If the first law of motion addresses the issue of balanced forces then the second law must obviously address the issue of unbalanced forces. The trouble with the second law is that the unbalanced forces can be parallel/antiparallel to the velocity vector or perpendicular to the velocity vector. In this unit you will concentrate on the former. The latter will be studied in the next unit.

Both the $1^{\text {st }}$ and $2^{\text {nd }}$ laws consider a single particle or object. But there is more than one particle in the universe. Eventually two objects will meet. Because of the meeting there needs to be a third law. This unit will consider Newton's Second and Third Laws of Motion. Before taking the next test you should be familiar with problems and concepts from the outline below.

## I. Newton's Second Law of Motion

A. Define the Law Verbally
B. Apply the Law Mathematically

## 1. Single object

2. Multiple Masses with strings and pulleys

## II. Newton's Third Law of Motion

A. Define the Law Verbally
B. Apply the Law Mathematically for direct contact

1. Stacked horizontally and moved horizontally
2. Stacked vertically and moved vertically
3. Stacked vertically and moved horizontally

This unit will take approximately seven days. The first day will address I.A when the net force is given. The second day will study using free-body diagrams for I.B.1. The third day considers I.B.2. The fourth and fifth days will be used to investigate II with most of the fourth day emphasizing II.A and II.B.1. On the fifth day II.B. 2 and II.B. 3 will be considered. A student may be adequately prepared for college physics without being exposed to II.B.3.

Recall that the $1^{\text {st }}$ Law of Motion considered types of possible motion an object may have if the forces are balanced. In this packet we will study Newton's Second Law of Motion and Newton's Third Law of Motion. Before we get to either law we must first take pause and address the concept of mass.

## Mass

Although we have already defined mass as something that is measured in Kilograms and designated with the variable " $m$ ", we have yet to give it a good definition. We could say that mass is the ratio of an object's weight to its gravitational acceleration. In this case, $\mathrm{m}=\mathrm{W} / \mathrm{g}$. This would be a nice definition of mass since it would work equally well on different planet surfaces. This definition of mass is known as "gravitational mass" for obvious reasons. There is another, more important definition of mass.

With a ball and some ramps, Galileo was able to prove that an object in motion would remain in motion forever, unless acted upon by an outside force. This sounds so much like Newton's $1^{\text {st }}$ Law of Motion that some argue that Newton stole it from Galileo. Galileo recognized that all objects at rest tend to remain at rest. All objects in motion tend to remain in the same state of motion as long as there was no outside influence. He called this property of an object, "inertia". Which has more inertia, a bowling ball or a ping-pong ball? Which is easier to get started moving from rest? Which, if both are moving at 20 mph , would you rather stop with your hand? Here we have our second definition of mass. Mass is the measure of an object's inertia. A 5 Kg object is five times as difficult to move from rest as a 1 Kg object. An 8 Kg object moving at 20 mph is four times more difficult to stop than a 2 Kg object moving at the same speed. The more mass an object has the greater will be its resistance to change in motion. When mass is defined in this manner it is known as "inertial mass". A question that we still ponder today: "Is an object's inertial mass equal to its gravitational mass?"

## Second Law of Motion

Since the first law addressed balanced forces acting on an object the second law must address unbalanced forces. Each box below discusses part of the Second Law of Motion.

| An object must |
| :---: |
| change speed or |
| curve or both if... |


| ...the forces are un- |
| :---: |
| balanced, no longer |
| in equilibrium. |


| $\mathrm{F}_{\mathrm{NET}}=\mathrm{ma}$ |
| :---: |
| or |
| $\Sigma \mathrm{F}=\mathrm{ma}$ |

## Newton's Second Law of Motion

When the forces acting upon an object become unbalanced the object will change speed, curve or do both. The value of the net force will always equal the product of mass and acceleration.
In this unit we will focus on half of the second law. Unbalanced forces will create a net force that is parallel or anti-parallel to the direction of motion. This causes changes in speed without curving. Unbalanced forces that make an object curve will be addressed in another unit. The forces acting on an object can demonstrate both the $1^{\text {st }}$ and $2^{\text {nd }}$ Laws of motion at the same time. Consider the problems in our first assignment. When we sum forces perpendicular to the surface they will always equal zero since the object is at rest
along that dimension. In contrast, the sum of the forces parallel to the surface may not cancel. The sum of the forces parallel to the surface will equal ma if not 0 .

