AQA Qualifications

Level 2 Certificate

Further Mathematics

Paper 2

Mark scheme

83602

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Version: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

M	Method marks are awarded for a correct method which could lead to a correct answer.
M dep	A method mark dependent on a previous method mark being awarded.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
ft	Follow through marks. Marks awarded following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
oe	Or equivalent. Accept answers that are equivalent. eg, accept 0.5 as well as $\frac{1}{2}$
[*a*, *b*]	Accept values between *a* and *b* inclusive.
3.14…	Accept answers which begin 3.14 eg 3.14, 3.142, 3.1416

Examiners should consistently apply the following principles

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a candidate has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the candidate. In cases where there is no doubt that the answer has come from incorrect working then the candidate should be penalised.

Questions which ask candidates to show working

Instructions on marking will be given but usually marks are not awarded to candidates who show no working.

Questions which do not ask candidates to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Candidates often copy values from a question incorrectly. If the examiner thinks that the candidate has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Continental notation

Accept a comma used instead of a decimal point (for example, in measurements or currency), provided that it is clear to the examiner that the candidate intended it to be a decimal point.

Q	Answer	Mark	Comments
1	**Alternative method 1**		
	$\frac{1}{2} \times 5 \times 6$ or $6 \times 5 - \frac{1}{2} \times 6 \times 3 - \frac{1}{2} \times 6 \times 2$ or $\frac{1}{2} \times 6 \times 3 + \frac{1}{2} \times 6 \times 2$	M2	Fully correct method M1 Plots correct three points or draws correct triangle
	15	A1	
	Alternative method 2		
	$\frac{1}{2} \times 5 \times [6.1, 6.5] \times \sin [70, 74]$ or $\frac{1}{2} \times 5 \times [6.5, 6.9] \times \sin [61, 65]$ or $\frac{1}{2} \times [6.1, 6.5] \times [6.5, 6.9]$ $\times \sin [43, 47]$	M2	Fully correct method with tolerances on measurements M1 Plots correct three points or draws correct triangle
	15 with no evidence that rounding has been applied	A1	eg 14.9 seen in working
	Additional Guidance		
	15 from counting squares		M2 A1
	Incorrect triangle drawn is zero unless recovered		
	Answer only of 15		M2 A1

Q	Answer	Mark	Comments
2(a)	$x = 2$	B1	
	Additional Guidance		
	$2 = x$		B1
	$y = 2$		B0
	2		B0

Q	Answer	Mark	Comments
2(b)	–0.8 and 4.8 with no other answers	B2	Both correct in either order B1 One correct and one incorrect or missing
	Additional Guidance		
	The word 'and' is not needed If their answer has both –0.8 and 4.8 with no other solutions award B2 eg 4.8, –0.8 with no other solutions		B2

Q	Answer	Mark	Comments
2(c)	$-3 \le f(x) \le 6$ or $6 \ge f(x) \ge -3$ or [–3, 6]	B2	oe B1 $f(x) \ge -3$ or $f(x) \le 6$ on their own or embedded within an interval for $f(x)$ or only –3 and 6 chosen
	Additional Guidance		
	Allow as two inequalities $f(x) \ge -3$ $f(x) \le 6$		B2
	–3 to 6 inclusive		B2
	$f(x)$ may be replaced by f or y or $x^2 - 4x + 1$ for B2 and B1		
	B1 may be seen with an incorrect inequality eg1 $-3 < f(x) \le 6$ eg2 $-3 \le f(x) < 6$ eg3 $-3 \le f(x) \le 5$		 B1 B1 B1
	For B1 ignore incorrect notation if only –3 and 6 chosen eg1 $-3 \le x \le 6$ eg2 $-3 < x \le 6$ eg3 –3 to 6 eg4 –3 to 6 = 9 (working out a statistical range) eg5 $-3 \ge f(x) \ge 6$ eg6 –3, 6		 B1 B1 B1 B1 B1 B1
	{–3, –2, –1, 0, 1, 2, 3, 4, 5, 6}		B0

Q	Answer	Mark	Comments
3(a)	$(\frac{c}{a}, 0)$	B1	
3(b)	$-\frac{a}{b}$	B1	

Q	Answer		Mark	Comments
4	**Alternative method 1 Equates coefficients and eliminates a variable**			
	$4x + 6y = 22$ and $9x + 6y = 39$	$6x + 9y = 33$ and $6x + 4y = 26$	M1	oe Equates coefficients of one variable Allow one term error
	$4x - 9x =$ $22 - 39$ or $-5x = -17$	$9y - 4y =$ $33 - 26$ or $5y = 7$	M1dep	oe Eliminates a variable Must be correct method for their equations Unless correctly eliminated, intention to subtract (or add if appropriate) must be seen with the result of their subtraction
	$(x =)$ 3.4	$(y =)$ 1.4	A1	oe eg $x = \frac{17}{5}$
	(3.4, 1.4)		A1	oe eg $(\frac{17}{5}, \frac{7}{5})$ SC3 (1.4, 3.4) oe
	Alternative method 2 Makes a variable the subject in first equation			
	$y = \frac{11}{3} - \frac{2}{3}x$	$x = 5.5 - 1.5y$	M1	oe eg $x = \frac{11-3y}{2}$ Makes y or x the subject Allow one term error
	$2(\frac{11}{3} - \frac{2}{3}x) =$ $13 - 3x$	$2y =$ $13 - 3(5.5 - 1.5y)$	M1dep	oe Eliminates a variable Must be correct method for their equations
	$(x =)$ 3.4	$(y =)$ 1.4	A1	oe eg $x = \frac{17}{5}$
	(3.4, 1.4)		A1	oe eg $(\frac{17}{5}, \frac{7}{5})$ SC3 (1.4, 3.4) oe

MARK SCHEME CONTINUES ON THE NEXT PAGE

Q	Answer		Mark	Comments
4	**Alternative method 3 Makes a variable the subject in second equation**			
	$y = 6.5 - 1.5x$	$x = \frac{13}{3} - \frac{2}{3}y$	M1	oe eg $y = \frac{13-3x}{2}$ Makes y or x the subject Allow one term error
	$2x + 3(6.5 - 1.5x) = 11$	$2(\frac{13}{3} - \frac{2}{3}y) + 3y = 11$	M1dep	oe Eliminates a variable Must be correct method for their equations
	$(x =)$ 3.4	$(y =)$ 1.4	A1	oe eg $x = \frac{17}{5}$
	(3.4, 1.4)		A1	oe eg $(\frac{17}{5}, \frac{7}{5})$ SC3 (1.4, 3.4) oe
	Alternative method 4 Makes same variable the subject in both equations			
	$y = \frac{11}{3} - \frac{2}{3}x$ or $y = 6.5 - 1.5x$	$x = 5.5 - 1.5y$ or $x = \frac{13}{3} - \frac{2}{3}y$	M1	oe eg $y = \frac{13-3x}{2}$ Makes y or x the subject Allow one term error
	$\frac{11}{3} - \frac{2}{3}x =$ $6.5 - 1.5x$	$5.5 - 1.5y =$ $\frac{13}{3} - \frac{2}{3}y$	M1dep	oe Eliminates a variable Must be correct method for their equations
	$(x =)$ 3.4	$(y =)$ 1.4	A1	oe eg $x = \frac{17}{5}$
	(3.4, 1.4)		A1	oe eg $(\frac{17}{5}, \frac{7}{5})$ SC3 (1.4, 3.4) oe

ADDITIONAL GUIDANCE FOR Q4 IS ON THE NEXT PAGE

Q	Answer	Mark	Comments

<table>
<tr><td rowspan="7">4</td><td colspan="2">Additional Guidance (Q4)</td></tr>
<tr><td>Answer only (3.4, 1.4)</td><td>M2 A2</td></tr>
<tr><td>One value correct (possibly by drawing) with no incorrect working seen for that variable</td><td>M2 A1 A0</td></tr>
<tr><td>If the same method is used for both x and y (eg equates coefficients and eliminates a variable), mark the attempt that favours the student</td><td></td></tr>
<tr><td>Alt 1 $6x + 9y = 33$
$\underline{6x + 4y = 26}$
$4y = 6$
is M1 M0 unless intention to subtract is seen (eg a subtraction symbol is seen or the word subtract is seen) which would then get M1 M1</td><td></td></tr>
<tr><td>Alts 2, 3 and 4
Allow rounding or truncating to 1dp or better for up to M1 M1
eg (Alt 2) $y = 3.6 - 0.6x$
$2(3.6 - 0.6x) = 13 - 3x$</td><td>

