

FSMQ

Additional Mathematics

Unit 6993: Additional Mathematics

Free Standing Mathematics Qualification

Mark Scheme for June 2018

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Annotations and abbreviations

Annotation in scoris	Meaning
√and ×	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
<u>۸</u>	Omission sign
MR	Misread
Highlighting	
Other abbreviations in	Meaning
mark scheme	
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Marking Instructions

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

Μ

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

Α

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

В

Mark for a correct result or statement independent of Method marks.

6993/01

Mark Scheme

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise;
- e The abbreviation **ft** implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.
- h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Section A

)uest	ion	Answer	Marks	Guidance
1			2-x < 1+3(x-2) $\Rightarrow 2 < 4x-5$ $\Rightarrow 4x > 7$ $\Rightarrow x > \frac{7}{4}$	B1 B1 B1	Remove brackets giving rhs $1 + 3x - 6$ or better Ft Result in the form $ax > b$ oe
				[3]	

Question	Answer	Marks	Guidance		
2	$\frac{dy}{dx} = 2 + 2x - 3x^{2}$ $\Rightarrow (y =) 2x + x^{2} - x^{3}(+c)$ Through (2, 3)	M1 A1 M1	Attempt to integrate – at least two powers increased by 1 Coefficients unsimplified Dep. Substitute in <i>their</i> function	Beware of mult by <i>x</i>	
	$\Rightarrow 3 = 4 + 4 - 8 + c$ $\Rightarrow c = 3$ $\Rightarrow y = 2x + x^{2} - x^{3} + 3$	A1	Equation must be given		
		[4]			

	Question	Answer	Marks	Guidance	
3	(i)	(x =) -3, 1	B1	Both, by any means	Allow (-3, 0) and (1, 0)
			[1]		
	(ii)	The line is $(y =) 3x + 3$ The solution is 3, -2	B1 B1	Equation (soi by plot) Correctly plotted	Algebraic soln not acceptable.
		The solution is 5, 2	B1	Dep on previous 2	Allow (3, 12) and (-2, -3)
			[3]		

(Question	Answer	Marks	Guidar	nce
4		$\cos^{2} \theta = 1 - \sin^{2} \theta = 1 - \frac{1}{25}$ $\Rightarrow \cos \theta = \sqrt{\frac{24}{25}} \text{ or } \frac{2}{5}\sqrt{6}$	M1 A1	Use of Pythagoras Isw allow $\sqrt{0.96}$	
		$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{1}{5} \times \frac{5}{2\sqrt{6}}$ $= \frac{1}{\sqrt{24}} \text{ or } \frac{1}{2\sqrt{6}} \text{ or } \frac{1}{12}\sqrt{6}$	M1 A1	Use of tan ratio Isw allow $\frac{1}{5\sqrt{0.96}}$	M0 if approximate values used
			[4]		
		Alternatively: Find third side of triangle $=\sqrt{24}$ giving $\cos\theta = \sqrt{\frac{24}{25}}$		M1 A1isw	
		giving $\cos \theta = \sqrt{\frac{25}{25}}$ and $\tan \theta = \sqrt{\frac{1}{24}}$		A1 isw A1 isw	

Question	Answer	Marks	Gui	dance
5	$\int_{0}^{3} (6x^{2} - 2x^{3}) dx$ $= 2x^{3} - \frac{x^{4}}{2}$	M1	Integration – ignore limits	i.e. both powers increased by 1
	2	A1		Allow unsimplified
	$\Rightarrow A = \left[2x^3 - \frac{x^4}{2}\right]_0^3$			
	$=\left(54-40\frac{1}{2}\right)-0$	M1	Dep. Substitution of $x = 3$ (and $x = 0$ soi). Any other limits M0	
	$=13\frac{1}{2}$	A1		SC Answer only or www seen B4
		[4]		

()uesti	ion	Answer	Marks	Guidance	
6	(i)		(i) \times 3: 9x+12y = 54	M1	Making a coefficient the same	Alternatively soln by substitution
			(ii) \times 4: $28x - 12y = 20$ Add: $37x = 74$	M1	Elimination	
			$\Rightarrow x = 2$	A1		SC Answer only or www seen B4
			$\Rightarrow y = 3$	A1		
				[4]		
	(ii)		Sketch to show two lines, one +ve gradient	B1	Two lines	
			and one –ve,			
			intersecting at <i>their</i> point from (i)	B1	Dep. Their intersection	
				[2]		

