Physics A Unit 2 pp. 39 to 75 Linear Motion

You will study the motion of objects stuck on a line in this unit. The objects can move forward and backwards but cannot get off the line, hence the adjective "linear". The first part of the unit will consider objects that move at constant speed. The latter half of the unit will begin to consider objects that move with constantly changing speed such as objects in free-fall. It is the second half of this unit that is emphasized in many college courses. Before taking your second physics test you should be prepared to work problems and answer questions from the following outline.

I) Constant Speed Motion
   A. Analytical
      1. Distance & Speed
      2. Displacement & Velocity
   B. Graphical
      1. Reading Position vs. Time Graphs
      2. Reading Velocity vs. Time Graphs
      3. Constructing Position vs. Time Graphs
   C. Relative Velocity
      1. Algebraic Definition
      2. Graphical Analysis

II) Constant Acceleration along a Straight Line
   A. Defined
      1. Verbally (both types)
      2. Algebraically
      3. Graphically
   B. Numerical Analysis
   C. Algebraic Analysis (kinematics equations)
   D. Graphical Analysis

This unit will take 8 standard class periods from beginning to testing. Day 1 will cover I.A, B.1-2. Day 2 will explore I.B.3& I.C. Day 3 will consider the ideas of II.A&B. Day 4 will be a reading exercise over II.C and day 5 will be the algebraic use of the kinematics equations. Day 6 will demonstrate the functional graphing ability of the TI83 calculator in analyzing linear motion. Day 7 will be practice and review in small groups. Day 8 is test day.

Day 2 is an optional lesson for some schools. It may be omitted and still leave students adequately prepared for college physics.

Lesson 1-07 **Constant Speed (Part I)**

Consider a bug that can walk back and forth along the x-axis. Think about a car that can move along a long section of interstate with no entrance or exit ramps. Both of these are examples of an object that will exhibit linear motion because they are confined to move along a single line. In this lesson you will study the easiest motion an object can have, moving at constant speed.

Distance, rate and time

All students should have been exposed to "distance equals rate times time" by now. The equation that goes along with that is supposed to be something like "d=rt". In physics we would frown upon such a statement and also shudder at the equation. The statement is poor from a physics point of view because there are many different rates. Which one of those rates does the statement really mention? And the variable "r" in physics usually means radius. We would prefer the statement, "Distance equals speed times time." Now everyone knows exactly what rate we are wish to discuss. As mentioned in the previous unit, speed is the scalar part of velocity or in other words, velocity is speed and direction. So the true word for the rate is velocity rather than speed. This is why, in physics, the working equation uses the letter "v" for the rate. V is for velocity or for speed depending upon how you use the equation. We will return to the versatility of the equation after a few definitions. The most critical idea to know at this point is that the above equation is always used for an object moving at constant speed.

$$\boxed{d = v\,t}$$

Distance and Displacement

The "d" in the above equation can stand for either distance or displacement. But there is a sharp contrast in the two words. Displacement is a vector measurement of how an object changes position. Displacement is complicated by the need to include a direction. Distance is more literal in exactly how far an object travels. A car odometer measures distance rather than displacement. We finish the definitions with several examples.

Suppose you stand on the ground and toss a ball straight up into the air. The ball leaves your hand, rises 3 meters upward before momentarily stopping and then falls back into your hand. The distance traveled by the ball is 6 meters. The displacement of the ball is zero because it is at the same place that it started. Suppose that you stand on the top-edge of a building that is 10 m high. The previous toss is repeated but on the way down the ball misses your hand and the edge of the building. Upon striking the ground the ball has traveled a distance of 16 meters. The displacement is 10 meters down. You can get displacement from distance by doing the following simple trick. In this course we will make up and right positive directions for displacement; down and left will be negative directions for displacement. For the second toss the distance was found using
3+3+10. The displacement for the second toss was found using +3 + -3 + -10 = -10. We see that mathematically speaking, the displacement of the ball could also be stated as
-10 m instead of 10 m down.

Checkpoint Problem

A car leaves home and drives at 60 mph east for three hours. The car stops for one hour and then drives west at 80 mph for one hour. What distance has the car traveled from home? What is the displacement for the car from home?

Checkpoint Solution: The car moved 180 miles plus 0 miles plus 80 miles for a total of 260 miles. The distance is 260 miles. The displacement is +180 mi + -80 miles or +100 miles. The displacement could also be stated as 100 miles to the east of home.

