Physics Unit 3 Projectile Motion

In this unit we turn your attention from objects moving in a straight line to objects that move in a plane. There are a variety of two dimensional motions that we could consider. Objects moving along a circle are stuck in a geometrical plane. Objects moving through elliptical paths are also two dimensional in nature. In this unit we will stick to the special case of parabolic paths. The circular and elliptical motions will be reserved for later units that will also include circular forces and gravity.

Any object will exhibit parabolic motion if the following two conditions are met: 1) the object must have constant speed along one axis (use d=vt) and 2) the object must have constant acceleration along a perpendicular axis (use $d = v_o t + \frac{1}{2} at^2$). All objects in the presence of gravity have the potential for this to occur. Consider a car driving off the top edge of a cliff. The sideways motion is pure constant speed as long as we neglect air resistance. The vertical motion is pure free fall as long as air resistance is also minimal. In this case the car will take an approximately parabolic path. There are numerous examples as you shall soon see in unit three. Before taking the next text be sure that you are prepared to work problems and answer concept questions from the following outline.

I. Projectile Motion in General

A. Horizontal Motion

B. Vertical Motion

II. Special Cases of Projectile Paths

A. Starts Horizontal (second half of the symmetric case)

B. Symmetrical Path (begins and ends at same height)

C. Asymmetrical Path (Begins and ends at different height)

III. Problem Solving Techniques

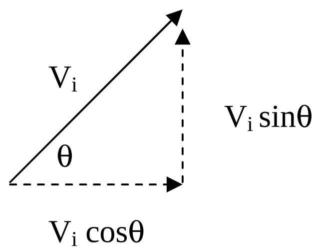
A. Algebraic approach

B. Graphing Calculator Approach

This unit should take no more than 6 days to complete. Day 1 will introduce the use of parametric equations and the graphing calculator to solve projectile motion problems. Day 2 will focus on II.A. Day three covers II.B. Day four will look into II.C. Day five will be group work and practice test. Day six should be test day.

Lesson 1-13 **Parametric Motion & Graphing Calculator**

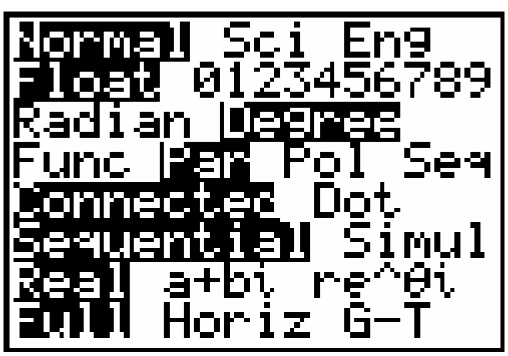
In this worksheet you will learn how to analyze projectile motion with the aid of a TI-83 graphing calculator. Initial velocity must be indicated as shown below.



Consider for example a ball being kicked at 13 m/s at 67.4° above the horizontal from the edge of a 24m high cliff.

The 13 m/s is the hypotenuse for the above diagram. The ball is initially travelling 5 m/s sideways and 12 m/s up. Gravity will change the upward speed at 10 m/s^2 while the 5 m/s will remain constant throughout the flight. The calculator can be used to express both vertical and horizontal motion simultaneously.

Mode:



Set the mode as shown above.

While in **par**ametric mode you can express motion in two dimensions. What you see as a path on the screen is a scaled version of what actually happens in space.

[y =] button

If you select the y= button at this time you will notice that the calculator calls for both the x equation and the y equation.

For the horizontal motion use x = vt which you should recognize as the old distance equals rate times time.

X_{1T} = 5T or X1T = 13cos (67.4°) T

Either of the above can be used to enter the horizontal motion of the example in the previous column. In the latter step notice how you can let the calculator find the horizontal speed for you but be sure to close the parenthesis, [ )].

For the vertical motion use the same equation as last week. In the example for the ball kicked from a height of 24 meters above the ground.

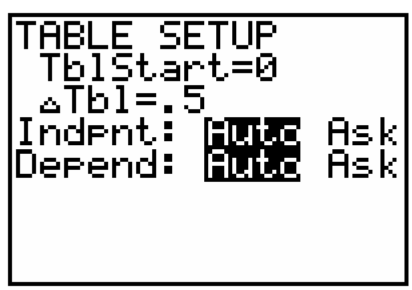
Y_{1T} = 24 + 12T – 5T^2 or

24 + 13sin (67.4°) T – 5T^2.

