AQA

LEVEL 2 CERTIFICATE
Further Mathematics

Paper 1 8360/1 Non-calculator
Mark scheme

8360
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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

M	Method marks are awarded for a correct method which could lead to a correct answer.
M dep	A method mark dependent on a previous method mark being awarded.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
ft	Follow through marks. Marks awarded following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
oe	Or equivalent. Accept answers that are equivalent. eg, accept 0.5 as well as $\frac{1}{2}$
[a, b]	Accept values between *a* and *b* inclusive.
3.14…	Accept answers which begin 3.14 eg 3.14, 3.142, 3.1416

Examiners should consistently apply the following principles.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a candidate has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the candidate. In cases where there is no doubt that the answer has come from incorrect working then the candidate should be penalised.

Questions which ask candidates to show working

Instructions on marking will be given but usually marks are not awarded to candidates who show no working.

Questions which do not ask candidates to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Candidates often copy values from a question incorrectly. If the examiner thinks that the candidate has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Continental notation

Accept a comma used instead of a decimal point (for example, in measurements or currency), provided that it is clear to the examiner that the candidate intended it to be a decimal point.

Q	Answer	Mark	Comments
1	Any straight line of gradient $\frac{3}{4}$ or a correct point plotted, other than (2, 1)	M1	
	Line through (2, 1) and (−2, −2) or line through (2, 1) and (6, 4)	A1	this is the minimum length required
	Additional guidance		
	$y = \frac{3}{4}x - \frac{1}{2}$ oe scores SC1 A line of gradient $-\frac{3}{4}$, through (2, 1), tolerance as below, scores SC1 If they draw the correct line **and** the $-\frac{3}{4}$ line then award 1 mark only Tolerance of $\frac{1}{4}$ cm square at two of the three points (2, 1), (6, 4) and (−2, −2)		

Q	Answer	Mark	Comments
2	$2ax$ **or** +3	M1	either term correct
	their $2a(-1) + 3 = -5$	M1dep	oe two terms needed here ... an x term with −1 substituted and a constant term
	$(a =)$ 4	A1	
	Additional guidance		
	If $dy/dx = 5$ is used (misread) then $-2a + 3 = 5$ scores M1 M1 A0 A 1st line of $2a + 3$ followed by $2a + 3 = -5$ can only score M1 M0 A0 Condone $y = 2ax + 3$ for the 1st M1 ... they have differentiated but used the wrong notatioin		

3(a)	Fully correct curve with all intersections labelled ie. −9 and 2 on the x-axis and −18 on the y-axis	B3	B2 for two correct x-axis points of intersection labelled (they must have a quadratic graph drawn in the correct orientation) B1 for U shaped curve, in the correct orientation, crossing y-axis at −18, (the x-axis crossing points not labelled) **or** two of the three x or y axis crossing points marked or stated ... eg this could be (2, 0) and (0, −18) seen in a table of values **or** $(x + a)(x + b)$ with $ab = -18$ **or** $a + b = +7$
	Additional guidance		
	Table of values, points plotted and graph drawn, fully correct, scores all 3 marks Minimum point must be to the left of the y-axis to score full marks. Both sides of the graph must be drawn above the x-axis to score full marks Maximum of B1 if no graph drawn eg $x = -9$ and $x = 2$ stated, but no graph		

3(b)	$x = -3.5$	B1	oe
	Additional guidance		
	It must be $x = -3.5$, do not accept $y = -3.5$ or −3.5 on its own		

