Physics A Unit 7 Work, Energy & Power

In unit 7 you attention is directed to an entirely new way of looking at the mechanics of our universe and how to go about solving some of the problems. The main idea of this unit is one of the most powerful ideas in all of science- conservation of energy. The methods of this unit are faster and easier to work with yet a bit more abstract than the forces methods of previous units. Before taking the next test be prepared to explain both concepts and solve problem from the topics in the following outline.

I. Work

A. Definition and Units

B. Three Definitions of Net Work

C. Work, Conservative Forces and Potential Energy

II. Conservation of Energy

A. With Conservative Forces Only

B. With Centripetal Forces

C. With Non-Conservative Forces

III. Power

A. Units

B. Average Power

C. Instantaneous Power

This unit should last 8 days. The first day will cover I.A&B. The second day will explore the potential energy of gravity and springs in I.C. The third day will investigate II.A. Day four will look into II.B. Day five will address the issues of II.C and all of III. Day six is a practice test day in small groups. Day seven is the day of the unit test.

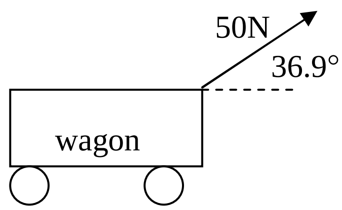
We begin what is perhaps the most important topic in all of mechanics. The concepts developed in this unit are used throughout all of the sciences.

Work Defined

- Work takes place only when a force acts over a distance.
- The amount of work done is calculated using the following formula:

$$W = \mathbf{F} \bullet \mathbf{S} = |F||S|\cos\theta$$

Example #1



A force of 50 N at 36.9° pulls a 16 Kg wagon from rest over a distance of 6 meters. How much work is done?

W = 50N*6m* cos36.9° = 240 J

- When calculating work you multiply only the part of the force that is parallel to displacement. The previous example demonstrates this idea. By recognizing that the 50N is part of a 3-4-5 right triangle you can see that you are really multiplying 40N*6m.
- Work has units of Newtons times meters which are actually Joules.

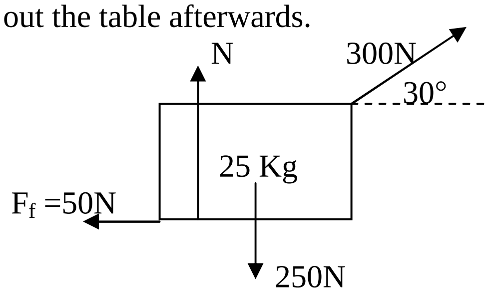
Net Work on a System

"Net Work" on a system means the total work done by all forces. There are three definitions of net work.

- $W_{net} = W_1 + W_2 + W_3 + \ldots = \Sigma W_i$
- $W_{net} = F_{net} \bullet S$
- $W_{net} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \Delta KE$

Example #2

Use the free-body diagram below to fill out the table afterwards.



A 300N applied force at 30° above the horizontal moves a 25 Kg box from rest over a distance of 5 meters. Use three definitions of net work on this problem.

Force	Work Calculated	value
Applied	300N*5m*cos30°	+1300J
Normal	100N*5m*cos90°	0J
Weight	250N*5m*cos90°	0J
Friction	50N*5m*cos180°	-250J
total	210N*5m*cos0°	1050J

By adding the last column you are using the first definition of net work. By working across the last row you are using the second definition of net work. The second method is definitely easier to use. You can take the last definition of net work and the value of W_{net} = 1050J to find final speed of the object.

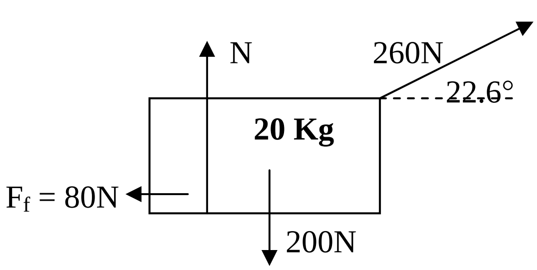
1050 J = ½ (25Kg)v_f^2 – 0 or v_f = 9.2 m/s

You see that we can now define work in general as follows:

Applying a parallel force over a distance does work. A positive work is done to increase the energy in the system. A negative work will take energy out of the system.