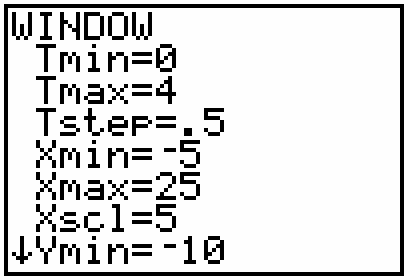
Again note that you can let the calculator find the vertical part of the initial velocity. The equations will always have a single term in the X and three terms in the Y. The constant term (24 in the above example) will be zero if the ball is kicked from the ground. The linear term (12T in the above example) will be zero if the projectile starts out horizontally. Enter the above equations in the [y =] button.

Window

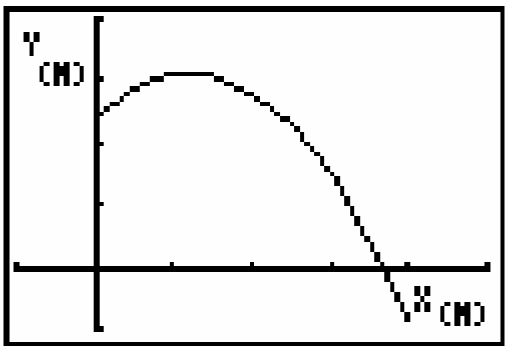
First fix the [Table Setup] as below:



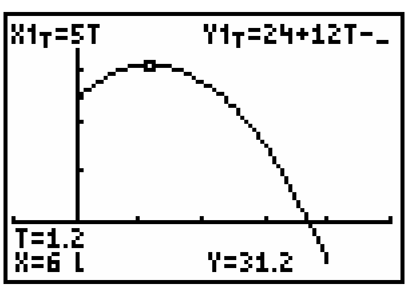
This will make the table start at T=0sec and show ½ second increments. Now use the [Table] to fix your window. From the table you see the first negative height at T=4 seconds so the time part of the window should go from 0 seconds to 4 seconds. Let the X part range from -5 for X_{min} to 25 for $X_{max.}$ The highest Y value is 31 meters but that may not quite be the maximum value. Set Y_{min}= -10 and Y_{max} = 40 and Yscl = 10.



You should get a graph like the one below without labels.



To get the maximum height the best you can do is trace. To get a more accurate trace you will need to change your T_{step} in the window to smaller times. Try 0.1 sec instead of 0.5 sec and see a difference.



You can see that the maximum height is at 31.2 meters above the ground. The ball reaches that height 1.2 seconds after being kicked. The speed at maximum height is not zero meters/second since the ball is still moving sideways.

[2nd] → [Trace]

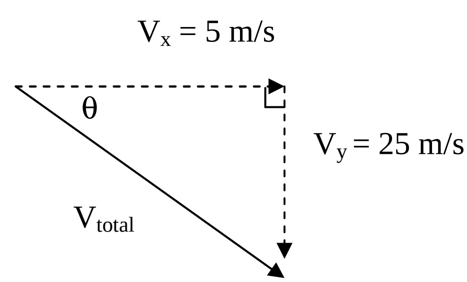
Since you are in parametric mode the menu for the [CALC] has changed. The third selection on the menu is the change in height over change in time or vertical speed. Try the following:

[2nd]→[Trace]→[3]→[1.2]→[Enter]
[2nd]→[Trace]→[4]→[1.2]→[Enter]

The first row asks the calculator to give the velocity in the y direction at 1.2 sec. The second row asks for the horizontal velocity at 1.2 seconds. Are you surprised by the result?[2nd]→[Trace]→[2]→[1.2] will give you the slope of the path at 1.2 seconds.

Find a) time of flight to impact, b) vertical speed at impact, c) horizontal speed at impact, d) impact angle measured from the horizontal and total impact velocity. For parts (b) through (e) it is advised to construct a triangle.

t_{impact} = 3.7 seconds



Tan θ = - 5 so θ = 78.7° below horizontal

Total Impact Speed is

$V_{tot} = \sqrt{(5^2 + 25^2)}$ = 25.5 m/s

At 3.7 seconds after being kicked the ball strikes the ground at 25.5 m/s at 78.7° below the horizontal.