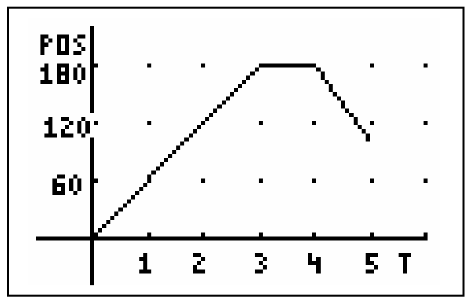
Average Speed & Average Velocity

As previously stated, "Speed is the scalar part of velocity." We will define average speed as distance divide by time. Average velocity is displacement divided by time. With these two definitions you can use the equation from the last page in two different ways. If the "d" is going to represent distance then the "v" will really stand for speed. If the "d" is going to stand for displacement then the "v" will represent velocity.

The checkpoint problem at the bottom of the previous page can be used to demonstrate the difference in speed and velocity. The total trip was 5 hours. Dividing distance by time leads to an average speed of 260 miles/5 hours or 52 miles per hour. To get the average velocity one merely divides displacement by time. Average velocity is (100 miles east) / 5 hours or 20 miles per hour east. In this calculation there were three different mathematical operations. The first step was to "do the math" of 100/5 to get the value of 20. The second step was to apply the math to the units. The third step was to carry the direction forward to the final answer.

The first problem on your next test will most likely be given a problem similar to the checkpoint problem. You should be able to determine distance, displacement, average speed and average velocity.

Position vs. Time Graphs



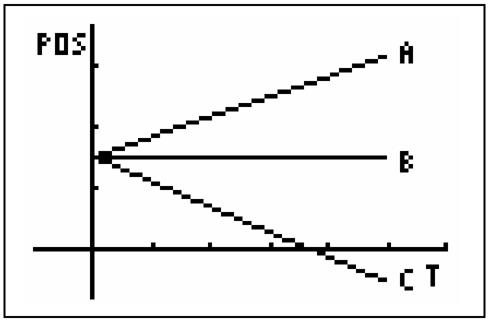
The motion of the car from the checkpoint problem is shown in a graph to the right. The horizontal axis shows time in hours. The vertical axis shows position in miles. The grid has been turned on under [2nd]. [ZOOM] in order to facilitate reading the graph. The initial starting point is assumed to be at the origin of the coordinate system. There are numerous ideas that could be pointed out at this time. In today's lesson we focus upon the most critical. Notice that during the interval when the car is moving in the east or positive direction the graph has a positive slope. While the car is at rest the slope is zero. When the car starts moving to the west or in the negative direction the graph has a negative slope. Is it coincidence that slope is positive when velocity is positive? Is it coincidence that slope is zero when velocity is zero and slope is negative when velocity is negative? No!

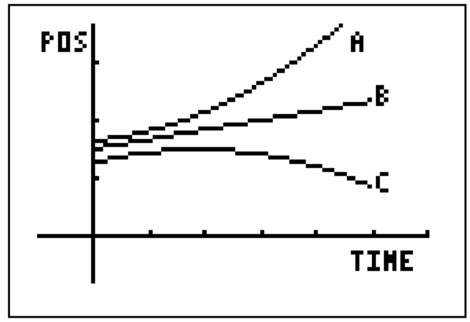
- **The value of the slope of a position vs. time graph at any instant is the velocity of the object in motion at that instant.**
- **If the slope of a position vs. time graph is constant then the velocity is constant; if slope is curved then an object is changing velocity.**

In the above graph the slope is constant from 0hrs < t < 3hrs. The slope of that interval is rise/run = $\Delta y/\Delta x$ = Δposition/Δtime = +180 miles / 3 hrs = +60mph. Notice that in science the rise and run of a graph must have the units of the y-axis divided by the x-axis.

Checkpoint Question #1

In the left-hand figure at the top of the next page are shown three position vs. time graphs. Which graph shows an object moving to the left? Which graph shows an object at rest?





Checkpoint Question #2

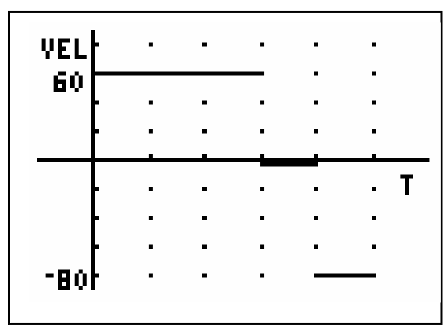
In the figure shown above and to the right are shown three position vs. time graphs. Which graph shows an object moving with constant speed? Which graph shows an object in motion with increasing speed?

