Physics A | Unit 6 | Centripetal Forces & Circular Motion

In previous units you have considered how a net force parallel to an object's velocity vector will cause the object's speed to increase. You have also seen a net force antiparallel to the velocity vector cause an object's speed to decrease. How does a net force change the velocity vector if the net force is perpendicular to the velocity vector? In this position the magnitude cannot increase or decrease but velocity must change. The theme of this entire unit has to do with objects that have net forces perpendicular to the velocity vector. As a result the objects will curve into a circular path. Before taking the unit six test, students should be aware of all topics listed below.

I. Circular Motion

A. Period, Frequency and Angular Frequency

B. Kinematics

1. Tangential Velocity
2. Centripetal Acceleration (Radial Acceleration)

II. Newton's Laws

A. The Second Law for Horizontal Circular Paths

B. The Third Law and Centripetal vs. Centrifugal

III. Gravity

A. Newton's Law of Universal Gravity

B. Gravitational Acceleration of a Planet

C. Orbital Mechanics (Circular orbits only)

1. Orbital Velocity
2. Orbital Period
3. Escape Velocity

D. Kepler's Laws of Planetary Motion

E. Shoulder of Giants- Copernicus, Tycho & Galileo

Lesson 2-01 Circular Kinematics

Consider observing any object going around a circular path at constant speed. Now suppose that you wanted to describe the motion of this object to someone who is somewhere else. You are communicating by phone, computer etc. How can you convey the rate that the object moves along the circle in a manner that the other person would have the complete picture? First you would describe the radius or the diameter of the circular path. Now that the other person has established the size of the circular path you can describe how fast the object goes around the circle. There are 4 unique ways to describe the rate of motion along the circular path.

Period (τ)

The period is the time to complete one cycle. You could use a stopwatch to see how long it takes the object to go around the circle one time and report that time as the orbital period. The period is measured in units of time since it is a time. The symbol for period uses the Greek letter for time, tau. Note that measuring the time for one circle will be experimentally error prone. It would be better to measure the time for 10 circles and then divide that time by 10. This gives you 8 exact periods averaged in with 2 error prone periods. The orbital period for Earth around the Sun is 365 days for example.

Frequency (f)

Frequency is the number of cycles completed in one second. It is the inverse of period. Rather than describing how many seconds to complete one circle you could tell how many circles are completed in one second. Frequency is measured in s^{-1} or Hertz. Suppose that an object completes 4 circles in 20 seconds. The period would be 5 seconds per circle or $\tau = 5$ sec. The frequency is 0.2 s^{-1} or 0.2 Hz or 0.2 cycles per second.

Angular Frequency (ω)

Angular frequency describes the rate at which the radius sweeps out an angle as the object moves around the edge of the circle. This is also known as the angular velocity. Angular frequency is measured in units of radians per second or rad/sec. Since there are 2π radians in each cycle the angular frequency is related to the previous two variables as shown in the above box. Many consider these nothing more than conversion relationships. For that reason it is easier to understand the physical relationship than it is to memorize them.

$$\boxed{\boldsymbol{\omega} = 2\pi \boldsymbol{f} \text{ or } \omega = 2\pi/\boldsymbol{\tau}}$$

Tangential Velocity (v)

The first three items are all independent of the size of the circle. Yet all three can be related to the speed of the object as long as you know the radius of the circle. The adjective "tangential" is added to velocity to note that the direction of the velocity vector is always tangent to the circle. You can always use d=vt as long as the object moves along the circle at constant speed. Applying this relation for the special case of one exact circle brings the following result: $2\pi r = v*\tau$. Now the relations among all four can be found. Since the first of the three is merely nothing more than "speed equals distance divided by time" there should be no trouble picking that as a starting point.

$$\boxed{\mathbf{v} = 2\pi r/\boldsymbol{\tau} \quad \text{or} \quad \mathbf{v} = 2\pi r\boldsymbol{f} \quad \text{or} \quad \mathbf{v} = r\boldsymbol{\omega}}$$

