Momentum & Collisions

Newton's 3rd Law of Motion describes what will happen when two objects meet. The second of three very big ideas concerning conservation laws comes from the 3rd Law. In the first of five lessons we will merely define momentum and kinetic energy in terms of mass and velocity. Before taking the next test students should be sure that they are familiar with all of the material from the following outline.

I. Momentum Definitions

- A. With velocity
- B. Momentum and Force
- C. Momentum and Kinetic Energy

II. Momentum and Impulse

- A. Graphical Analysis
- B. Algebraic Analysis

III. Collisions

- A. One Dimensional Totally Inelastic
- B. One Dimensional Elastic
- C. Two Dimensional Totally Inelastic

IV. Explosions

V. Ballistic Pendulum & Variations on Theme

This unit should take about seven days from beginning to end. The first lesson will cover I.A-C and II.A&B. Lesson 2 will explore III.A. Lesson 3 studies III.B. Lesson 4 looks into III.C and IV. Lesson 5 will consider V. The sixth day will be a practice test in small groups. The last day of the unit will be test day.

Lesson 2-12 **Momentum & Impulse**

Any object in motion will have a quantity called "momentum". Momentum is a vector quantity so that you will need to indicate both an amount and a direction. The direction of the momentum vector is in the same direction as an object's velocity vector. Momentum has a value that is the product of an object's mass times its velocity. The symbol for momentum is "p". Momentum is measured in the units of Kg∗m / sec or N∗sec. Consider the following example. A 4 kg object is moving to the right at 5 m/s. The momentum is p = + 20 N∗sec.

$$\boxed{\mathbf{p} = m\,\mathbf{v}}$$

Kinetic Energy

An object in motion also has energy of motion known as kinetic energy. Kinetic energy is a scalar quantity that is measured in Joules. One Joule is equivalent to1 N∗m or 1 Kg∗m^2/s^2. The kinetic energy can be calculated as shown in the box to the right. The 4 Kg object from the previous example has an energy of 50 Joules.

$$\boxed{KE = p^2/\,(2m)}$$

There is also a direct relationship between an object's momentum and its kinetic energy. Kinetic energy is also found by using $p^2/\,(2m)$. Find answers to the following:

1. Find the momentum and kinetic energy of a 5 Kg object moving left at 6 m/s.
2. Find the kinetic energy and the velocity of a 3 Kg object if it has +24 N∗sec of momentum.
3. Find the magnitude of the momentum and speed of a 6 Kg object that has 300 J of kinetic energy.
4. An object has a mass of M and a velocity of v. If you double the object's speed while keeping mass constant you will find the kinetic energy has increased by what factor?
5. A 20 N force acts on a 10 Kg mass moving it from rest. If the applied force acts on the mass for a total of 4 seconds then what will be the final velocity? What will be the final momentum?

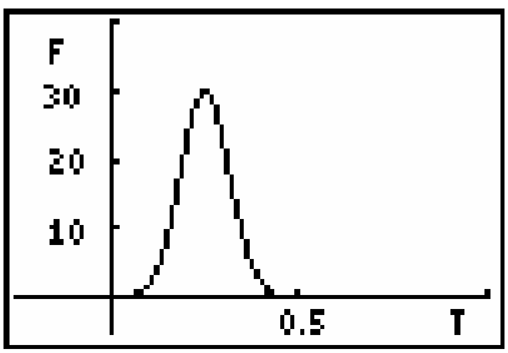
- ✓ Define momentum in terms of mass and velocity.
- ✓ Define kinetic energy in terms of mass and velocity.
- ✓ Write the correct units of both momentum and energy.

