Level 2 Certificate
FURTHER MATHEMATICS
8365/2

Paper 2 Calculator

Mark scheme

June 2023

Version: 1.0 Final

2 3 6 G 8 3 6 5 / 2 / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

M	Method marks are awarded for a correct method which could lead to a correct answer.
M dep	A method mark dependent on a previous method mark being awarded.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
ft	Follow through marks. Marks awarded following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
oe	Or equivalent. Accept answers that are equivalent. eg accept 0.5 as well as $\frac{1}{2}$
[a, b]	Accept values between *a* and *b* inclusive.
3.14…	Accept answers which begin 3.14 eg 3.14, 3.142, 3.1416

Examiners should consistently apply the following principles.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a candidate has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the candidate. In cases where there is no doubt that the answer has come from incorrect working then the candidate should be penalised.

Questions which ask candidates to show working

Instructions on marking will be given but usually marks are not awarded to candidates who show no working.

Questions which do not ask candidates to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Candidates often copy values from a question incorrectly. If the examiner thinks that the candidate has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Continental notation

Accept a comma used instead of a decimal point (for example, in measurements or currency), provided that it is clear to the examiner that the candidate intended it to be a decimal point.

Q	Answer	Mark	Comments
1	$2(8d-3)$ or $5(3d-7)$ or $\frac{5}{2}(3d-7)$ or $\frac{2}{5}(8d-3)$	M1	oe eg $16d-6$ or $15d-35$ may be seen in an equation
	$16d-6=15d-35$ or $8d-3=\frac{15}{2}d-\frac{35}{2}$ or $\frac{16}{5}d-\frac{6}{5}=3d-7$	M1dep	oe equation with brackets expanded
	-29	A1	
	Additional Guidance		
	Up to M2 may be awarded for correct work with no answer or incorrect answer, even if this is seen amongst multiple attempts		
	Missing brackets must be recovered		
	Embedded answer		M2A0
	$2\left(\frac{8d-3}{3d-7}\right)=5$ with no further correct work		M0

Q	Answer	Mark	Comments
2(a)	$3.5n + 11.5$	B2	oe eg $\frac{7n}{2} + \frac{23}{2}$ or $15 + (n-1) \times 3.5$ B1 linear expression with 3.5 oe as coefficient of n
	Additional Guidance		
	For B1 linear expressions may or may not have a constant term eg1 $3.5n$ eg2 $15 + 3.5(n + 1)$ ($3.5n$ implied)		 B1 B1
	Do not allow the n term to have the coefficient after n for B2 eg1 $n3.5 + 11.5$ eg2 $n3.5 + 12$		 B1 B1
	Allow unambiguous notation eg $T_n = n \times 3.5 + 11.5$		 B2
	Condone $n =$ or $= 0$ eg1 $n = 3.5n + 11.5$ eg2 $3.5n + 12.5 = 0$		 B2 B1
	Condone use of a different variable eg N or x		
	Only identifying common difference of 3.5		B0

<table>
<tr><th>Q</th><th>Answer</th><th>Mark</th><th>Comments</th></tr>
<tr><td rowspan="11">2(b)</td><td>–6</td><td>B2</td><td>B1 $318 - 9n < 0$ oe inequality
or $318 - 9n \leqslant -1$ oe inequality
or
$318 \div 9$ oe calculation
or $319 \div 9$ oe calculation
or $35\frac{1}{3}$ or 35.3(…)
or $35\frac{4}{9}$ or 35.4(…)
or 36(th)</td></tr>
<tr><td colspan="3">Additional Guidance</td></tr>
<tr><td colspan="2">B1 may be awarded for correct work with no answer or incorrect answer, even if this is seen amongst multiple attempts</td><td></td></tr>
<tr><td colspan="2">Answer –6</td><td>B2</td></tr>
<tr><td colspan="2">Condone $n =$
eg1 $n = -6$
eg2 $n = 35\frac{1}{3}$</td><td>
B2
B1</td></tr>
<tr><td colspan="2">Condone 36(th) and –6 on answer line</td><td>B2</td></tr>
<tr><td colspan="2">–6 in working with 36(th) on answer line</td><td>B1</td></tr>
<tr><td colspan="2">Calculations or values that score B1 may be seen in an equation or an inequality (even if incorrect)
eg1 $n = 106 \div 3$
eg2 $n < 35\frac{1}{3}$</td><td>
B1
B1</td></tr>
</table>

Q	Answer	Mark	Comments
3	$(t =)\ 3 \times 1 + 5 \times 4$ or $(t =)\ 23$ or $u \times 1 + 2 \times 4$ or $u + 8$ or $u = -2$	M1	oe may be seen in a matrix or an equation eg $\begin{pmatrix} 23 \\ ... \end{pmatrix}$ or $u + 8 = 6$
	$t = 23$ and $u = -2$	A1	SC1 $t = -2$ and $u = 23$
	Additional Guidance		
	Condone both answers unambiguously seen in working but transposed on answer line		M1A1

Q	Answer	Mark	Comments
4	**Alternative method 1** Works out r and uses it to work out gradient expression for PQ		
	$\frac{1+r}{2} = 5$ or $(r =)\ 5 \times 2 - 1$ or $(r =)\ 1 + 4 + 4$ or $(r =)\ 9$	M1	oe equation or calculation eg $\frac{r-1}{2} + 1 = 5$ or $(r =)\ 5 + 5 - 1$ may be seen on a sketch implied by $Q(9, 6)$ implied by correct gradient expression in k
	$\frac{6-k}{\text{their } 9-1}$ or $\frac{6-\frac{6+k}{2}}{\text{their } 9-5}$	M1	oe expression their 9 must be a value their 9 cannot be 1 or 5
	$\frac{6-k}{\text{their } 9-1} = 2$ or $\frac{6-\frac{6+k}{2}}{\text{their } 9-5} = 2$	M1dep	oe equation eg $\frac{k-6}{1-\text{their } 9} = 2$ or $6 - k = 16$ or $\frac{6+k}{2} = -8 + 6$ dep on 2nd M1
	-10	A1	