Practice Problems for Net Work

1. A 3Kg object is held 2 m above the ground. How much work is being done on the object while it is being held? The object is released from rest. How much work is done by gravity while pulling it to the ground? What is the final speed when it hits the ground?
2. A 500 grams ball is held exactly one meter above the ground with the intentions of tossing it straight up. The hand exerts a force of 20N up through a distance of 0.7 m during the throw. How much work is done by the hand during the toss? How much work is done by gravity during the toss? How much kinetic energy does the ball have at the end of the toss? What is the speed of the ball when it leaves the hand?
3. A 6 Kg chair has been pushed across a floor from rest. An applied force of 80N sideways acts on the chair over a distance of 2m. How much work was done on the chair by the applied force? At the same time 30N of friction was acting against the chair's motion. How much work was done by friction? What is the chair's kinetic energy and speed at the end of the push? How much farther will it slide before coming to rest? Hint: Consider the work that friction has to do after the applied force is removed to stop the chair.
4. Use the free-body diagram to answer the following questions.

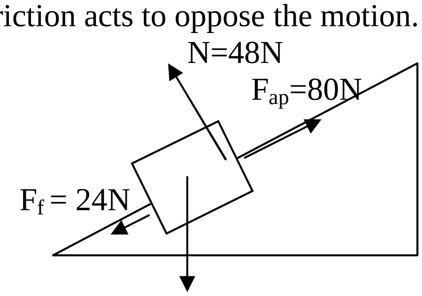


An applied force of 260 N at 22.6° above the horizontal moves a 20 Kg box from rest. The force acts over a distance of 4m. The force of friction is 80 N while the applied force is present.

Force	Work Calculated	Value
Applied		
Normal		
Weight		
Friction		
total		

What is the kinetic energy of the box at the end of the 4m? What is the speed of the box at the end of the 4m? When the applied force is removed the friction force changes to a new value of 160N. How much farther will the box have to slide before the new value of friction can remove all of the kinetic energy?

5. A 6 Kg block is placed at the bottom of a 5m long ramp that has been raised 36.9° above the horizontal. An applied force of 80 N moves the block up the ramp from rest at the bottom to the very top. A 24 N force of friction acts to oppose the motion.



Fill in table below to find work done by each force.

Force	Work Calculated	Value
Applied		
Normal		
Weight		
Friction		
total		

How much kinetic energy does the block have at the top of the incline? What is the speed of the block at the top?

Answers to Practice Problems

1. 0J ; +60J ; 6.32 m/s
2. +14J ; -3.5 J ; +10.5 J ; 6.48 m/s
3. +160 J ; -60 J ; 100 J ; 5.77 m/s ; 3.3m
4. + 640 J ; 8.0 m/s ; 4.0 m
5. 100 J ; 5.77 m/s

Lesson 2-08 Potential Energy

As long as the only forces doing work on a system are *conservative* forces then there is a special technique that can be used to solve problems. Conservative forces include gravity and spring forces. Non-conservative forces include air resistance, applied forces like a push from a hand and friction. In this part of the unit we consider only conservative systems.

Work done by a conservative force will store energy for later use. This work can be converted to kinetic energy at some later point in time. When work is done against a gravity force the amount of stored energy can be calculated.

$$\boxed{P.E._g = m|g|h}$$

In the above formula "h" is the height above some referenced line. It is usually wise to make the <u>lowest point</u> in the problem the h=0 m line. This eliminates the chance of losing negative signs.

Potential energy stored in a spring can also be calculated provided that you know how strong the spring is. The relative strength of a spring is indicated by the spring constant, "k".

$$\boxed{P.E._{SPRING} = \tfrac{1}{2} k x^2 \text{ where k is found using } F_{SPR} = -kx}$$

Lesson 2-09 Conservation of Energy

Now you have the possibility of having both kinetic and potential energy in a system at the same time. The total mechanical energy of the system is E.

$$\boxed{E_{mech} = K.E. + P.E.}$$

If energy is conserved in a system then E cannot increase or decrease. The total energy at the beginning will equal the total energy at the end. This leads to two different ways of writing a conservation of energy statement.

$$\boxed{E_f = E_i \quad \text{or} \quad \Delta P.E. = -\Delta K.E.}$$

The first statement merely says that the energy before equals the energy after. The second statement stresses the fact that as potential increases kinetic must decrease or vice-versa so that the sum of the two can remain constant. In either event you must consider terms for both before and after and in this case both potential and kinetic energy.

$$mgh_f + \tfrac{1}{2} mv_f^2 = mgh_i + \tfrac{1}{2} mv_i^2$$

The above expression would be typical for a system with an object changing height if there are no springs involved.