There is need to realize one final concept before starting the problem solving. Many engineering programs fail to see this point which causes some conceptual trouble later in the course. The product, "ma", has units of $\mathrm{Kgm} / \mathrm{s}^{2}$ or Newtons. The product of "ma" is not however a force. You will not draw "ma" on a free-body diagram. It is not the force due to a surface or a rope or a hand pushing on a block or so forth. The forces will add up to "ma" but "ma" is not counted as a force.

## Unbalanced Forces on a Single Mass

I) Draw a free-body diagram and label all of the forces acting on the object.
II) Sum the forces perpendicular to the surface and set them equal to zero.
A) $\Sigma \mathrm{F}_{\perp}=0$
B) Solve for the normal force. $\mathbf{n}=$ ?
C) Use the $\mathrm{F}_{\mathrm{f}}=\mu \mathbf{n}$ to get a value for the friction term if mu is given.
III) Sum the forces parallel to the surface and set them equal to "ma". $\Sigma \mathrm{F}_{\|}=\mathrm{ma}$
A) Solve for the acceleration or
B) Use the given acceleration to solve for another unknown force.
IV) Use the acceleration from step III as one of the three out of five givens in the kinematics equations.
Note: sometimes you use three given kinematics values to solve for acceleration before you do step III. Depending upon the problem, steps III and IV may be reversed in order.

## Single Mass Problems with Net Force Given

Sometimes you do not have to draw a free-body diagram since the net force is given directly in the problem. When this occurs use $\mathrm{F}_{\text {net }}=\mathrm{ma}$ to find the acceleration. Use it to get acceleration and analyze motion problems. Consider the following examples.

1. A net force of 72 N acts on a 9 Kg object for three seconds from rest. Find the object's acceleration, the distance traveled while being pushed and the final speed at the end of the three seconds.
2. An 8 Kg block is sliding at $+12 \mathrm{~m} / \mathrm{s}$ across a smooth level surface when it encounters some friction. The friction creates a net force of -24 N . How much is the deceleration rate? How long will it take to stop the block?
3. A 5 Kg block is moving at $4 \mathrm{~m} / \mathrm{s}$ to the right when $\mathrm{a}+30 \mathrm{~N}$ net force acts on the block over a distance of 10.67 meters. What is the block's acceleration rate? What is the block's final speed? How long does the net force act on the block?
4. A 2 Kg ball is tied to a 1.4 m long string and whirled in a circle at a constant speed of $5.4 \mathrm{~m} / \mathrm{s}$. What is the acceleration of the ball? What is the net force on the ball?
5. A 50 Kg student is riding on a merry-go-round in a circle of radius 4.8 meters. The net force pulling them into a circle is 196 N . Find the acceleration and the speed of the student.

## Checkpoints

$\checkmark$ State Newton's $2^{\text {nd }}$ Law of Motion. Compare and contrast it with the $1^{\text {st }}$ Law.
$\checkmark$ Express the $2^{\text {nd }}$ Law in equation form and apply it to objects changing speed.
$\checkmark$ Define both gravitational and inertial mass.

If net force is not given find it by drawing a free-body diagram and summing the forces parallel to the motion. For objects changing speed usually $\Sigma \mathrm{F}_{\| \mid}=$ma and $\Sigma \mathrm{F}_{\perp}=0$. When writing the equation for the forces parallel to the surface use the following questions to construct your equation.
A. Which way is the object moving? (This establishes the positive direction.)
B. What driving forces create motion in that direction?
C. What retarding forces oppose the motion?
D. How is the object accelerating?
i) if it is not then use " 0 " ii) if changing speed then use " $m a$ "
iii) if curving we will address this issue later
E) Construct your equation by writing the answers to $\{B\}-\{C\}=\{D\}$

Practice Problems

1. A 10 Kg block is placed on a level surface where the kinetic coefficient of friction is 0.5 . A 70 N applied force moves the block from rest over a distance of 9 meters. Find a) the
 net force acting on the block, the acceleration and final speed.
2. A 26 kg block is placed on a level surface where $\mu_{\mathrm{K}}=2 / 3$. An applied force of 333.4 N at $17.46^{\circ}$ below the horizontal moves the block from rest to a final speed of $9 \mathrm{~m} / \mathrm{s}$. Find a) the normal force, the force of friction, acceleration and distance pushed.
3. An 83.33 kg crate is placed on a level surface where $\mu_{\mathrm{K}}=3 / 4$. An applied force of 792.3 N at $22.25^{\circ}$ above the horizontal acts on the block from rest for 3 seconds. Find a) the normal force, b) force of friction, c) acceleration and
 d) final speed.
4. A 10 Kg box is placed at the top of a $38.66^{\circ}$ inclined plane. The coefficient of friction is 0.281 between box and ramp. Find a) force of friction, b) net force, c) acceleration and speed at the bottom of the incline.

