AQA

Level 2 Certificate
FURTHER MATHEMATICS
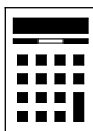
Paper 2   Calculator

Thursday 21 June 2018   Afternoon   Time allowed: 2 hours

Materials
For this paper you must have:
- a calculator
- mathematical instruments.

Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 105.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.
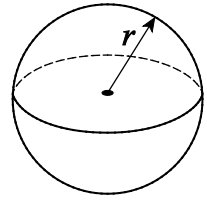- The use of a calculator is expected but calculators with a facility for symbolic algebra must **not** be used.

For Examiner's Use	
Pages	Mark
3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
26–27	
28–29	
TOTAL	

Formulae Sheet

Volume of sphere $= \dfrac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$

Volume of cone $= \dfrac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$
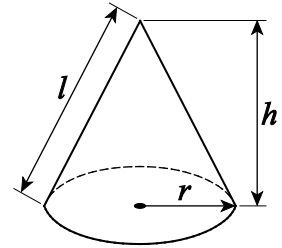
In any triangle ABC

Area of triangle $= \dfrac{1}{2}ab \sin C$
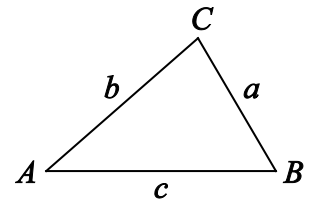
Sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

$$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

Trigonometric Identities

$\tan \theta \equiv \dfrac{\sin \theta}{\cos \theta} \qquad \sin^2 \theta + \cos^2 \theta \equiv 1$

Answer **all** questions in the spaces provided.

1   The nth term of a sequence is $\dfrac{1420 - 5n}{1420 + 5n}$

1 (a)   Work out the **position** of the term that has the value zero.

[2 marks]

_____

_____

_____

_____

Answer _____

1 (b)   Write down the limiting value of the sequence as $n \rightarrow \infty$

[1 mark]

_____

_____

_____

Answer _____

Turn over for the next question

2   *P* (−3, −10) and *Q* (*a*, *b*) are points on a straight line with gradient 12

Work out one possible pair of integer values for *a* and *b*.

[2 marks]

a = _____   *b* = _____

3 $p = \dfrac{m+2}{m^2+1}$

3 (a) Work out the value of p when $m = -5.5$

[1 mark]

Answer _____

3 (b) Work out the values of m when $p = 2$

[3 marks]

Answer _____

Turn over for the next question

6

Turn over ▶

4   *A*, *B*, *C* and *D* are points on a circle, centre *O*.
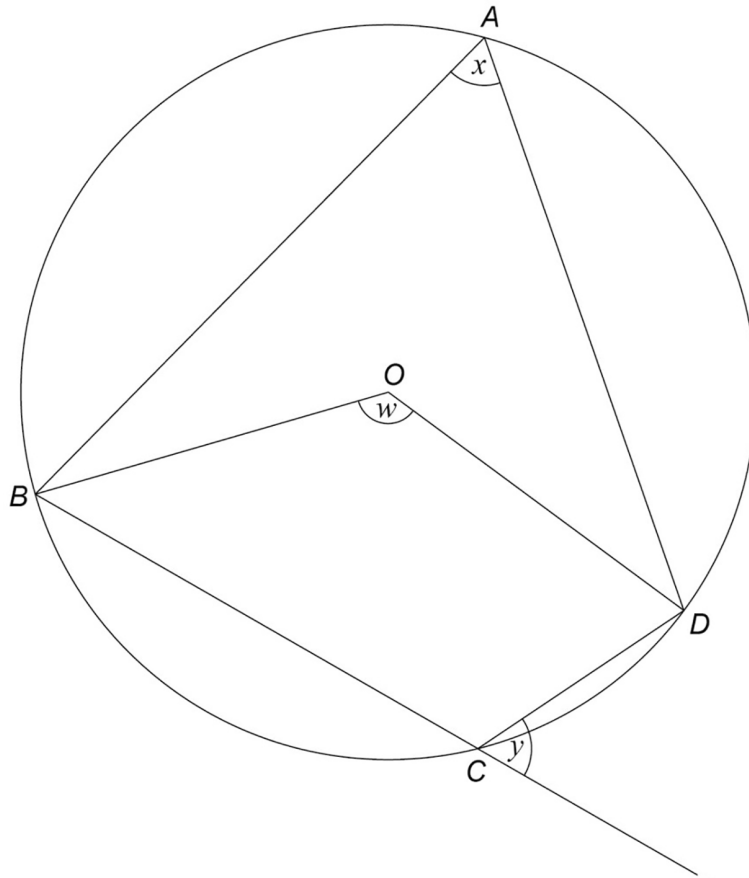
Which statement is correct?