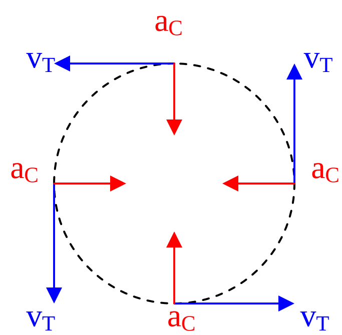
Centripetal Acceleration

As already mentioned, an object moving around a circle has acceleration even if it moves at constant speed. Recall that acceleration is how fast velocity changes. As the object moves along the circle it has a constantly changing direction for the tangential velocity. So the object accelerates while moving at constant speed. The direction of the acceleration vector is always from the object towards the center of the circle. The adjective, "centripetal", means center-seeking.

$$\boxed{a_C = a_R = \mathbf{v}^2/r = r\,\boldsymbol{\omega}^2}$$

The acceleration is known as centripetal acceleration. The direction is always along the radius so it can also be called "radial acceleration". Any of the previous measurements can be used to determine the centripetal acceleration. The above box shows the two that are most commonly used and needed.

Both acceleration and velocity are vectors. Each has a magnitude and a direction. As an object moves along the circle at constant speed the magnitude of both vectors remains constant but the direction is continually changing. The best that you can say for the direction of the velocity vector is that it is always tangent to the circle. For acceleration vector the best that you can describe is that it is always pointing towards the center of the circle. In the figure to the right, tangential velocity vectors are shown labeled in blue. The acceleration vectors are shown labeled in red.



- ✓ Given the radius and any one of τ, f, ω, v_T or a_C find the remaining four values for an object moving along a circle at constant speed.
- ✓ Describe the direction of the velocity vector and the acceleration vector for an object moving along a circle.
- ✓ State the dimensions or units τ, f, ω, v_T and a_C.

<u>Homework Problems:</u> Find τ, f, ω, v_T and a_C

1. A ball on the end of a 1.2 m long string is moving around a circular path at a constant rate of 6.8 radians/sec.
2. A buzzing bee flies in a circle that has a diameter of 3.0 m. The fly completes 10 circles in a half minute at a uniform rate.
3. A student is on an amusement park ride that moves along a circular path at constant speed with a radius of 6.4 meters. The student experiences a sideways acceleration of 2.0 g's (20 m/s^2).
4. A person stands on the equator of Earth with a radius of 3960 miles (6.34E6m). The Earth completes one revolution every 24 hours.
5. A point on the surface of a neutron star is 32,000 m from the spin axis. The star rotates at 180 revolutions per minute.

Homework Answers: Given values are shown in **Bold**

	τ (sec)	f (rev/sec)	ω (rad/sec)	v_T (m/s)	a_C (m/s^2)	Radius (m)
1	0.924	1.082	**6.8**	8.16	55.5	**1.2**
2	3.0	0.333	2.09	3.14	6.58	**1.5**
3	3.55	0.281	1.77	11.3	**20**	**6.4**
4	**8.64E4**	1.16E-5	7.27E-5	461	0.034	**6.34E6**
5	0.333	**3.0**	18.85	603200	1.14E7	**32,000**

Note for a person standing on the equator the speed is 2π(3960mi)/24hr = 1037 mph. For people at any latitude other than at the equator the tangential speed about the Earth's spin axis is 1037∗cosine (latitude). For a person who is at latitude 35° the tangential speed is down to 849 mph.

Lesson 2-02 Centripetal Force

We have seen to this point in mechanics how a force can accelerate an object by changing the speed. Newton realized that forces also accelerate an object by curving an object off a straight-line path. He also had a model for how gravity can attract an object to the earth. When he chanced upon an apple falling as the moon was rising he realized both events were dictated by the same force, the gravitational pull of Earth.