Problems from a text book

1. A stone is thrown horizontally at a speed of +5 m/s from the top of an 80 m high cliff.
   a) How long does it take to reach the bottom of the cliff?
   b) How far from the base of the cliff does the stone land?
   c) What are the vertical and horizontal components of the impact velocity?

Let $X_{1T} = 5T$; $Y_{1T} = 80 + 0T - 5\,T^2$

2. For part (a) the book asks what would happen to the path for the rock if it was thrown from the same cliff but twice as fast? Use the same window and do not change X_{1T} & Y_{1T}.

   $X_{2T} = 10T$; $Y_{2T} = 80 + 0T - 5\,T^2$

   For part (b) the book asks what will happen if the rock is thrown with the same speed as in the first problem but with a cliff that is twice as high? You will have to change the maximum values of X, Y and T.

   $X_{3T} = 5T$; $Y_{3T} = 160 - 5.0\,T^2$

5. A player kicks a football from ground level with a velocity of 27 m/s @ 30° above the horizontal. Find the following:
   a) Hang time if it hits ground
   b) Horizontal distance it travels
   c) Maximum height

Let $X_{1T} = 27\cos(30)T$ and
$Y_{1T} = 0 + 27\sin(30)T - 4.9T^2$

Don't forget to reset your window to these formulas.

6. Suppose that the kicker kicks the ball at the same speed but at the comp-limentary angle of 90° – 30° = 60°? Here retype the same equations as before but replace the angle.
   $X_{2T} = 27\cos(60)T$;
   $Y_{2T} = 0 + 27\sin(60)T - 4.9T^2$

You may need to adjust your window in the Y direction. Also you may wish to experiment with X_{3T} and Y_{3T}at 45°. This will give you the maximum range for a given speed.

8. A rude tourist throws a peach pit horizontally at 7.0 m/s out of an elevator cage.
   a) If the cage is at rest at 17m above the ground what is the time of flight and horizontal range for the pit in the air?
   c) Suppose that the elevator cage had been rising at 8.5 m/s when the pit is thrown from the same height. How would the answers change?

<u>Answers Only:</u>

1. 4 sec; 20 m ;V_x = 5m/s &V_y = –39 m/s
2. 4 sec; 40 m ; V_x=10m/s& V_y=–39 m/s
5. 2.75 sec; 64.3 m; 9.3m
6. 4.78 sec; 64.3 m; 27.9 m
8. 1.86 sec; 13 m; 2.92 sec; 20.4 m

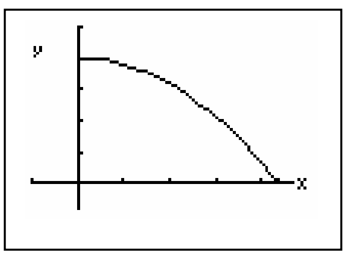
Lesson 1-14 Algebraic Method of Projectile Motion (Part I)

In this unit we will consider the motion of particles that are thrown at an angle rather than straight up. You may be surprised to find out that object thrown at an angle other than 90° above the horizontal is merely a combination of two motions that you have already studied. The horizontal (sideways) motion acts according to d=vt or x=vt. The vertical (up-down) motion behaves according to the kinematics equations such as $d=v_ot+½at^2$ or $y=v_ot+½at^2$. We have to modify the d to either x or y according to horizontal or vertical displacements but the equations are same as before.

There are two approaches to solving projectile motion problems. The *algebraic* method will work for every problem that we will consider. The *graphing calculator* method will work only if the initial conditions of height, speed and angle are given. The algebraic approach has the advantage that the steps are consistent for every problem no matter how difficult. The graphing approach has the advantage that all of the math is done by the calculator as long as you can set a window, trace and use 2nd trace.

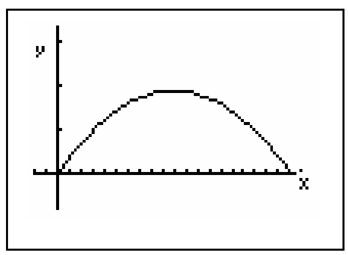
There are three types of projectile motions that we will consider. The first kind of motion is for a projectile that starts sideways. Consider a bullet fired from a gun or a person running horizontally from a cliff. The second type of projectile starts from a height and comes back to the same height. The path is perfectly symmetric about the y-axis. Consider a football kicked from the ground or a baseball hit and caught at the same height above ground. The third type of motion starts at an angle either above or below the horizontal and ends at some height different than the initial height.