Tick **one** box.

[1 mark]

$x + y = 180°$ and $w = 2x$ ☐

$x + y = 180°$ and $x = 2w$ ☐
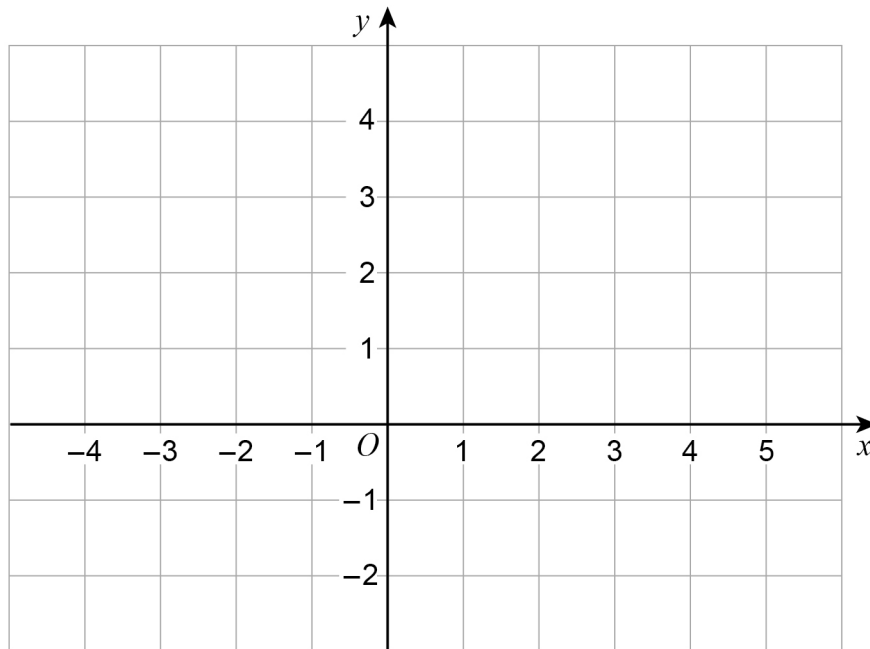
$x = y$ and $w = 2x$ ☐

$x = y$ and $x = 2w$ ☐

5   On the grid, draw the graph of $y = f(x)$

$$f(x) = x + 4 \qquad -4 \leqslant x < 0$$

$$= 4 - 3x \qquad 0 \leqslant x < 2$$

$$= -2 \qquad 2 \leqslant x \leqslant 5$$

[4 marks]

Turn over for the next question

5

Turn over ▶

6 $f(x) = x^2 - 7$ for all values of x

   $g(x) = 1 - 3x$ for $-4 \leqslant x \leqslant 4$

6 (a)   Work out the range of f(x).

Give your answer as an inequality.

[1 mark]

Answer _____

6 (b)   Work out the range of g(x).
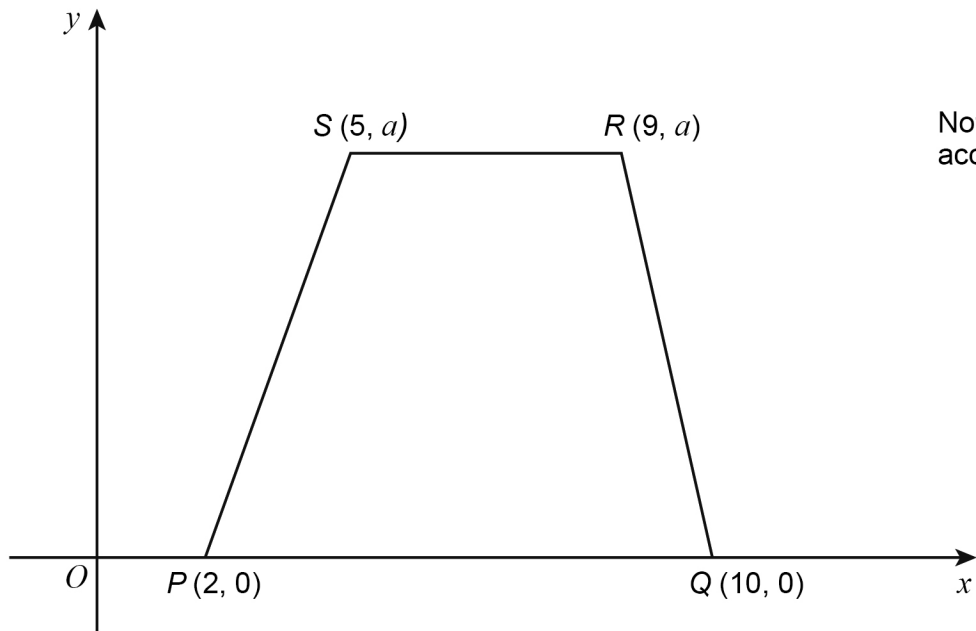
Give your answer as an inequality.