<u>Example #1</u>

Consider dropping a 2 Kg ball from rest at a height of 8 meters above the ground. Using the above expression would give:

$$0 + \tfrac{1}{2}(2Kg)v_f^2 = (20N)8m + 0$$

You can solve for the impact speed. The beauty of this method is that the problem is path independent where previous methods required a straight line and constant acceleration.

Practice Problems II

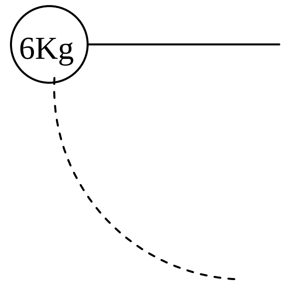
1. A 4 Kg object is dropped from rest 5m above the ground. Let h=0m be at ground level. What is the initial mechanical energy of the object? What is the impact speed?
   a) 200 J b) 10 m/s

2. A 3 Kg object is thrown straight up at a speed of 20 m/s. The object is tossed upward at an initial height of 12 m above the ground. Let h=0 m be at ground level. What is the initial potential energy of the object? What is the initial kinetic energy? What is the potential energy at max height? How high is maximum height above ground? What will be the impact speed of the object?
   a) 360 J b) 600J c) 960 J d) 32 m
   e) 25.3 m/s

3. A 2 Kg object is thrown sideways at a speed of 8 m/s from the top of a 11.25 m high building. h=0 m is at ground level. What is the potential energy at the moment the object is thrown? What is the kinetic energy at the instant the object is thrown? How much kinetic energy does the object have at impact? What is the speed of the object at impact? What is the horizontal impact angle?
   a) 225 J b) 64 J c) 289 J
   d) 17 m/s e) 61.9°

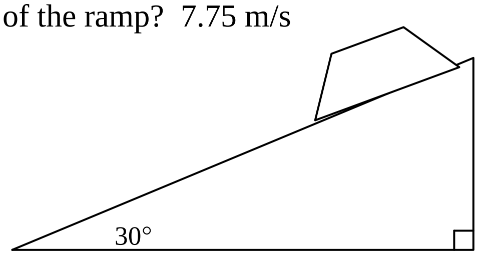
4. An object of unknown mass is thrown upward from an initial height of 12 m above the ground. The initial speed is 16 m/s. How high does it rise? 12.8m

5. A 6 Kg bowling ball is tied to the end of a 4m long string. The other end of the string is tied to the ceiling. The ball is pulled back until the string is horizontal and tight. The ball is released from rest. What is the speed of the ball as it passes through the bottom of the arc? 8.94 m/s



6. Suppose that the ball in the previous problem had been pulled back from the vertical until the string formed only a 37° with the vertical rather than a full 90°. If the ball is released from rest then what will be its speed as it passes through the lowest point? 4m/s

7. A 10 Kg block of ice is placed at the top of a 6 m long ramp that has been raised 30° above the horizontal. The block is released from rest. What is the speed of the block at the bottom of the ramp? 7.75 m/s



8. A 4Kg block is placed on top of a level, frictionless surface. A string passes from this block over a pulley and down to a 2 Kg block that is suspended 1.6 m above the ground. With the system released from rest what is the potential energy of the hanging block? What is the impact speed of the hanging block?
   a) 32 J b) 3.27 m/s

Lesson 2-10 Centripetal Forces & Vertical Circular Paths

There are numerous examples of objects moving along vertical circular paths. Consider riding on a Ferris wheel or what about a ball tied to a string and rotated vertically around your hand. Roller coasters usually include a loop-the-loop part to the ride. Even the idea of a car topping a hill so fast that car and road take different paths is an example.