M1
M1</td></tr>
<tr><td>(1.4, 3.4) is SC3 or M2 A2 if $x = 3.4$ and $y = 1.4$ seen in working</td><td></td></tr>
</table>

Q	Answer	Mark	Comments
5	Always true Sometimes true Never true Sometimes true	B4	B1 for each correct answer
	Additional Guidance		
	More than one box selected in a row is B0 for that row		
	Allow any unambiguous indication of a selection in a row eg uses crosses instead of ticks		
	Ignore working seen and mark the boxes		

Q	Answer	Mark	Comments
6	$\frac{3}{2} \times (-2) - k \times (-2)^4 + k$ or $-3 - 16k + k$ or $-3 - 15k$	M1	oe Allow missing brackets even if not recovered eg $\frac{3}{2} \times -2 - k \times -2^4 + k$ or $-3 + 16k + k$ or $-3 + 17k$
	$-3 - 16k + k = 12$ or $-3 - 15k = 12$ or $-15k = 15$	A1	oe correct equation (brackets may be recovered) $\frac{3}{2} \times (-2)$ and $(-2)^4$ must be evaluated Implied by $k = -1$
	-1	A1	SC2 $\frac{15}{17}$ or 0.88... or 0.9
	Additional Guidance		
	-1 with no errors seen (recovered bracket is not an error)	M1 A2	
	Substituting $x = 2$	M0 A0	

Q	Answer	Mark	Comments
7	$\frac{8}{27}x^9y^3$ or $\frac{8x^9y^3}{27}$	B2	oe B1 Two of the three components correct and simplified
	Additional Guidance		
	Allow multiplication signs for B2 and B1		
	Allow $0.\dot{2}9\dot{6}$ or $0.\dot{2}\dot{9}\dot{6}$ as a correct component		
	$0.296x^9y^3$		B1
	$\frac{8}{27}x^9y^3$ followed by incorrect further work (only penalise B2 responses)		B1
	$8x^9y^3 \div 27$		B1
	$(\frac{2}{3})^3x^9y^3$		B1
	$\frac{8}{27}x^9$		B1
	$8x^9 \times 27y^3$		B1
	$\frac{8}{27}x^9 + y^3$		B0

Q	Answer	Mark	Comments
8	**Alternative method 1**		
	$\frac{5\times -6+3\times -2}{5+3}$ or $\frac{5\times 4+3\times 9}{5+3}$	M1	oe
	–4.5 or 5.875	A1	oe
	(–4.5, 5.875)	A1	oe eg $(-\frac{9}{2}, \frac{47}{8})$ SC2 (5.875, –4.5)
	Alternative method 2		
	$\frac{3}{5+3}$ × (–2 – – 6) or 1.5 or $\frac{3}{5+3}$ × (9 – 4) or 1.875	M1	oe eg $\frac{3}{8}$ × 4 or $\frac{3}{8}$ × 5 or $\frac{4}{8}$ × 3 or $\frac{5}{8}$ × 3
	–4.5 or 5.875	A1	oe
	(–4.5, 5.875)	A1	oe eg $(-\frac{9}{2}, \frac{47}{8})$ SC2 (5.875, –4.5)

MARK SCHEME CONTINUES ON THE NEXT PAGE

Q	Answer		Mark	Comments
8	**Alternative method 3**			
	$\frac{5}{5+3} \times (-2 - -6)$ or 2.5 or $\frac{5}{5+3} \times (9-4)$ or 3.125		M1	oe eg $\frac{5}{8} \times 4$ or $\frac{5}{8} \times 5$ or $\frac{4}{8} \times 5$
	–4.5 or 5.875		A1	oe
	(–4.5, 5.875)		A1	oe eg $(-\frac{9}{2}, \frac{47}{8})$ SC2 (5.875, –4.5)
	Alternative method 4			
	$\frac{x - -6}{-2 - -6} = \frac{3}{5+3}$ or $\frac{-2-x}{-2 - -6} = \frac{5}{5+3}$ or $\frac{x - -6}{-2-x} = \frac{3}{5}$	$\frac{y-4}{9-4} = \frac{3}{5+3}$ or $\frac{9-y}{9-4} = \frac{5}{5+3}$ or $\frac{y-4}{9-y} = \frac{3}{5}$	M1	oe eg both fractions inverted
	–4.5	5.875	A1	oe
	(–4.5, 5.875)		A1	oe eg $(-\frac{9}{2}, \frac{47}{8})$ SC2 (5.875, –4.5)

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Q	Answer	Mark	Comments
8	**Alternative method 5**		
	$\frac{3}{5+3} \times \sqrt{(9-4)^2 + (-2--6)^2} \times \sin(\tan^{-1} \frac{9-4}{-2--6})$ or 1.875 or $\frac{3}{5+3} \times \sqrt{(9-4)^2 + (-2--6)^2} \times \cos(\tan^{-1} \frac{9-4}{-2--6})$ or 1.5	M1	$\tan^{-1} \frac{9-4}{-2--6}$ is the angle *DE* makes with the horizontal (= 51.3...) $\sqrt{(9-4)^2 + (-2--6)^2}$ is *DE* (= $\sqrt{41}$ or 6.4...)
	–4.5 or 5.875	A1	oe
	(–4.5, 5.875)	A1	oe eg $(-\frac{9}{2}, \frac{47}{8})$ SC2 (5.875, –4.5)

MARK SCHEME CONTINUES ON THE NEXT PAGE

Q	Answer	Mark	Comments
8	**Alternative method 6**		
	$\frac{5}{5+3} \times \sqrt{(9-4)^2+(-2--6)^2} \times \sin(\tan^{-1}\frac{9-4}{-2--6})$ or 3.125 or $\frac{5}{5+3} \times \sqrt{(9-4)^2+(-2--6)^2} \times \cos(\tan^{-1}\frac{9-4}{-2--6})$ or 2.5	M1	$\tan^{-1}\frac{9-4}{-2--6}$ is the angle *DE* makes with the horizontal (= 51.3...) $\sqrt{(9-4)^2+(-2--6)^2}$ is *DE* (= $\sqrt{41}$ or 6.4...)
	–4.5 or 5.875	A1	oe
	(–4.5, 5.875)	A1	oe eg $(-\frac{9}{2}, \frac{47}{8})$ SC2 (5.875, –4.5)
	Additional Guidance		
	(–4.5, 5.9) or (–4.5, 5.88) is M1 A1 A0 unless 5.875 seen in working		
	(5.875, –4.5) is SC2 or M1 A1 A1 if x = –4.5 and y = 5.875 seen in working		
	–4.5 in working that becomes 4.5 on answer line should not be regarded as choice so gains at least M1 A1		
	2 marks if one coordinate correct and 3 marks if both correct (possibly from accurate drawing or working with midpoints) with no incorrect working for that coordinate		
	Alts 5 and 6 also have equivalents where the angle *DE* makes with the vertical (= 38.6... or 38.7) is used. Mark using the principles of alts 5 and 6 or escalate		

Q	Answer	Mark	Comments
9	**Alternative method 1** ***PBC* in terms of x and in terms of y**		
	$180 - x$ or $360 - (y + 100 + 2y + 80)$ or $360 - (3y + 180)$	M1	May be on diagram *PBC* (allow *B*)
	$180 - x$ and $360 - (y + 100 + 2y + 80)$ or $360 - (3y + 180)$	M1	
	$180 - x$ and $180 - 3y$ and $x = 3y$	A1	Must have seen correct working for M1 M1
	Both reasons given	A1	Must have M1 M1 (Co-)interior angles or allied angles (add up to 180°) and angles at a point (add up to 360°)
	Alternative method 2 ***PBC* in terms of x + reflex *PBC* in terms of y = 360**		
	$180 - x$ or $y + 100 + 2y + 80$ or $3y + 180$	M1	May be on diagram *PBC* (allow *B*) or reflex *PBC*
	$180 - x + y + 100 + 2y + 80 = 360$ or $180 - x + 3y + 180 = 360$	M1	oe unsimplified correct equation
	Simplifies to $x = 3y$	A1	Must have seen correct working for M1 M1
	Both reasons given	A1	Must have M1 M1 (Co-)interior angles or allied angles (add up to 180°) and angles at a point (add up to 360°)