5. A 30 kg block is placed at the bottom of a $60^{\circ}$ incline. The coefficient between block and surface is $5 / 6$. An applied force of 474.5 N parallel to the plane moves the block up the incline. Find a) force of friction, b) acceleration rate and time to reach the top
 of the incline.

| 1 | 20 N | $2 \mathrm{~m} / \mathrm{s}^{2}$ | $6 \mathrm{~m} / \mathrm{s}$ |  |
| :--- | :--- | :--- | :--- | :--- |
| 2 | 360 N | 240 N | $3 \mathrm{~m} / \mathrm{s}^{2}$ | 13.5 m |
| 3 | 533 N | 400 N | $4 \mathrm{~m} / \mathrm{s}^{2}$ | $12 \mathrm{~m} / \mathrm{s}$ |
| 4 | 21.9 N | 40.6 N | $4.1 \mathrm{~m} / \mathrm{s}^{2}$ | $8.1 \mathrm{~m} / \mathrm{s}$ |
| 5 | 125 N | $3 \mathrm{~m} / \mathrm{s}^{2}$ | 2.58 sec |  |

## Lesson 1-23

## Pulleys, Strings and Multiple Mass Systems

In this lesson we will consider how two or masses connected by a systems of strings and pulleys will accelerate. We will assume that all pulleys have perfectly frictionless spin axes and that the mass of the pulleys is insignificant compared to mass of other objects. Two examples are shown below. The first diagram is known as an "Atwood's Machine". An Atwood's machine is a pulley mounted to the ceiling with two weights and a rope.


Objects are released from rest with the 7 Kg cylinder 1 m above the ground. Find the tension in the lines, the acceleration of the objects and the impact velocity of the cylinder with the ground.


A lightweight, frictionless pulley is mounted on the corner of a tall cabinet. A 7 Kg cylinder is released from rest 2.4 m above the ground. The cylinder is attached by string to a 3 Kg block on a cart with lightweight wheels. Find the acceleration of the objects, the tension in the line and the impact speed of the cylinder.

We will use these as examples to demonstrate how to solve multiple mass problems. Before solving the problems however, we must consider what type of approach is best.

## Problem Solving Technique

I. Draw a free-body diagram for each individual mass. Put a "+" next to force arrows to identify driving forces and a "-" next to force arrows to identify retarding forces for each object.
II. Use the steps at the top of page 3 to construct a single $\mathrm{F}=$ ma equation for each object in the system.
III. Add the equations to cancel out internal forces such as tension.
IV. Solve for the acceleration.
V. Substitute the acceleration back into the individual equations of step II to solve for the internal forces like tension.
VI. Use the value for acceleration from step IV for kinematics analysis.

The previous two examples will be used to demonstrate the above steps.

$\Sigma \mathrm{F}_{7}: 70-\mathrm{T}=7 \mathrm{a}$
$\Sigma \mathrm{F}_{3}: \mathrm{T}-30=3 \mathrm{a}$
Adding left-hand sides and right-hand sides gives the single equation-

$$
70-30=10 a
$$

Solving for "a" gives $a=4 \mathrm{~m} / \mathrm{s}^{2}$.
Substitution in either equation gives $\mathrm{T}=$ 52 N . If you don't get the same answer in both equations something is wrong. Also recognize that T must greater than the small weight in order to lift it and less than the heavy weight in order to lower it. ( $30 \mathrm{~N}<\mathrm{T}<70 \mathrm{~N}$ ) Since our answer is in that range it also makes sense.
Using $\mathrm{v}_{\mathrm{f}}^{2}=\mathrm{v}_{\mathrm{o}}{ }^{2}+2 \mathrm{ad}$ and starting from rest gives $\mathrm{v}_{\mathrm{f}}=\sqrt{ }\left[0^{2}+2\left(4 \mathrm{~m} / \mathrm{s}^{2}\right)(1 \mathrm{~m})\right]$ or the impact velocity is $2.82 \mathrm{~m} / \mathrm{s}$ down.