Since the aforementioned objects are moving along circular paths centripetal forces are encountered:

$$\sum F_R: F_{IN} - F_{OUT} = mv^2/r$$

But these objects tend to change speed as potential and kinetic energies are exchanged (except for the Ferris wheel). One must also consider conservation of energy as the object changes height.

$$½ m v_o^2 + mgh_o = ½ mv_F^2 + mgh_F$$

It should be noted upon closer inspection that opportunity for two equations with two unknowns can occur. In most instances the equations are decoupled so that you use the energy equation to get the speed at a critical point. Then centripetal forces can be used.

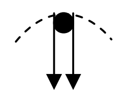
<u>Critical Speed</u>

If the ball on the string moves too slowly through the top of the circular path it will collapse out of a circle and into a parabola. There is a minimum speed in order for the ball on string to remain in a circle. The same is true for a passenger of a roller coaster at the top of the loop-the-loop. For a passenger on a Ferris wheel or in a car at the top of a hill there is a maximum speed that can be traveled through the top of the circle without having passenger and seat part paths. In both extremes the critical speed is determined using the equation to the right.

$$\boxed{v = \sqrt{rg}}$$

<u>Example 1</u>

A 2 Kg ball is tied to a 0.8 m long string and whirled in a vertical circle. The ball passes through the top of the circle with a speed of 3 m/s. The ball has a speed of 5m/s halfway between top and bottom of the circle. At the bottom of the circle the ball has a speed of 6.4 m/s. Find the tension in the string at all three positions.



T + mg

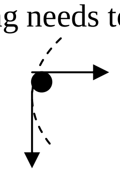
At the top of the circle both weight and tension point toward center of the path.

$\Sigma F_R = T + mg = m(v^2)/R$

$T + 20 = 2(3^2)/ 0.8 \quad T = 2.5$ N

Here string tension is only 2.5N.

A total force of 22.5 N is needed to curve the ball along the circle of radius 0.8m at a speed of 3 m/s. Since the earth is already pulling with a force of 20N the string needs to make up the difference.



T

W=mg

Since the weight is perpendicular to the radius of the circle it will not show up in the radial equation. The tension is along the radius.

$\Sigma F_R = T – 0 = m(v^2) / R$

$T = 2(5^2)/ 0.8 \quad T = 62.5$ N

It takes 62.5N to curve the ball along the circle of radius 0.8 m at a speed of 5 m/s. Since the string gets no help from gravity the tension provides the entire force.

At the bottom of the circle the weight pulls outward. The string must now provide tension to cancel the weight and tension to curve the ball.

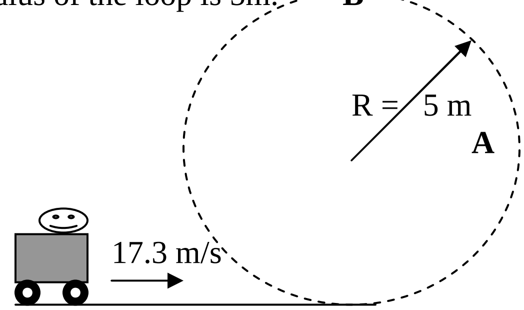
$\Sigma F_R = T – 20 = 2(6.4^2)/0.8$

$T = 122.4$ N

In the previous example the speeds were provided in the problem. Now that you have learned conservation of energy that will not always be true.

Example 2

A cart and child have a combined mass of 140 Kg. The cart and child pass through the bottom of a circular loop-the-loop with a speed of 17.3 m/s. The radius of the loop is 5m.



Point A is at height even with the center of the loop. Find the speed of cart and rider at point A. Find the force of track pushing inward on the cart at point A.

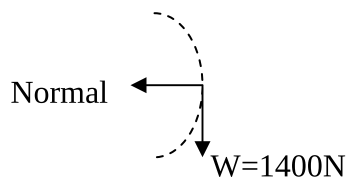
Using Conservation of Energy:

$\frac{1}{2}$ (140Kg)(17.3^2) + 0

= 1400N(5m) + $\frac{1}{2}$ (140Kg)(v_A^2)

v_A = 14.1 m/s

Using Centripetal Forces:



There is only one force along the radius-

$\sum F_R$: Normal – 0 = 140Kg (14.1^2)/5m

Normal = 5580 N

Point B is at the top of the loop. Find the values for speed and normal force between track and cart at point B.