MARK SCHEME CONTINUES ON THE NEXT PAGE

Q	Answer	Mark	Comments
9	**Alternative method 3** $x = 180 - PBC$ **in terms of** y		
	$360 - (y + 100 + 2y + 80)$ or $360 - (3y + 180)$	M1	May be on diagram *PBC* (allow *B*)
	$x = 180 - (360 - (y + 100 + 2y + 80))$ $x = 180 - (360 - (3y + 180))$	M1	oe unsimplified correct equation
	Simplifies to $x = 3y$	A1	Must have seen correct working for M1 M1
	Both reasons given	A1	Must have M1 M1 (Co-)interior angles or allied angles (add up to 180°) and angles at a point (add up to 360°)
	Alternative method 4 $x + PBC = 180$ **and reflex** *PBC* **in terms of** $y + PBC = 360$		
	$x + PBC = 180$ or $y + 100 + 2y + 80 + PBC = 360$ or $3y + 180 + PBC = 360$	M1	*PBC* (allow *B*)
	$x + PBC = 180$ and $y + 100 + 2y + 80 + PBC = 360$ or $3y + 180 + PBC = 360$	M1	
	$x + PBC = 180$ and $3y + PBC = 180$ and $x = 3y$	A1	Must have seen correct working for M1 M1
	Both reasons given	A1	Must have M1 M1 (Co-)interior angles or allied angles (add up to 180°) and angles at a point (add up to 360°)

MARK SCHEME CONTINUES ON THE NEXT PAGE

Q	Answer	Mark	Comments
9	**Alternative method 5 (Produces *CB* to *X*) *PBX* = reflex *PBC* in terms of y – 180**		
	$y + 100 + 2y + 80$ or $3y + 180$	M1	May be on diagram reflex *PBC*
	$y + 100 + 2y + 80 – 180$ or $3y + 180 – 180$	M1	*PBX*
	Simplifies to $3y$ and states $x = 3y$	A1	Must have seen correct working for M1 M1
	Both reasons given	A1	Must have M1 M1 Angles on a (straight) line (add up to 180) and alternate angles (are equal)
	Alternative method 6 (Produces *PB* to *Y*) *CBY* = reflex *PBC* in terms of y – 180		
	$y + 100 + 2y + 80$ or $3y + 180$	M1	May be on diagram reflex *PBC* (allow reflex *B*)
	$y + 100 + 2y + 80 – 180$ or $3y + 180 – 180$	M1	*CBY*
	Simplifies to $3y$ and states $x = 3y$	A1	Must have seen correct working for M1 M1
	Both reasons given	A1	Must have M1 M1 Angles on a (straight) line (add up to 180) and corresponding angles (are equal)

MARK SCHEME CONTINUES ON THE NEXT PAGE

Q	Answer	Mark	Comments
9	**Alternative method 7 (Produces *CB* to *X*) *PBX* = 180 – *PBC* in terms of *y***		
	360 – (y + 100 + 2y + 80) or 360 – (3y + 180)	M1	May be on diagram *PBC* (allow *B*)
	180 – (360 – (y + 100 + 2y + 80)) or 180 – (360 – (3y + 180))	M1	*PBX*
	Simplifies to 3y and states x = 3y	A1	Must have seen correct working for M1 M1
	All reasons given	A1	Must have M1 M1 Angles at a point and angles on a (straight) line (add up to 180) and alternate angles (are equal)
	Alternative method 8 (Produces *PB* to *Y*) *CBY* = 180 – *PBC* in terms of *y*		
	360 – (y + 100 + 2y + 80) or 360 – (3y + 180)	M1	May be on diagram *PBC* (allow *B*)
	180 – (360 – (y + 100 + 2y + 80)) or 180 – (360 – (3y + 180))	M1	*CBY*
	Simplifies to 3y and states x = 3y	A1	Must have seen correct working for M1 M1
	All reasons given	A1	Must have M1 M1 Angles at a point and angles on a (straight) line (add up to 180) and corresponding angles (are equal)

ADDITIONAL GUIDANCE FOR Q9 IS ON THE NEXT PAGE

Q	Answer	Mark	Comments

Q	Additional Guidance (Q9)	
9	Recovery of brackets is not allowed as it is a proof	
	Acceptable reasons must include the word 'angles' Angles at a point can be angles round a point	
	These reasons are not allowed: Alternating angles, alternative angles, angles in a circle, straight line, at a point, round a point, parallel lines	
	Supplementary angles are not allowed unless accompanied by an acceptable reason eg Allied angles are supplementary is allowed	
	Other variations on these methods will be seen. Escalate if necessary.	
	Starting with $x = 3y$ or substituting values for x and y is zero unless M marks seen in working	

Q	Answer	Mark	Comments
10(a)	$(x-5)(x-2)$ or $(x-5)(x+3)$ or $(5-x)(2-x)$ or $(5-x)(-x-3)$	M1	oe factorisation
	$\frac{x-2}{x+3}$ or $\frac{2-x}{-x-3}$	A1	oe numerator and denominator both linear
	Additional Guidance		
	Correct answer followed by incorrect further work		M1 A0
	$\frac{x-2}{x+3}$ or $\frac{2-x}{-x-3}$ from incorrect method		M0 A0
	Allow fraction with correct factorisations and common factors crossed out		M1 A1
	Allow $x - 2 / x + 3$		M1 A1

Q	Answer	Mark	Comments
10(b)	$w^2x^3y^2(w^3 + x^3y)$	B2	B1 A correct partial factorisation with at least one variable fully factorised eg1 $y^2(w^5x^3 + w^2x^6y)$ eg2 $w^2x(w^3x^2y^2 + x^5y^3)$ or a correct partial factorisation with all three variables as factors eg $wxy\,(w^4x^2y + wx^5y^2)$ or full common factor with one term in brackets correct $w^2x^3y^2(w^3 + ...)$ or $w^2x^3y^2(... + x^3y)$ Must be two terms in each bracket
	Additional Guidance		
	Allow multiplication signs and 1s and unnecessary brackets for B2 and B1		
	Use of negative or fractional or zero powers is a maximum of B1		
	$w^2x^3y^2(w^3 + x^3y)$ followed by incorrect further work		B1
	Answer line incorrect, check working lines for possible B1		

Q	Answer	Mark	Comments
11	$4x^2$ or $3px^2$ or $4 + 3p$	M1	May be seen in an expansion or a grid Allow unsimplified eg $3x \times px$
	their $4(x^2)$ + their $3p(x^2) = –23(x^2)$	M1dep	Correct or ft their expansion ft is equating their terms in x^2 to $–23x^2$ Must be at least two terms with at least one linear term in p Allow unsimplified eg $3x \times px + 4x^2 = –23x^2$
	–9	A1	
	Additional Guidance		
	In this question, only consider terms in x^2		
	If only one term in x^2 the maximum mark is M1		
	$4 + 3p = –23$ followed by $7p = –23$		M1 M1 A0

Q	Answer	Mark	Comments
12(a)	$2n + 5$	B2	B1 $2n + k$ $k \neq 5$
	Additional Guidance		
	Allow any letter eg $2x + 5$		B2
	Allow $n =$ eg $n = 2n + 5$ or $n = 2x + 5$		B2
	$2n$		B1
	$2n + 5 = 0$ is B2 unless also seen with answer of –2.5 which then scores B1		