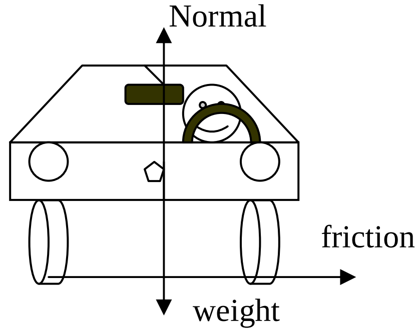
Centripetal Forces

$$\boxed{\Sigma F_{radius} = F_{in} - F_{out} = m\,(v^2)/r}$$

When an object moves along a circular path there is a net force directed towards the center of the circle. The net force is determined by Newton's 2nd Law of motion according to $F_{net} = ma$. The acceleration is the product of v^2/r or $r(\omega^2)$. Notice that in the above box the forces are summed along the radius. Any force off angle to the radius must be broken into parts that are parallel and perpendicular to the radius. The positive direction will be inward, toward the center of the circle. Examples are shown below.

Example #1

A car rounding a flat circular curve of radius R requires friction to turn. What maximum speed can be used in the turn?



The car is facing toward you with wheels turned to your right and the driver's left.
The weight and normal force are perpen-dicular to the circle and are not considered to be radial forces.

$\Sigma F_{\perp} = N - mg = 0$

Static friction is the only radial force turning the car.

$F_{fric} = \mu_s N = \mu_s mg$

Now we can sum the radial forces.

$\Sigma F_{radius} = \mu_s mg = m(v^2) / R$

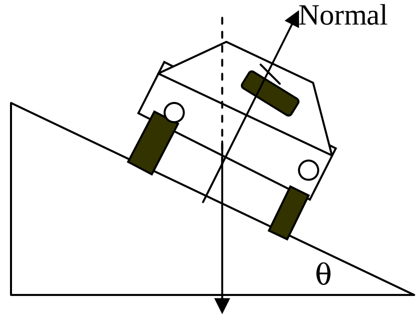
Canceling masses and isolation of the maximum velocity gives the upper limit to the speed.

$$\boxed{V_{max} = \sqrt{R \mu_s g}}$$

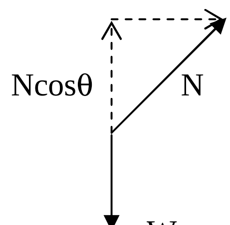
Caution! Do not do lab experiments with your car and a local curve. If you take a curve too fast the tires will slip. At the instant the tires slip your coefficient of friction changes from the static case to the kinetic case. Since the kinetic case has less friction your conditions at slipping change instantly from "having almost enough friction to make the turn" to "no where near enough friction for the turn".

Example #2

A curve can be banked so that no friction is needed to make the turn. Locally you will see this on entrance and exit ramps for interstate "clover-leafs".



The outside of the curve has been raised higher than the inside of the curve. This car is turning to the driver's left. In this case the banking of the road tilts the normal force towards the center of the curve. Normal now has a part that is along the radius, $N_R = N \sin\theta$. This part of the normal force is what turns the car. The normal force also has a part that is perpen-dicular to the circular path, $N_{\perp} = N \cos\theta$. This part of the normal force allows the road to hold the car up against gravity.



The up-down force must balnace to hold the car in the horizontal plane.

$\Sigma F_{\perp} = N\cos\theta - mg = 0$

The normal force for a car on a banked curve is now greater than a car on a flat road. The steeper the bank the greater the normal force. This can be shown by isolating the normal force above.

$N = mg / \cos\theta$

Now the radial forces can be summed.

$$\Sigma F_R = N\sin\theta = m (v^2) / R$$

The normal force can be replaced with the previous expression to eliminate N. Canceling of the mass and isolation of v gives an upper limit to the speed of a banked curve.

$$\boxed{V_{max} = \sqrt{R g \tan\theta}}$$

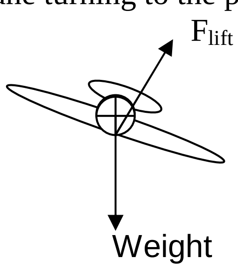
Shown in the box below is the maximum speed for a banked curve with friction.

$$\boxed{V_{max} = \sqrt{Rg (\tan\theta + \mu_s)/ (1 - \mu_s\tan\theta)}}$$

The angle θ is the banking angle and μ is the static coefficient of friction.