$\Sigma \mathrm{F}_{7}: 70-\mathrm{T}=7 \mathrm{a}$
$\Sigma \mathrm{F}_{3}: \mathrm{T}-0=3 \mathrm{a}$
Adding equations yields $70-0=10 \mathrm{a}$ or $\mathrm{a}=7 \mathrm{~m} / \mathrm{s}^{2}$.

Notice that the wheels prevent friction from causing a retarding force on the 3 Kg block. Also, notice that the weight of the block is pulling perpendicular to the direction of motion and thus cannot be a retarding force.

In this case the tension has only the single requirement that it is less than 70 N in order for the cylinder to descend. Substitution of $a=7$ into either equation leads to $\mathrm{T}=21 \mathrm{~N}$.

Use of the same kinematics equation as before leads to an impact speed of 5.8 $\mathrm{m} / \mathrm{s}$ in 0.82 seconds after release.

Homework Problems for Multiple Mass Systems
(All pulleys are lightweight \&frictionless.)

1. An Atwood's machine is made of a 6 Kg object, a 4 Kg object and pulley. Find the tension in the line, the acceleration of the weights and the time for the 6 Kg object to fall 1.8 m from rest. See the first figure in this section as needed.
2. Consider the second example of this section. Suppose that the cart had a mass of 7 Kg and that the hanging weight had a mass of only 3 Kg . How would this change the problem? Find the new tension and new acceleration for this system. Note: a lighter weight can move a heavier weight since it is friction at the top versus weight on the side that sets acceleration for this system.
3. A 6 Kg block is placed at rest on a flat level surface where $\mu=0.50$. A string runs from the block, over a pulley and down to a 4 Kg ball. The system is released from rest with the ball 2.2 m above the ground. Find a) the acceleration of the system, b) the tension in the line, c) speed of the block when the ball hits the ground. Also, state what minimum hanging weight could replace the ball and still move the block across the surface.

4. A pulley is mounted at the top of a 5 m long ramp that is inclined $36.9^{\circ}$ above the horizontal. A 5 Kg box is placed at the bottom of the ramp. The coefficient of friction between box and ramp is $\mu=0.6$. A string runs from the box, up the ramp and over the pulley to an 8 Kg object suspended 2.6 m above the ground. Determine the acceleration and tension. If the system is released from rest how long will it take the hanging weight to reach the ground?

5. Repeat problem \#4 if the ramp is frictionless. Before doing the problem predict if acceleration will increase or decrease and why. Also, before doing the problem predict if tension will increase or decrease.
6. The system shown below has two different ropes. The tension in rope $\# 1$ is $\mathrm{T}_{1}$ while tension in rope \#2 is $\mathrm{T}_{2}$. Find the tension in each rope and the acceleration of the system.

Answers to Multiple Mass Homework

|  | $\mathrm{a})$ | $\mathrm{b})$ | $\mathrm{c})$ | $\mathrm{d})$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $\mathrm{~T}=48 \mathrm{~N}$ | $\mathrm{a}=2 \mathrm{~m} / \mathrm{s}^{2}$ | $\mathrm{t}=1.34 \mathrm{~s}$ | - |
| 2 | $\mathrm{~T}=21 \mathrm{~N}$ | $\mathrm{a}=3 \mathrm{~m} / \mathrm{s}^{2}$ | - | - |
| 3 | $\mathrm{a}=1 \mathrm{~m} / \mathrm{s}^{2}$ | $\mathrm{~T}=36 \mathrm{~N}$ | $\mathrm{v}=2.1 \mathrm{~m} / \mathrm{s}$ | 30 N |
| 4 | $\mathrm{a}=2 \mathrm{~m} / \mathrm{s}^{2}$ | $\mathrm{~T}=64 \mathrm{~N}$ | $\mathrm{t}=1.6 \mathrm{~s}$ |  |
| 5 | $\mathrm{a}=3.85 \mathrm{~m} / \mathrm{s}^{2}$ | $\mathrm{~T}=49 \mathrm{~N}$ | $\mathrm{t}=1.2 \mathrm{~s}$ |  |
| 6 | $\mathrm{a}=4 \mathrm{~m} / \mathrm{s}^{2}$ | $\mathrm{~T}_{1}=48 \mathrm{~N}$ | $\mathrm{~T}_{2}=28 \mathrm{~N}$ |  |

