OCR
Oxford Cambridge and RSA

Tuesday 18 June 2019 – Morning

Level 3 Free Standing Mathematics
Qualification: Additional Mathematics

6993/01 Paper 1

Time allowed: 2 hours

You may use:
- a scientific or graphical calculator

Please write clearly in black ink. **Do not write in the barcodes.**

Centre number | | | | | | Candidate number | | | | |

First name(s) _____

Last name _____

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully before you start to write your answer.
- Write your answer to each question in the space provided.
- Where appropriate, your answer should be supported with working.
- Additional paper may be used if necessary, but you must clearly show your candidate number, centre number and question number(s).
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question or part question are shown in brackets [ ].
- You are reminded of the need for clear presentation in your answers.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- This document consists of **24** pages.

Formulae
FSMQ Additional Mathematics (6993)

Binomial series

When n is a positive integer

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$, $r \leqslant n$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$

Numerical methods

Trapezium rule: $\displaystyle\int_a^b y \, \mathrm{d}x \approx \tfrac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

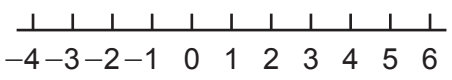
Answer **all** the questions.

1 A committee consists of five people. The roles of Chairman, Secretary and Treasurer are to be allocated at random from the committee with no one person taking more than one role.
In how many ways can this allocation of roles be made? **[2]**

1	

2 **(a)** Solve the inequality $x^2 - x - 12 \leqslant 0$. **[3]**

(b) Illustrate your answer to part **(a)** on the number line provided. **[2]**

2(a)	
2(b)	

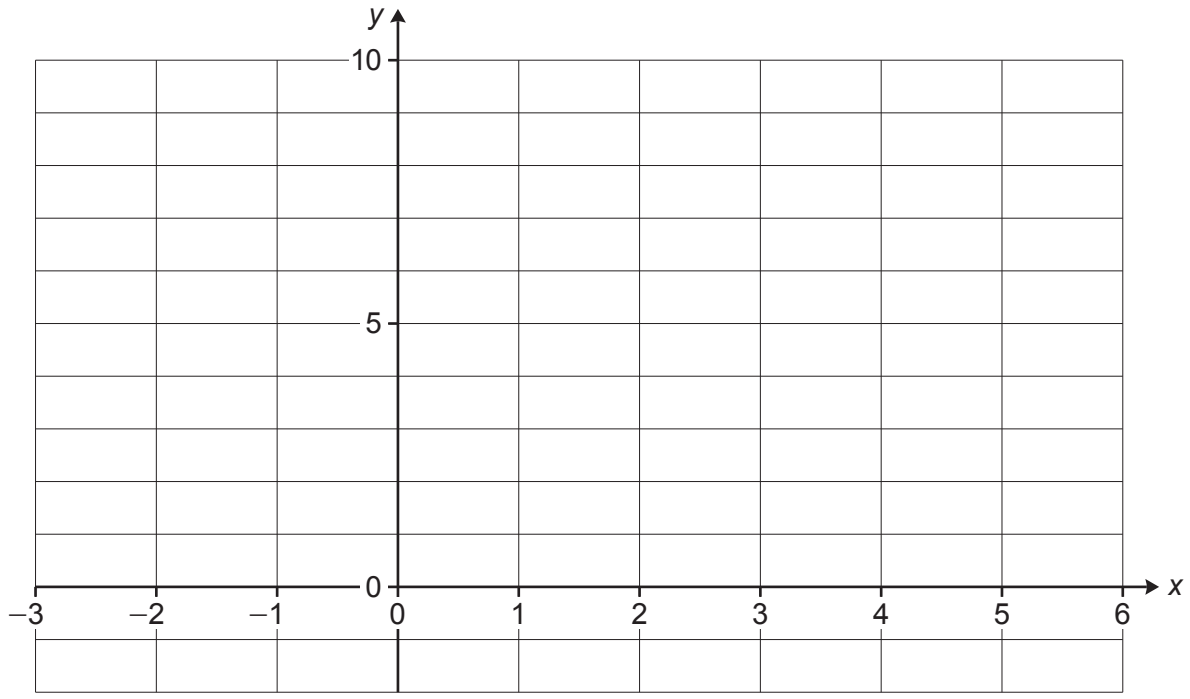
3 Find the equation of the normal to the curve $y = x^3 - 2x^2 + 2x + 4$ at the point (2, 8). **[6]**