Question 4 continues on the next page

Q	Answer	Mark	Comments
4 cont	**Alternative method 2** Works out r and uses it to work out equation of line PQ		
	$\frac{1+r}{2} = 5$ or $(r =)\ 5 \times 2 - 1$ or $(r =)\ 1 + 4 + 4$ or $(r =)\ 9$	M1	oe equation or calculation eg $\frac{r-1}{2} + 1 = 5$ or $(r =)\ 5 + 5 - 1$ may be seen on a sketch implied by $Q(9, 6)$
	$6 = 2 \times$ their $9 + c$ or $c = -12$ or $y = 2x - 12$	M1	oe their 9 must be a value their 9 cannot be 1 or 5
	$2 \times 1 + 6 - 2 \times$ their 9	M1dep	oe eg $2 - 12$ dep on 2nd M1
	-10	A1	
	Alternative method 3 Works out r and uses it to work out difference in y-coordinates of P and Q		
	$\frac{1+r}{2} = 5$ or $(r =)\ 5 \times 2 - 1$ or $(r =)\ 1 + 4 + 4$ or $(r =)\ 9$	M1	oe equation or calculation eg $\frac{r-1}{2} + 1 = 5$ or $(r =)\ 5 + 5 - 1$ may be seen on a sketch implied by $Q(9, 6)$ implied by difference in x-coordinates of P and Q is 8
	$(y_Q - y_P =)\ 2 \times$ (their $9 - 1$) or 16	M1	oe eg 2×8 may be seen on a sketch as difference of y-coordinates of P and Q their 9 must be a value their 9 cannot be 1 or 5
	$6 - 2 \times$ (their $9 - 1$)	M1dep	oe eg $6 - 16$ dep on 2nd M1
	-10	A1	

Question 4 continues on the next page

Q	Answer	Mark	Comments
4 cont	**Alternative method 4** Works out difference in y-coordinates of P and midpoint (M) and uses it to work out y-coordinate of M		
	$(y_M - y_P =)\ 2 \times (5 - 1)$ or 8	M1	oe eg 2×4 implied by difference in y-coordinates of P and M is 8 seen on a sketch
	(y-coordinate of $M =$) $6 - 2 \times (5 - 1)$ or -2	M1dep	oe eg $6 - 8$ may be seen on a sketch
	their $-2 - 2 \times (5 - 1)$	M1dep	oe eg $-2 - 8$ or $6 - 2 \times (5 - 1) - 2 \times (5 - 1)$
	-10	A1	
	Alternative method 5 Works out an expression for y-coordinate of midpoint (M) and uses it to work out gradient expression for PM		
	(y-coordinate of $M =$) $\frac{6+k}{2}$	M1	oe may be seen on a sketch
	$\frac{\frac{6+k}{2} - k}{5-1}$	M1dep	oe expression eg $\frac{k - \frac{6+k}{2}}{1-5}$ or $\frac{6-k}{8}$
	$\frac{\frac{6+k}{2} - k}{5-1} = 2$	M1dep	oe equation eg $6 - k = 16$
	-10	A1	
	Additional Guidance		
	Up to M3 may be awarded for correct work with no answer or incorrect answer, even if this is seen amongst multiple attempts		
	In all alts a fully correct 2nd M implies the 1st M		
	NOTE In Alts 4 and 5 the value of r is not needed		

Q	Answer	Mark	Comments
5	$2x^3$	M1	oe eg $4 \times 0.5x^{4-1}$
	$x^3 = \frac{6.75}{2}$ or $x^3 = 3.375$ or $\sqrt[3]{3.375}$	M1dep	oe eg $\frac{27}{8} = x^3$ or $\sqrt[3]{\frac{27}{8}}$
	1.5 or $\frac{3}{2}$	A1	oe value eg $1\frac{1}{2}$
	Additional Guidance		
	Up to M1 may be awarded for correct work with no answer or incorrect answer, even if this is seen amongst multiple attempts		
	Condone incorrect notation eg $y = 2x^3$		1st M1
	Ignore higher derivatives		
	Ignore attempt at y-coordinate		
	$2x^3 = 6.75$ or $8x^3 = 27$ with no further correct work		M1M0

Q	Answer	Mark	Comments
6	(–7, 4)	B1	
	6	B1	ignore units do not allow $\sqrt{36}$ or ± 6
	Additional Guidance		
	(–7, +4)		1st B1
	$\sqrt{36}$ and 6		2nd B1

Q	Answer	Mark	Comments
7(a)	5	B1	condone (5, 0) but not (0, 5)
	Additional Guidance		
	Condone 5 or (5, 0) next to p on sketch if answer line blank		B1

Q	Answer	Mark	Comments
7(b)	Either of $x < -4$ $x >$ their 5	M1	oe eg $-4 > x$ correct or ft their 5 in (a) their 5 in (a) must be greater than 0.5 condone $\leqslant$ for $<$ condone $\geqslant$ for $>$
	Both of $x < -4$ $x >$ their 5	A1ft	correct or ft their 5 in (a) their 5 in (a) must be greater than 0.5 must be two separate inequalities
	Additional Guidance		
	Ignore the use of or/and eg1 $x < -4$ or $x > 5$ eg2 $x < -4$ and $x > 5$		 M1A1 M1A1
	Condone embedded even if in an invalid statement for M1 but not A1 eg1 $-4 > x > 5$ eg2 $-4 > x > 6$		 M1A0 M1A0
	If answer to (a) is 4 $x < -4$ $x > 4$		 M1A1ft
	If answer to (a) is blank eg1 $x < -4$ eg2 $x < -4$ $x > 5$		 M1A0 M1A1
	$-4 < x < 5$		M0A0
	If answer to (a) is eg (0, 4) take their 5 to be 4		
	Condone eg $x = < -4$ for M1 but not A1		
	Allow eg $(-\infty, -4)$ for $x < -4$ (condone eg $[-\infty, -4)$ for M1 but not A1)		