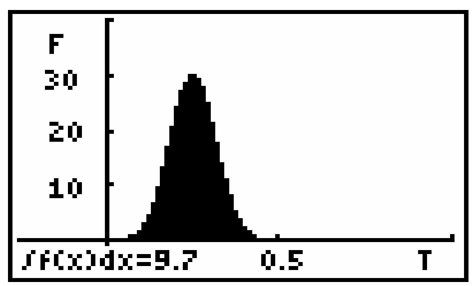
Momentum and Force

When two objects come into contact with one another there will be a force present due to action and reaction. As the objects push back and forth they will experience a change in their own momentum. The quicker an object changes its momentum then greater will be the amount of force the object experiences. Consider the following question- “If you come to rest from a speed of 150 mph will it hurt?” This question has a variety of answers that all depend on how quickly the momentum changes. Drive into a brick wall at 150 mph and the rapid change in momentum is devastating. On the other hand, airplanes generally land at similar speeds without losing a single customer since they take a longer time to slow to rest.

The relationship among force, change in momentum and time are expressed in the box to the right. With the equation written in the box you can see that force is always reduced if the stopping time is increased. When a baseball or softball player catches a line drive they know that it could possibly cause injury. To minimize the chance of that happening you will notice that they will catch the ball to the side and “give” with the catch. Raw eggs are caught in a similar manner. A study of cats that fell from tall buildings in New York City found that cats falling from more than seven stories high had a better chance of living when compared to cats falling from less than five stories. It was proposed that the cats tended to relax after about seven stories so that their impact time with pavement was increased.

$$\boxed{\mathbf{F} = \frac{\Delta \mathbf{p}}{\Delta t}}$$





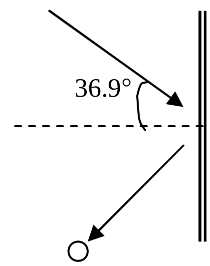
Consider the impulsive force in the above graph. A force goes from nothing to 30 N to nothing again in less than half a second. This could represent a bat, club or racquet hitting a ball. Since the force continues to vary there is no way to easily analyze the accelerations of any object experiencing this type of force. However, we can look at the area under the curve to determine the change in momentum. From the second figure you can see that the momentum changed by 9.7 Nsec. For a 150 gram ball at rest before this impulse acted you would see a final speed of 64.7 m/s.

Checkpoints for this section:

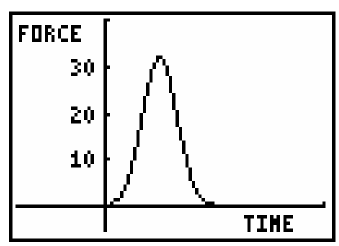
- ✓ Use the boxed equation above to analyze an object in motion algebraically.
- ✓ Use graphical analysis of force vs. time to determine the change in momentum.

Practice Problems for Force and Momentum

1. A 250 gram mud ball is thrown at a speed of 12 m/s perpendicular to a brick wall. Upon making contact with the wall the mud ball comes to rest in 0.08 seconds. Find the change in momentum and the average force acting on the ball. What was the average acceleration?
2. A 200 gram rubber ball is thrown at a speed of 16 m/s normal to a brick wall. The ball rebounds off the wall at a speed of 10 m/s. If the ball is in contact with the wall for only 0.09 seconds what is the average force exerted on the ball by the wall?
3. A 500 gram mass is dropped from a height of 2 meters above the floor. How fast is it traveling when it makes contact with the ground? If the mass is in contact with the ground for only 0.2 seconds before coming to rest then what is the average impulsive force acting on the floor? What is the average total force on the floor during impact? Sketch a rough graph of the total force vs. time.
4. A 125 gram rubber ball impacts a wall at 25 m/s at an angle of 36.9° from the normal to the wall. The elastic ball bounces off the wall at the same speed and angle. What is the change in momentum for the ball during the bounce? The diagram to the right is a top view of the ball and wall.



5. If the angle in the previous problem is doubled will the ball experience more or less force than before? Assume approximately the same impact time in either case.
6. An impulsive force of F = -450t (t – 0.5) acts on a 200 gram ball for ½ second. Graph this force with time on the x-axis and force on the y-axis. At what time does the maximum force occur on the ball? What is the maximum force acting at that instant? What is the total change in momentum for the ball? If the ball is initially at rest what will be its final speed?
7. An impulsive force of F = 32N * e ^ (- 80(t – 0.25)^2) acts on a 240 gram ball that is initially at rest. The force is in contact for 0.6 seconds. What is the change in the ball's momentum? What is the final speed?