(Question		Answer	Marks	Guida	nce
7	(i)		$7x-9 = x^2 + 2x - 5$	M1	Equate	Alt: Make x subject and substitute
			$\Rightarrow x^2 - 5x + 4 = 0$	A1	Correct quadratic	to give $y^2 - 17y - 38 = 0$
			$\Rightarrow (x-4)(x-1) = 0$ $\Rightarrow x = 1, 4$ $\Rightarrow y = -2, 19$	A1	Both <i>x</i> -values (or both <i>y</i> -values or one pair)	
			$\Rightarrow y = -2, 19$ $\Rightarrow (1, -2), (4, 19)$	A1	Both coordinates	Allow $x = 1, y = -2$ and $x = 4, y = 19$
						SC. Answer (i.e. both pairs) only or www B4
				[4]		
	(ii)		$\left(\frac{dy}{dx}\right) = 2x + 2$ At (1,-2), $\frac{dy}{dx} = 2 + 2 = 4$	M1 A1	Diffn Both values	Alternative: Diffn M1 One value seen and correct numeric comparison A1
			At (4,19), $\frac{dy}{dx} = 8 + 2 = 10$ Grad normals = $-\frac{1}{4}$ and $-\frac{1}{10} \neq 7$ so no. oe	A1	Correct comparison	

(Questi	ion	Answer	Marks	Guida	nce
8	(i)		$\frac{x+a}{x} + \frac{x-2}{4} = 0$ $\Rightarrow 4x + 4a + x^2 - 2x = 0$	M1	Clear fractions on lhs	
			$\Rightarrow x^{2} + 2x = -4a$ $\Rightarrow x^{2} + 2x + 1 = 1 - 4a$ $\Rightarrow (x+1)^{2} = 1 - 4a$	M1	Collection of terms to a 3 term quadratic and attempt to complete the square	"Attempt" means make lhs include $x^2 + 2px + p^2$
				A1	Correct final form	
				[3]		
	(ii)		(Roots if) their $q \ge 0$ $\Rightarrow a \le \frac{1}{4}$	M1 A1	<pre>Soi. Allow use of > ft their q . correct inequality.</pre>	Allow = here only if ans is correct. Allow expansion of quadratic and use of discriminant
				[2]		
	(iii)		$(x+1)^2 = 5$ $\Rightarrow x = -1 \pm \sqrt{5}$	M1 A1	Substitute to obtain quadratic in form $(x+p)^2 = n$ Both required isw	Allow use of formula
				[2]		

(Quest	ion	Answer	Marks	Guidance	
9	(a)	(i)	$\left(\frac{4}{5}\right)^{10} = 0.107(4)$	M1 A1	Correct power and <i>p</i> Awrt isw	One term only
				[2]		
		(ii)	$\binom{10}{4} \left(\frac{1}{5}\right)^4 \left(\frac{4}{5}\right)^6 = 210 \times 0.00041943$ $= 0.088(08)$	M1 A1 A1	Includes correct powers and a coefficient 210 soi Awrt isw	
	(b)		Fixed number of trials Each trial has two outcomes Fixed probability for success Independent trials	[3] B1 B1	Any one correct Another correct	Ignore incorrect answers or other answers
				[2]		

(Juestion	Answer	Marks	Guidance
10	(i)	Because $AP = XP = GP$ (= radius)	B1	
			[1]	
	(ii)	Angle GAX is angle in semicircle and since	B1	Accept any valid method
		BA is vertical, XA must be horizontal.		
			[1]	
	(iii)	Finding AG = $120\cos 40$ (= 91.93) oe	M1	Distance of Y below AX =
		Finding depth of Y below G =80cos40	M1	200cos40 M1(correct triangle) M1
		(=61.28)		(correct ratio)
		Ht of Y above ground = $200 - AG - depth$	M1	Need not be numeric $= 153.2$ cm
		of Y below G		So height above ground $= 200 -$
		=46.8 cm	A1	153.2 M1
				= 46.8 cm A1
			[4]	

Section B

Q	uestio	n Answer	Marks	Guidan	ce
11	(i)	$x^{2} + (y - 3)^{2} = 9$	M1		
		$\Rightarrow x^2 + y^2 - 6y + 9 = 9$			
		$\Rightarrow x^2 + y^2 - 6y = 0$	A1	Isw	
		(k=6)			
			[2]		
	(ii)	They meet when $x^{2} + (mx-2)^{2} - 6(mx-2) = 0$	M1	Substitute line into <i>their</i> curve	Or substitute for <i>x</i> to give quadratic in <i>y</i> :
		$\Rightarrow (1+m^2)x^2 - 10mx + 16 = 0 \qquad \mathbf{oe}$	A1	ft Allow bracket expanded	$y^{2}(1+m^{2}) + y(4-6m^{2}) + 4 = 0$
		Tangent if coincident roots	M1	Dep. Attempt to find coincident roots	$\Rightarrow 36m^4 - 64m^2 = 0$
		$\Rightarrow (-10m)^2 = 4.16(1+m^2) \qquad \text{oe}$	A1	using " b^2-4ac "	
		$\Rightarrow 36m^2 = 64$	A1		
		$\Rightarrow m = \pm \frac{4}{3}$	A1		
			[6]		
	(iii)	In triangle PCA, $PC = 5$	B1	soi	
		CA = 3	B1	soi	
		By Pythagoras:	M1		
		PA = PB = 4	A1	Both	
			[4]		