Q	Answer	Mark	Comments
12(b)	**Alternative method 1**		
	$(3n - 1)$ (×) their $(2n + 5)$	M1	ft their (a) Brackets needed but may be recovered
	$6n^2 + 15n - 2n - 5$	M1dep	4 terms with at least 3 correct Implied by $an^2 + 13n - 5$ $(a \neq 0)$ or $6n^2 + 13n + b$ $(b \neq 0)$ ft their two term linear expression in (a)
	$6n^2 + 13n - 5$	A1ft	ft their two term linear expression in (a)
	Alternative method 2		
	(Second differences =) 12 or $6n^2$ or $a = 6$	M1	Seen at least once and not contradicted
	14 – 6 45 – 24 88 – 54 (143 – 96) or 8 21 34 (47)	M1dep	Subtracts $6n^2$ from the terms in sequence Z
	$6n^2 + 13n - 5$	A1	
	Alternative method 3		
	Any two of $a + b + c = 14$ $4a + 2b + c = 45$ $9a + 3b + c = 88$ $16a + 4b + c = 143$	M1	
	$3a + b = 45 - 14$ and $5a + b = 88 - 45$	M1dep	oe Obtains two correct equations in same two variables from their equations
	$6n^2 + 13n - 5$	A1	

MARK SCHEME CONTINUES ON THE NEXT PAGE

Q	Answer	Mark	Comments
12(b)	**Alternative method 4**		
	(Second differences =) 12 or $6n^2$ or $a = 6$	M1	Seen at least once and not contradicted
	$3a + b = 45 - 14$ and substitutes $a = 6$	M1dep	oe eg1 $5a + b = 88 - 45$ and substitutes $a = 6$ eg2 $7a + b = 143 - 88$ and substitutes $a = 6$
	$6n^2 + 13n - 5$	A1	
	Alternative method 5		
	(Second differences =) 12	M1	Seen at least once and not contradicted
	$14 + (45 - 14)(n - 1) +$ $0.5 \times 12(n - 1)(n - 2)$	M1dep	Using $p + q(n - 1) + 0.5r(n - 1)(n - 2)$ p is 1st term q is 2nd term – 1st term r is second differences
	$6n^2 + 13n - 5$	A1	
	Additional Guidance		
	Allow any letter or mixed letters eg $6x^2 + 13x - 5$ or $6n^2 + 13x - 5$		M1 M1 A1
	Allow $n =$ eg $n = 6n^2 + 13x - 5$		M1 M1 A1
	$6n^2 + 13n - 5 = 0$ is M1 M1 A1 unless also seen with solutions which then scores M1 M1 A0		

Q	Answer	Mark	Comments
13	**Alternative method 1**		
	$a - b - 2 = 0$ or $a + \frac{7}{2}b - \frac{49}{2} = 0$	M1	oe equation Allow an unsimplified equation eg $a + b \times (-1) - 2(-1)^2 = 0$ Missing brackets can be recovered
	$a - b - 2 = 0$ and $a + \frac{7}{2}b - \frac{49}{2} = 0$	M1	oe two equations Allow unsimplified equations
	$\frac{7}{2}a + a = 7 + \frac{49}{2}$ or $\frac{7}{2}b - -b = \frac{49}{2} - 2$ and a – their $b - 2 = 0$	M1dep	oe dep on first M1 Correct method to form an equation in a or correct method to form an equation in b and substitutes to form an equation in a Their two equations must both contain a and b
	(0, 7)	A1	SC4 Answer (0, 7) from $(2x - 7)(1 + x)$ or $2x^2 + 2x - 7x - 7$ or $2x^2 - 5x - 7$

MARK SCHEME CONTINUES ON THE NEXT PAGE

Q	Answer	Mark	Comments
13	**Alternative method 2**		
	$(7 - 2x)$ or $(1 + x)$ or $(2x - 7)$ or $(-1 - x)$ or $(x - 3.5)$ or $(3.5 - x)$ or $2x^2 + 2x - 7x - 7$ or $2x^2 - 5x - 7$	M1	oe Brackets not needed
	$(7 - 2x)(1 + x)$ or $(2x - 7)(-1 - x)$ or $7 + 7x - 2x - 2x^2$ or $7 + 5x - 2x^2$	M1	oe eg $-(2x^2 - 5x - 7)$ $y =$ not needed Expansion not needed
	Substitutes $x = 0$ in their quadratic or selects the constant term from their quadratic	M1dep	dep on first M1 Expansion not needed for their quadratic but must be correct if attempted May be implied by the final answer
	(0, 7)	A1	SC4 Answer (0, 7) from $(2x - 7)(1 + x)$ or $2x^2 + 2x - 7x - 7$ or $2x^2 - 5x - 7$
	Alternative method 3		
	$b - 2 \times 2x$ or $b - 4x$	M1	Differentiates correctly
	$b - 2 \times 2 \times 1.25 = 0$ or $b - 4 \times 1.25 = 0$ or $b = 5$	M1	
	$a +$ their $b \times (-1) - 2 \times (-1)^2 = 0$ or $a +$ their $b \times (\frac{7}{2}) - 2 \times (\frac{7}{2})^2 = 0$	M1dep	oe dep on first M1 Must have substituted a value into $b - 4x$ and equated to 0 Missing brackets can be recovered
	(0, 7)	A1	SC4 Answer (0, 7) from $(2x - 7)(1 + x)$ or $2x^2 + 2x - 7x - 7$ or $2x^2 - 5x - 7$

MARK SCHEME CONTINUES ON THE NEXT PAGE

Q	Answer	Mark	Comments
13	**Alternative method 4**		
	$-2\ \{(x - \frac{b}{4})^2\ .....\ \}$	M1	oe completed square
	$\frac{b}{4} = 1.25$ or $b = 5$	M1	
	a + their $b \times (-1) - 2 \times (-1)^2 = 0$ or a + their $b \times (\frac{7}{2}) - 2 \times (\frac{7}{2})^2 = 0$	M1dep	oe dep on first M1 Must have equated $\frac{b}{4}$ to a value Missing brackets can be recovered
	(0, 7)	A1	SC4 Answer (0, 7) from $(2x - 7)(1 + x)$ or $2x^2 + 2x - 7x - 7$ or $2x^2 - 5x - 7$
	Additional Guidance		
	Answer (7, 0)		A0
	Answer only (0, 7)		M3 A1
	Alt 1 3rd M1 Follow correct method rules as in Q4 Alt 1 2nd M1		
	Answer (0, 7) with working may not score full marks Check the working each time eg (Alt 2) $2x^2 - 3x + 7$ (no method seen) Answer (0, 7)		Zero

Q	Answer	Mark	Comments
14	**Alternative method 1**		
	180 ÷ (7 + 5) or 15	M1	oe
	(w =) 7 × their 15 or 105 or (y =) 5 × their 15 or 75	M1dep	oe May be seen on diagram M2 105 : 75
	$\frac{180 - \text{their } w}{2}$ or $\frac{\text{their } y}{2}$	M1dep	oe dep on M1 M1
	37.5	A1	oe SC2 52.5
	Alternative method 2		
	$w + y = 180$ and $5w = 7y$ or $w + \frac{5}{7}w = 180$ or $y + \frac{7}{5}y = 180$	M1	oe
	$(w =)\frac{180 \times 7}{12}$ or 105 or $(y =)\ \frac{180 \times 5}{12}$ or 75	M1dep	oe May be seen on diagram M2 105 : 75
	$\frac{180 - \text{their } w}{2}$ or $\frac{\text{their } y}{2}$	M1dep	oe dep on M1 M1
	37.5	A1	oe SC2 52.5

MARK SCHEME CONTINUES ON THE NEXT PAGE

Q	Answer	Mark	Comments
14	**Alternative method 3**		
	$\frac{w}{180-w} = \frac{7}{5}$ or $\frac{y}{180-y} = \frac{5}{7}$	M1	oe
	$(w =)\frac{180 \times 7}{12}$ or 105 or $(y =)\frac{180 \times 5}{12}$ or 75	M1dep	oe May be seen on diagram M2 105 : 75
	$\frac{180 - \text{their } w}{2}$ or $\frac{\text{their } y}{2}$	M1dep	oe dep on M1 M1
	37.5	A1	oe SC2 52.5
	Alternative method 4		
	$y = 2x$ and $w = \frac{7}{5} \times 2x$	M1	oe
	$2x + \frac{7}{5} \times 2x = 180$ or $4.8x = 180$	M1dep	oe
	180 ÷ their 4.8	M1dep	oe dep on M1 M1
	37.5	A1	oe SC2 52.5
	Additional Guidance		
	75 : 105 implies M1 M0 M0 unless recovered (award SC2 if answer 52.5)		