## Lesson 1-24

Comments for Some Problems
2. The acceleration decreased since there is a lighter hanging weight but the tension did not change. Curious!
3. The minimum hanging weight is what is needed to cancel out friction which in this case was only 30N. You would still have to give the system a push to get started.
4. There are two retarding forces on the sliding box.
5. Removing one of the retarding forces has allowed the system to accelerate faster and with less tension.
6. You should have three equations, one for each object, with three unknowns.

Newton's Third Law of Motion

The first two laws of motion are concerned with the forces acting on a single object or particle. In contrast to the first two laws, the third law acknowledges that there is more than one object in the universe. In fact, the third law addresses what will happen when any two objects meet. Many people have memorized, "For every 'action force' there is an equal and opposite 'reaction force'." But that is not good enough. Newton's Third Law of Motion is-

When object $\boldsymbol{A}$ exerts a force on object $\boldsymbol{B}$ then object $\boldsymbol{A}$ will experience a force that is equal in value but opposite in direction.
If you punch me in the face your hand will experience just as much force as does my face. I may scream louder but the forces will be equal. A 20 ton bus hits a fly. The bus and fly will experience equal and opposite forces. Of course the force does much more damage to fly since it has less mass. This is why the reaction force of the fly hitting the bus may go unnoticed. We can write the third law in equation form if we define some notation. The force that $\mathbf{A}$ exerts on $\mathbf{B}$ will be $\mathrm{F}_{\mathbf{A B}}$. The force that $\mathbf{B}$ exerts

$$
\mathrm{F}_{\mathrm{AB}}=-\mathrm{F}_{\mathrm{BA}}
$$

on $\mathbf{A}$ will be designated $\mathrm{F}_{\mathbf{B A}}$. The boxed equation is meant to say that both objects experience the same amount of force while the negative sign implies that the forces are oppositely directed. This law will be demonstrated with a few examples.

Before getting to the examples the idea of action and reaction must be explored. When we defined weight there was really an action/reaction pair present. If the Earth pulls down on an object with a force of 200 \# we would call that the "weight" of the object. The reaction would be that the object pulls up on the Earth with 200 \# of force. As a box is pulled across a floor from right to left we learned to recognize that friction acts on the box from left to right. While the floor pulls backward on the box (action), the box is pulling forward on the floor (reaction). This is why a carpet will bunch up if a heavy object is scooted across it. When learning to walk you do one of the most counterintuitive things in your life. To make yourself go forward you must push backwards with
your feet. You push backwards on the floor with your foot. The reaction is that the floor pushes forward on your foot. Your car moves in a similar manner. The tire pushes backwards on the pavement (action) while the pavement pushes forward on the tire (reaction). Without friction the action and reaction cannot occur. Strange as this seems there are times like this one when friction is not a retarding force. In this last example friction is a driving force. (Pardon the pun!) In this final part of the unit we will see times when both friction and normal forces can become driving forces.
Problem Solving Technique
I) Recognize that this is a "Third Law" problem by seeing multiple masses in direct contact.
II) Draw a free-body diagram for each individual mass in the following order:
A. Draw the figure for each individual object.
B. Draw the driving force for each individual object. These are the action forces with one exception. That exception is some external force such as an applied force or weight.
C. Go back and draw the reaction forces for each action force.
III) Use the steps at the top of page 3 to construct a single $F=$ ma equation for each object in the system.
IV) Add the equations to cancel out internal forces such as tension or normal force. This step will not work if the objects slip while moving. The exception occurs for objects that are stacked horizontally but are moved vertically.
V) Solve for acceleration.
VI) Substitute the acceleration value back into individual equations to solve for internal forces such as a tension or normal force.
This is the same outline used for the previous lesson with a few changes. First you must recognize when reaction forces need to be considered. Second, instead of drawing the total free-body diagram for object draw them all together so that you become more aware of action and reaction forces.
Example \#1: Objects stacked horizontally and moved horizontally.
A 600 N applied force moves the three carts shown in the figure below. Find the acceleration of all three carts. How much force does the 70 Kg cart exert on the 30 Kg cart? How much force does the 30 Kg cart exert on the 50 Kg cart?