3

4 **(a)** On the grid provided, sketch the curve $y = \frac{1}{5} \times 2^x$. **[2]**

(b) Solve algebraically the equation $\frac{1}{5} \times 2^x = 3$, giving your answer correct to 3 significant figures. **[3]**

4(a)



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4(b)

5 In this question you must show detailed reasoning.

Solve the equation $\log_{10} x + \log_{10}(x+2) = 3\log_{10} 2$. **[5]**

5	

6 Angle θ is such that $\tan\theta = 1.5$.

(a) Find the two values of θ in the range $0° \leqslant \theta \leqslant 360°$. **[2]**

(b) **In this question you must show detailed reasoning.**

Find the **exact** values of $\sin\theta$. **[3]**

6(a)	
6(b)	

7 In this question you must show detailed reasoning.

The equation $x^3 - 3x + k = 0$, where k is a constant, has a root $x = 2$.

Find the numerical value(s) of the other roots of this equation. **[5]**

7	

8 Each of five students has a fair coin. They play a game in which each student tosses their coin and when the result of their toss is a head then that student is eliminated from the game. The game continues with the remaining students tossing their coin again. As before, any student who tosses a head is eliminated. The game continues until all the students have been eliminated or there is a single winner.

Calculate the probability that

(a) all students are eliminated after their first toss of the coin, **[2]**

(b) exactly two students are eliminated after their first toss and exactly two after their second toss, leaving one winner. **[4]**

8(a)	
8(b)	

9 The equation $x^3 + 2x^2 - x - 1 = 0$ has two negative roots, α and β, and one positive root, γ.

(a) By considering a change of sign, show that γ lies in the interval [0, 1]. **[2]**

(b) Show that $x = \sqrt{\dfrac{x+1}{x+2}}$ is a rearrangement of the equation. **[2]**

(c) Using the iterative formula $x_{r+1} = \sqrt{\dfrac{x_r + 1}{x_r + 2}}$ with $x_0 = 0.8$, find γ correct to 3 decimal places, showing the result of each iteration. **[3]**

9(a)	
9(b)	
9(c)	

10 You are given that the line $y = 2x + k$ cuts the circle $x^2 + y^2 = 5$ in two points, A and B.

(a) Show that the x-coordinates of A and B satisfy the equation

$$5x^2 + 4kx + (k^2 - 5) = 0.$$ **[2]**

(b) Hence find the values of k for which the line is a tangent to the circle. **[2]**

10(a)	
10(b)	

11 John makes wooden toys in his workshop at home. He classifies the toys as small or large.
It takes 5 hours to make a small toy and 8 hours to make a large toy. He works for a maximum of 60 hours each week.
Let x be the number of small toys and y be the number of large toys he makes each week.

(a) Write down an inequality giving the time constraint. **[1]**

John knows from experience that
- he needs to make at least 3 large toys each week,
- the number of large toys should be no more than double the number of small toys.

He never leaves any toys unfinished at the end of the week.

(b) From this information, write down two more inequalities in x and y. **[2]**

(c) On the grid provided, illustrate these three inequalities.
Shade the region that is **not** required. **[4]**