Q	Answer	Mark	Comments
15(a)	$\sqrt{x} = \frac{5}{2}$ or $\sqrt{x} = 2.5$ or $\sqrt{x} = \frac{1}{0.4}$ or $\frac{4}{25}x = 1$ or $4x = 25$	M1	oe Must have $\sqrt{x} = ...$ or have eliminated $\sqrt{\ }$ with no errors
	$\frac{25}{4}$ or 6.25	A1	oe
	Additional Guidance		
	$2\sqrt{x} = 5$ (no further correct work)		M0
	Allow unprocessed values for M1 eg $\frac{4}{25}x = 1^2$ or $\left(\frac{2}{5}\right)^2 x = 1$		M1
	$\sqrt{x}$ may be seen as $x^{\frac{1}{2}}$ or $x^{0.5}$		

Q	Answer	Mark	Comments
15(b)	$x^2(x - 5)$ (= 0) or $x^2(5 - x)$ (= 0) or (x =) 0 or (x =) 5	M1	oe factorisation eg1 $(x^2 - 0)(x - 5)$ eg2 $x\,(x^2 - 5x)$
	0 and 5 with no other solutions	A1	
	Additional Guidance		
	For A1, the word 'and' is not needed If their answer has both 0 and 5 with no other solutions award M1 A1 eg 0, 5 with no other solutions		M1 A1
	0, 5, –5		M1 A0
	Either or both solutions seen embedded		M1 A0

Q	Answer	Mark	Comments
16	**Alternative method 1**		
	$yx = 8(w - x)$ or $y = \frac{8w - 8x}{x}$	M1	
	$yx = 8w - 8x$	M1dep	oe eg $yx - 8w + 8x = 0$ Implies M1 M1
	$yx + 8x = 8w$ or $x(y + 8) = 8w$ or $\frac{8w}{y+8}$	M1dep	oe dep on M1 M1 Implies M1 M1 M1
	$x = \frac{8w}{y+8}$	A1	oe eg $\frac{-8w}{-y-8}$ Must have $x =$ SC2 $x = \frac{8w}{y+1}$ SC1 $\frac{8w}{y+1}$
	Alternative method 2		
	$y = \frac{8w}{x} - 8$ or $y = \frac{8w}{x} - \frac{8x}{x}$	M1	
	$y + 8 = \frac{8w}{x}$	M1dep	oe eg $y + 8 - \frac{8w}{x} = 0$ Implies M1 M1
	$yx + 8x = 8w$ or $x(y + 8) = 8w$ or $\frac{1}{y+8} = \frac{x}{8w}$ or $\frac{8w}{y+8}$	M1dep	oe dep on M1 M1 Implies M1 M1 M1
	$x = \frac{8w}{y+8}$	A1	oe eg $\frac{-8w}{-y-8}$ Must have $x =$ SC2 $x = \frac{8w}{y+1}$ SC1 $\frac{8w}{y+1}$

MARK SCHEME CONTINUES ON THE NEXT PAGE

Q	Answer	Mark	Comments
16	**Alternative method 3**		
	$yx = 8(w - x)$	M1	
	$\frac{yx}{8} = w - x$	M1dep	oe eg $\frac{yx}{8} - w + x = 0$ Implies M1 M1
	$\frac{yx}{8} + x = w$ or $x(\frac{y}{8} + 1) = w$ or $\frac{w}{\frac{y}{8}+1}$	M1dep	oe dep on M1 M1 Implies M1 M1 M1
	$x = \frac{w}{\frac{y}{8}+1}$	A1	oe eg $x = \frac{-w}{-\frac{y}{8}-1}$ Must have $x =$ SC2 $x = \frac{8w}{y+1}$ SC1 $\frac{8w}{y+1}$
	Additional Guidance		
	$x = \frac{8w}{y+8}$ in working with $\frac{8w}{y+8}$ on answer line		M3 A1
	$x = \frac{8w}{y+1}$ in working with $\frac{8w}{y+1}$ on answer line		SC2
	3rd M1 is for collecting terms in x (or x in numerator in Alt 2)		
	Allow multiplications signs and 1s throughout		
	Correct answer followed by incorrect further work		M3 A0

Q	Answer	Mark	Comments
17	$\pi x^2 y$	M1	oe
	$\frac{1}{2} \times \frac{4}{3}\pi(6y)^3$ or $\frac{2}{3}\pi(6y)^3$ or $144\pi y^3$	M1	oe Only allow missing brackets if recovered in subsequent working M2 $\frac{2}{3}(6y)^3 = x^2 y$ oe equation eg $\frac{x^2}{y^2} = 144$ M2 $\sqrt{144}$
	12 or $\frac{12}{1}$	A1	SC2 $\sqrt{288}$ or $12\sqrt{2}$ or 16.97...
	Additional Guidance		
	12 and –12 or –12		A0
	Allow multiplication signs in M marks		
	Answer only of 12		M2 A1
	Answers only of 12 and –12		M2 A0
	Answer only of –12		M0 A0

Q	Answer	Mark	Comments
18(a)	$y > 45°$	B1	

Q	Answer	Mark	Comments
18(b)	$(p + 1)^2 + (p - 1)^2$	M1	oe May be within a square root
	$p^2 + p + p + 1 + p^2 - p - p + 1$ or $p^2 + 1 + p^2 + 1$	M1	oe May be within a square root Implies M1 M1
	$2p^2 + 2$ or $2(p^2 + 1)$	A1	May be within a square root Must be simplified May be implied by final mark
	$\dfrac{p+1}{\sqrt{2p^2+2}}$	A1	Allow $a = b = 1$ $c = d = 2$ SC2 $\dfrac{p+1}{\sqrt{(p+1)^2+(p-1)^2}}$
	Additional Guidance		
	$\dfrac{p+1}{\sqrt{2p^2+2}}$ and further incorrect work		M2 A1 A0
	Allow $1p$ for p		
	Use of $\dfrac{\sin y}{\cos y} = \dfrac{p+1}{p-1}$ can be marked by the scheme eg $(p - 1)^2 \sin^2 y = (p + 1)^2 \cos^2 y$ $(p - 1)^2 \sin^2 y = (p + 1)^2 (1 - \sin^2 y)$ $((p - 1)^2 + (p + 1)^2) \sin^2 y = (p + 1)^2$ First M1 gained here (M1 A1 A1 may subsequently be gained)		

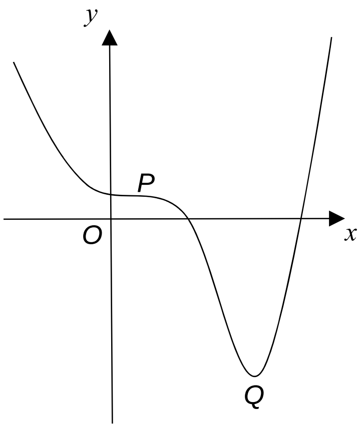
<table>
<tr><th>Q</th><th>Answer</th><th>Mark</th><th>Comments</th></tr>
<tr><td>19</td><td>Continuous curve with

point of inflection, labelled P or (1, 2), in first quadrant

and

minimum point, labelled Q or (a, b), in fourth quadrant, with x-coordinate of Q > x-coordinate of P

eg


</td><td>B3</td><td>For B3, allow the labelling of one coordinate as sufficient for each point