The 600 N applied force acts only on the first cart. The action force that moves the 30 Kg cart is the 70 Kg cart acting on the 30 Kg cart. We designate that force as $\mathrm{F}_{73}$. The 30 Kg cart pushes on the 50 Kg cart in a similar manner. We designate that force as $\mathrm{F}_{35}$. The individual free-body diagrams and the corresponding equations are shown on the next page.


The action forces are shown as solid arrows and the reaction forces as dashed arrows. Note the variable designated for the reaction forces is the same symbol as used for the action forces according to Newton's Third Law. The negative sign is accounted for when you draw the arrow in the opposite direction. The following three equations can be generated:

$$
\left.\begin{array}{c}
\Sigma \mathrm{F}_{\| 70}: 600-\mathrm{F}_{73}=70 \mathrm{a} \\
\Sigma \mathrm{~F}_{\| 30}: \mathrm{F}_{73}-\mathrm{F}_{35}=30 \mathrm{a} \\
\Sigma \mathrm{~F}_{\| 50}: \mathrm{F}_{35}-0=50 \mathrm{a}
\end{array}\right\} \begin{gathered}
\text { Summing these equations gives: } \\
600 \mathrm{~N}-0=150 \mathrm{a} \text { or that } \\
\mathrm{a}=4 \mathrm{~m} / \mathrm{s}^{2}
\end{gathered}
$$

You may be asking yourself, "Why bother with all of this when I can look at the problem as a single 150 Kg object being pushed by a single 600 N force? I get the same acceleration either way." The argument is made even stronger by realizing that the summation equation tells one exactly what the short cut implies. The short-cut will work but as problems get more involved the short-cut becomes more difficult to use while the strategy outlined above will always work as long as there is no slipping between objects. By substitution of " $a=4 \mathrm{~m} / \mathrm{s}^{2}$ " back into the other equations you find that the 30 Kg cart must push on the 50 Kg cart with a force of 200 N . Also, the 70 Kg cart pushes on the 30 Kg cart with a force of 320 N . There is one other fact that is more than just coincidence. The force, $\mathrm{F}_{73}$, is moving $8 / 15$ of the total mass and the value for $\mathrm{F}_{73}$ is $8 / 15$ of the applied force. The force, $\mathrm{F}_{35}$, is moving $5 / 15$ of the total mass and has a value that is $5 / 15$ of the applied force. This is due to Newton's Second Law of Motion.
Example \#2: Objects stacked vertically and moved vertically
A force of 195 N up is exerted on the bottom of the three stacked blocks as shown in the figure to the right.
a) Find the acceleration of the stack.
b) Find the force that the top of the 8

Kg block exerts on the bottom of the 4 Kg block.
c) Find the force that the top of the 4 Kg block exerts on the bottom of the 3 Kg block.


This problem was demonstrated on "America's Funniest Home Videos" where a caterer went to move a three tiered wedding cake. Lifting on the bottom layer of the cake increased the normal forces at the top of each layer. The increased normal force allowed
a column under the middle layer to break through the icing on the top of the bottom layer. But only one of the five columns punched through the icing causing the entire wedding cake to topple sideways. This demonstrates that even cake designers should know Newton's Laws of Motion! Who would have figured?

If the stacked blocks in example \#2 were at rest or moving at constant speed the normal force of the 8 Kg block pushing upward on the 4 Kg block would be 70 N . The 4 Kg block pushing upward on the bottom of the 3 Kg block would be only 30 N . But these blocks are accelerating. Consider the solution and results below.

## Free-body Diagrams



The action forces are shown in bold, solid arrows. The reaction forces are shown in dashed arrows.

$$
\begin{aligned}
& \Sigma \mathrm{F}_{\mathrm{Y} 8}: 195-\mathrm{F}_{84}-80=8 \mathrm{a} \\
& \Sigma \mathrm{~F}_{\mathrm{Y} 4}: \mathrm{F}_{84}-\mathrm{F}_{43}-40=4 \mathrm{a} \\
& \Sigma \mathrm{~F}_{\mathrm{Y} 3}: \mathrm{F}_{43}-30=3 \mathrm{a} \\
& \hline
\end{aligned}
$$

add. eqtns: $195 \mathrm{~N}-150 \mathrm{~N}=(15 \mathrm{Kg}) \mathrm{a}$

$$
\begin{gathered}
45 \mathrm{~N}=(15 \mathrm{Kg}) \mathrm{a} \\
\mathrm{a}=3 \mathrm{~m} / \mathrm{s}^{2}
\end{gathered}
$$

The value of $\mathrm{F}_{43}$ is 39 N which is $30 \%$ higher than the at rest value. The value of $\mathrm{F}_{84}$ is 91 N which is also $30 \%$ higher than the value at rest.