Using Conservation of Energy:

$\frac{1}{2}$ (140Kg)(17.3^2) + 0

= 1400N(10m) + $\frac{1}{2}$ (140Kg)(v_A^2)

v_A = 9.96 m/s

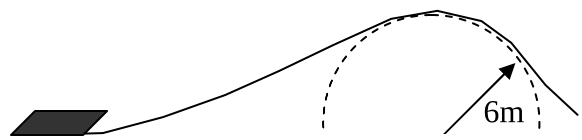
Both weight and normal are radial at top:

$\sum F_R$:Normal + 1400 = 140Kg (9.96^2)/5m

Normal = 1380 N

Example 3

A 740 Kg car coasts along a level road at a speed of 12 m/s. The car encounters a 6 m high rise that sits at the top of a hill with an equivalent radius of 6m.



Find the speed of the car at the top of the hill. What is the normal force on the car from the road at the top of the hill? At what speed at the top of the hill will the car and road separate?

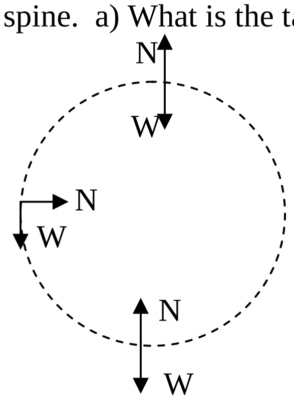
a) $\frac{1}{2}\cancel{m}\,v_o^2 + 0 = \frac{1}{2}\cancel{m}\,v_T^2 + \cancel{m}gh$
   $12^2 - 2(10)6 = v_T^2$ or v_T = 4.9 m/s
b) At the top of the hill the weight is an inward force and the normal is outward: W – N = mv^2/r
   7400N – N = 740(4.9^2)/6
   7400 N – N = 2960 or
   N = 4440N
c) Critical speed occurs when gravitational acceleration is not sufficient enough to provide all of the centripetal acceleration or when $g \le v^2 / r$. For this case $10m/s^2 \le v^2$ / 6m. The car will leave the road for any speed greater than or equal to 7.74 m/s.

This lesson combines the ideas of lesson 2-02 in the previous unit and the ideas of lesson 2-09. There is no new learning here except to draw the free body diagrams and correctly find the forces that are inward, outward of partially radial. Practice with the following problems. Free-body diagrams will not be provided for the test. Study these well.

Homework Problems

1. A 400 gram ball is tied to the end of a 1.2m long string and turned in a vertical circle. The ball passes through the top of the circle with a speed of 4.0 m/s. a) What is the tension in the string at the top of the circle? b) What is the speed of the ball when it is halfway between the top and bottom of the circle? c) What is the tension at the midpoint of the circle? d) What is the speed of the ball at the bottom of the circle? e) What is the tension in the string at the bottom of the circle? f) What minimum speed at the top of the circle keeps the ball moving in a circular path?
   a) 1.33 N b) 6.3 m/s c) 13.3N d) 8.0 m/s e) 25.3 N f) $\sqrt{(12)}$

2. A 40 Kg child rides on a Ferris wheel that turns at a constant rate of 0.5 radians per second. The child rides 9 m from the center of the wheel. There are actually two normal forces acting on the child. The seat pushes up on the child through her posterior. The back pushes in on the child along her spine. a) What is the tangential speed of the child at all times? b) What is the value of the normal force of the seat acting on the child at the top of the ride? c) What is the value of the normal force of the back acting on the child at the mid-point of the ride? d) What is the value of the normal force of the seat acting on the child at the bottom of the ride? e) How many radians per second should the Ferris wheel rotate to create a weightless sensation at the very top of the ride?