[2 marks]

Answer _____

6 (c) Solve $2f(x) = g(x)$

You **must** show your working.

Give your answers to 3 decimal places.

[4 marks]

Answer _____

Turn over for the next question

7

Turn over ▶

7   *PQRS* is a trapezium.

S (5, *a*)   R (9, *a*)   Not drawn accurately

O   P (2, 0)   Q (10, 0)   *x*

The area of the trapezium is 63 square units.

Work out the value of *a*.

[2 marks]

Answer _____

8 Here is a sketch of triangle *ABC*.

P is a point on *AB*.

Not drawn accurately
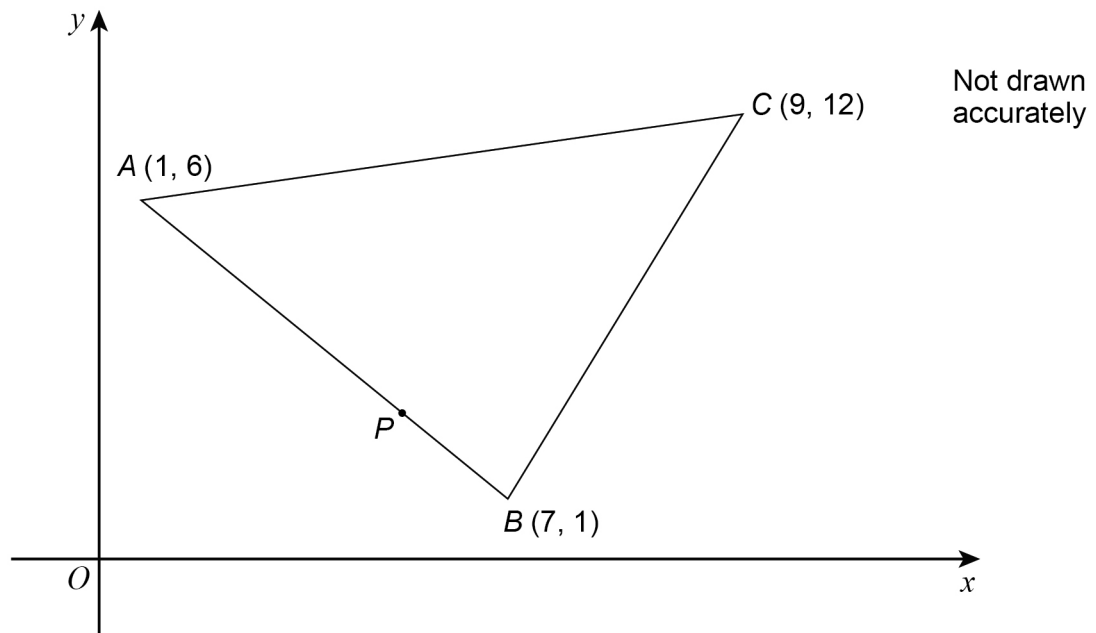
AP : *PB*   is   3 : 1

Work out the length *PC*.

Give your answer to 4 significant figures.

[4 marks]

Answer _____ units

6

Turn over ▶

9 $y = \dfrac{2x^7 + 15x^2}{3x}$

Work out the value of x when $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 133$

[4 marks]

Answer _____

10 The transformation matrix $\begin{pmatrix} a & b \\ 2a & 3b \end{pmatrix}$ maps the point (1, −3) onto the point (1, 4)

Work out the values of a and b.

You **must** show your working.

[5 marks]

$a = $ _____ $b = $ _____

Turn over for the next question

9

Turn over ▶

11 Expand and simplify fully $(x + 2)(x + 3)(x + 4)$

[3 marks]

Answer _____

12 (a) Write $\dfrac{7}{9x} + \dfrac{2}{3x^2}$ as a single fraction in its simplest form.