When blocks are stacked vertically but moved horizontally the action-reaction pairs that cause the horizontal motion are the friction forces. Depending upon how hard you pull sideways on a vertical stack will determine if the stack moves as a single unit or if the blocks will shear into different layers. We will look at some of these problems in class to illustrate Newton's Third Law some more. Since this course does not emphasize the difference between static and kinetic friction we will not look at the case where vertical stacks move horizontally with slipping. However, it is worth mentioning that if you move too fast sideways slipping does occur. Have you ever pulled a child's wagon out from under the child because they slipped? Or have you ever been in a truck where a person in the back slid towards the tailgate because the truck accelerated forward too much? (Actually, the tailgate moves forward faster than the person moves forward and it just appears that the person slides backwards.) The slipping case will be left for a second year physics problem.
Checkpoints
$\checkmark$ Given net force and mass on an object, find acceleration and use it in kinematics.
$\checkmark$ Construct a free-body diagram for unbalanced forces acting on a single object. Sum the forces to determine the net force and acceleration of the object.
$\checkmark$ Construct multiple free-body diagrams for objects joined by a system of pulleys and ropes. Sum the forces on individual objects to determine tensions, normal forces and accelerations from several equations.
$\checkmark$ Write Newton's Third Law of motion. Identify action and reaction forces and recognize that these forces will always be equal in magnitude.
$\checkmark$ Recognize when the $3^{\text {rd }}$ Law must be considered in problem solving situations.
$\checkmark$ For objects in direct contact be able to draw the action/reaction pairs of forces as necessary to construct proper free-body diagrams. From these diagrams be able to write a system of equations to find normal forces and accelerations.

## Homework Problems for Multiple Masses in Direct Contact

1. An 8 Kg block, 2 Kg block and 6 Kg block are placed on a level, frictionless surface. An applied force of 48 N acts on the 8 Kg block to move all three as shown in the figure below. Determine the acceleration of the masses, the normal force between the 8 Kg and 2 Kg surface. Determine the force between the 2 Kg and 6 Kg surface.
$\mathrm{F}_{\mathrm{AP}}=48 \mathrm{~N}$

2. The blocks from the previous problem are now stacked vertically as shown in the figure below. An upward force of 240 N acts on the bottom of the 8 Kg block. Find the acceleration of the system as well as the force between each pair of surfaces.

3. Suppose that the applied force in problem 2 is only 128 N up on the bottom of the 8 Kg block. What would be the acceleration and forces between surfaces in this situation? Hint: acceleration is down.
4. Three slabs are stacked vertically with the lowest slab being on wheels as shown below. An applied force of 44 N acts horizontally on the bottom slab. What is the acceleration of the system if the slabs do not slip? How much friction acts between the bottom and middle slab? What coefficient of friction is necessary to prevent slipping between middle and bottom slab? How much friction is needed between top and middle slab to prevent slipping?

5. Suppose the same applied force in the previous problem had acted on the top block rather than the bottom. The acceleration and normal forces would be the same. What would be the new friction values and new $\mu$ needed to prevent slipping?

|  | $(\mathrm{a})$ | (b) | (c) | (d) |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $3 \mathrm{~m} / \mathrm{s}^{2}$ | 24 N | 18 N | - |
| 2 | $+5 \mathrm{~m} / \mathrm{s}^{2}$ | 120 N | 30 N | - |
| 3 | $-2 \mathrm{~m} / \mathrm{s}^{2}$ | 64 N | 16 N | - |
| 4 | $4 \mathrm{~m} / \mathrm{s}^{2}$ | 24 N | 0.4 | 8 N |
| 5 | $4 \mathrm{~m} / \mathrm{s}^{2}$ | 20 N | 0.6 | 36 N |

Notice in problem 5 as compared to problem 4 there is an increased need for friction since values went up. This is why you push/pull sleds and wagons from the bottom so riders don't slip off the ride!