Q	Answer	Mark	Comments
8	**Alternative method 1**		
	$\frac{1}{2} \times BC \times 4 = 25$ or $(BC =)\ 25 \times 2 \div 4$ or $(BC =)\ 12.5$	M1	oe eg $\frac{1}{2} \times BD \times 4 + \frac{1}{2} \times DC \times 4 = 25$ allow any unambiguous indication of *BC* or *BD* or *DC* may be seen on diagram
	$(DC =)\ \frac{3}{3+2} \times$ their 12.5 or 7.5	M1dep	oe eg $(DC =)$ their $12.5 \div 5 \times 3$ may be seen on diagram
	$\tan w = \frac{4}{\text{their } DC}$ or $\tan w = \frac{8}{15}$ or $\tan^{-1} 0.53(\ldots)$	M1	oe eg $\sin w = \frac{4}{\sqrt{4^2 + (\text{their } DC)^2}}$ or $\cos w = \frac{\text{their } DC}{\sqrt{4^2 + (\text{their } DC)^2}}$ or $90 - \tan^{-1} \frac{\text{their } DC}{4}$
	[27.9, 28.4]	A1	SC3 [46.6, 46.94] or 47
	Alternative method 2		
	Expression for area = 25 with *BD* : *DC* = 2 : 3	M1	eg $\frac{1}{2} \times 2x \times 4 + \frac{1}{2} \times 3x \times 4 = 25$ or $\frac{1}{2} \times y \times 4 + \frac{1}{2} \times \frac{3}{2}y \times 4 = 25$ implied by *BD* = 5 may be seen on diagram
	Method to work out *DC* or $(DC =)\ 7.5$	M1dep	eg $(DC =)\ 2.5 \times 3$ or $(DC =)\ 5 \times \frac{3}{2}$ may be seen on diagram
	$\tan w = \frac{4}{\text{their } DC}$ or $\tan w = \frac{8}{15}$ or $\tan^{-1} 0.53(\ldots)$	M1	oe eg $\sin w = \frac{4}{\sqrt{4^2 + (\text{their } DC)^2}}$ or $\cos w = \frac{\text{their } DC}{\sqrt{4^2 + (\text{their } DC)^2}}$ or $90 - \tan^{-1} \frac{\text{their } DC}{4}$
	[27.9, 28.4]	A1	SC3 [46.6, 46.94] or 47

Question 8 continues on the next page

	Additional Guidance	
8 cont	Up to M3 may be awarded for correct work with no answer or incorrect answer, even if this is seen amongst multiple attempts	
	$BC = 12$ (no working seen) $\quad \frac{3}{5} \times 12 = 7.2$ $\tan w = \frac{4}{7.2} \quad w = 29.1$	M0M0 M1A0
	SC3 is from omitting the $\frac{1}{2}$ from area of triangle formula	
	Alt 1 Using $\frac{2}{3+2}$ instead of $\frac{3}{3+2}$ is not a misread	2nd M0
	3rd M1 oe must be of the form $\tan w =$ (or $\sin w =$ or $\cos w =$) or be a calculation that evaluates to [27.9, 28.4] eg applying cosine rule to triangle *BAC* $\cos w = \frac{12.5^2 + 72.25 - 41}{2 \times 12.5 \times \sqrt{72.25}}$	M3
	3rd M1 Allow $\sin w = 0.47(\ldots)$ or $\cos w = 0.88(\ldots)$ or $90 -$ [61.6, 62.1]	M3
	3rd M1 Condone their $DC = 3$ if not contradicted	

Q	Answer	Mark	Comments
9(a)	Expression for length of all edges = 300 or expression for length of the 3 given edges = $\frac{300}{4}$	M1	must have terms in x and y on one side and 300 or $\frac{300}{4}$ on the other side eg $4(3x + 2x + x + 2y) = 300$ or $12x + 8x + 4x + 8y = 300$ or $24x + 8y = 300$ or $6x + 2y = \frac{300}{4}$ do not award M1 if only seen from working back from $y = \frac{75 - 6x}{2}$ only $6x + 2y = 75$ is M0
	M1 seen and shows correct simplification to $y = \frac{75 - 6x}{2}$	A1	must see at least one intermediate equation after M1 with terms in x and y fully simplified and no incorrect equations seen

Question 9(a) continues on the next page

9(a) cont	**Additional Guidance**	
	Condone eg $\frac{8y = 300 - 24x}{8}$ as not being an incorrect equation	
	Missing brackets must be recovered	
	Ignore units and working involving areas or volumes	
	$12x + 8x + 4x + 8y = 300$ $8y = 300 - 24x$ $y = \frac{75 - 6x}{2}$	M1 A1
	$24x + 8y = 300$ $y = \frac{75 - 6x}{2}$	M1 A0
	$24x + 8y = 300$ $8y = 300 + 24x$ (incorrect equation seen) $y = \frac{75 - 6x}{2}$	M1 A0
	NOTE $y = \frac{75 - 6x}{2}$ may be seen with no correct working eg $6x + 2y = 300$ $y = \frac{75 - 6x}{2}$	M0A0
	$12x + 8x + 4x + 8y = 300$ $8y = 300 - 24x$ $y = 37.5 - 3x$ and $\frac{75 - 6x}{2} = 37.5 - 3x$	M1 A1