Type I Projectile



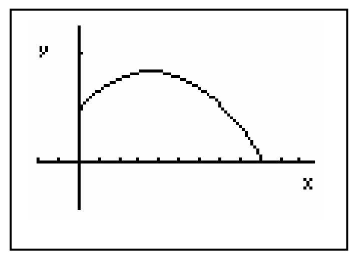
Starts Horizontal

Type II Projectile



Symmetric Path

Type III Projectile



Asymmetric Path

We shall devote one lesson to the algebraic approach for each path. Finally, we will devote one lesson to the graphing calculator approach of all three. Before considering the first type of projectile we should consider the algebraic approach.

<u>Algebraic Approach</u>

Students must recognize that the horizontal equation is $x= v_xt$. Given any two of the three variables you can solve for the other one. The vertical equations are the kinematics where given any three of the five equations you can solve for the other two. The trouble with projectile motion problems is when to stop reading for information and when to start "doing the math". I have found the table to the right to be helpful. Use this table to work the following three problems.

	vertical 3/5	horizontal 2/3
d		
v_o		
v_f		
a	$-10\ m/s^2$	###########
t		

Type I Projectile Motion

1. A car moves horizontally at 8 m/s as it leaves the roof of a parking garage. The street is 11.3m below. How long is the car in the air? How far from the base of the building does the car land? What is the total speed of the car at impact? What is the impact angle from the horizontal?

	Y direction (3/5)	X direction (2/3)
d		
v_o		
v_f		
a	- 10 m/s^2	##########
t		

2. A ball is thrown horizontally from the top of a 28.8 m high building. The ball lands 16.8 m from the base of the building. How long is the ball in the air? How fast was the ball thrown sideways? How fast was the ball traveling when it hit the ground? What was the impact angle from the vertical?

	Y direction (3/5)	X direction (2/3)
d		
v_o		
v_f		
a	- 10 m/s^2	##########
t		

3. A child aims a pellet gun horizontally out of the window of a building. The pellet is in the air for 3.6 seconds before landing 54 m from the edge of the building. How high was the gun above the ground? What was the initial speed of the pellet? What was the final speed of the pellet? What was the horizontal impact angle of the pellet with the ground?

	Y direction (3/5)	X direction (2/3)
d		
v_o		
v_f		
a	- 10 m/s^2	##########
t		

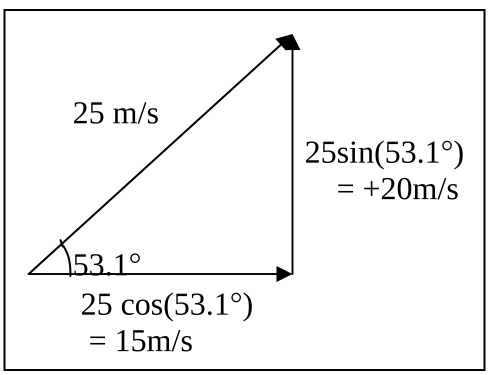
Lesson 1-15 Projectile Motion, Part II

In the last lesson we considered the path of a projectile that starts horizontally. In this lesson we consider another special case, the symmetric path. The projectile starts at an angle above the horizontal. The initial velocity must be broken into horizontal and vertical components or parts. This special case is special in that the projectile will return to its original height at the end of the path. Students will encounter two groups of questions concerning the flight of the projectile. The first set of questions will be in regard to values at maximum height. The latter set of questions will be concerned with values at impact. One may think that you need to work two charts; however, due to the symmetry of the path you need only to work a chart from start to maximum height. Consider the following example:

<u>Example Problem</u>

A soccer ball is kicked from ground level. The ball leaves the ground at a velocity of 25 m/s @ 53.1° above the horizontal. How long is the ball in the air? What is the maximum height of the ball above the ground? How far from the kicking point does the ball first land?

The first thing that must be done is to find the horizontal and vertical parts of the initial velocity. We do this by forming a triangle with the initial velocity being the hypotenuse.