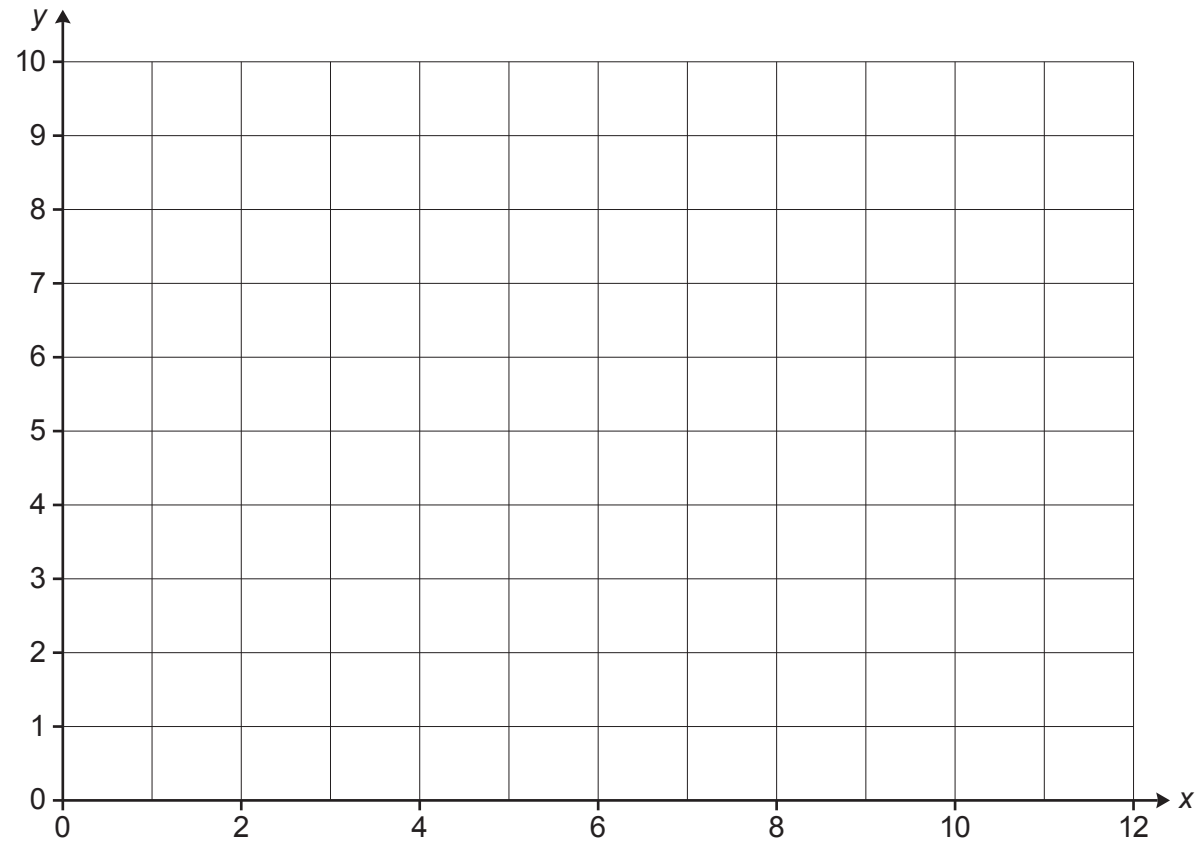
(d) Find the maximum number of toys that John can make in a week and the number of hours he would take to make them. **[2]**

The price for which John sells his wooden toys is such that the profit made is £28 for each small toy and £60 for each large toy.

(e) Assuming that at the end of each week he sells all the toys, find the number of each type of toy he should make to maximise his profit and calculate the profit in this case. **[3]**

11(a)	
11(b)	

11(c)



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11(d)

11(e)

12 The curve C_1 has equation $y = 10x - x^2 + k$ and passes through the point (5, 10).

(a) Show that $k = -15$. **[2]**

(b) Show that there is a maximum value at the point (5, 10). **[3]**

The curve C_2 with equation $y = (x - 3)^2$ has been plotted on the grid provided.

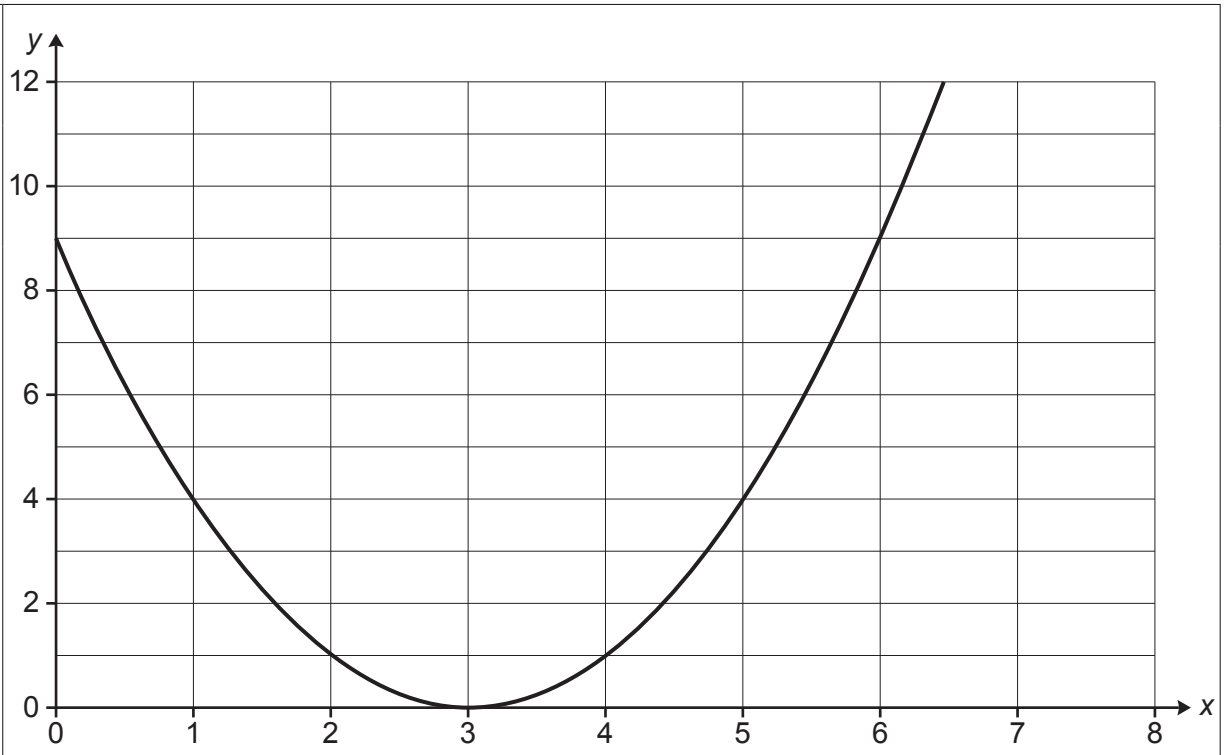
(c) On the same grid, sketch the curve C_1. **[2]**