Q	Answer	Mark	Comments
9(b)	$3x \times 2x \times (x + 75 - 6x)$ or $6x^2(75 - 5x)$	M1	oe expression only in terms of x eg $3x \times 2x \times (75 - 5x)$ or $6x^3 + 12x^2 \times \frac{75 - 6x}{2}$
	M1 seen and $450x^2 - 30x^3$ with no incorrect expressions seen	A1	
	Additional Guidance		
	For M1 the expression cannot be $450x^2 - 30x^3$		
	$V =$ is not required		
	M1 may be seen in stages		
	Missing brackets must be recovered before the final answer eg1 $3x \times 2x \times x + 2y$ $6x^3 + 12x^2 \times \frac{75 - 6x}{2} = 450x^2 - 30x^3$ eg2 $6x^2 \times 75 - 5x = 450x^2 - 30x^3$	 M1A1 M0
	Using the given answer and working back	M0	
	$6x^2(x + 2y)$ with no further correct work	M0	
	$6x^2(75 - 6x)$	M0	
	NOTE $450x^2 - 30x^3$ may be seen with no correct working eg $6x(75 - 5x) = 450x^2 - 30x^3$	 M0A0	

Q	Answer	Mark	Comments
9(c)	$900x$ or $-90x^2$	M1	oe eg $2 \times 450x^{2-1}$ or $3 \times -30x^{3-1}$
	their $(900x - 90x^2) = 0$	M1dep	oe must have exactly two terms
	15 000 with $900x - 90x^2$ seen	A1	ignore units SC1 15 000
	Additional Guidance		
	Up to M2 may be awarded for correct work with no answer or incorrect answer, even if this is seen amongst multiple attempts		
	Condone incorrect notation eg $V = 900x - 90x^2$		1st M1
	Answer $x = 10$ with $900x - 90x^2$ and 15 000 in working lines		M2A0
	Answer $x = 10$ with 15 000 in working lines		M0
	Condone (10, 15 000) for 15 000		
	Ignore higher derivatives		
	Ignore any work investigating the nature of the 'turning point'		

Q	Answer	Mark	Comments
10	$y = \frac{4}{5}x \ldots$ or $\frac{16-6}{-5-3}$	M1	oe eg $y = \frac{-4}{-5}x \ldots$ or $\frac{6-16}{3--5}$ equation of K must have y as the subject but the intercept can be ignored allow eg $y = \frac{4x-17}{5}$
	(gradient of K =) $\frac{4}{5}$ or (gradient of L =) $-\frac{5}{4}$	M1dep	oe value eg (gradient of K =) 0.8 do not allow a gradient if only seen embedded in an equation unless unambiguously selected (eg circled) condone inclusion of x
	Both gradients correct and valid explanation and middle box ticked	A1	eg $\frac{4}{5} \times \frac{10}{-8} = -1$ and middle box ticked do not allow a gradient if only seen embedded in an equation unless unambiguously selected (eg circled) do not allow inclusion of x in either gradient

Additional Guidance

0.8 and –1.25 and states gradients multiply to make –1 and middle box ticked	M1M1A1
$\frac{4}{5}$ and $-\frac{5}{4}$ and states negative reciprocals and middle box ticked	M1M1A1
Condone eg $\frac{10}{-8}$ for the value $-\frac{5}{4}$	
M1 may be embedded in equation of line L eg $y - 6 = \frac{16-6}{-5-3}(x-3)$	M1
(gradient of K =) $\frac{4}{5}x$ (gradient of L =) $-\frac{5}{4}x$ $\frac{4}{5} \times -\frac{5}{4} = -1$ and middle box ticked (recovers to use numerical gradients)	M1M1 A1
(gradient of K =) $0.8x$ (gradient of L =) –1.25 $0.8x \times -1.25 = -1$ (includes x)	M1M1 A0

Q	Answer	Mark	Comments
11	3 correct from $6x^5 + 8x^3 - 27x^2 - 36$	M1	expansion of $(2x^3 - 9)(3x^2 + 4)$ may be seen in a grid
	$6x^5 + 8x^3 - 27x^2 - 36$	M1dep	may be seen in a grid
	$x^3 - 4x^2 - 4x^2 + 16x$ or $x^3 - 8x^2 + 16x$	M1	expansion of $x(x-4)^2$ may be seen in a grid
	$6x^5 + 9x^3 - 35x^2 + 16x - 36$	A1	
	Additional Guidance		
	A correct term includes the sign (in a grid allow eg $8x^3$ for $+8x^3$)		
	Terms may be in any order throughout		
	Terms must be fully processed eg do not allow $2x^3 \times 3x^2$ for $6x^5$		
	Allow eg $+ -27x^2$ for $-27x^2$ for M marks		
	$6x^5 + 8x^3 - 27x^2 - 36$ scores the 1st M1 and 2nd M1 even if incorrectly simplified (or differentiated) eg $6x^5 + 8x^3 - 27x^2 - 36 = 6x^5 - 19x^2 - 36$		M1M1
	$x^3 - 4x^2 - 4x^2 + 16x$ (or $x^3 - 8x^2 + 16x$) scores the 3rd M1 even if incorrectly simplified (or differentiated) eg $x^3 - 4x^2 - 4x^2 + 16x = x^3 - 16x^2 + 16x$		3rd M1
	Correct answer then incorrect further work eg differentiation		M3A0
	$6x^5 + 9x^3 - 35x^2 + 16x - 36 = x(6x^4 + 9x^2 - 35x + 16) - 36$		M3A0