4	**Alternative method 1**		
	Intention to work out gradient or reciprocal of gradient **or** Intention to work out the equation of the straight line	M1	Condone **one** sign error in the calculation eg $\frac{-5-7}{6--4}$ **or** $\frac{-5-t}{6-8}$ **or** $\frac{7-t}{-4-8}$ **or** −1.2 oe eg $7 = -4m + c$ **or** $-5 = 6m + c$ eg $y - 7 = m(x - -4)$
	A correct value for m or a correct expression for m **and** an expression to calculate the value of t or the value of c **or** $m = -1.2$ **and** $c = 2.2$	M1dep	eg. $(m =)\ \frac{7--5}{-4-6}$ **or** $(m =)\ \frac{-6}{5}$ oe **and** eg $\frac{t--5}{8-6} = \frac{-5-7}{6--4}$ **or** $t = \frac{-6}{5}(8) + (7 - \frac{24}{5})$ **or** $7 = \frac{-6}{5}(-4) + c$ **or** $-5 = \frac{-6}{5}(6) + c$
	$(t =)\ -7.4$ **or** $-7^2/_5$ **or** $\frac{-37}{5}$	A1	
	Alternative method 2		
	−4 to 6 is +10 **and** 7 to −5 is −12	M1	oe Condone a sign error
	6 to 8 is +2 **and** −5 to t is $\frac{-12}{5}$	M1	oe
	$(t =)\ -7.4$ **or** $-7^2/_5$ **or** $\frac{-37}{5}$	A1	
	Alternative method 3		
	$\sqrt{[(-4-6)^2 + (7--5)^2]}\ (=\sqrt{244})$ **and** stating −4 to 6 is 10 **and** 6 to 8 is 2	M1	Correct use of Pythagoras and identifying the correct displacements
	$\sqrt{[(6-8)^2 + (-5-t)^2]} = (\sqrt{244}) \div 5$	M1	ft their 244
	$(t =)\ -7.4$ **or** $-7^2/_5$ **or** $\frac{-37}{5}$	A1	

	Additional guidance	
	−7.4 seen on answer line is 3 marks −7.4 seen in the working but sign error on answer line is 3 marks 'Algebraic method' means the question must not be done graphically ... although a diagram is fine when used to do the gradient calculations $\frac{t - -5}{8 - 6} = \frac{t - 7}{8 - -4}$ seen implies M1 M1 Look at any diagram they may have drawn for evidence of the alt 2 method $\frac{7 - -5}{-4 - 6}$ (correct expression) = 1.2 (error) followed by 7 = (1.2)(−4) + c scores M1 M1 but will not lead to a correct final answer, so A0 m = − 1.2, but they use 1.2 instead ... 7 = 1.2(−4) + c giving c = 11.8 is M1 M1 A0 m = − 1.2, then t = −1.2 + 11.8 = 2.2 scores M1 M1 A0 because this is a correct method for calculating c, and so scores the 2nd M1,even though they think they are calculating t $m = \frac{-5 - 7}{6 - -4} = \frac{-12}{10}$ $\frac{-12}{10} \times 2 = \frac{-24}{20} = \frac{-6}{5}$ = −1.2 so t = −5 − 1.2 = − 6.2 M1 M1 A0 ... because the only error is $\frac{-12}{10} \times 2 = \frac{-24}{20}$ ... if this had been −2.4 then t = −7.4	

5	(x^3 +) $4x^2 - kx^2 - 4kx - 5x$ (− 20)	M1	or $4 - k$ and $-4k - 5$ seen as coefficients
	$4 - k = 2(-4k - 5)$	M1dep	ft their expansion if first M mark earned
	(k =) −2	A1	
	Additional guidance		
	Condone one **sign** error in the first two steps Ignore errors in x^3 and −20 for the first M1		

6	$(x + 6)^3\ [x + 6 + 3x + 4]$ **or** $(x + 6)^2[(x + 6)^2 + (x + 6)(3x + 4)]$ **or** $(x + 6)[(x + 6)^3 + (x + 6)^2(3x + 4)]$	M1	for sight of $(x + 6)^3$, $(x + 6)^2$ or $(x + 6)$ taken out as a common factor
	$(x + 6)^3[4x + 10]$	A1	
	$2(x + 6)^3(2x + 5)$	A1	
	Additional guidance		
	$(x + 6)^3(x + 6)(3x + 4)$ implies M1 SC1 for all correct factors seen in working but never written as a product of terms An attempt to expand brackets will be M0 unless the expansion leads to a correct solution worth 2 or 3 marks $(x + 6)^3\ [x + 6 + 4x + 3]$ scores M1 ... ignore the error in the 2nd bracket		

7(a)	$x \geqslant \frac{5}{2}$	B1	
	Additional guidance		

7(b)	$1.2^2 = 2x - 5$ **or** $1.44 = 2x - 5$	M1	oe
	$(x =)\ 3.22$	A1	oe eg $\frac{161}{50}$
	Additional guidance		