Question		Answer	Marks	Guidance	
12	(i)	p = 2y + 6x + AE + DE $AE = DE = 5x$	M1 B1	Adding www soi	N./B. $\sqrt{3^2 x + 4^2 x} = \sqrt{25x} = 5x$ or $\sqrt{3x^2 + 4x^2} = \sqrt{25x^2} = 5x$
		Giving $p = 2y + 6x + 5x + 5x$ p = 2y + 16x	A1	www AG algebra must be correct	could earn M1 B0 A0
		$p - 2y + 10\lambda$	[3]		
	(ii)	$A = 6xy + 12x^2$	M1 M1	Calculate the area Substitute correct	
		$= 3x(96-16x) + 12x^{2}$ = 288x - 36x^{2}		expression for y	
		$= 288x - 36x^{-1}$	A1	AG	
			[3]		
	(iii)	$A = 288x - 36x^{2}$ $\Rightarrow \frac{dA}{dx} = 288 - 72x$ $= 0 \text{ when } x = \frac{288}{72} = 4$ $\Rightarrow A = 288 \times 4 - 36 \times 16 = 576 \text{ cm}^{2}$ $\Rightarrow y = \frac{96 - 64}{2} = 16$ Alternatively: $A = 288x - 36x^{2} = 36(8x - x^{2}) = 36(16 - (16 - 8x + x^{2})) \text{ M1 A1}$ $= 36(16 - (x - 4)^{2})$ which has its greatest value when $x = 4$ M1 A1 $\Rightarrow A = 36 \times 16 = 576$ Alternatively: $A = 36 \times $	M1 A1 M1 A1 A1 A1	Diffn - reduce each power by 1 Set = 0 x area y	SC Graph of fn goes through (0,0) and (8,0) so being quadratic means max value at <i>x</i> = 4 B4 Area B1 <i>y</i> value B1 Other symmetrical points may be used.
		y = 48 - 8x = 16 A1			
			[6]		

Question		on	Answer	Marks	Guidance		
13	(i)		$AC^2 = 25 + 4 - 2.2.5 \cos 40$	M1	Cos formula		
			=13.68	A1	Correct subs soi		
			\Rightarrow AC = 3.70 km	A1	AC^2 soi		
			\rightarrow AC = 5.70 KIII	A1	cao		
				[4]			
	(ii)		$\frac{5}{3}$ hrs(=100 mins)			J = 100 mins	
			$3^{\text{ms}(-100\text{ mms})}$	B1		$\mathbf{B} = 111 \text{ mins}$	
			$\frac{3.7}{2}$ hrs (=111 mins)	B1			
			$\frac{1}{2}$ ms(-111 mms)	DI			
			111 - 100 = 11 mins	B1	www AG		
				D1			
				[3]			
	(iii)		$\frac{\sin\theta}{\sin\theta} = \frac{\sin 40}{\cos\theta}$ oe				
			$\frac{1}{3} = \frac{1}{2}$ oe	M1	Sin rule with denominators in	Solution using cosine rule	
			3		proportion 3:2	acceptable.	
			$\Rightarrow \sin \theta = \frac{3}{2} \sin 40 \ (= 0.9642)$	A1	Soi		
			$\Rightarrow \theta = 74.6^{\circ}$				
				A1	One angle awrt	Alternatively 2nd on ele	
			Bearing 345 [°]	A1	Correct bearing awrt	Alternatively 2nd angle	
			or 105.4°	A1	2nd angle plus bearing A1awrt	Then both bearings	
			Bearing 015 [°]		(Allow 15°) awrt	Then both bearings	
			Dearing 015				
				[5]			
				[ວ]			

Question		ion	Answer	Marks	Guidance		
14	(a)	(i)	$s = \frac{1}{2}2t^2 \left(=t^2\right)$	B1			
				[1]			
		(ii)	90 km h ⁻¹ = 25 m s ⁻¹ or $2ms^{-2} = 25920$ km hr ⁻²	B1	Units must be given - others are possible	Beware mixing of units which could give 12.5	
			$v = 2t \Longrightarrow 25 = 2t$	M1	Application of $v = u + at$ with consistent units		
			$\Rightarrow t = 12.5 \text{ secs}$	A1	Units must be given		
				[3]			
	(b)	(i)	When $t = 10 \ s_P = 100$ For Q:	B1	Seen anywhere		
			a = 1 + kt	M1	Integrating wrt <i>t</i> - both powers increased by 1		
			$\Rightarrow v = t + \frac{kt^2}{2}$	A1	Must not include c .		
			$\Rightarrow s_Q = \frac{t^2}{2} + \frac{kt^3}{6}$	A1	Must not include $ct + d$.		
			At $t = 10 \ s_Q = 50 + \frac{1000k}{6} = 100$				
			$\Rightarrow k = \frac{300}{1000} = 0.3$	A1			
				[5]			
		(ii)	When $t = 12.5$				
			$v_Q = t + \frac{kt^2}{2} = 12.5 + \frac{0.3 \times 12.5^2}{2}$	M1	Inserting <i>their t</i> and <i>their k</i> into velocity eqn		
			$= 35.94 \text{ m s}^{-1}$	A1			
			$= 35.94 \times \frac{60 \times 60}{1000} = 129.4 \text{ km h}$	A1	AG		
				[3]			

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