(d) Find the coordinates of the points of intersection of the curves C_1 and C_2. **[2]**

(e) Find the area between the curves C_1 and C_2. **[5]**

12(a)	
12(b)	

12(c)



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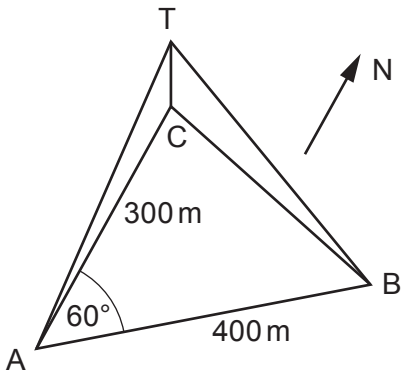
12(d)

12(e)

13 A straight road runs on a bearing of 060° from a point A to a point B, 400 m from A.
A vertical mast, CT, stands at a point C, 300 m due north of A. From the point A the angle of elevation of the top of the mast, T, is 7°. The triangle ABC is on horizontal ground.



(a) Find the height of the mast. **[2]**

(b) Find the angle of elevation of the top of the mast from point B. **[5]**

(c) Find the bearing of the base of the mast from point B. **[5]**

13(a)	
13(b)	

13(c)	

14 Speed bumps are designed to encourage drivers to drive slowly. On a particular road, the bumps put onto the road are designed to give minimum discomfort and damage at a speed of $9\,\text{m s}^{-1}$. Paul is driving along the road at a speed of $14\,\text{m s}^{-1}$ when he sees the warning sign, he is 50 m before the first bump. He immediately slows down with uniform deceleration so that when he reaches the first bump he is travelling at a speed of $9\,\text{m s}^{-1}$.

(a) Calculate the uniform deceleration **and** the time taken for Paul to reach the first bump. **[3]**

Immediately after the bump he accelerates such that at t seconds after leaving the bump his speed, $v\,\text{m s}^{-1}$, is given by $v = \frac{1}{100}(15t^2 - t^3) + 9$.

(b) Show that he reaches his original speed of $14\,\text{m s}^{-1}$ in 10 seconds. **[1]**

(c) Find the distance travelled from the speed bump by the time he reaches this speed. **[4]**

(d) Find the maximum acceleration in this period. **[3]**

(e) If all drivers decelerate and accelerate in the same way as Paul, suggest a maximum distance between bumps to ensure that drivers do not exceed a speed of $14\,\text{m s}^{-1}$ when driving down the road. **[1]**