[3 marks]

Answer _____

12 (b) Show that $\quad \dfrac{x^4}{x+4} \times \dfrac{x+2}{x} \div \dfrac{x^2}{3x+12}$

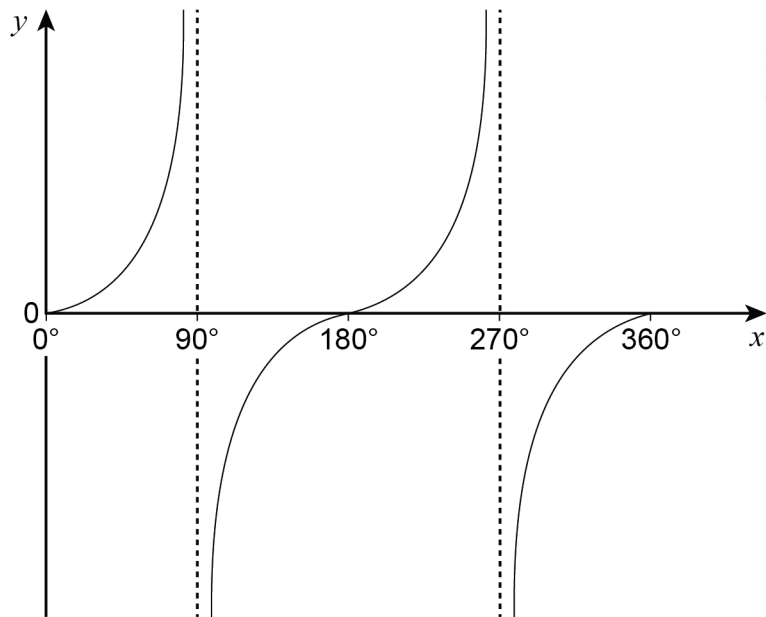simplifies to the form $ax^2 + bx$ where a and b are integers.

[4 marks]

Turn over for the next question

10

Turn over ▶

13 (a) Here is a sketch of $y = \tan x$ for $0° \leqslant x \leqslant 360°$

Not drawn accurately

How many solutions of $\tan x = k$ where $k > 0$ are between 90° and 360° ?

[1 mark]

Answer _____

13 (b) $0 < p < 1$

How many solutions of $\sin x = p - 1$ are between 0° and 180° ?

You may use a sketch graph to help you.

[1 mark]

Answer _____

13 (c)   State the coordinates of each point where the graph

  $y = \cos x$ for $0° \leqslant x \leqslant 360°$

meets or intersects an axis.

[2 marks]

Answer _____

Turn over ▶

4

14 (a)   Factorise fully $12pq^3r - 18pq^2r^2 + 24pq^2r$

[2 marks]

Answer _____

14 (b)   Factorise fully $6(y + 3)^5 + 4(y + 3)^4$

Give your answer in its simplest form.

Do **not** attempt to expand $(y + 3)^5$ or $(y + 3)^4$

[3 marks]

Answer _____

14 (c)   Factorise fully $48 - 75x^2$

[2 marks]

Answer _____

15   Work out the rate of change of y with respect to x at the point on the curve

$y = x^2(x^2 - 9)$ where $x = -2$

You **must** show your working.

[4 marks]

Answer _____

Turn over for the next question

11

16 $A = 2 - 5x$ $B = 3x - 1$ $C = x^2$

Show that $(2A + 3B)^2 \equiv A + B + C$

[4 marks]

_____

_____

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17   A circle has equation $x^2 + y^2 = 29$

P is the point $(-5, 2)$

17 (a)   Show that P is on the circle.

[1 mark]

_____

_____

17 (b)   The tangent to the circle at *P* intersects the *x*-axis at point *Q*.

Work out the *x*-coordinate of *Q*.

You **must** show your working.

[4 marks]

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Answer _____

Turn over ▶

18 (a) Work out all the **integer** values of x for which

$$-5 < 4x + 3 \leqslant 13$$

[3 marks]

Answer _____

18 (b) Work out the range of values of x for which

$$x^2 - 11x + 28 > 0$$

You **must** show your working.

[3 marks]

Answer _____

19 Use **matrix multiplication** to show that, in the x–y plane,

- a reflection in the line $y = -x$, followed by
- a rotation, 90° anticlockwise about the origin, followed by
- a reflection in the x-axis

is equivalent to a transformation by the identity matrix.