<u>Example #3</u>

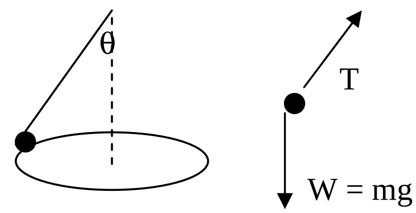
The previous example of a banked curve without friction shows up in two other places that may be of interest. Consider an airplane turning to the pilot's left.



Instead of using the rudder at the back of the plane to turn left, the pilot can rotate the fuselage about the central axis. By dropping his left wing and raising his right wing the lifting force becomes tilted. It now has a part of the lift for turning and a part of the lift for holding the plane up against gravity. Since the true lifting part is now less than with wings level a slight increase in speed is often used for more airflow and thus more lift.

<u>Example #4</u>

A string attached to the ceiling with a weight on the other end can be used to make a conical pendulum. The weight moves along a flat circular path while the string sweeps out a cone.



Here the tension in the string serves the same purpose for the weight as the normal force did for the car on the banked curve. The solutions are identical except that N must be replaced with T. Notice that for this solution as the speed of the weight increases the angle θ also increases. You cannot however get the angle to form 90° because the string will always break first due to an infinite tension. Also be aware that the radius of the circle can be found using geometry since the radius is the length of the string, L, times the sine of angle θ. In other words, radius = L sinθ.

<u>Vertical Circular Paths (Warning!)</u>

Objects moving along vertical circular paths have a tendency to change speed due to gravitational accelerations. You need to learn a nice method for changing speeds before you can attempt objects moving along vertical circles.

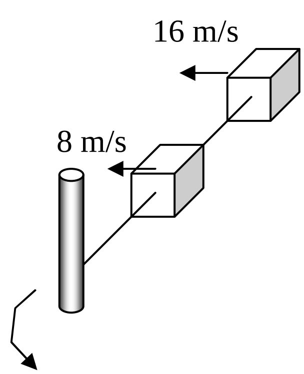
Centripetal forces for vertical circles will be introduced in the next unit. This means that you will see all of this again. This makes it advisable to study this lesson very well since it will soon be coming back for a visit in the following unit. You have been warned!

Exercises for Centripetal Forces

Section A: Horizontal Circles

For each of these problems you should start with the drawing of a free-body diagram. Write the equations for the $\Sigma F_{\perp} = 0$ and $\Sigma F_{rad} = mv^2 / r$. Then do the math.

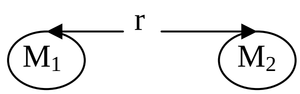
1. On a dry day the coefficient of friction between tires and pavement is 0.80. On a wet day $\mu = 0.10$. If a car can take a particular flat curve at a speed of 45 mph (20 m/s) on a dry day what is the radius of the curve? What would be the maximum safe speed for the same curve on a rainy day?
   a) 50 m b) 7.1 m/s or 16 mph
2. A flat curve with radius of 60 m has experienced multiple accidents dues to frequent icy conditions. Engineers decide to reconstruct the curve with a bank in order that friction will not be needed. If the curve is banked to an angle of 12° then what will be the maximum safe speed for this curve on an icy day? What would be the maximum safe speed on the same banked curve on a dry day if $\mu = 0.8$?
   a) 11.3 m/s or 25 mph b) 27 m/s or 60 mph (when you include friction)
3. A 4 Kg bowling ball is tied to a 2.6 m piece of piano wire. The opposite end of the wire is attached to the ceiling. The ball is pushed sideways in order to create a conical pendulum. The piano wire forms an angle of 22.6° with the vertical. Determine a) the radius of the ball's circular path, b) tension in the piano wire, c) speed of the ball and d) period of the orbit.
   a) r = 2.6m sin(22.6) = 1.0 m b) 43.3 N c) 2.04 m/s 3.08 sec
4. The piano wire of the previous problem can take a maximum tension of 107 N before breaking. What is the largest angle that the wire can make with the vertical before the wire is in danger of snapping? What is the speed of the ball at maximum tension? What is the rotational period under these conditions?
   a) 68° b) 7.7 m/s c) 1.96 sec; this is the minimum time to go around w/o breaking
5. The very small blocks in the figure below are placed on an air hockey table. A peg mounted in the center of the table allows rod #1 to go from the peg to the 3 Kg block. Rod #2 couples the inner block to an outer, 2 Kg block. The inner block is 25 cm from the peg and moves at 8 m/s. The outer block is another 25 cm from the inner block and moves at 16 m/s. Determine the tension on each of the rods.
   a) $T_1 =$ 1792 N b) $T_2 =$ 1024 N