   a) 4.5 m/s b) 310 N c) 90 N d) 490 N e) 1.05 radians/second

3. A 50Kg rider in a roller coaster cart enters a vertical, circular loop-the-loop at the bottom with a speed of 20 m/s. The radius of the loop is 6m. a) What is the speed of the cart at the midpoint of the loop? b) What is the force of the seat acting on the rider at the midpoint of the loop? c) What is the speed of the cart at the top of the loop? d) What is the force of the seat acting on the rider at the top of the loop? e) What minimum speed must the rider have at the bottom of the loop in order to keep the rider in the cart at the top of the loop?
   a) 16.7 m/s b) 2330 N c) 12.6 m/s d) 833 N e) 17.3 m/s
4. A 70 Kg daredevil ties himself to a lightweight, stretch less rope that is 12 m long. The other end of the rope is tied to the center of a bridge, 14 m above a river. The daredevil walks towards the end of the bridge until the rope is approximately taut. Before stepping off the edge of the bridge and swinging through a semicircular arc over the water he tells you that his rope can withstand a maximum tension of 2000N before breaking. a) What is the speed of the daredevil when his rope forms an angle of 30° with the bridge? b) What is the tension in the rope for part (a)? c) Is his stunt safe or dangerous? Why?
   a) 11.0 m/s b) $T - mg\cos(60) = mv^2/r$ or T = 1050 N c) At the bottom of the swing the rope would experience 2100 N of tension and break. The daredevil become a sideways projectile at a speed of 15.5 m/s at a height of 2m above the water.

Lesson 2-11A

Non-Conservative Forces & Energy

A slightly modified form of the conservation of energy can be used even when friction or applied forces are present.

$E_f = E_i + \vec{F} \bullet \vec{S}$ where the last term is work from ncf.

In the above box the work done by the non-conservative force can be positive and increase the total energy in the system. This typically happens when an applied force is pushing on an object to accelerate it from rest. The work done by the non-conservative force can also be a negative term that reduces the total energy in the system. Air resistance and sliding friction are examples that generally will reduce the energy in the system.

Example:

A 10 Kg object is placed at the top of a 5m long ramp that has been raised 37° above the horizontal. As the object slides down the incline a 20N force of friction acts on the object.

$\frac{1}{2}\, m\, v_f^2 = mgh - F_f\,(S)$

The final form of energy at the bottom of the ramp is kinetic. At the top of the ramp the energy started as potential. Friction converted some of the initial energy to heat. What remains shows up as kinetic.

$\frac{1}{2}\,(10\text{ Kg})\, v_f^2 = 100\text{N}(3\text{m}) - 20\text{N}(5\text{m})$

$5\text{Kg}(\, v_f^2\,) = 300\text{ J} - 100\text{ J}$

$v_f = 6.32\text{ m/s}$

Lesson 2-11B **Power**

Power is the rate at which work gets done. It is measured in Joules per second.

$3\text{ W} = 3\text{ J/s} = 3\text{ N-m/s} = 3\text{ Kg }(\text{m}^2\,)\,/\,\text{s}^3$

Power can be calculated for an average value or an instantaneous value. The instantaneous power tells us how fast energy is being generated or reduced at a specific point in time.

$P_{ave} = W\,/\,t$ or $\Delta E\,/\,\Delta t$

For the average power you need merely find the total energy before and after and divide by the time needed to change the energy.

$\vec{P}_{inst} = \vec{F} \bullet v$

The instantaneous power requires that you know the velocity at a particular instant. Note the dot product which will mathematically allow for both positive and negative powers.

Example:

In the example problem of the previous column it takes 1.58 seconds to go from top of the ramp to the bottom. During this time the average power into the object by gravity is found to be -

$P_{ave} = +300\text{J}\,/\,1.58\text{ sec} = 190\text{ W}$

The average power due to friction is found to be

$P_{ave} = -100\text{ J}\,/\,1.58\text{ sec} = -\,63.2\text{ W}$

At the instant the object is sliding down the ramp at 6 m/s the instantaneous power due to friction is found to be-

$P_{inst} = (\,20\text{N}\,)(\,6\text{ m/s}\,)\cos(180°\,)$

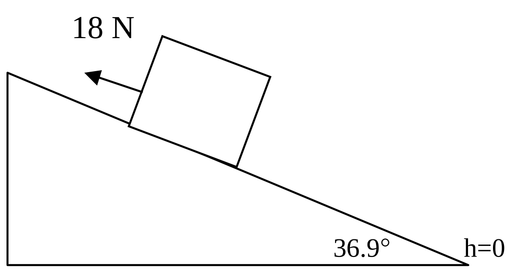
$= -\,120\text{ Watts}$

Practice Problems III

1. A 4Kg object is dropped from rest at a height of 5 m above the ground. While falling the object experiences an average force of 14 N due to air resistance. What is the initial energy of the object at release if h=0m at ground level? How much work is done by air resistance during the complete fall? What is the kinetic energy at impact? Impact speed?
   a) 200 J b) –70 J c) +130J d) 8.1m/s