7(c)	$\sqrt{5\frac{1}{4} - 5}$ **or** $\sqrt{\frac{21}{4} - 5}$	M1	oe $\sqrt{\frac{2(21)}{8} - 5}$ $\sqrt{\frac{42}{8} - 5}$ $\sqrt{2\left(2\frac{5}{8}\right) - 5}$ $\sqrt{5.25 - 5}$ $\sqrt{2(2.625) - 5}$
	$\sqrt{\frac{1}{4}}$ **or** $\sqrt{(0.25)}$	A1	oe
	$\frac{1}{2}$ **or** 0.5	A1	Condone $\pm\frac{1}{2}$ but not $-\frac{1}{2}$ on its own
	Additional guidance		
	Condone decimals throughout An answer of $\frac{\sqrt{1}}{2}$ is M1 M1 A0		

8	2nd difference = 8 **or** $a = 4$	M1	sight of $4n^2$ implies this mark
	subtract their $4n^2$ **or** sight of three of 6 17 28 39	M1	subtracting 4 16 36 64 the coefficient of their $4n^2$ will come from half the value of their 2nd difference
	subtract their $11n$ **or** $b = 11$ **or** tests $4n^2 + 11n$ and compares to original sequence **or** sight of three of 15 38 69 108	M1dep	dep on 2nd M mark
	$4n^2 + 11n - 5$	A1	
	Alternative method 2		
	Any three of these $a + b + c = 10$ $4a + 2b + c = 33$ $9a + 3b + c = 64$ $16a + 4b + c = 103$	M1	
	Any two of these $3a + b = 23$ $5a + b = 31$ $7a + b = 39$	M1dep	
	$a = 4$ and $b = 11$	A1	
	$4n^2 + 11n - 5$	A1	
	Alternative method 3		
	$a = 4$	M1	
	$3a + b = 33 - 10$ **and** substitutes their a in this equation	M1	oe
	$b = 11$	A1	
	$4n^2 + 11n - 5$	A1	
	Additional guidance		
	SC3 for $4n^2 - 11n + 5$ Condone $4x^2 + 11x - 5$ **or** eg $4x^2 + 11n - 5$ (mixed letters)		

9(a)	$2x^2 - 3x + 2x - 3$	B1	terms can be written in any order
	Additional guidance		
	Must show all four terms.		

9(b)	$2x^2 - x - 10$ (> 0)	M1	oe must have three terms eg $2x^2 > x + 10$ Condone $2x^2 - x - 10$ $(= 0)$ oe eg $2x^2 - x = 10$
	correct factors $(2x - 5)(x + 2)$	M1dep	For use of quadratic formula ... condone **one** numerical or sign error
	Sight of 2.5 **and** −2	A1	
	$x > 2.5$	A1	oe Must have seen $x = 2.5$ and $x = -2$ and reject the negative solution
	Additional guidance		
	This is a quadratic inequality so we need to see an attempt at finding the two critical values then making a decision as to the correct solution SC1 for $x > 2.5$ with no working SC1 for $x > 2.5$ from T&I If T&I done such that both critical values (2.5 and −2) have been identified and tested (eg they give 7 when substituted in the expression $2x^2 - x - 3$) then it is possible to score 3 marks or 4 marks		

10	(3, 0) marked **or** used	B1	
	radii 6 and 8 identified	B1	oe
	$\sqrt{(6^2 + 8^2)}$ **or** 6, 8, 10 triangle **or** 10	M1	
	$(h =)$ 13 **or** M = (13, 0)	A1	might be seen in the working or on the diagram
	Additional guidance		
	(3, 0) can be implied eg $LM = h - 3$ **or** $OM = 3 +$ their LM Look on the diagram for evidence of the B marks $(h =)$ 13 with no working is 4 marks (−13 with no working is 0 marks)		