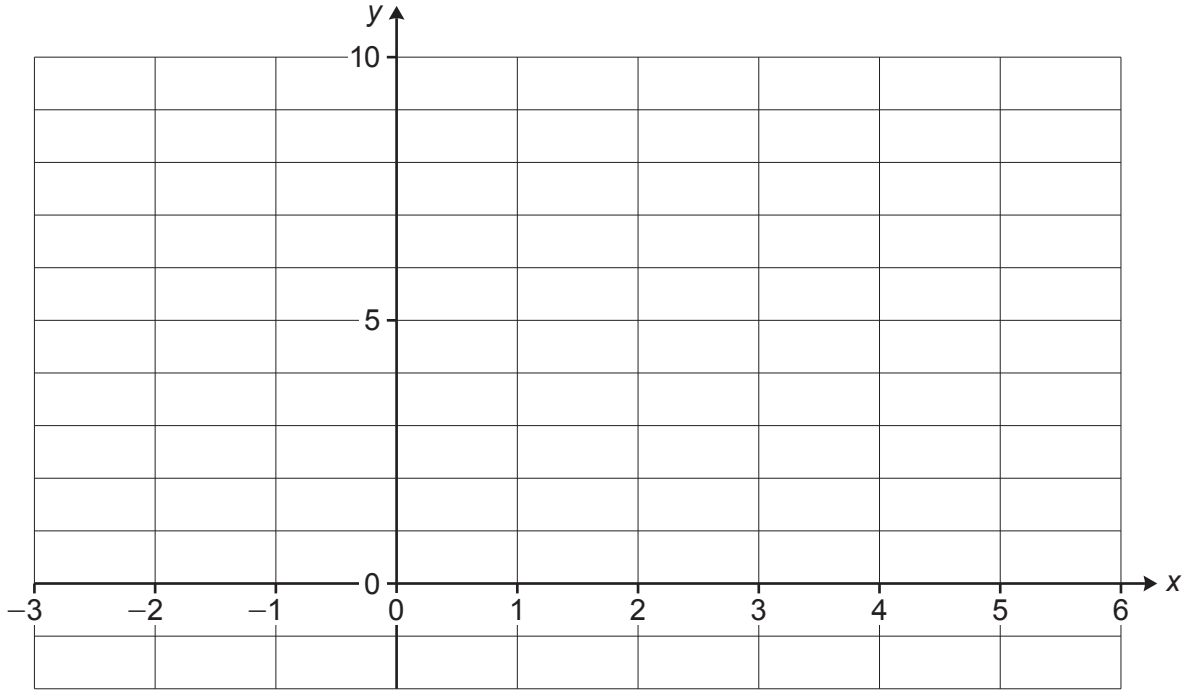
14(a)	
14(b)	

14(c)

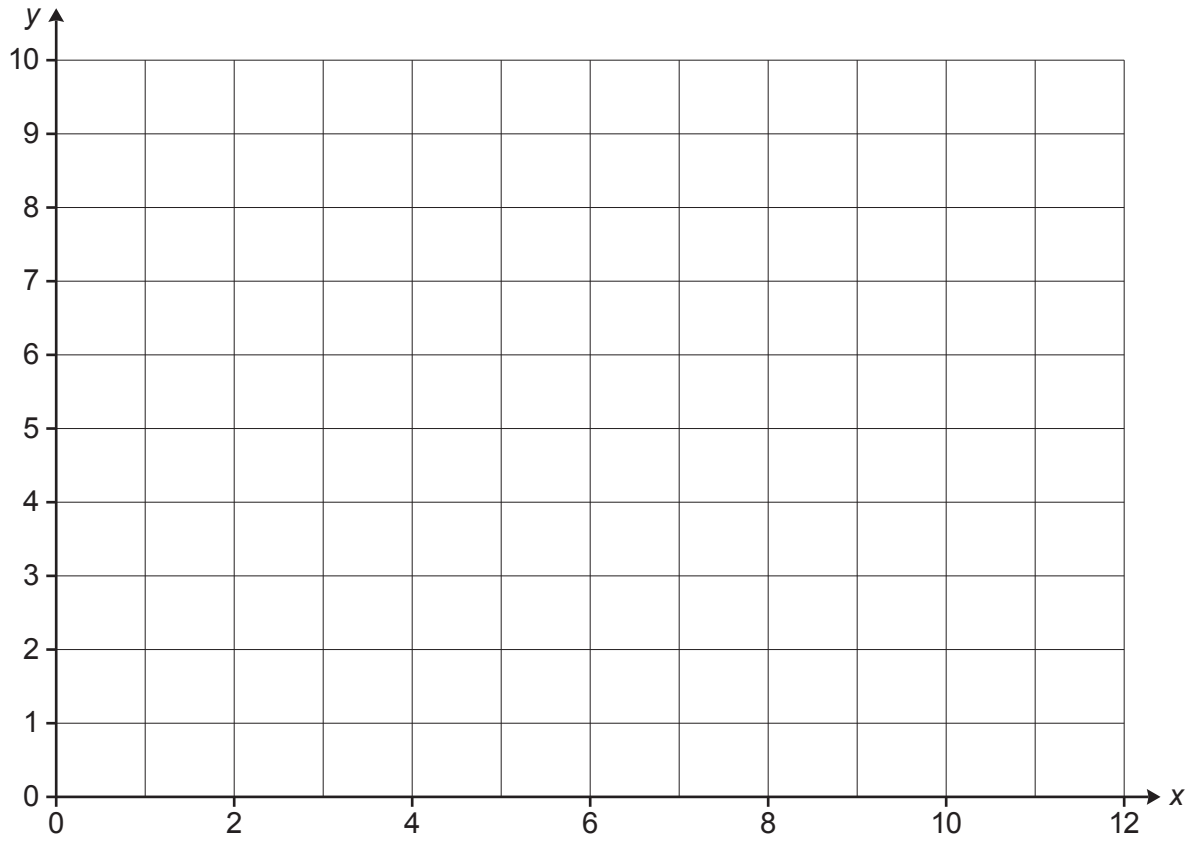
14(d)