We have used the definitions from right triangles as follows-

Adjacent side = hypotenuse*cosine ∠

Opposite side = hypotenuse*sine ∠

Now we can fill in numbers in the chart(s):

AtoB	Y direction (3/5)	X direction (2/3)
d		
v_o	+20 m/s	15 m/s
v_f	0 m/s	
a	- 10 m/s^2	##########
t		

AtoC	Y direction (3/5)	X direction (2/3)
d		
v_o	+20 m/s	15 m/s
v_f	-20 m/s	
a	- 10 m/s^2	##########
t		

The first chart goes from start to max height while the second chart goes from start back to ground. Without a calculator I can tell the total time of flight is 4 seconds and the total range is 60m. How do I get these without a calculator?

More Practice problems:

1. A baseball is struck at an initial velocity of 26 m/s @ 22.6° above the horizontal. The ball is caught at the same height from which it was hit. How long is the ball in the air? How far did it travel horizontally? What was the maximum height above the hitting point? What is the speed of the ball at maximum height?

AtoB	Y direction (3/5)	X direction (2/3)
d		
v_o		
v_f		
a	- 10 m/s^2	##########
t		

2. A golf ball is struck from a tee with an initial velocity of 19.2 m/s @ 38.66° above the horizontal. Assuming a level fairway, how far away did the ball land? How long was it in the air? What was the maximum height of the ball above the fairway? What was the total speed of the ball 1 second after being struck?

AtoB	Y direction (3/5)	X direction (2/3)
d		
v_o		
v_f		
a	- 10 m/s^2	##########
t		

3. A football is kicked from the ground at an initial velocity of 22.8 m/s at an angle of 37.87° above the horizontal. How high does the ball rise before falling? How long is the ball in the air? How far away does it land? At the same initial speed what other angle will yield the same horizontal range?

AtoB	Y direction (3/5)	X direction (2/3)
d		
v_o		
v_f		
a	- 10 m/s^2	##########
t		

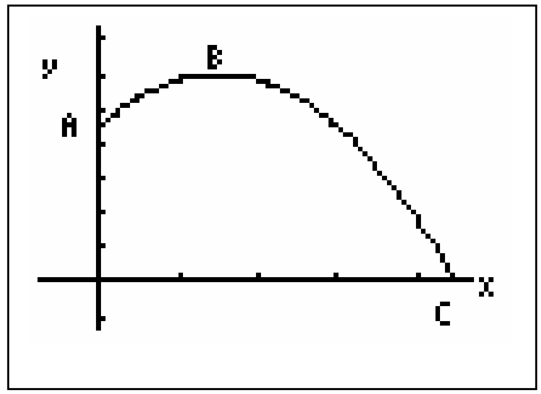
Lesson 1-16 Projectile Motion, Part III

(Asymmetrical Paths)

In the third type of projectile we see objects that begin their path at angles either above or below the horizontal. This requires that we once again break up the initial velocity into vertical and horizontal parts. In contrast to the previous lesson we now see a path that does not end at the same height as it started. Because the total, vertical displacement is no longer zero at the end of the path you may now be forced to make two distinct charts. One chart is for the path that leads from the initial point to maximum height and another chart is from initial point to final point. Consider the following example problem.

<u>Example Problem:</u>

A projectile is fired from the top edge of a 135 m high cliff at an initial velocity of 50m/s at an angle of 36.9° above the horizontal. What is the maximum height of the projectile above the bottom of the cliff? What is the time to reach maximum height? How long is the projectile in the air? What is the impact velocity? How far from the base of the cliff does the projectile land?



The path is shown to the right.

The initial velocity must be broken into parts using sine and cosine. The initial vertical velocity is found using $v_y = v_o\sin\theta$ or $v_y = 50\sin(36.9) = +30$ m/s. The horizontal velocity is found $v_x = v_o\cos\theta$ or $v_x = 50\cos(36.9) = +40$ m/s. Now we can construct charts for both the A-B path as well as the A-C path.

A-B	Y direction	X direction
d		
v_o	+ 30 m/s	40 m/s
v_f	0	
a	- 10 m/s^2	##########
t		

There is enough information given to complete this table.