B2 As B3 but not sufficiently labelled

B1 Curve with point of inflection, labelled P or (1, 2), in first quadrant

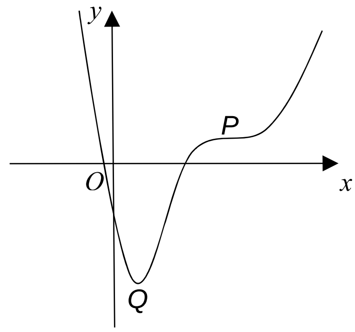
or

curve with minimum point, labelled Q or (a, b), in fourth quadrant

For B1, allow labelling using one coordinate as sufficient

SC2 As B3 but x-coordinate of Q < x-coordinate of P

eg


</td></tr>
</table>

ADDITIONAL GUIDANCE FOR Q19 IS ON THE NEXT PAGE

Q	Answer	Mark	Comments

Q	Additional Guidance (Q19)	
19	For B3, curve does not have to cross the x-axis after *Q* and does not have to cross the y-axis before *P*	
	For a stationary point, curve must not stop at the point	
	At *P*, the curve must change from concave upward to concave downward for B3 or B2 or vice-versa for B1 or SC2	
	Note that other non-stationary points of inflection may also be seen (up to B3 possible)	
	Curve may have horizontal asymptotes as $x \to \pm\ \infty$ (up to B3 possible)	
	Mark intention for stationary points, positioning of labels and smoothness of curve	
	More than 1 stationary point of inflection and/or more than 1 minimum point and/or maximum point(s) can score a maximum of B1	
	Labelling using a coordinate or coordinates may be seen by labelling on an axis or on axes (axes may also show other numbers)	

Q	Answer	Mark	Comments
20	$\begin{pmatrix} -1 & -3 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} a \\ 2 \end{pmatrix}$ or $\begin{pmatrix} -a-6 \\ 2a+8 \end{pmatrix}$	M1	Allow $(-a-6 \quad 2a+8)$
	$-a-6=a$ or $2a+8=2$	M1	oe linear equation(s) (not $a=-3$) Implies M1 M1
	$-a-6=a$ and $2a+8=2$	A1	oe equations (not $a=-3$)
	Shows both equations have a common solution ($a=-3$) and Yes	A1ft	ft M1 M1 A0 Must show that their two linear equations do not have a common solution and No SC4 $\begin{pmatrix} -1 & -3 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} -3 \\ 2 \end{pmatrix}=\begin{pmatrix} -3 \\ 2 \end{pmatrix}$ and Yes SC3 $\begin{pmatrix} -1 & -3 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} -3 \\ 2 \end{pmatrix}=\begin{pmatrix} -3 \\ 2 \end{pmatrix}$
	Additional Guidance		
	$\begin{pmatrix} a \\ 2 \end{pmatrix}\begin{pmatrix} -1 & -3 \\ 2 & 4 \end{pmatrix}$ is first M0 unless recovered		
	In matrices, allow missing brackets or inclusion of 'fraction' lines		
	Only one equation can score a maximum of M1 M1 A0 A0		
	$a=-3$ with no correct working		Zero
	$\begin{pmatrix} -a-6 \\ 2a+8 \end{pmatrix}=\begin{pmatrix} a \\ 2 \end{pmatrix}$ with no further valid work		M1 M0 A0 A0
	The final A mark may be seen in various ways eg1 Solves both equations obtaining $a=-3$ each time and Yes (or shows that both equations simplify to $2a=-6$ and Yes) eg2 Solves one equation obtaining $a=-3$ and shows by substitution that $a=-3$ satisfies the other equation and Yes eg3 Adds the two equations to obtain a correct statement and Yes $-2a-6=0$ $\underline{2a+8=2}$ $2=2$		

Q	Answer	Mark	Comments
21	$3(x-1)$ or $3x-3$ or $2(x-2)$ or $2x-4$	M1	
	$3x-3$ and $2x-4$ or $5x-7$	M1	Implies M1 M1
	$5(x-1)(x-2)$ or $5(x^2-2x-x+2)$ or $5x^2-15x+10$ or $(x-1)(x-2)$ expanded and multiplied by 5	M1	oe Allow one error in four term expansion of $5(x-1)(x-2)$ Implied by $5(x^2-3x+k)$ or $5(ax^2-3x+2)$
	$5x^2-20x+17$ (= 0)	M1dep	dep on 3rd M1 oe 3-term quadratic equation eg $5x^2-20x=-17$ Correctly collects terms in their expansion
	$\dfrac{--20\pm\sqrt{(-20)^2-4\times5\times17}}{2\times5}$ or $\dfrac{10\pm\sqrt{15}}{5}$ or $(x-2)^2-4=-\dfrac{17}{5}$ or $5[(x-2)^2-4]=-17$	M1	oe Correct use of quadratic formula for their 3-term quadratic eg $(-20)^2$ can be 20^2 or correct factorisation of their 3-term quadratic or attempt to complete the square for their 3-term quadratic Must be correct up to form $(x-a)^2+b=c$ or $k[(x-d)^2+e]=f$
	1.23 and 2.77	A1	Must be 3 significant figures

Additional Guidance

For A1, the word 'and' is not needed eg 1.23, 2.77 (with method seen)	M5 A1
Brackets may be recovered throughout	
5th M1 may be implied by solutions of their quadratic equation seen	
M0 M0 M0 M0 M1 A0 is possible if they have a 3-term quadratic equation	
Answers only	Zero

Q	Answer	Mark	Comments
22	**Alternative method 1 Triangles *VMB and VXM* (*M* is the midpoint of *BC*)**		
	$17^2 - (16 \div 2)^2$ or 225 or $17^2 = VM^2 + (16 \div 2)^2$	M1	oe
	$(VM =) \sqrt{17^2 - (16 \div 2)^2}$ or $\sqrt{225}$ or 15	M1	Implies M1 M1 May be seen on diagram
	$\cos x = \dfrac{22 \div 2}{\text{their } 15}$ or $\sin x = \dfrac{\sqrt{15^2 - (22 \div 2)^2}}{\text{their } 15}$ or $\tan x = \dfrac{\sqrt{15^2 - (22 \div 2)^2}}{22 \div 2}$	M1dep	x is required angle dep on M1 M1 oe eg correct method using cosine rule or sine rule simplified to $\cos x =$ or $\sin x =$ or $90 - \sin^{-1} \dfrac{22 \div 2}{\text{their } 15}$
	42.8…	A1	Allow 43 with correct working SC2 Answer 36.8... or 36.9

MARK SCHEME CONTINUES ON THE NEXT PAGE

Q	Answer	Mark	Comments
22	**Alternative method 2 Triangles *BXM* and *VXB and VXM* (*M* is the midpoint of *BC*)**		
	$BX^2 = (16 \div 2)^2 + (22 \div 2)^2$ or 185 and 17^2 – their BX^2 or $17^2 = VX^2$ + their BX^2	M1	oe eg for BX^2 $BX^2 = (\frac{1}{2}BD)^2 = \frac{1}{4}(16^2 + 22^2)$
	$(VX =) \sqrt{17^2 - (\text{their } BX)^2}$ or $\sqrt{104}$ or $2\sqrt{26}$ or [10.19, 10.2]	M1	Implies M1 M1 May be seen on diagram
	$\tan x = \frac{\text{their } [10.19, 10.2]}{22 \div 2}$ or $\sin x = \frac{\text{their } [10.19, 10.2]}{\text{their } VM}$ or $\cos x = \frac{22 \div 2}{\text{their } VM}$	M1dep	x is required angle dep on M1 M1 oe eg correct method using cosine rule or sine rule simplified to $\cos x =$ or $\sin x =$ or $90 - \tan^{-1} \frac{22 \div 2}{\text{their } [10.19, 10.2]}$
	42.8…	A1	Allow 43 with correct working SC2 Answer 36.8... or 36.9

MARK SCHEME CONTINUES ON THE NEXT PAGE

Q	Answer	Mark	Comments
22	**Alternative method 3 Triangles *VMB* and *VMN* (*M* is the midpoint of *BC*, *N* is the midpoint of *AD*)**		
	$17^2 - (16 \div 2)^2$ or 225 or $17^2 = VM^2 + (16 \div 2)^2$	M1	oe
	$(VM =)\sqrt{17^2 - (16 \div 2)^2}$ or $\sqrt{225}$ or 15	M1	Implies M1 M1 May be seen on diagram
	$\frac{1}{2} \times (180 - \cos^{-1} \frac{\text{their } 15^2 + \text{their } 15^2 - 22^2}{2 \times \text{their } 15 \times \text{their } 15})$	M1dep	dep on M1 M1
	42.8…	A1	Allow 43 with correct working SC2 Answer 36.8... or 36.9
	Additional Guidance		
	Alt 2 3rd M1 their *VM* must be from correct method		