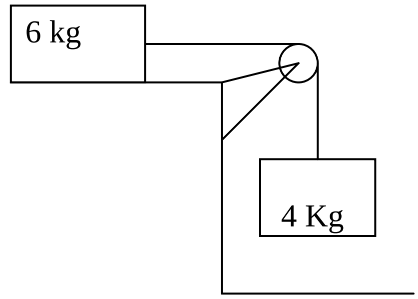
2. A 3 Kg object is launched from the ground at a speed of 20 m/s. Due to air resistance the object rises to a height of only 12 meters. What is the initial kinetic energy of the object? What is the maximum potential energy? How much work is done by air resistance? What is the average force of air resistance?
   a) 600 J b) 360 J c) –240J d) -20N

3. A 9 Kg object is placed at the top of a 10m long ramp that has been raised 36.9° above the horizontal. The object is released from rest and slides against an 18 N force of friction to the bottom.



   What is the initial potential energy of the block at the top of the incline? How much work is done by friction? What is the kinetic energy at the bottom of the incline? What is the speed of the object at the bottom of the incline?
   a) 540J b) –180J c) 360 J d) 8.9m/s

4. A 4 kg block is suspended below a lightweight pulley. A string runs from the block over the pulley and attaches to a 6 Kg object at rest on a level table.



   The suspended mass starts from rest 2m above the floor where h=0m. How much potential energy does the 4Kg block have? What will be the impact speed if the 6Kg block slides without friction? Suppose that the 6Kg block slides with a force of 12N of friction. How much work is done by friction? What will be the impact speed of the hanging block? a) 80 J b) 4 m/s c) –24J d) 3.35 m/s

5. A 7 Kg monkey climbs 5m up a tree in 3 seconds. How much work did the monkey do? What average power was developed by the monkey while climbing? a) 350 J b) 117 Watts

6. A 100 Watt light bulb is turned on for 10 minutes. How much electrical energy is used by the bulb? If 25% of the energy goes to light while the rest goes to heat then how much heat is generated? a) 60,000 Joules or 60 kJ b) 45 kilojoules

Checkpoints for each lesson

Lesson #1: Work Defined

- ✓ Given a force and a displacement, use F dot S to get the amount of work done.
- ✓ Define a dot product both mathematically and physically.
- ✓ Define the unit of a Joule in terms of Newtons & meters or Kilograms, meters and seconds.
- ✓ Know the three definitions of net work.
- ✓ Given a labeled free-body diagram be able to fill in a table for the work done by each individual force and the net work done.
- ✓ Use the net work and mass to find a change in kinetic energy and the final speed of an object.

Lesson #2: Conservation of Energy

- ✓ Compare and contrast kinetic energy, potential energy and total mechanical energy.
- ✓ List examples of conservative forces; list examples of non-conservative forces.
- ✓ Compare and contrast conservative and non-conservative forces.
- ✓ Define conservation of energy in words using both equations in the middle of the 2nd column on page 3.
- ✓ Use conservation of energy equations to analyze motion for final speed or height for objects that are changing potential energy under the following conditions:
  - a) Objects in vertical motion only
  - b) Projectile motion
  - c) An object moving in a vertical, circular path with centripetal forces
  - d) Objects on a frictionless track such as a roller coaster or inclined plane
  - e) Multiple mass problems with systems of pulleys and strings

Lesson #3: Energy Equations with Non-Conservative Forces

- ✓ When non-conservative forces are present be able to modify the conservation of energy equations to account for changes in total mechanical energy.
- ✓ Apply the modified form of the energy equation in the following situations:
  - a) Objects are in motion under the influence of applied forces
  - b) Objects are in vertical motion under the influence of air resistance
  - c) Objects are sliding across a surface where friction is present

Lesson #4:

- ✓ Define average and instantaneous power.
- ✓ Compare and contrast average and instantaneous power.
- ✓ Define the unit of a Watt in terms of Joules and seconds, in terms of Newtons, meters and seconds and in terms of Kilograms, meters and seconds.
- ✓ Calculate the average power over a given time interval.
- ✓ Determine the instantaneous power using $P = \mathbf{F} \bullet \mathbf{v}$.