Q	Answer	Mark	Comments
12	$(AC =)\ \sqrt{15^2 + 15^2}$ or $\sqrt{450}$ or $15\sqrt{2}$ or 21.2(…) or $(AX =)\ \sqrt{7.5^2 + 7.5^2}$ or $\sqrt{112.5}$ or $\frac{15\sqrt{2}}{2}$ or [10.6, 10.61]	M1	oe eg $(DB =)\ \sqrt{225 + 225}$ or $(BX =)\ \sqrt{56.25 + 56.25}$ or $(AX =)\ \sqrt{\frac{15^2}{2}}$ or $(AX =)\ 15\sin 45$ allow any unambiguous indication of side may be seen on diagram
	$\cos y = \frac{0.5\sqrt{15^2 + 15^2}}{28}$ or $\cos y = \frac{\sqrt{7.5^2 + 7.5^2}}{28}$ or $\cos y = \frac{15\sqrt{2}}{56}$ or $\cos^{-1}$ [0.37, 0.38]	M1dep	oe eg $\sin y = \frac{\sqrt{28^2 - 7.5^2 - 7.5^2}}{28}$ or $\tan y = \frac{\sqrt{28^2 - 7.5^2 - 7.5^2}}{\sqrt{7.5^2 + 7.5^2}}$ or $90 - \sin^{-1}\frac{\sqrt{7.5^2 + 7.5^2}}{28}$
	[66.9, 68.435]	A1	SC2 [21.565, 23.1]
	Additional Guidance		
	Up to M2 may be awarded for correct work with no answer or incorrect answer, even if this is seen amongst multiple attempts		
	M1dep y may be x or *A* or *VAX* etc		
	M1dep oe must be of the form $\cos y =$ (or $\sin y =$ or $\tan y =$) or be a calculation that evaluates to [66.9, 68.435] eg applying cosine rule to triangle *VAC* $\cos y = \frac{28^2 + 450 - 28^2}{2 \times 28 \times \sqrt{450}}$		M2
	M1dep allow $\sin y =$ [0.92, 0.93] or $\tan y =$ [2.44, 2.45] or 90 – [21.565, 23.1]		M2

Q	Answer	Mark	Comments
13(a)	$\frac{3}{x^7}$	B1	

Q	Answer	Mark	Comments
13(b)	$\frac{3w^2}{4}$ or $0.75w^2$	B2	B1 unsimplified equivalent fraction with brackets processed eg $\frac{12w^8}{16w^6}$ or $\frac{12w^2}{16}$ or $\frac{6w^2}{8}$ or $\frac{3w^5}{4w^3}$ SC1 $\frac{3w^3}{4}$ or $0.75w^3$ or $3w^2$
	Additional Guidance		
	B1 may be awarded for correct work with no answer or incorrect answer, even if this is seen amongst multiple attempts		
	$\frac{3}{4}w^2$ is equivalent to $\frac{3w^2}{4}$ etc		

Q	Answer	Mark	Comments
13(c)	$c = 6$ and $d = 5$	B3	B2 $y^{\frac{5k}{6k}}$ or $\sqrt[6k]{y^{5k}}$ or $\left(y^{5k}\right)^{\frac{1}{6k}}$ or $\left(y^{\frac{1}{6k}}\right)^{5k}$ where k is an integer $\geqslant 1$ or $\left(\frac{1}{2}+\frac{1}{3}=\right)\frac{5}{6}$ oe fractions or $\left(\frac{c}{2}+\frac{c}{3}=\right)\frac{5c}{6}$ oe fractions or $5c = 6d$ oe equation B1 $y^{\frac{1}{2}}$ or $y^{\frac{1}{3}}$ or $y^{\frac{c}{2}}$ or $y^{\frac{c}{3}}$ or $\left(y^{c}\right)^{\frac{1}{2}}$ or $\left(y^{c}\right)^{\frac{1}{3}}$ or $y^{\frac{d}{c}}$ or $\left(y^{\frac{1}{c}}\right)^{d}$ or $\left(y^{d}\right)^{\frac{1}{c}}$ or $\frac{1}{2}+\frac{1}{3}$ oe sum of fractions or $\frac{c}{2}+\frac{c}{3}$ oe sum of fractions SC2 $c = 6k$ and $d = 5k$ where k is an integer $\geqslant 2$

Question 13(c) continues on the next page

13(c) cont	**Additional Guidance**	
	Up to B2 may be awarded for correct work with no answer or incorrect answer, even if this is seen amongst multiple attempts	
	$\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$	B2
	$\frac{3}{6}+\frac{2}{6}$	B1
	$\frac{10}{12}$ or $y^{\frac{10}{12}}$	B2
	$c=18$ and $d=15$	SC2
	$\frac{1}{2}$ may be 0.5	
	Only trialling numerical values for y is B0 or B3	

Q	Answer	Mark	Comments
14	Simplifies $15a^2$ and $3a$ to $5a$ or $\frac{120a^2 - 60a^3}{3a^3 + 18a^2 - 48a}$	M1	eg seen as cancelling or single fraction with numerator and denominator expanded may be implied
	Correct numerator with factor $(a - 2$ #) or $(2 - a)$	M1	eg $(8 - 4a =) 4(2 - a)$ or $-4(a - 2)$ or $(40a - 20a^2 =) 20a(2 - a)$ or $(120a^2 - 60a^3 =) 60a^2(2 - a)$ or $(40a^2 - 20a^3 =) 20a^2(2 - a)$ or $(120a - 60a^2 =) -60a(a - 2)$
	Correct denominator with factor $(a - 2)$ or $(2 - a)$	M1	eg $(a^2 + 6a - 16 =) (a + 8)(a - 2)$ or $(-a - 8)(2 - a)$ or $(a^3 + 6a^2 - 16a =) a(a + 8)(a - 2)$ or $a(-a - 8)(2 - a)$ or $(3a^3 + 18a^2 - 48a =) 3a(a + 8)(a - 2)$ or $(3a^2 + 24a)(a - 2)$ or $(3a^2 + 18a - 48 =) 3(a + 8)(a - 2)$ or $(3a + 24)(a - 2)$
	$-\frac{20a}{a+8}$	A1	oe simplest form eg $\frac{-20a}{a+8}$ or $\frac{20a}{-a-8}$