A-C	Y direction	X direction
d	-135 m	
v_o	+ 30 m/s	40 m/s
v_f		
a	- 10 m/s^2	##########
t		

Again, there is enough information to complete the table.

After completion of the problem on the previous page get into groups and work on the following three problems. In order to save space the charts have not been provided. Draw your own and good luck.

1. A snow skier leaves the top of a jump at a velocity of 17 m/s at 28.1° above the horizontal. The top of the jump is 9.6 m above a level plane of packed snow. What is the maximum height of the skier above the packed snow? How long does it take the skier to reach maximum height? How far does the skier land horizontally from the base of the jump? What is the impact velocity? What is the time of flight?

   a) 12.8 m b) 0.8 s c) 36 m d) 21.9 m/s @ 46.8° below horizontal e) 2.4 s

2. An airplane is climbing from take-off at a velocity of 65 m/s at an angle of 22.6° above the horizontal. At an altitude of 88.8 m above the ground a bolt drops from the plane. How long does the bolt rise before it begins to fall? What is the maximum height of the bolt above the ground? What is the total time of flight for the bolt? What is the impact velocity of the bolt?

   a) 2.4 s b) 120 m c) 7.4 s d) 77.5 m/s @ 39.2° below the horizontal

3. Suppose that the plane in the previous problem had been coming in for final approach to land rather than taking off. Assume that the velocity was 65 m/s at an angle of 22.6° <u>below</u> the horizontal at an altitude of 88.8 m when the bolt fell from the plane. What would be the time of flight? What would be the impact velocity? What would be the horizontal range for the path of the bolt? Why are there no questions about maximum height now?

   a) 2.4 s b) same as before, 77.5 m/s @ 39.2° below the horizontal c) 144 m

Projectile Motion Practice Problems

1. A car drives horizontally off a cliff. After being in the air for 2.45 seconds the car lands 36.75 m from the base of the cliff. A) How fast was the car moving when it left the cliff? B) How high is the cliff? C) What is the impact speed of the car with the ground? D) At what angle below the horizontal was the car moving when it struck the ground?

2. A truck tire launches a rock from the road into the air at 21.6 m/s @ 56.3° above the horizontal. The rock travels through the air before once again landing on the road. A) How long is the rock in the air? B) How far does it travel through the air? C) What is the maximum height of the rock above the road? D) What is the speed of the rock at maximum height?

3. A ball is thrown at 16.2 m/s @ 60.3° above the horizontal from the edge of a12 m high building. A) What is the maximum height of the ball above the ground? B) How long is the ball in the air? C) What is the impact speed of the ball? D)What is the impact angle for the ball with the ground as measured below the horizontal? E) When the ball is once again level with the building how far is it horizontally from the building?

4. A seagull is flying with an oyster in its beak. The gull goes into a <u>dive</u> at 13 m/s at 67.4° below the horizontal. When the gull is at an altitude of 44 m above a parking lot the oyster is released. A) How long is the oyster in the air after release? B) How far does the oyster free-fall horizontally? C) What is the oyster's impact speed with the pavement in the parking lot?

5. A ball is kicked from the ground at an angle of θ above the horizontal. The ball travels 38.4 m horizontally through before landing on the ground. The time of flight is 2.4 seconds. A) What is the horizontal speed of the ball? B) What is the initial vertical speed of the ball? C) What is the initial total speed of the ball? D) What is the initial angle, θ? E) What is the maximum height of the ball above the ground?

Answers:

1. 15 m/s ; 30 m high ; 28.7 m/s ; 58.5°

2. 3.6s ; 43.2 m ; 16.2 m ; 12 m/s

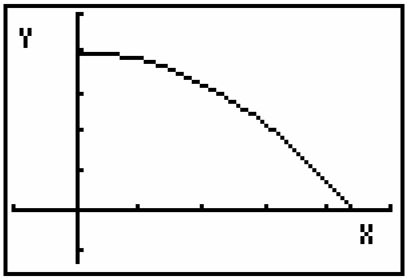
3. 21.8 m ; 3.5 sec ; 22.4 m/s ; 69.1° ; 22.4 m