Lesson 2-03

Newton's Law of Gravity

Any object that has mass will attract any other object that has mass. The strength of the force of attraction depends upon the mass of either object as well as the separation distance between the objects.



The strength of the force can be found by using the following equation.

$$F_G = G\, M_1 M_2 \,/\, (r^2)$$

where $G = 6.67 \times 10^{-11}\ N(m^2)/Kg^2$

At the time Newton came up with this expression the value of G was not known.

Gravity Field of a Planet

The gravitational acceleration of an object near a planetary mass can be determined. In this section we will determine the gravitational acceleration above the surface ($g_>$) and below the surface ($g_<$)of planet when given mass and radius of the planet.

Always determine the gravitational acceleration at the surface of a planet first. Set the old equation for an object's weight equal to the Newtonian expression.

$$mg = GmM \,/\, R^2$$

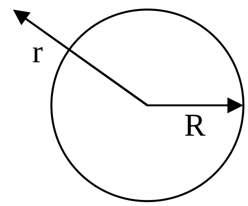
The lower case m is the mass of any object on the surface of the planet. The mass "M" is the mass of the planet. R is the radius of the planet. Notice that the mass of the object on the surface cancels out of the equation so that <u>all</u> objects fall at the same rate in the absence of air resistance.

$$g_{surface} = GM \,/\, R^2$$

For the earth $M = 5.98 \times 10^{24}$ Kg and the radius at the equator is 6.37×10^6 m. Find the value of g at the surface of earth.

When an object is above the surface of a planet the gravitational force is weaker since the distance between the objects is larger. The solution is the same as before except that the value used for r is the distance from the center of planet to object rather than the actual radius of the planet.

$$g_{>R} = GM \,/\, r^2$$



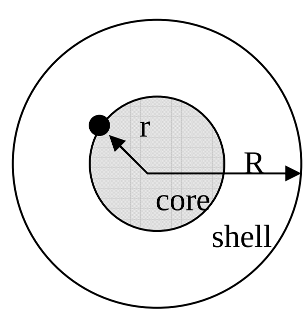
The space shuttle orbits about 200 miles above the surface of the earth. What is the gravitational acceleration for the shuttle and its occupants? The altitude is 320 km or 320,000 m which must be added to the earth radius to find r.

r = altitude + R

$= 320{,}000 + 6.37 \times 10^6\ m = 6.69 \times 10^6 m$

After substitution you find that the gravitational acceleration is 8.9 m/s^2 which is no where near weightless!

For objects below the surface the gravitational acceleration is also weaker. You could argue that an object below the surface is closer to the center of the earth's mass so gravity should be stronger. While this is true one must also realize that part of the earth now pulls in the opposite direction. The only part of planetary mass that pulls on an object is the mass closer to the center than the actual object.



The planetary mass closer to the center than the object is designated as the core. The mass between the surface and the core is a spherical shell. If the mass in the shell is uniformly distributed it will cancel itself out. How much mass is in the core?