Question 14 continues on the next page

14 cont	**Additional Guidance**	
	Up to M3 may be awarded for correct work with no answer or incorrect answer, even if this is seen amongst multiple attempts	
	NOTE 'crossed out' work may be the student 'cancelling'	
	2nd M1 denominator may be incorrect or missing	
	3rd M1 numerator may be incorrect or missing	
	2nd M1 and 3rd M1 may not necessarily be seen in a fraction	
	Correct answer followed by further incorrect work	M3A0

Q	Answer	Mark	Comments
15	$\frac{1}{2} = a \times b^0$ or $a = \frac{1}{2}$ or $\frac{3}{2} = a \times b^{(1)}$	M1	oe eg $a \times 1 = 0.5$ or $ab = 1.5$
	$(a =) \frac{1}{2}$ and $(b =) 3$	A1	oe values may be implied eg $y = \frac{1}{2} \times 3^x$
	their $\frac{1}{2}$ × (their 3)$^{-1}$ or $\frac{1}{2} \times \frac{1}{3}$ or $\frac{1}{6}$ **and** their $\frac{1}{2}$ × (their 3)^2 or $\frac{1}{2} \times 3^2$ or $\frac{9}{2}$	M1	oe neither their $\frac{1}{2}$ nor their 3 can be 0 or 1 $\frac{1}{6}$ and $\frac{9}{2}$ is M1A1M1 ignore other calculations or values
	$\frac{1}{6} \leqslant g(x) \leqslant \frac{9}{2}$ or both of $g(x) \geqslant \frac{1}{6}$ $g(x) \leqslant \frac{9}{2}$	A1ft	oe eg $\left[\frac{1}{6}, \frac{9}{2}\right]$ only ft M1A0M1

Question 15 continues on the next page

15 cont	**Additional Guidance**	
	Ignore words if answer given as separate inequalities eg1 $g(x) \geqslant \frac{1}{6}$ or $g(x) \leqslant \frac{9}{2}$ eg2 $g(x) \geqslant \frac{1}{6}$ and $g(x) \leqslant 4.5$	M1A1M1A1 M1A1M1A1
	$a = \frac{1}{2}$ and $b = 2$ $\frac{1}{4} \leqslant g(x) \leqslant 2$	M1A0 M1A1ft
	Condone $g(x)$ replaced by eg y or g or gx or $f(x)$ or f or fx or $a \times b^x$	
	Do not allow $g(x)$ to be replaced by x for the final mark eg $\frac{1}{6} \leqslant x \leqslant \frac{9}{2}$	M1A1M1A0
	Equivalent inequalities may be seen eg $\frac{9}{2} \geqslant g(x) \geqslant \frac{1}{6}$	M1A1M1A1
	Inequality symbols must be correct eg $\frac{1}{6} < g(x) \leqslant \frac{9}{2}$ or $\frac{1}{6} \leqslant g(x) \geqslant \frac{9}{2}$	M1A1M1A0
	$\frac{1}{6} \leqslant g(x) \leqslant \frac{9}{2}$ in working with list of integers on answer line	M1A1M1A0
	Only a list of integers	M0A0M0A0
	Allow 0.16(…) or 0.17 for $\frac{1}{6}$	
	Correct answer followed by incorrect further work eg $\frac{1}{6} \leqslant g(x) \leqslant \frac{9}{2}$ and $\left(\frac{9}{2} - \frac{1}{6} =\right) \frac{13}{3}$	M1A1M1A0

Q	Answer	Mark	Comments
16	$(2x-3)(3x^2-8x\ \ldots)$ or $(2x-3)(3x^2\ \ldots+2)$	M1	implied by algebraic division eg $2x-3\overline{)6x^3-25x^2+28x-6}$ $3x^2-8x\ \ldots$
	$3x^2-8x+2\ (=0)$ or $3x^2-8x=-2$	M1dep	
	$\frac{--8\pm\sqrt{(-8)^2-4\times3\times2}}{2\times3}$ **or** $\frac{4}{3}\pm\sqrt{\frac{10}{9}}$ **or** [2.38, 2.39] and [0.27, 0.28]	M1	oe use of formula or completing the square or factorising their 3-term quadratic eg $\frac{8\pm\sqrt{40}}{6}$ or $\frac{4}{3}\pm\frac{\sqrt{10}}{3}$ [2.38, 2.39] and [0.27, 0.28] implies M3 $\frac{8\pm\sqrt{40}}{6}$ oe implies M3
	$\frac{3}{2}$ and $\frac{4}{3}+\frac{\sqrt{10}}{3}$ and $\frac{4}{3}-\frac{\sqrt{10}}{3}$ with no other solutions	A1	oe values eg 1.5 and $\frac{8\pm\sqrt{40}}{6}$ **or** $1\frac{1}{2}$ and $\frac{4}{3}+\sqrt{\frac{10}{9}}$ and $\frac{4}{3}-\sqrt{\frac{10}{9}}$