4. 2 sec ; 10 m ; 32.4 m/s

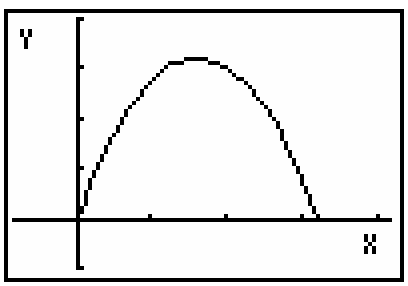
16 m/s ; 12 m/s ; 20 m/s ; 36.9° above horizontal ; 7.2

More Projectile Motion Practice Problems

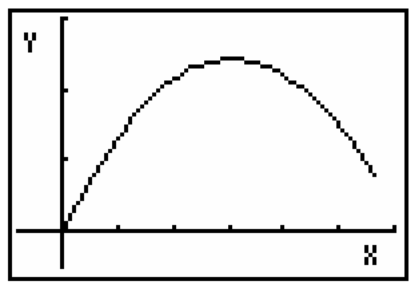
6. A car traveling at 12 m/s leaves the top edge of an embankment. The car falls 24.2 m vertically before striking the ground below. A) How long is the car in the air? B) How far does it travel horizontally before landing? C) What is the vertical speed of the car at impact? D) What is the total speed of the car at impact?



7. While playing kickball Big John kicks a ball off the ground. The initial speed of the ball is17.1 m/s @ 69.4° above the horizontal. The ball travels through the air without being caught before once again landing on the ground. A) How long is the ball in the air? B) How far from kicking point does the ball land? C) What is the maximum height of the ball above the ground? D) What is the minimum speed of the ball in the air?



Problem 7



problem 8

8. A ball is hit at a speed of 26.1 m/s @ 57.5° above the horizontal. Assume that the very low, pitched ball was approximately just above ground level when hit. At exactly 4.0 seconds after being hit the ball strikes a seagull. A) How high above ground was the bird flying? B) What horizontal distance was the bird from the batter when struck? C) What was the speed of the ball at impact? D) What was the direction of impact velocity? E) What was the maximum height of the ball above the seagull?

Answers:

6. 2.2 sec; 26.4 m; 22 m/s; 25 m/s
7. 3.2 sec;19.3 m; 12.8 m; 6 m/s
8. 8 m; 56.1m; 22.8 m/s; 52° below horizontal; 16.1m

9. A person stands on the top-edge of a 50 m high building and throws a penny at 17 m/s @ 61.9° above the horizontal. A) How long is the penny in the air? B) How far from the base of the building does the penny land? C) What is vertical speed at impact? D) What is the total speed at impact? E) What is the maximum height of the coin above the ground?

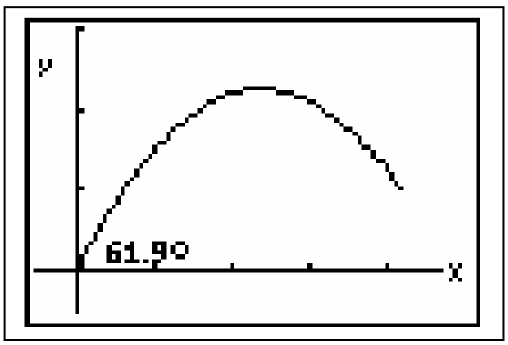
5 sec; 40 m; 35 m/s; 35.9 m/s; 61.2m

Practice Test 103 A **Projectile Motion**

Use g = - 10 m/s^2 and place calculator in degree mode

Problem 1

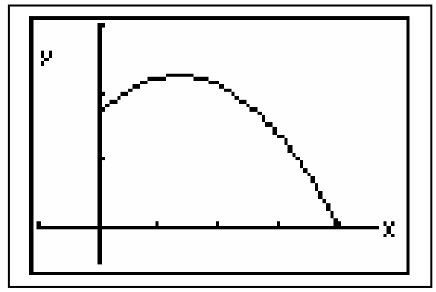
A raw egg is launched from ground at a speed of 34 m/s at 61.9° above horizontal. After five seconds of flight the egg strikes a bird in flight. The egg falls directly below the collision point with bird.