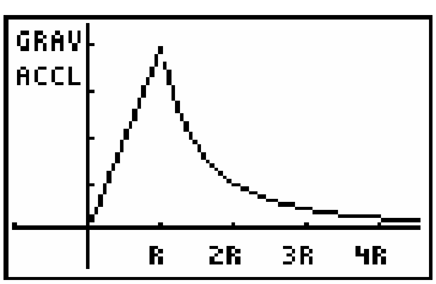
$$M_{core} = M_p \, (r / R)^3$$

To get the gravitational acceleration below the surface use the equation for above the surface with the total mass of the planet replaced with the core mass. As a result the gravitational acceleration is found according to the expression below.

$$\boxed{g_{<R} = (G\,M / R^3)\, r}$$

In the above equation M is the total mass of the planet and R is the radius of the planet. Notice that the acceleration is linear. This allows one to easily find g below the surface if you know the value at the surface. For the earth $g \cong 10$ m/s^2 while the radius of the planet is about 4000 miles. At 1000 miles from the center g=2.5 m/s^2. The logic follows from the fact that 1000 miles is ¼ of the way to the surface so g has to be ¼ of the surface value. At 2000 miles from the center g is ½ of the surface value or about 5 m/s^2.

A typical graph of g vs. r is shown below. Notice that the maximum value of g happens at the surface where r = R.

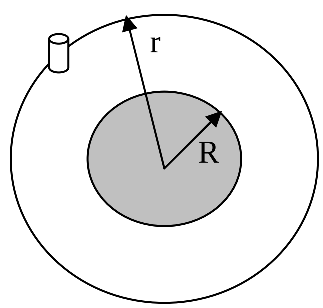


Homework exercise for gravitational acceleration lesson

1. Place the average radius for the Sun and planets in L_1. Place the mass for the Sun and planets in L_2. Move the cursor to highlight L_3 and enter 6.67E-11*L_2 / (L_1)^2. This will give you the gravitational acceleration at the surface of each planet and Sun.
2. Construct a graph for g vs r for the planets Mars and Venus for the range of $0 \le r \le 4R$. Refer to the bottom of page 4 as needed.
3. Our moon has a mass of 7.36E+22 Kg and a radius of 1.74E+6 m or about 1050 miles. Find g at the surface. While Apollo astronauts worked on the surface of the moon a command module orbited 60 miles above the lunar surface. Find g for com. module.

Lesson 2-04 Orbital Mechanics

When a satellite is in a circular orbit about a planet there are several key indicators of the motion. Orbital radius can be used to find the speed needed to stay in orbit as well as time to complete one orbit. How fast does a space shuttle have to go to stay in orbit? How long will it take to complete one orbit?



For a satellite in a circular orbit at a distance of "r" from the center of a planet of mass M and radius R the speed and period of the orbit can be determined. Newton realized that his expression for the gravitational pull is an inward centripetal force that holds the satellite in orbit.

$$\Sigma F_{rad} = G M m_S / r^2 = m_S (v^2) / r$$

Notice that the mass of the satellite, m_S, will fall out of the equation. This means that a big, blue whale must go just as fast as a mouse to stay in orbit at a given value of r.

$$\boxed{V_{orbit} = \sqrt{G M / r}}$$

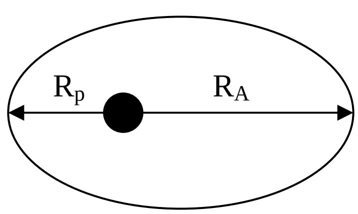
Determine the orbital velocity of the space shuttle in an orbit of 6.7 x10^6 m from the center of the earth. The answer is 7715 m/s or about 17, 150 mph.

Once the orbital velocity is known orbital period can be easily found. Recall that speed is distance divided by time. The distance around a circle is circumference or $2\pi r$.

$$\boxed{V_{orb} = 2\pi r / \tau}$$

Using the speed from above and the given value of r you can find the orbital period for a satellite. Using the numbers for the shuttle you get an orbital time of 5456 sec or about 91 minutes.

There is also the possibility of elliptical orbits. In an elliptical orbit objects will speed up and slow down. As the value of r decreases the value of v increases proportionately.