Question 16 continues on the next page

<table>
<tr><td rowspan="12">16
cont</td><td colspan="2">Additional Guidance</td></tr>
<tr><td>Up to M3 may be awarded for correct work with no answer or incorrect answer, even if this is seen amongst multiple attempts</td><td></td></tr>
<tr><td>Up to the first two marks may be seen in a grid
eg
<table>
<tr><td></td><td>$3x^2$</td><td>$-8x$</td><td>$(+)2$</td></tr>
<tr><td>$2x$</td><td>$6x^3$</td><td>$-16x^2$</td><td>$4x$</td></tr>
<tr><td>-3</td><td>$-9x^2$</td><td>$24x$</td><td>-6</td></tr>
</table></td><td>M1M1</td></tr>
<tr><td>3rd M1 8^2 is equivalent to $(-8)^2$</td><td></td></tr>
<tr><td>3rd M1 Use of -8^2 for $(-8)^2$ must be recovered</td><td></td></tr>
<tr><td>3rd M1 Completing the square must get as far as numerical answers which must be correct for their quadratic</td><td></td></tr>
<tr><td>3rd M1 3-term quadratic means $ax^2 + bx + c$ with no zero coefficients</td><td></td></tr>
<tr><td>3rd M1 Allow correct substitution into quadratic formula for their 3-term quadratic even if it has no real solutions</td><td></td></tr>
<tr><td>3rd M1 can be implied by correct solutions of their 3-term quadratic truncated or rounded to 2 dp or better</td><td></td></tr>
<tr><td>A quadratic from differentiation can be used to score the 3rd M1</td><td></td></tr>
<tr><td>Non-exact solutions cannot score full marks
eg 1.5 and 2.38 and 0.28</td><td>M3A0</td></tr>
<tr><td>Non-exact solutions on answer line is A0</td><td></td></tr>
</table>

Q	Answer	Mark	Comments
17	$3ax^3 - 2ax$	M1	oe with brackets expanded may be seen in an inequality or equation
	$9ax^2 - 2a + 5$	M1	oe eg $3 \times 3ax^{3-1} - 2ax^0 + 5x^0$ differentiates their expression derivative must contain 3 terms may be seen in an inequality or equation
	$9ax^2 - 2a + 5 > 0$ or $9ax^2 - 2a + 5 \geqslant 0$	M1dep	oe eg $9ax^2 - 2a > -5$ must be correct implied by $-2a + 5 > 0$ or $-2a + 5 \geqslant 0$ with no incorrect working dep on M2
	$(0 <) a < \frac{5}{2}$ or $(0 <) a \leqslant \frac{5}{2}$	A1	oe eg $(0 <) a < 2\frac{1}{2}$ or $(0 <) a < 2.5$ or $(0 <) a \leqslant 2\frac{1}{2}$ or $(0 <) a \leqslant 2.5$

Question 17 continues on the next page

	Additional Guidance	
17 cont	Up to M3 may be awarded for correct work with no answer or incorrect answer, even if this is seen amongst multiple attempts	
	Condone incorrect notation eg $h = 9ax^2 - 2a + 5$	M1M1
	Working with $= 0$ does not score 3rd M unless recovered to the correct answer eg $3ax^3 - 2ax + 5x$ $\quad 9ax^2 - 2a + 5 = 0$ $\quad a = 2.5$ Answer $a < 2.5$	M1M1 M1A1
	Working with < 0 (or $\leqslant 0$) does not score 3rd M and cannot be recovered eg $3ax^3 - 2ax + 5x$ $\quad 9ax^2 - 2a + 5 < 0$ $-2a + 5 < 0$ $\quad$ Answer $a < 2.5$	M1M1M0 A0
	Correct answer may come from other incorrect methods eg $3ax^3 - 2ax + 5x$ $\quad -2ax + 5x > 0$ $-2a + 5 > 0$ $\quad$ Answer $a < \frac{5}{2}$	M1M0 M0A0
	Allow the first two marks for fully correct use of the product rule for differentiation (M0M0 if not fully correct) eg $ax \times 6x + (3x^2 - 2) \times a + 5$	M1M1
	$3ax^3 - 2ax + 5x$ $9x^2 - 2a + 5$	M1 M0
	$3ax^2 - 2ax + 5x$ $6ax - 2a + 5$	M0 M1
	$3ax^3 - 2ax$ $9ax^2 - 2a$ (their derivative does not contain 3 terms)	M1 M0
	Ignore higher derivatives	
	Allow interval notation eg (0, 2.5]	
	Condone $0 \leqslant a < 2.5$ or $0 \leqslant a \leqslant 2.5$	

Q	Answer	Mark	Comments
18(a)	$360° - \alpha$ and $360° + \alpha$ with no other answers	B2	B1 $360° - \alpha$ or $360° + \alpha$ with at most one other answer
	Additional Guidance		
	Allow any unambiguous indication eg crosses in the two correct boxes with all other boxes blank	B2	
	Mix of ticks and crosses – mark the ticks eg ticks only in the two correct boxes with crosses in some or all the other boxes	B2	
	A tick in one of the two correct boxes with crosses in some or all the other boxes	B1	
	Marks cannot be awarded from the diagram		

Q	Answer	Mark	Comments
18(b)	$90° + \alpha$	B1	

Q	Answer	Mark	Comments
19	**Alternative method 1** Eliminates y then uses substitution		
	Eliminates y	M1	eg $3x + 4(2kx) = k$ or $3x = k - 8kx$ or $\frac{k-3x}{4} = 2kx$ or $x = \frac{k}{3+8k}$
	Substitutes their x into one of the equations	M1dep	eg $y = 2k \times$ their $\frac{k}{3+8k}$ or $3 \times$ their $\frac{k}{3+8k} + 4y = k$
	$x = \frac{k}{3+8k}$ $y = \frac{2k^2}{3+8k}$	A1	
	Alternative method 2 Eliminates x then uses substitution		
	Eliminates x	M1	eg $y = 2k \times \frac{k-4y}{3}$ or $y = \frac{2k^2 - 8ky}{3}$ or $3 \times \frac{y}{2k} + 4y = k$ or $y = \frac{2k^2}{3+8k}$
	Substitutes their y into one of the equations	M1dep	eg their $\frac{2k^2}{3+8k} = 2kx$ or $3x + 4 \times$ their $\frac{2k^2}{3+8k} = k$
	$x = \frac{k}{3+8k}$ $y = \frac{2k^2}{3+8k}$	A1	