14(e)

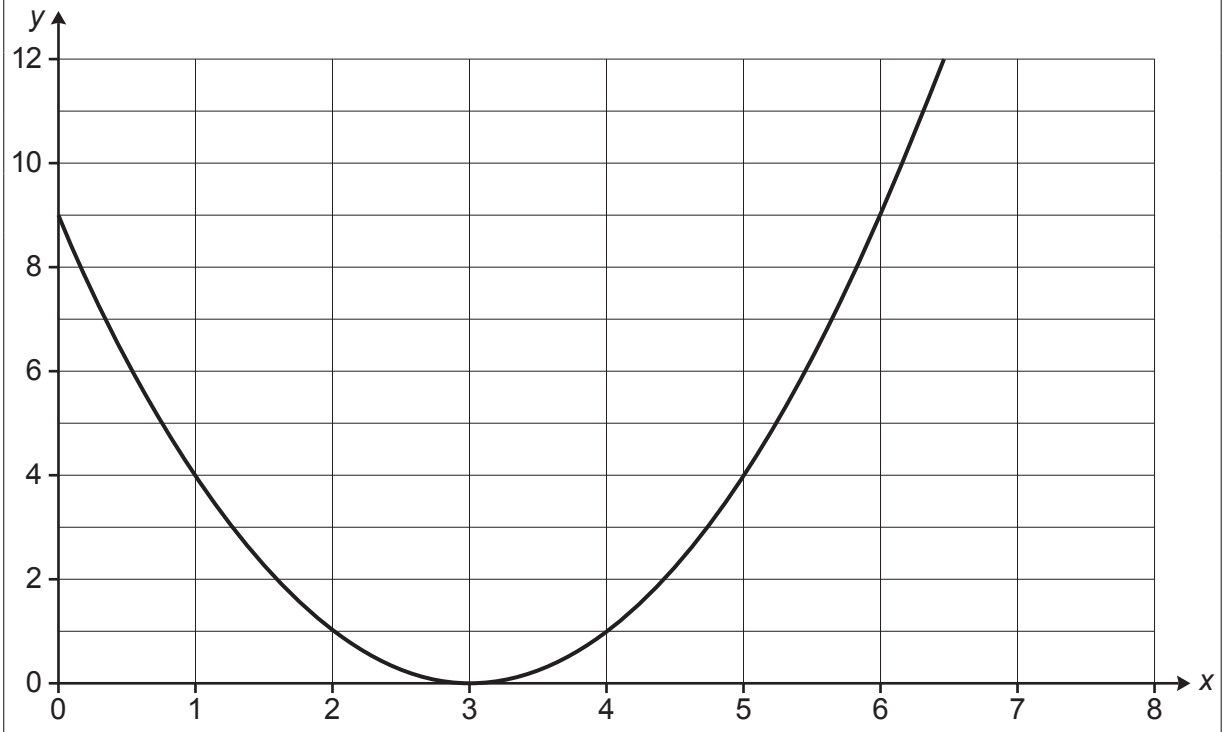
4(a) *This is a spare copy of the grid for question **4(a)**. Only write on this page if you want to offer a second attempt. If you do so, then you must cross through the first attempt on page 5.*



11(c) *This is a spare copy of the grid for question **11(c)**. Only write on this page if you want to offer a second attempt. If you do so, then you must cross through the first attempt on page 13.*



12(c) *This is a spare copy of the graph for question **12(c)**. Only write on this page if you want to offer a second attempt. If you do so, then you must cross through the first attempt on page 15.*



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