	a	b	c	d
1	3 Ns	37.5 N	150 m/s^2	
2	57.8 N			
3	6.3 m/s	15.8 N	20.8 N	
4	5 Nsec			
5	less			
6	0.25 sec	28.1 N	9.375 Ns	46.9 m/s
7	6.34 Ns	26.4 m/s		

Lesson 2-13 **One Dimensional Inelastic Collisions**

In this section you will consider what happens when two objects meet in a head-on type collision (1 dimensional) and stick together. When objects stick together the maximum amount of kinetic energy is converted to heat or other forms. Although the kinetic energy of the system decreases the amount of momentum for the combination will remain the same both before and after the collision.

Suppose that the two objects in the above diagram meet and stick together. If the collision occurs in 0.25 seconds then what will be the final speed of the combination? What force did each block experience during the collision? How much kinetic energy was lost during the collision? To answer these questions one needs to understand a law that springs from Newton's 3rd Law of Motion. "The momentum of the system before an event will equal the momentum of the system after the event in the absence of external forces." As long as the above blocks encounter only forces between each other the momentum before will equal the momentum after.

$$\boxed{m_a\vec{v}_{ai} + m_b\vec{v}_{bi} = m_a\vec{v}_{af} + m_b\vec{v}_{bf}}$$

For the above example problem you can see that the total momentum before the collision is + 90 Ns for the block moving to the right and – 30 Ns for the block moving to the left. Before the collision the net momentum of the system is +60 Ns. This will also be true after they stick together. Since they will have the same final speed the math is simplified to only one equation with one unknown.

$$(5\text{Kg})(+18\text{m/s}) + (10\text{ Kg})(-3\text{ m/s}) = (15\text{ Kg})V_f$$

The final speed of the combined masses is 4 m/s to the right. Using yesterday's equation we can determine the force that either block experienced during the collision:

$$(5\text{ Kg})\{+4\text{ m/s} - 18\text{ m/s}\} / 0.25\text{sec}) = -280\text{ N}$$

$$(10\text{ Kg})\{+4\text{ m/s} - (-3)\text{m/s}\} / 0.25\text{ sec}) = +280\text{ N}$$

You can also determine the amount of kinetic energy lost in the collision:

$K.E._f = \frac{1}{2}(15\text{ Kg})(4\text{ m/s})^2 = 120\text{ J}$ remains after the collision.

$K.E._i = \frac{1}{2}(5\text{Kg})(18\text{m/s})^2 + \frac{1}{2}(10\text{Kg})(3\text{m/s})^2 = 855\text{ J}$ before the collision.

$\Delta K.E. = KE_f - KE_i = 120\text{ J} - 855\text{ J} = -735\text{ J}.$

The negative sign in the final answer indicates that energy decreased rather than increased.

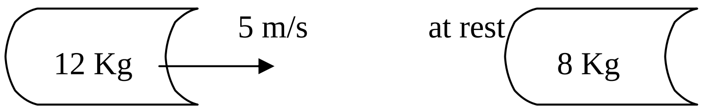
- ✓ State conservation of momentum in verbal or mathematical form.
- ✓ Recognize the single conditional requirement for momentum to be conserved.
- ✓ Define the specifics in "one-dimensional" and "totally inelastic".
- ✓ Calculate the final velocity, exchange force and kinetic energy loss when given the initial velocities, masses for two objects and collision time.

Practice Problems for Head-on Totally Inelastic Collisions

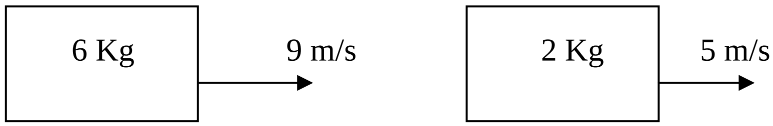
Find for the following: a) final velocity, b) kinetic energy loss and c) exchange force.