If the apogee distance is three times the perihelion distance then the speed at apogee will be 1/3 the speed at perihelion.

Now you see how to calculate the minimum speed needed to stay in an orbit without falling back to the surface. How fast do you need to travel if you wish to break totally free of the gravitational pull of a planet? If you know the particular orbital velocity then the "escape velocity" is easily determined.

$$\boxed{V_{escape} = \sqrt{2}\ V_{orb}}$$

To escape from an orbit the speed needs to be increased to the square root of two times the orbital velocity. For Apollo astronauts to go to the moon they had to break out of a low earth orbit (LEO) at about 17,600 mph and accelerate up to a speed of about 1.41*17,600mph which is about 25,000 mph. This is the fastest speed that a human being has traveled and lived to talk about.

Homework Assignment

1. Place the values for the planetary & solar masses in L_1. Place the corresponding radii in L_2. Calculate orbital velocity and escape velocity for objects orbiting just above the surface of each planet.
2. Find orbital velocity and escape velocity for the command module discussed for lunar orbit on last page.

Lesson 2-05 Shoulders of Giants

In a fit of modesty that was most rare for Isaac Newton he was heard to state, "If I have peered farther 'tis because I stood upon the shoulders of giants." The giants to whom Newton was referring included the names of Copernicus, Brahe, Kepler and Galileo. In conclusion of the unit on gravity one must review the contributions of these giants.

- Copernicus was not the first to propose a heliocentric universe but he was among the few to actively promote the idea by publishing a book on the matter (posthumously of course).
- Tycho Brahe accurately measured the positions of stars, planets and comets using large scale protractors and sextants. The accuracy of his measurements allowed his understudy to arrive at three laws of planetary motion. The accuracy also led to a notable flaw in Newtonian gravity theory in the year of 1800. Mercury was misbehaving! See page 156 text.
- Kepler was the understudy of Tycho. When his boss died Kepler became unemployed. Inheriting the data from his mentor's work Kepler came up with The Three Laws of Planetary Motion that are shown on the next page.
- Galileo Galilee did not invent the telescope but rather was the first person to use it for astronomical purposes. With his telescope he observed
    a) mountains of the moon
    b) spots of the sun
    c) moons of Jupiter
    d) phases of Venus
    e) rings of Saturn

With the geographical features on the moon of mountains, valleys and "seas" Galileo argued that the planets are perhaps other worlds similar to earth. The church was not pleased. With the phases of Venus and Mercury Galileo claimed that the easiest way to explain why only those two planets had phases was if Copernicus was correct. The church was upset. "If the earth is the center of the universe then why are the four moons of Jupiter, (Ganymede, Callisto, Europa and Io) circling about another planet?" asked Galileo. The church arrested him.

Lesson 2-06
Three Laws of Planetary Motion

All three of Kepler's Laws were contrary to Catholic Church doctrine.

1) All planets move in elliptical orbits with the sun at one of the focal points.
2) The radius from the sun to a planet sweeps out equal areas in equal time intervals.
3) The average orbital radius for a planet about the sun cubed is proportional to the average orbital period squared.

$$R_{AVE}^{3} = \text{constant} * \tau^{2}$$

The first law claims a heliocentric rather than a geocentric universe. It is worse than Copernican view since it has "God's" world traveling along a flawed elliptical path rather than a perfect circular path.

According to the second law planets must speed up when closer to the sun and slow down when farther from the sun. People asked sarcastically, "Are there little planet fairies pushing the planets to and fro' along these so called ellipses?"

The third law was the worst since it placed the planets in order from the center of the universe outward. With Earth having the third smallest orbital period then it must be the third planet out from the Sun. Not only did Kepler remove "God's" world away from the center of the universe he placed it third in line. Blasphemy!