Question 19 continues on the next page

Q	Answer	Mark	Comments	
19 cont	**Alternative method 3** Eliminates y and eliminates x			
	Eliminates y	M1	eg $3x + 4(2kx) = k$ or $3x = k - 8kx$ or $\frac{k-3x}{4} = 2kx$ or $x = \frac{k}{3+8k}$	
	Eliminates x	M1	eg $y = 2k \times \frac{k-4y}{3}$ or $y = \frac{2k^2 - 8ky}{3}$ or $3 \times \frac{y}{2k} + 4y = k$ or $y = \frac{2k^2}{3+8k}$	
	$x = \frac{k}{3+8k}$ $y = \frac{2k^2}{3+8k}$	A1		
	Additional Guidance			
	Up to M2 may be awarded for correct work with no answer or incorrect answer, even if this is seen amongst multiple attempts			
	Correct answers followed by further incorrect work			M2A0
	Missing brackets must be recovered			
	Alt 1 M1 scored then rearrangement errors before substituting can score the second M mark if their x is in terms of k (their expression for x must be seen) eg $3x + 8kx = k$ $x = \frac{k}{3-8k}$ $y = \frac{2k^2}{3-8k}$			M1 M1depA0
	Alt 2 M1 scored then rearrangement errors before substituting can score the second M mark if their y is in terms of k (their expression for y must be seen) eg $y = 2k \times \frac{k-4y}{3}$ $y = \frac{2k}{3+8k}$ $x = \frac{2k}{3+8k} \div 2k$			M1 M1depA0
	A correct equation is needed for elimination marks to be awarded eg Alt 1 $3x + 4y = k$ $\underline{4y = 8kx}$ $-$ is M0 unless $3x = k - 8kx$ also seen			

Q	Answer	Mark	Comments
20	$2\sin x(\sin^2 x + \cos^2 x)$ or $\sin x(2\sin^2 x + 2\cos^2 x)$ or $2\sin x(1 - \sin^2 x)$ or $2\sin x(1 - \cos^2 x)$	M1	oe writing first two terms using $\sin^2 x + \cos^2 x$ or $2\sin^2 x + 2\cos^2 x$ or replacing $\cos^2 x$ with $(1 - \sin^2 x)$ in 2nd term or replacing $\sin^2 x$ with $(1 - \cos^2 x)$ in 1st term ignore other terms
	$5\dfrac{\sin x}{\cos x}$ $(\times)\cos x$ or $5\sin x$	M1	oe replacing $\tan x$ with $\dfrac{\sin x}{\cos x}$ in 3rd term ignore other terms
	$7\sin x$ with M2 seen	A1	allow $p = 7$ with M2 seen
	Additional Guidance		
	Up to M2 may be awarded for correct work with no answer or incorrect answer, even if this is seen amongst multiple attempts		
	M marks are independent and can be scored in either order eg $2\sin^3 x + 2\sin x\cos^2 x + 5\sin x$ $\sin x(2\sin^2 x + 2\cos^2 x + 5)$ $7\sin x$		M1 (2nd) M1 (1st) A1
	Allow eg $7 \times \sin x$ or $\sin x \times 7$ for $7\sin x$		
	Missing brackets must be recovered		
	NOTE $7\sin x$ may come from incorrect or incomplete working eg $2\sin^2 x + 2\cos^2 x + 5\sin x = 7\sin x$		M0M1A0
	Condone unambiguous notation eg1 condone s or sin for $\sin x$ eg2 condone $\cos\theta$ for $\cos x$		

Q	Answer	Mark	Comments
21	angle $AOD = 12x - (180 - 4x)$ or angle $AOD = 16x - 180$ **or** angle $BOD = 14x - (180 - 4x)$ or angle $BOD = 18x - 180$ **or** angle $AOB = 180 - 4x$ and angle $BOD = 360 - 12x$ and angle $AOD = 360 - 14x$ **or** angle $AOB = 180 - 4x$ and reflex angle $BOD = 12x$ and reflex angle $AOD = 14x$	B2	oe B1 angle $AOB = 180 - 2x - 2x$ or angle $AOB = 180 - 4x$ or reflex angle $BOD = 12x$ or angle $BOD = 360 - 12x$ or reflex angle $AOD = 14x$ or angle $AOD = 360 - 14x$ all angles for B2 or B1 may be seen on diagram
	$14x + 12x - (180 - 4x) = 360$ or $14x + 16x - 180 = 360$ or $12x + 18x - 180 = 360$	M1	oe equation eg $180 - 4x + 16x - 180 + 18x - 180 = 360$ or $180 - 4x + 360 - 12x + 360 - 14x = 360$
	18	A1	
	Additional Guidance		
	Up to B2 may be awarded for correct work with no answer or incorrect answer, even if this is seen amongst multiple attempts		
	An equation scoring M1 implies B2		
	Incorrect use of obtuse/acute must be recovered		
	Missing brackets must be recovered		
	Condone eg angle $BOD = 12x$ if marked as reflex angle $BOD = 12x$ on diagram		B1
	NOTE 18 may be seen from incorrect working eg (interior angle of regular pentagon =) $540 \div 5 = 108$ $108 \div 6 = 18$ (no other correct working seen)		B0M0A0

Q	Answer	Mark	Comments
22	42	B3	B2 18 and 12 and 12 **or** 18 and 24 **or** a calculation that would evaluate to 42 B1 18 or 12 or 24 or 54 or 72
	Additional Guidance		
	Up to B2 may be awarded for correct work with no answer or incorrect answer, even if this is seen amongst multiple attempts		
	Answer 42		B3
	7×6		B2
	$7 \times 3!$		B2
	$30 + 12$		B2