1. The collision occurs in 0.20 seconds.

2. The collision takes place in 0.33 seconds.

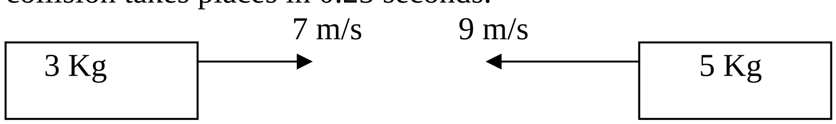


3. The collision happens in 0.167 seconds.



4. The collision lasts only 0.125 seconds.

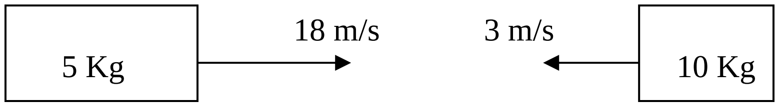
5. The collision takes places in 0.25 seconds.



	Final velocity	Δ Kinetic Energy	± Force
1	+ 5 m/s	- 192 J	160 N
2	+3 m/s	- 60 J	73 N
3	+8 m/s	- 12 J	36 N
4	0 m/s	- 216 J	288 N
5	- 3 m/s	- 240 J	120 N

Lesson 2-14 **Perfectly Elastic Head-on Collisions**

Suppose that two objects meet along a line as before except that this time there is no way for the kinetic energy to be converted to heat. Perhaps there is a spring or two magnets in between the objects so that they do interact but do not come into direct contact. This is what is known as a "perfectly elastic" collision. There will be two final velocities so that you will have two equations for two unknowns. Recall the example from the previous lesson.



How could these objects bounce and have the same total momentum as well as the same total kinetic energy before and after the collision? The only possible way for both momentum and energy to be conserved would be the following:

Conservation of Momentum $(5Kg)(+18m/s) + (10Kg)(-3m/s) = (5Kg)V_5 + (10Kg)V_{10}$

Conservation of Kinetic Energy $\frac{1}{2}(5)(18^2) + \frac{1}{2}(10)(3^2) = \frac{1}{2}(5)V_5^2 + \frac{1}{2}(10)V_{10}^2$

You should notice that you have two equations and two unknowns which can be solved algebraically or graphically to determine the final velocity of either object. After doing the algebra which involves the quadratic formula you find that the 5 Kg block must bounce backwards at 10 m/s. The 10 Kg block is now moving forward at 11 m/s. The algebraic method will always lead to a mess since you have the velocities being to the first power in the momentum equations while the velocities are quadratic in the energy expression. There is another way…

You can combine the momentum and energy equations to get a third equation that is also linear. The result is shown below without proof.

$$\boxed{\vec{V}_{Ai} + \vec{V}_{Af} = \vec{V}_{Bi} + \vec{V}_{Bf}}$$

By using this result along with the momentum equation you can always get quick results that will ensure that KE is conserved without having to do the messy algebra. For the above problem you get the following results:

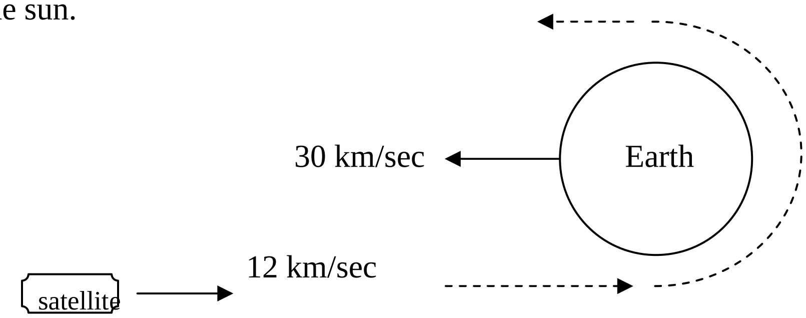
1) $60 = 5\,V_5 + 10\,V_{10}$ momentum
2) $342 = V_5^2 + 2\,V_{10}^2$ kinetic energy
3) $18 + V_5 = -3 + V_{10}$ relative velocity

From the third equation you can isolate $V_{10} = 21 + V_5$ and substitute back into equation 1. $60 = 5\,V_5 + 10\{21 + V_5\}$ After distributing the 10 and collecting like terms on either side you can solve for the final velocity of the 5 Kg block. $V_5 = -10$ m/s. By plugging that result back into the isolation step you see that $V_{10} = +11$ m/s.