1. The egg lands __ m horizontally from the launch point.
   A) 25 B) 40 C) 65 D) 80
2. The bird was flying at an altitude of ___ m above the ground.
   A) 25 B) 40 C) 65 D) 80
3. The egg collides with the bird at a speed of ___ m/s.
   A) 16 B) 20 C) 26 D) 32
4. The egg collides with the bird at an angle of ___° below horizontal.
   A) 21.3 B) 31.3 C) 41.3 D) 51.3
5. The maximum height of the egg above the ground is ___ m.
   A) 45 B) 50 C) 55 D) 60
6. The speed of the egg at maximum height is ___ m/s.
   A) 0 B) 8 C) 16 D) 30

Problem 2

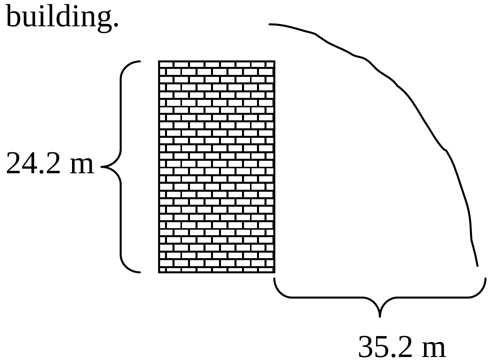
A small ball is tossed from the edge of a 33.8 m high cliff at a velocity of 39 m/s @ 22.6° above the horizontal.



7. The time of flight is ___ seconds.
   A) 4.0 B) 4.5 C) 5.0 D) 5.5
8. The ball lands ___ m from the base of the cliff.
   A) 132 B) 142 C) 152 D) 162
9. The vertical speed of the ball at impact is ___ m/s.
   A) 24 B) 30 C) 36 D) 42
10. The ball strikes the ground at a speed of ___ m/s.
    A) 42.5 B) 46.9 C) 48.7 D) 51.2
11. The ball impacts the ground at an angle of ___° below the horizontal.
    A) 19.8 B) 29.8 C) 39.8 D) 49.8
12. The maximum height of the projectile above the launch point is ___ m.
    A) 11.2 B) 22.2 C) 35 D) 45
13. The ball reaches maximum height at ___ seconds after launch.
    A) 1.0 B) 1.5 C) 2.6 D) 3.6

Problem 3

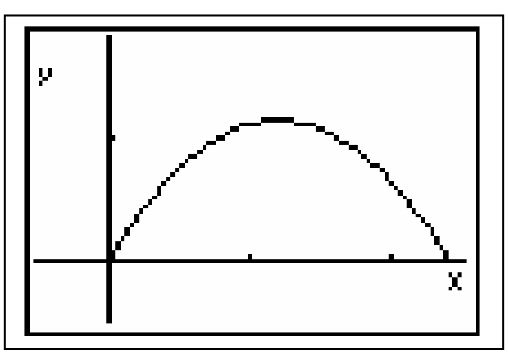
A car drives horizontally off the top edge of a 24.2 m high building. The car lands 35.2 m from the base of the building.



14. The car is in the air for __seconds.
    A) 1.2 B) 1.6 C) 2.2 D) 2.6
15. The car left the top of the building at a speed of ___ m/s.
    A) 13 B) 16 C) 19 D) 22
16. The car strikes the ground at a speed of ___ m/s.
    A) 21 B) 24 C) 27 D) 30
17. The car impacts at an angle of ___° below the horizontal.
    A) 24 B) 34 C) 44 D) 54

Problem 4

A soccer ball is kicked at 17 m/s at 61.9° above the horizontal from the ground. The ball travels through the air until once again landing on the ground.



Problem 4 (continued)

18. The ball is in the air for ___ seconds.
    A) 2 B) 3 C) 4 D) 5
19. The ball lands ___ m from where it was kicked.
    A) 12 B) 18 C) 24 D) 30
20. The maximum height of the ball is ___ m above the ground.
    A) 11.2 B) 13.2 C) 15.2 D) 17.2
21. The speed of the ball two seconds after being kicked is ___ m/s.
    A) 5 B) 8 C) 9.4 D) 11.6
22. To make the ball have the same range when kicked at the same initial speed the kicker could have used an angle of 61.9° above the horizontal or ___° above the horizontal.
    A) 16.9 B) 19.6 C) 21.8 D) 28.1

Answers

1. D
2. A
3. C
4. D
5. A
6. C
7. B
8. D
9. B
10. B
11. C
12. A
13. B
14. C
15. B
16. C
17. D
18. B
19. C
20. A
21. C
22. D