Q	Answer	Mark	Comments
23	**Alternative method 1**		
	$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ or $3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	B1	
	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	B1	
	their $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ (×) their $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$	M1	Either order This mark cannot be implied Must have scored B1 or B2
	$\begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$ or $-3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ or $3\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	M1dep	Correctly multiplies their pair of 2 by 2 matrices
	$\begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$ or $-3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ or $3\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ and scale factor –3	A1	Must gain B1 B1 M1 M1

MARK SCHEME CONTINUES ON THE NEXT PAGE

Q	Answer		Mark	Comments
23	**Alternative method 2 Algebraic method**			
	$\begin{pmatrix}3 & 0\\0 & 3\end{pmatrix}$ or $3\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$		B1	
	$\begin{pmatrix}-1 & 0\\0 & -1\end{pmatrix}$		B1	
	their $\begin{pmatrix}3 & 0\\0 & 3\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$ = $\begin{pmatrix}3x\\3y\end{pmatrix}$	their $\begin{pmatrix}-1 & 0\\0 & -1\end{pmatrix}$ (×) $\begin{pmatrix}x\\y\end{pmatrix}$ = $\begin{pmatrix}-x\\-y\end{pmatrix}$	M1	This mark cannot be implied Must have scored B1 or B2 Multiplications must be correctly worked out
	their $\begin{pmatrix}-1 & 0\\0 & -1\end{pmatrix}$ (×) their $\begin{pmatrix}3x\\3y\end{pmatrix}$ = $\begin{pmatrix}-3x\\-3y\end{pmatrix}$	their $\begin{pmatrix}3 & 0\\0 & 3\end{pmatrix}$ (×) their $\begin{pmatrix}-x\\-y\end{pmatrix}$ = $\begin{pmatrix}-3x\\-3y\end{pmatrix}$	M1dep	Multiplications must be correctly worked out
	$\begin{pmatrix}-3x\\-3y\end{pmatrix}$ and scale factor –3		A1	Must gain B1 B1 M1 M1

MARK SCHEME CONTINUES ON THE NEXT PAGE

Q	Answer		Mark	Comments
23	**Alternative method 3 Unit square method**			
	$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ or $3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$		B1	
	$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$		B1	
	their $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ (×) $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ $= \begin{pmatrix} 3 & 0 & 3 \\ 0 & 3 & 3 \end{pmatrix}$	their $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ (×) $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ $= \begin{pmatrix} -1 & 0 & -1 \\ 0 & -1 & -1 \end{pmatrix}$	M1	This mark cannot be implied Must have scored B1 or B2 Multiplications must be correctly worked out May be seen as three products
	their $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ (×) their $\begin{pmatrix} 3 & 0 & 3 \\ 0 & 3 & 3 \end{pmatrix}$ $= \begin{pmatrix} -3 & 0 & -3 \\ 0 & -3 & -3 \end{pmatrix}$	their $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ (×) their $\begin{pmatrix} -1 & 0 & -1 \\ 0 & -1 & -1 \end{pmatrix} = \begin{pmatrix} -3 & 0 & -3 \\ 0 & -3 & -3 \end{pmatrix}$	M1dep	Multiplications must be correctly worked out May be seen as three products
	$\begin{pmatrix} -3 & 0 & -3 \\ 0 & -3 & -3 \end{pmatrix}$ and scale factor –3		A1	Must gain B1 B1 M1 M1 May be seen as three 2 by 1 matrices

ADDITIONAL GUIDANCE FOR Q23 IS ON THE NEXT PAGE

Q	Answer	Mark	Comments

Q	Additional Guidance (Q23)	
23	If both matrices are incorrect	Zero
	Matrices must be used - ignore diagrams	
	In matrices, allow missing brackets or inclusion of 'fraction' lines	
	Alt 1 B2 gained then $\begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$ stated	B2 M0 M0 A0
	Allow 'enlargement –3' for 'scale factor –3' Do not allow '–3' for 'scale factor –3'	
	Scale factor –3 with no valid working	Zero
	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ scores B1 but does not score M1 M1 for the multiplication of two matrices with B1 scored	
	Alt 3 May also see working for $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	

Q	Answer	Mark	Comments
24	$\frac{x+1}{2(x+1)+1}$ or $\frac{x+1}{2x+3}$	M1	oe
	$(2x + 1)(2x + 3)$	M1	oe Correct common denominator for their two fractions Their two fractions must have different algebraic denominators
	$(2x + 1)(x + 1)$ and $x(2x + 3)$	M1dep	oe Correct expressions for their numerators dep on 2nd M1
	$2x^2 + 2x + x + 1 – 2x^2 – 3x$	M1dep	Subtracts their numerators Must expand all brackets Allow one sign error dep on 2nd M1
	$\frac{1}{(2x+3)(2x+1)}$ or $\frac{1}{4x^2+8x+3}$	A1	oe fraction in its simplest form
	Additional Guidance		
	Correct answer followed by incorrect further work However, $\frac{1}{(2x+3)(2x+1)}$ with an attempt to expand the denominator should not be penalised even if their expansion is not correct	M4 A0	
	2nd and 3rd M1 $(2x + 1)$ must be used		
	$\frac{2x^2+3x+1}{(2x+3)(2x+1)} - \frac{2x^2+3x}{(2x+3)(2x+1)}$ (no further correct work)	M1 M1 M1 M0 A0	
	Allow = 0 throughout, including in the answer		

Q	Answer	Mark	Comments
25	y-coordinate of C is –5 or (0, –5)	B1	May be on diagram or implied by final answer
	$3 \times 2x^2$ or $6x^2$	M1	Differentiates correctly
	$6 \times 2 \times 2$ or 24	M1	Substitutes $x = 2$ in their $\frac{dy}{dx}$ their $\frac{dy}{dx}$ must be a function of x but cannot be $2x^3 - 5$
	$y - 11$ = their 24 $(x - 2)$ or y = their $24x + c$ and substitutes (2, 11) or $y = 24x - 37$	M1dep	dep on 2nd M1
	$y - 11$ = their 24 $(0 - 2)$ or $y = 0 - 37$ or $y = -37$ or (y-coordinate of D =) –37	M1dep	Substitutes $x = 0$ into their linear equation dep on 3rd M1 May be seen on diagram 11 – 2 × their 24 scores 3rd M1 and 4th M1
	32	A1ft	ft B0 M4 with a positive length from their y-coordinate of C – –37
	Additional Guidance		
	–32		A0
	(–5, 0) is B0 unless recovered		
	1st M1 Allow $6x^2 + c$ (c an unknown constant) if c subsequently rejected		
	Differentiates to $6x^2 - 5$ 1st M0 gradient = $6 \times 2^2 - 5 = 19$ 2nd M1 Could continue and gain 3rd M1 and 4th M1		
	3rd M1 Cannot be the equation of the normal at (2, 11)		

Q	Answer	Mark	Comments
26(a)	**Alternative method 1 (LHS → RHS)**		
	$\sin^2 x - 3(1 - \sin^2 x)$	M1	Must see $(1 - \sin^2 x)$
	$\sin^2 x - 3 + 3\sin^2 x = 4\sin^2 x - 3$	A1	Must see correct expansion SC1 Correct rearrangement of given identity to $3\sin^2 x + 3\cos^2 x = 3$ and $3(\sin^2 x + \cos^2 x) = 3$ and $\sin^2 x + \cos^2 x = 1$
	Alternative method 2 (LHS → RHS)		
	$1 - \cos^2 x - 3\cos^2 x = 1 - 4\cos^2 x$ $= 1 - 4(1 - \sin^2 x)$	M1	Must see $(1 - \cos^2 x)$ and $(1 - \sin^2 x)$
	$1 - 4 + 4\sin^2 x = 4\sin^2 x - 3$	A1	Must see correct expansion SC1 Correct rearrangement of given identity to $3\sin^2 x + 3\cos^2 x = 3$ and $3(\sin^2 x + \cos^2 x) = 3$ and $\sin^2 x + \cos^2 x = 1$
	Alternative method 3 (RHS → LHS)		
	$4\sin^2 x - 3(\sin^2 x + \cos^2 x)$	M1	Must see $(\sin^2 x + \cos^2 x)$
	$4\sin^2 x - 3\sin^2 x - 3\cos^2 x$ $= \sin^2 x - 3\cos^2 x$	A1	Must see correct expansion SC1 Correct rearrangement of given identity to $3\sin^2 x + 3\cos^2 x = 3$ and $3(\sin^2 x + \cos^2 x) = 3$ and $\sin^2 x + \cos^2 x = 1$