The boxed equation also has a physical significance. It tells us that the relative velocity of approach is equal to the relative velocity of rebound. If for example you were riding on top of the 5 Kg block during this collision you would have seen the 10 Kg block coming at you at a relative speed of 21 m/s before the collision. You would also see that the 10 Kg block appears to move away from you at the same speed, 21 m/s.

NASA and the Gravitational "Slingshot" Effect

When a small satellite crosses the path of an orbiting planet and is also very near to the planet there is a collision. You don't really notice it because the interaction is gravitational rather than in direct contact, but momentum and energy are exchanged. Consider the following scenario where a satellite has been launched from earth, gone around the sun and is now coming back by earth in the opposite direction. The satellite has a speed of about 12 km/sec far from earth. The earth by the way is moving at 30 km/sec around the sun.



By interacting with the earth gravitationally the satellite will get a boost. The force of the interaction is gravity which is a conservative force so this is totally elastic. The relative approach speed is 42 km /sec. If the satellite whips 180° around the planet then it will have to recede from earth at 42 km/sec as seen by us. To do this the satellite must move at 72 km/sec afterwards! With only small rockets for getting into proper alignment NASA can get satellites to boost up to very high speeds. The method is cheap and always available but it does take longer to get satellites to their final objective.

This move was first used with the crippled Apollo 13 crew to help get them back to Earth. They used the moon for a slingshot ride. The Voyager satellites used the planets in succession. Mars got them to Jupiter which took them to Saturn and so forth.

Practice Problems for Totally Elastic Collisions

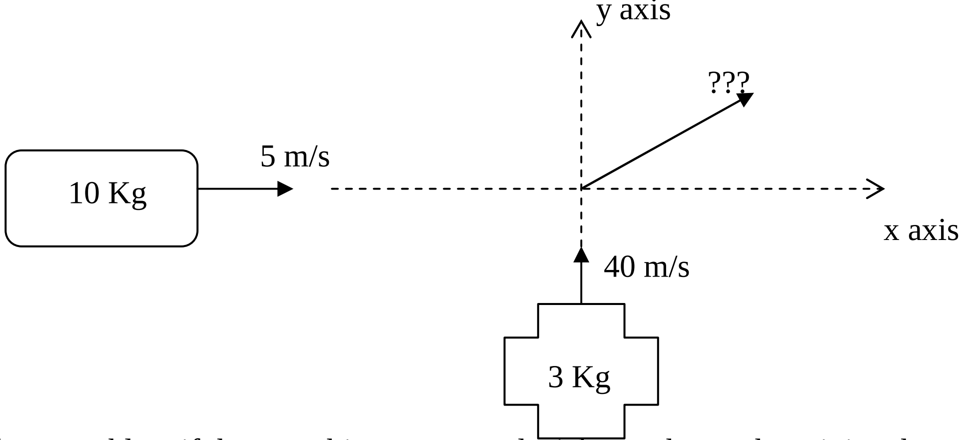
Repeat the five problems from the previous lesson assuming perfectly elastic collisions. Find the final velocity for each object. How much kinetic energy is lost during the collision? How much momentum is lost?

1	$V_8 = +1$ m/s	$V_4 = +13$ m/s
2	$V_{12} = +1$ m/s	$V_8 = 6$ m/s
3	$V_6 = 7$ m/s	$V_2 = 11$ m/s
4	$V_{12} = -3$ m/s	$V_4 = +9$ m/s
5	$V_3 = -13$ m/s	$V_5 = +3$ m/s

So far all of the collisions have been along a single line. In nature some of the collisions are at angles other than 0° or 180°.

Lesson 2-15 **Two Dimensional Totally Inelastic Collisions**

If objects meet and stick together at angles other than head-on then you can write a conservation of momentum equation for both the x direction and the y direction.



In the above problem if the two objects meet and stick together at the origin what will be the final speed of the combination? What will be the heading of the combination? How much energy is lost in the collision? To answer these questions you can write a conservation of momentum equation for two different directions.