[5 marks]

Turn over for the next question

11

Turn over ▶

20 PQRSTU is a triangular prism.

PQRS is a rectangle and angle QRU = 90°

PQ = 10 cm   QR = 12 cm   UR = 7 cm

M is the midpoint of PQ.
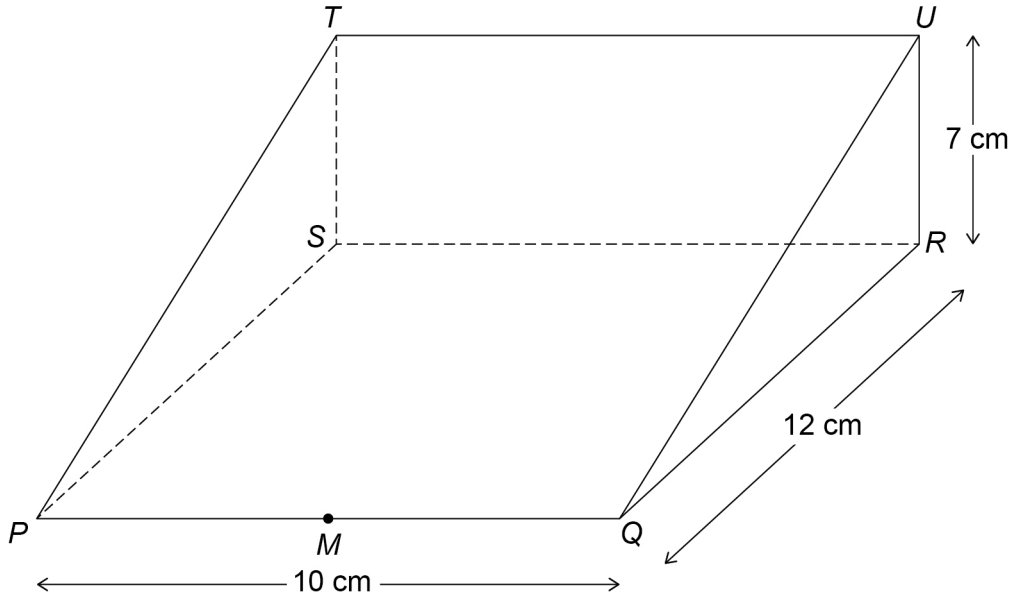
20 (a) Calculate the size of the angle between the line UM and the plane PQRS.

[4 marks]

Answer _____ degrees

20 (b) Calculate the size of the angle between the planes *UMR* and *UQR*.

[2 marks]

Answer _____ degrees

Turn over for the next question

6

Turn over ▶

21 The continuous curve $y = f(x)$ has exactly two stationary points.

Here is some information about the curve.

$x < -1$	$x = -1$	$-1 < x < 2$	$x = 2$	$x > 2$
$\dfrac{dy}{dx}$ is positive	$\dfrac{dy}{dx}$ is zero	$\dfrac{dy}{dx}$ is negative	$\dfrac{dy}{dx}$ is zero	$\dfrac{dy}{dx}$ is negative

$f(-1) = 3$ and $f(2) = 1$

State the coordinates **and** the nature of each of the stationary points.

[3 marks]

stationary point (_____ , _____) nature _____

stationary point (_____ , _____) nature _____

22 (a) $8 \cos x + 5 \sin x = 0$ where $90° < x < 180°$

Work out the size of angle x.

[3 marks]

Answer _____ degrees

22 (b) $6 \sin^2 x + 4 \cos^2 x \equiv A + B \cos^2 x$ where A and B are integers.

Work out the values of A and B.
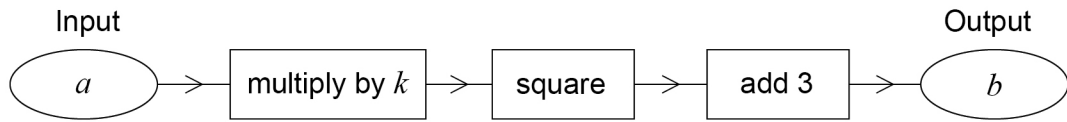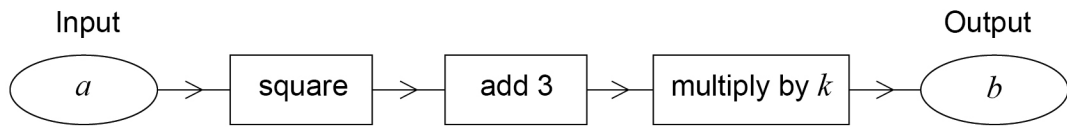You **must** show your working.

[2 marks]

A = _____   B = _____

Turn over ▶

23 For each of these two function machines, when the input is a the output is b.

$k > 0$ and $k \neq 1$ and $a > 0$

Input → a → square → add 3 → multiply by k → Output b

Input → a → multiply by k → square → add 3 → Output b

Work out an expression for a in terms of k.

Give your answer in its simplest form.

[6 marks]

Answer _____

24 Work out the value of p when

$$9^{0.5p} \times 81 = 27^{2p-1}$$

[4 marks]

Answer _____

END OF QUESTIONS

10

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