MARK SCHEME CONTINUES ON THE NEXT PAGE

Q	Answer	Mark	Comments
26(a)	**Alternative method 4 (RHS → LHS)**		
	$4(1 - \cos^2 x) - 3 = 4 - 4\cos^2 x - 3$ $= 1 - 4\cos^2 x$ $= \sin^2 x + \cos^2 x - 4\cos^2 x$	M1	Must see $(1 - \cos^2 x)$ and $\sin^2 x + \cos^2 x$ and correct expansion
	$= \sin^2 x - 3\cos^2 x$	A1	SC1 Correct rearrangement of given identity to $3\sin^2 x + 3\cos^2 x = 3$ and $3(\sin^2 x + \cos^2 x) = 3$ and $\sin^2 x + \cos^2 x = 1$
	Alternative method 5 (LHS and RHS → common expression)		
	$1 - \cos^2 x - 3\cos^2 x = 1 - 4\cos^2 x$ and $4(1 - \cos^2 x) - 3 = 4 - 4\cos^2 x - 3$ $= 1 - 4\cos^2 x$	B2	Must see $(1 - \cos^2 x)$ and correct expansion SC1 Correct rearrangement of given identity to $3\sin^2 x + 3\cos^2 x = 3$ and $3(\sin^2 x + \cos^2 x) = 3$ and $\sin^2 x + \cos^2 x = 1$

ADDITIONAL GUIDANCE FOR Q26(a) IS ON THE NEXT PAGE

Q	Answer	Mark	Comments

Q	Additional Guidance (Q26(a))	
26(a)	As shown in the mark scheme, allow = signs but they may be seen (correctly) as the identity symbol	
	= signs may be implied (eg working down the page, line by line)	
	To give M1 the working must not need any further identities applying	
	The other side of the identity may be seen throughout working in Alts 1 to 4 However, full working on one side of the identity is needed for M1 A1 eg (Alt 2) $1 - \cos^2 x - 3\cos^2 x = 4\sin^2 x - 3$ $1 - 4\cos^2 x = 4\sin^2 x - 3$ $1 - 4(1 - \sin^2 x) = 4\sin^2 x - 3$ $1 - 4 + 4\sin^2 x = 4\sin^2 x - 3$ (with $4\sin^2 x - 3 = 4\sin^2 x - 3$ it would be M1 A1)	M1 A0
	Other examples may be seen, escalate if necessary	
	Allow any variable or mixed variables or no variables	
	Allow $(\sin x)^2$ for $\sin^2 x$ and $(\cos x)^2$ for $\cos^2 x$ Allow s^2 for $\sin^2 x$ and c^2 for $\cos^2 x$	
	Do not allow $\sin x^2$ for $\sin^2 x$ (but could still gain M1) eg1 Alt 1 $\sin^2 x - 3(1 - \sin^2 x)$ $= \sin^2 x - 3 + 3\sin x^2 = 4\sin x^2 - 3$ eg1 Alt 1 $\sin x^2 - 3(1 - \sin^2 x)$ $= \sin^2 x - 3 + 3\sin x^2 = 4\sin x^2 - 3$	M1 A0 M0 A0
	Do not allow recovery of missing brackets as this is a proof	
	SC1 Instead of factorisation, they can divide by 3	
	Other examples of SC1 may be seen where the identity is assumed to be correct and correct working with use of $\sin^2 x + \cos^2 x = 1$ is seen	

Q	Answer	Mark	Comments
26(b)	**Alternative method 1**		
	$\sin^2 x = \frac{3}{4}$ or $\sin x = \frac{\sqrt{3}}{2}$ or $\sin x = \sqrt{\frac{3}{4}}$ or 60 or 120	M1	oe eg $(\sin x)^2 = \frac{3}{4}$ Allow 0.86... or 0.87 for $\frac{\sqrt{3}}{2}$ Must have $\sin^2 x =$ or $\sin x =$ or $\sin^{-1}$ Allow s for $\sin x$ Do not allow $\sin x^2$ for $\sin^2 x$ but may be recovered
	$\sin x = -\frac{\sqrt{3}}{2}$ or $\sin x = -\sqrt{\frac{3}{4}}$ or 240 or 300 or –60	M1	oe Allow –0.86... or –0.87 for $-\frac{\sqrt{3}}{2}$
	60 and 120 and 240 and 300 with no other angles in range	A2	A1 60 and 120 or 240 and 300
	Alternative method 2		
	$\tan^2 x = 3$ or $\tan x = \sqrt{3}$ or 60 or 240	M1	oe eg $(\tan x)^2 = 3$ Allow 1.73... for $\sqrt{3}$ Must have $\tan^2 x =$ or $\tan x =$ or $\tan^{-1}$ Allow t for $\tan x$ Do not allow $\tan x^2$ for $\tan^2 x$ but may be recovered
	$\tan x = -\sqrt{3}$ or 120 or 300 or –60	M1	Allow –1.73... for $-\sqrt{3}$
	60 and 120 and 240 and 300 with no other angles in range	A2	A1 60 and 240 or 120 and 300

MARK SCHEME CONTINUES ON THE NEXT PAGE

Q	Answer	Mark	Comments
26(b)	**Alternative method 3**		
	$\cos^2 x = \frac{1}{4}$ or $\cos x = \frac{1}{2}$ or $\cos x = \sqrt{\frac{1}{4}}$ or 60 or 300	M1	oe eg $(\cos x)^2 = \frac{1}{4}$ Must have $\cos^2 x =$ or $\cos x =$ or $\cos^{-1}$ Allow c for cos x Do not allow cos x^2 for $\cos^2 x$ but may be recovered
	$\cos x = -\frac{1}{2}$ or $\cos x = -\sqrt{\frac{1}{4}}$ or 120 or 240	M1	oe
	60 and 120 and 240 and 300 with no other angles in range	A2	A1 60 and 300 or 120 and 240

ADDITIONAL GUIDANCE FOR Q26b IS ON THE NEXT PAGE

Q	Answer	Mark	Comments

Q	Additional Guidance (Q26(b))	
26(b)	Ignore any solutions outside of $0 < x < 360$ ie 0 and 360 are outside the range and can be ignored	
	All four solutions with extra solutions in range, $0 < x < 360$,are penalised one accuracy mark eg 60 90 120 150 240 300 Only penalise extra solutions in range when all four correct solutions are given	M1 M1 A1
	Answer line blank, award any marks gained from working lines	
	If angles are found in working lines but only some are listed on answer line award any method marks gained from the working lines award any accuracy marks gained from the answer line eg1 Working lines $\sin x = \pm\frac{\sqrt{3}}{2}$ 60 and 120 and 240 and 300 Answer line 60 and 120 and 240 eg2 Working lines $\tan x = \sqrt{3}$ 60 240 Answer line 60 eg3 Working lines $\sin x = \frac{\sqrt{3}}{2}$ 60 120 $\sin x = -\frac{\sqrt{3}}{2}$ 300 Answer line 300	 M1 M1 A1 M1 M0 A0 M1 M1 A0
	Answers only can score up to 4 marks All 4 correct → 4 marks 3 correct → 3 marks 2 correct → 2 marks 1 correct → 1 mark	
	M1 M0 A1 or M0 M1 A1 are possible eg1 $\sin x = \frac{\sqrt{3}}{2}$ 60 120 eg2 $\sin x = -\frac{\sqrt{3}}{2}$ 240 300	 M1 M0 A1 M0 M1 A1
	Embedded answers can score up to M1 M1 A0	
	Working in rads or grads can score M marks if method seen	