$p_{ix} = p_{fx}$

$(10Kg)(5m/s) + (3Kg)(0m/s) = (13Kg)V_{fx}$

$50 = 13\ V_{fx}$

$3.846 m/s = V_{fx}$

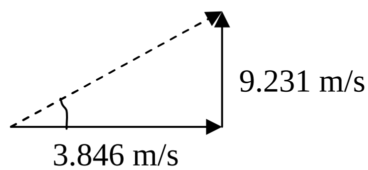
$p_{iy} = p_{fy}$

$(10Kg)(0m/s) + (3Kg)(40m/s) = (13Kg)V_{fy}$

$120 = 13\ V_{fy}$

$9.231\ m/s = V_{fy}$

These two speeds are perpendicular to each other. You can form a triangle to get the final speed and heading.



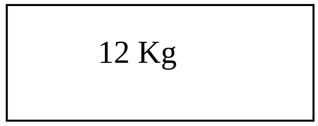
The final speed is 10.0 m/s and the final heading is 67.4° above the x axis.
You can now find the kinetic energy lost by using ½ mv^2.
$\Delta KE = ½ (13)10^2 – ½ (10)5^2 – ½ (3)40^2 = -1875$ J.

For the following problems determine a) the final speed, b) final heading and c) kinetic energy lost if the objects meet and stick together at the origin.

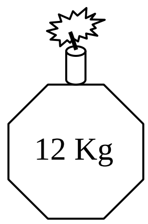
1. 12 Kg moves to the right at 25 m/s. 5Kg moves up the page at 32 m/s.
2. 10 Kg moves to the right at 7 m/s. 15 Kg moves up the page at 16 m/s.
3. 30 Kg moves to the right at 4 m/s. 10 Kg moves up the page at 16 m/s.
4. 20 Kg moves to the right at 18 m/s. 10 Kg moves up the page at 60 m/s.

	Final speed	Angle above x axis	Lost Kinetic Energy
1	20 m/s	28.1°	2910 J
2	10 m/s	73.7°	915 J
3	5 m/s	53.1°	760 J
4	23.3 m/s	59.0°	13,073 J

Explosions
When an object breaks into parts momentum is still conserved. We will focus on a special case where objects are initially at rest. For this special case the initial momentum is zero. Consider the following two examples-



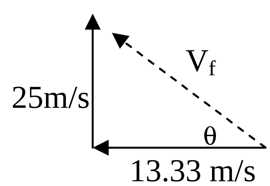
A 12 Kg package at rest explodes into two parts. An 8 kg part is blown left at 10 m/s. How much is the mass of the other part? What the resulting speed of the other part? How much energy is released in the explosion? From conservation of mass you can recognize that the other part is 12 Kg = 8 Kg + m or m = 4 Kg. From conservation of momentum you can determine that the final speed of the 4 Kg part is 20 m/s. The energy released in the explosion is $½(8)10^2 + ½(4)20^2 = 400\text{ J} + 800\text{ J} = 1200\text{ J}$.



A 12 Kg bomb initially at rest explodes into three parts. A 5 Kg part is blown down the page at 15 m/s. A 4 Kg part is blown to the right at 10 m/s. How much mass is the third part? What is the final speed of the third part? Which way is it blown after the explosion? How much energy is released in the explosion? If the detonation of a single TNT molecule releases 4.8×10^{-18} J then how many molecules would be needed to release the energy of the bomb?

$p_{ix} = p_{fx}$
$0 = (5Kg)0m/s + (4Kg)10m/s + (3Kg)V_{fx}$ $V_{fx} = -13.33$ m/s

$p_{iy} = p_{fy}$
$0 = 5Kg(-15m/s) + 4Kg(0m/s) + (3Kg)V_{fy}$ $V_{fy} = +25$ m/s



The final speed is 28.3 m/s at θ = 61.9°.

$\Delta KE = ½(5)15^2 + ½(4)10^2 + ½(3)28.3^2 – 0 = +1963.8$ J.