11	**Alternative method 1**		
	common denominator $(x - 3)(x - 5)$ oe	M1	allow $(x - 3)^2(x - 5)$ oe
	numerator $x(x - 5) + 6$ **or** $x^2 - 5x + 6$	M1dep	allow $x(x - 3)(x - 5) + 6(x - 3)$ oe
	$\frac{(x-3)(x-2)}{(x-3)(x-5)}$	A1	$\frac{(x-3)^2(x-2)}{(x-3)^2(x-5)}$
	$\frac{x-2}{x-5}$	A1	
	Alternative method 2		
	$\frac{1}{(x-3)}\left(x+\frac{6}{(x-5)}\right)$	M1	
	$\frac{1}{(x-3)}\left(\frac{x(x-5)+6}{(x-5)}\right)$ **or** $\frac{1}{(x-3)}\left(\frac{x^2-5x+6}{(x-5)}\right)$	M1	
	$\frac{(x-3)(x-2)}{(x-3)(x-5)}$	A1	
	$\frac{x-2}{x-5}$	A1	
	Additional guidance		
	Further work eg answer of $\frac{-2}{-5}$ means the final A1 must not be awarded eg $\frac{x(x-5)}{(x-3)(x-5)} + \frac{6}{(x-3)(x-5)}$ scores M1 M1 Either ... follow the LHS of the mark scheme for the first three steps Or ... follow the RHS ... do **not** mix expressions ... the numerators and denominators must match		

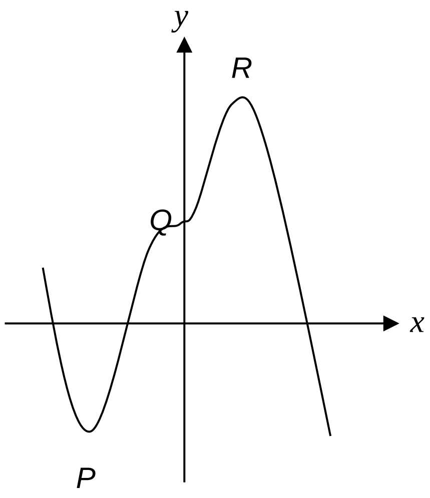
12(a)	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	B1	
	Additional guidance		

12(b)	Rotation of 180° about the origin Enlargement SF −1 centre the origin	B2	B1 if either the 180° or the origin is missing B1 if either the SF or the centre is missing
	Additional guidance		
	Ignore any reference to direction Accept 'Rotation of half a turn' for B1 Answers of Rotation or Enlargement with no other description attached score B0 Rotation 90° is B0 (incorrect angle, no centre of rotation) Enlargement SF2 is B0 (incorrect SF and no centre of enlargement)		

12(c)	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	B1	
	Additional guidance		
	If no working or answer seen in (c), look at (b) ... the matrix for M^2 might be written there, and, if correct, will score B1 in (c)		

13	Handling the negative power first ... $\frac{1}{x^{\frac{1}{4}}} = 0.2$ **or** $\frac{1}{x^{\frac{1}{4}}} = \frac{1}{5}$ **or** $\frac{1}{0.2} = x^{\frac{1}{4}}$ **or** $5 = x^{\frac{1}{4}}$ **or** $\frac{1}{\sqrt[4]{x}} = 0.2$ **or** $\sqrt[4]{x} = \frac{1}{0.2}$	M1	Handling the 4th root first ... $x^{-1} = 0.2^4$ **or** $x = 0.2^{-4}$
	All of these are valid 2nd steps following any of the above 1st steps $\frac{1}{x} = 0.2^4$ **or** $\frac{1}{x} = \frac{1}{5^4}$ **or** $x = 5^4$ **or** $x = \frac{1}{0.2^4}$ **or** $x = \left(\frac{1}{0.2}\right)^4$	M1	
	(x =) 625	A1	
	Additional guidance		
	The two method marks are for handling the negative power and for handling the 4th root ... and an error in one of the 1st steps does **not** mean that you cannot give credit for a correct 2nd step eg $-\sqrt[4]{x} = 0.2$ (incorrect) followed by $x = 0.2^4$ = 0.0016 scores M0 M1 $-\sqrt[4]{x} = 0.2$ (incorrect) followed by $-x = 0.2^4$ = 0.0016 , x = − 0.0016 is M0 M0 because the 4th power of $-\sqrt[4]{x}$ would be a positive quantity $\frac{1}{0.0016}$ scores M1 M1 A0 0.0016 **or** $\frac{1}{625}$ will score M1 M0 A0 unless other valid working seen		

