing $= 180 + 60 + their\ B$ $= 286^0$	**M1** **A1** **A1** **M1** **A1**	Either sin rule or cosine rule to find angle at B or C Alternative methods ok Inserting correct values into chosen formula correctly awrt 46 or 74 Correct attempt to find bearing awrt 286	rearranging correctly either sin = or cos = Or using C
				[5]		

Question			Answer	Marks	Guidance	
14	(a)		$\frac{dv}{dt} = a$ (constant) $\Rightarrow v = c + at \Rightarrow v = 14 + at$ $\frac{ds}{dt} = v = 14 + at \Rightarrow s = c + 14t + \frac{1}{2}at^2 = 14t + \frac{1}{2}at^2$ $\Rightarrow 50 = 14t + \frac{1}{2}at^2$ and $9 = 14 + at \Rightarrow at = -5$ $\Rightarrow 50 = 14t - \frac{5}{2}t = \frac{23}{2}t \Rightarrow t = \frac{100}{23} \approx 4.35$ $\Rightarrow a = -\frac{23}{20} = -1.15$	M1 A1 A1		
			Alternative method: $u = 14,\ v = 9,\ s = 50$ Use of $v^2 = u^2 + 2as \Rightarrow 100a = 9^2 - 14^2$ $\Rightarrow a = -1.15$ Use of $v = u + at \Rightarrow t = \frac{5}{1.15} = 4.35$ **Alternative method:** $s =$ (Average velocity) $\times$ time $\Rightarrow 50 = \frac{9+14}{2}t \Rightarrow t = \frac{100}{23} = 4.35$ $\Rightarrow a = \frac{9-14}{4.35} = -1.15$	M1 A1 A1 M1 A1 A1	Use of at least one correct suvat formulae to get a or t www www Use of at least one correct suvat formulae to get a or t www www	Suvat formulae can be quoted without derivation Allow 1.15 if it is clear that a is the deceleration Correct suvat formula is $v^2 = u^2 + 2as,\ v = u + at, s = \frac{(u+v)}{2}t$
				[3]		

	(b)		$v = 14$	**B1**	Substitute $t = 10$	Or substitute $v = 14$ to give cubic in t and $t = 10$ shown to be a root The cubic is $t^3 - 15t^2 + 500 = 0$
				[1]		
	(c)		$v = \frac{1}{100}\left(15t^2 - t^3\right) + 9$ $\Rightarrow s = \frac{1}{100}\left(5t^3 - \frac{t^4}{4}\right) + 9t$ $\Rightarrow s = \frac{1}{100}\left(5000 - 2500\right) + 90$ $= 115$	**M1** **A1** **M1*** **A1**	Integrate - at least two powers increased by 1. Beware multiplying by t Ignore c **Dep** Substitute $t = 10$	M0 if 1/100 integrated Beware: suvat formulae will give 115
				[4]		
	(d)		$v = \frac{1}{100}\left(15t^2 - t^3\right) + 9$ $\Rightarrow a = \frac{1}{100}\left(30t - 3t^2\right)$ $\Rightarrow \frac{\mathrm{d}a}{\mathrm{d}t} = \frac{1}{100}\left(30 - 6t\right) = 0$ when $t = 5$ $\Rightarrow a = \frac{1}{100}\left(150 - 75\right) = 0.75$	**M1** **A1** **A1**	Differentiate twice	
			Alternative method: Differentiate once and use symmetry of quadratic or complete square Correct diffn and completed square or symmetry around 5 $a = 0.75$	**M1** **A1** **A1**	Symmetry must be stated	
				[3]		
	(e)		Distance between bumps = *their* 115 + 50 = 165 m	**B1**	**FT** their answer to (c)	
				[1]		

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