Although Kepler's Laws came from data he had no theory to explain why they were true. That job was left to Newton. Kepler and Galileo lived at about the same time. Galileo lived from 1564 to 1642 while Kepler lived from 1571 to 1630. Near the end of his life Galileo was under house arrest and was forced to recant every statement he had made. In the year that Galileo died, Isaac Newton was born on Christmas Day. Newton lived from 1642 to 1727.

With Newton's discovery of The Calculus, his Three Laws of Motion and his Universal Theory of Gravity Newton went on to develop the theoretical reasons for the existence of all three of Kepler's laws. It can be shown that elliptical orbits are stable for planets about a star. The second law is a mere consequence of the Law of Conservation of Angular Momentum. This same law works on ice skaters. When they draw in their limbs they spin faster and when arms are extended they spin slower. The third law can be proved rather easily for the special case of circular orbits. Look back two pages and notice the two boxed equations in the right column. By setting the two equations equal you can eliminate V and get a relation between R and τ.

$$\sqrt{GM / r} = 2\pi r / \tau$$

Square both sides to eliminate $\sqrt{\ }$

$$GM / r = 4\pi^2 r^2 / \tau^2$$

Collect like factors and constants

$$\boxed{\tau^2 = (4\pi^2 / GM)\, r^3}$$

This is how Newton proved Kepler's 3rd Law of Planetary Motion. As a laboratory exercise we will use the orbital period for the six innermost planets as measured with their motion relative to the stars. We will also use their observed distance from the sun as determined by parallax shift. By doing a curve fit with the graphing calculator we will see how close the data agrees with Kepler's observations and Newtonian theory. We will also determine the mass of the sun from the data.

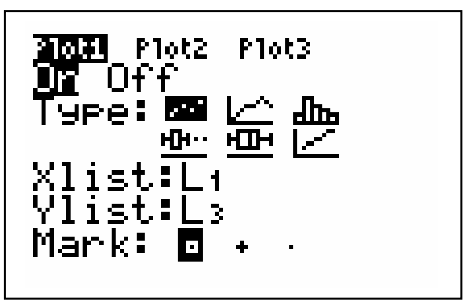
Newton made one other great contribution to astronomy. He designed the first reflecting telescope using mirrors rather than glass lenses.

Calculator Exercise For Kepler's Third Law of Planets

Use the lists in the STAT button to load the following numbers into your calculator. The first list contains the average distance for a planet from the sun. The second list contains the orbital period in years that must be converted to seconds since G is in seconds.

Planet	Radius (m)	Period (years)
Mercury	0.58E11	0.241
Venus	1.081E11	0.615
Earth	1.496E11	1.00
Mars	2.278E11	1.88
Jupiter	7.781E11	11.9
Saturn	14.27E11	29.5
	Place in L_1	Place in L_2

- Move your cursor to highlight L_3. Enter L_2 * 365.25 * 24 * 3600. This will convert the orbital period to seconds.
- Scatter Plot your Data by [2nd] [y=].



- Zoom Stat or Zoom 9.
- Now we wish to find the best fit curve for the data. Since the data seem to curve upward we will use a Power Regression mode.
  [Stat], [Calc],[A],[Enter],L_1,L_3,Y_1 and then enter.

The calculator will attempt to find the best fit line to the equation:

$$\tau = a\, r^{b}$$

If Kepler is correct then the value of b should be exactly b = 3/2. How close did you get to Kepler's theoretical value?

% error = $|b - 1.5| / 1.5 * 100$

% error = __________ %

If Newton is correct, the value of "a" can be used to determine the mass of our sun.

$$a = \sqrt{4\pi^2 / G\, M_{SUN}}$$

Use your value for "a" and the value of G as found by Cavendish (pp. 162 – 163) to determine the mass of our sun.

$M_{SUN} = 4\pi^2 / (a^2 G) =$ _____________Kg

Compare your value to the value in your book of M_{SUN} = 1.99E30 Kg.

%err = $|$Your Value –1.99E30$|$/1.99E28

% error = _____________ %

Be prepared on test day to take the data for orbital radius and orbital period of the moons of Jupiter or Saturn and determine the mass of either planet.