14	$AB = \sqrt{3}$	B1	
	Any one of these responses ... $\frac{BD}{2\sqrt{3}} = \cos 30°$ $\quad \frac{BD}{2\sqrt{3}} = \sin 60°$ $\frac{\sqrt{3}}{BD} = \tan 30°$ $\quad \frac{BD}{\sqrt{3}} = \tan 60°$ $BD^2 + (\sqrt{3})^2 = (2\sqrt{3})^2$ oe	M1	... or these ... $\frac{BD}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$ $\quad \frac{BD}{\sqrt{3}} = \sqrt{3}$ $\quad \frac{\sqrt{3}}{BD} = \frac{1}{\sqrt{3}}$ $\frac{BD}{\sin 60°} = \frac{\sqrt{3}}{\sin 30°}$ $\quad \frac{BD}{\sqrt{3}/2} = \frac{\sqrt{3}}{1/2}$
	$BD = 3$	A1	
	$CD = 3 - \sqrt{3}$	A1	oe
	Additional guidance		
	SC1 for a **final** answer of $\frac{2\sqrt{3}\sin 15°}{\sin 135°}$, possibly with $\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$ for sin 135°		

15		B4	There are **five** features ... minimum point in correct region ie the 3rd quadrant maximum point in correct region, ie the 1st quadrant, with y-coordinate > 3 point of inflection on positive y-axis with attempt to show decreasing then increasing gradient three distinct x-axis crossing points, a continuous curve and no more than three stationary points their minimum and maximum points (which need not be stationary points) labelled *P* and *R* B4 for all five features B3 for four features B2 for three features B1 for two features
	Additional guidance		
	Accept (a, b) and (c, d) instead of *P* and *R* respectively. Condone actual values suggested for the coordinates of *P* and *R*. SC2 for a completely correct shaped sketch, with all features, BUT a reflection of the correct graph in the y-axis		

16	$\sin 120° = \dfrac{\sqrt{3}}{2}$	B1	
	$\dfrac{y}{\sin 120°} = \dfrac{6}{\sin x°}$ **or** $\dfrac{y}{\sin 120°} = \dfrac{6}{1/\sqrt{12}}$	M1	oe
	$y = 6\sqrt{12} \times \sin 120°$ **or** $y = 12\sqrt{3} \times \sin 120°$	M1dep	dep on previous M mark earned
	18	A1	
	Additional guidance		
	They might use a wrong value for sin 120° ... eg $\sin 120° = 1/2$ then write eg $\dfrac{y}{1/2} = \dfrac{6}{1/\sqrt{12}}$ followed by $y = 6\sqrt{12} \times 1/2$, this scores M1 M1 because their method is correct ... they will already have lost the B1 mark and will be unable to score the A1 mark Do not condone the use of 120 instead of sin 120°		

17 (a)	$(2x + a)(x + b)$	M1	$ab = 5$ **or** $a + 2b = 7$
	$(2x + 5)(x + 1)$	A1	
	Additional guidance		
	$2(x + 2.5)(x + 1)$ and $(x + 2.5)(2x + 2)$ both score SC1 Ignore subsequent working ... eg solving		

17 (b)	$(2\sin\theta + 5)(\sin\theta + 1)$ (=0) **or** $2\sin\theta + 5 = 0$ **and** $\sin\theta + 1 = 0$	M1	ft their factors from part (a)
	$\sin\theta = -1$	M1	
	270°	A1	only 270° ... no extra (incorrect) solutions
	Additional guidance		
	In (b) they can work with x or s but must eventually use $\sin\theta = -1$		

18	**Alternative method 1**		
	$10\sqrt{3}$	B1	
	$\frac{(24 - \text{their } 10\sqrt{3})(4\sqrt{3} + 5)}{(4\sqrt{3} - 5)(4\sqrt{3} + 5)}$	M1	oe
	$96\sqrt{3} - 120 + 120 - 50\sqrt{3}$	M1dep	allow one sign error
	$48 - 25$ or 23	M1	
	$2\sqrt{3}$	A1	
	Alternative method 2		
	$\frac{(24 - \sqrt{300})(4\sqrt{3} + 5)}{(4\sqrt{3} - 5)(4\sqrt{3} + 5)}$	M1	
	$96\sqrt{3} + 120 - 4\sqrt{900} - 5\sqrt{300}$	M1dep	allow one sign error
	$96\sqrt{3} - 120 + 120 - 50\sqrt{3}$	M1	
	$48 - 25$ or 23	M1	
	$2\sqrt{3}$	A1	
	Additional guidance		
	For the 1st M1, multiplying numerator and denominator by $(4\sqrt{3} + 5)$ could legitimately be replaced by $-4\sqrt{3} - 5$ ... almost identical working ... it just changes all the signs on the next lines of working		