1963.8 J *(1 molecule)/ ($4.8\text{x}10^{-18}$J) = $4.09\text{x}10^{+20}$ molecules or 0.00068 moles

Conservation of Momentum
Momentum is always conserved for a system <u>unless outside forces are present</u>. All examples in this packet except for the first lesson exclude external forces from the situation.

Lesson 2-16 **Ballistic Pendulums and Variations**

Before radar guns and before high speed strobes there was a very simple way to estimate the muzzle velocity of a bullet fired from a gun. The technique came to be known as the ballistic pendulum. The idea was to fire the bullet into a block of wood. The block of wood was the bob of a pendulum. By measuring how high the block swung after absorbing the bullet you can calculate backwards to find the muzzle speed of the bullet.

A bullet of mass (m) with initial speed of v_o is fired towards a stationary block of wood of mass M. After the bullet sticks into the block the combined mass rises to a height of H before falling once again in pendulum fashion. This problem has two unique parts with momentum being conserved in the first part and energy being conserved in second half.

Part I – A collision

During the totally inelastic collision momentum is conserved so that-

$$mv_o + M(0) = (m + M)V$$

V is combined speed of block and bullet just after the collision with approximately no change in height of the block.

Part II- Conservation of Energy

After collision energy is once again conserved-

$$\tfrac{1}{2}\,\cancel{(m + M)}\,V^2 = \cancel{(m + M)}\,gH$$

Isolating V from the above equation gives $V = \sqrt{(2gH)}$. Now this can be inserted into the collision equation in the previous column.

The muzzle velocity can be found by using the equation shown in the box to the right. Since the masses show as ratios you can keep both in grams or kilograms as long as they have the same units.

$$\boxed{v_o = \frac{(m+M)}{m}\sqrt{2gH}}$$

As you can see from the above equation two types of measurements are needed. You must determine the mass of block and bullet. A complicated and messy determination of the height that the pendulum raises can lead to the speed of the bullet just after leaving the gun. Variations on the theme include different ways to convert the residual kinetic energy after the collision.

College professors love this problem because it is so simple to set the initial kinetic energy of the bullet to the final potential energy of the block. This is wrong of course because it ignores the idea that energy is lost during a collision. Also beware that the mass of the bullet is usually in grams and the mass of the block in Kilograms.

Practice Problems for Ballistic Pendulums

1. A 4 gram bullet is fired into a 2.2Kg block of wood that forms a ballistic pendulum. After the collision the block and bullet rise 25 cm above the initial height. a) What is the speed of the block and bullet just after the collision? b) What is the speed of the bullet just before the collision? c) What percent of the bullet's initial kinetic energy is lost during the collision?
   a) 2.24 m/s b) 1230 m/s c) 99.8%
2. A 3 Kg block is placed on a frictionless surface. A spring with constant of k= 30N/m is placed between the block and a wall. A 2 gram bullet is fired horizontally into the block. The spring is observed to compress 12 cm before expanding once again.
   a) What is the speed of the block and bullet just after the collision? b) What is the speed of the bullet just before the collision? c) How much energy is lost during the collision? Recall that Spring Potential Energy is ½ k x^2.
   a) 37.9 cm/s b) 569 m/s c) 324 J
3. A 1 Kg block is placed just on the edge of an 84 cm high table. A paint ball gun fires a paint ball with a mass of 10 grams at a speed of 30 m/s. Assume that the entire paint ball sticks to the block. a) What is the speed of block and ball just after the collision? b) How long will the paint ball and block be in the air? c) How far from the base of the table does the block and ball land?
   a) 29.7 cm/sec b) 0.41 sec c) 0.12 m
4. The previous experiment is done once again. This time the paint ball is replaced with a 20 gram, perfectly elastic, rubber ball. Answer the same questions as before.
   a) -28.8 m/sec for ball; +1.2 m/s for block b) 0.41 sec c) 0.48 m
5. A 2.4 gram bullet is fired into a 1.6 Kg block of wood resting on a level table. The coefficient of friction between block and table is μ= 0.6. The bullet sticks into the block. The block is found to slide 14 cm before coming to rest. a) What was the speed of the block and bullet just after the collision? b) What was the muzzle speed of the bullet?
   a) 1.30 m/s b) 865 m/s