For the left-hand graph object A is moving to the right, object B is at rest and object C is moving to the left. All three objects in the left-hand graph show constant speed (0 m/s is a perfectly good constant) since they have constant slope.

For the right-hand graph, object A shows increasing speed because it has increasing slope. Object B has constant speed to the right since it has constant, positive slope. Object C shows a motion that is initially moving to the right because the slope starts out positive. At some point the slope goes to zero indicating that object C is at rest. From the beginning to this point object C has decreasing speed. From the top of graph C forward, object C shows increasing speed but in the opposite (to the left) direction.

Velocity vs. Time Graphs

If given a position vs. time graph you should now be able to find the velocity at a given instant in time. Suppose you are given a velocity vs. time graph. Can you find position from that graph? The answer is “not exactly”. Review the initial checkpoint problem for the car that travels at 60 mph for $0 < t < 3$hrs and is at rest for 3hrs $< t < 4$hrs and is moving at 80 mph to the west for 4hrs $< t < 5$hrs. The velocities are shown in the figure to the right. You can find the displacements for each of the intervals by inspection of the graphs.

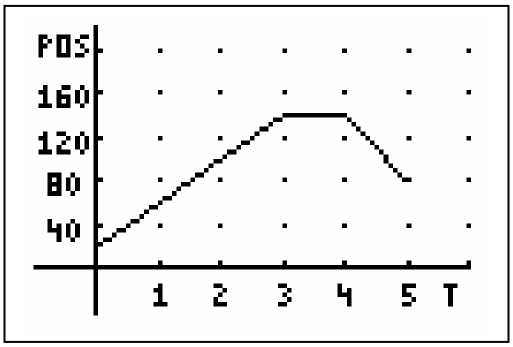


The displacement for an interval is represented by the area between the graph and the time axis.

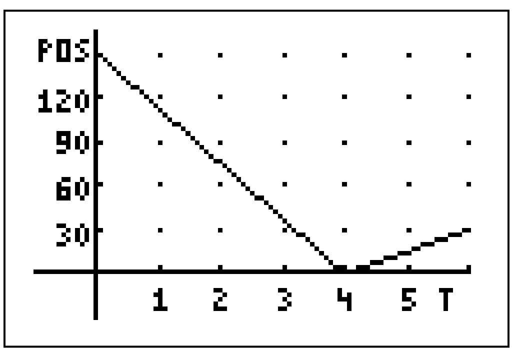
The 60 mph line forms a rectangle with the time axis. The rectangle has height = 60mph and width equal to 3 hrs. Since area of a rectangle is height x width the resulting area is +60 miles/~~hour~~ * 3 ~~hrs~~ = +180 miles. A second rectangle is formed for the time from t= 4hrs to 5hrs. This rectangle has a height of -80mph and a width of 1 hr. The area of the rectangle is again height x width or -80 miles/~~hour~~*1~~hour~~ = -80 miles. You can only find position by adding displacements to the initial position.

The position vs. time graphs shown below have time marked in hours and position in kilometers. The questions for each graph are written above the graph. If a line appears to split the tick marks on the side of the graph you must assume that it is at the midpoint between the marks. The initial position of the first graph is 20 km for example.

1. The velocity of object at t = 2hrs is ___ kph.
2. The speed of object at t = 3.5 hrs is ___ kph.
3. The velocity of object at 4.75 hours is ___ kph.
4. At what time did object slow down?
5. At what time did object gain speed?



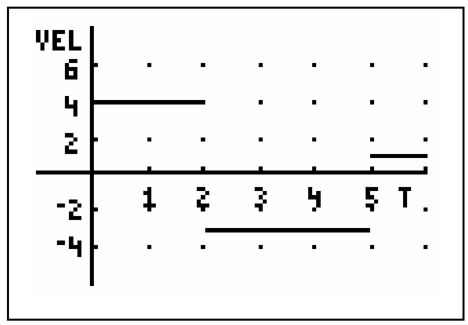
6. Initial position of object is ___ km.
7. Velocity of car at t = 2hrs is ___kph.
8. Position of car at t =2.5hrs is ___km.
9. Average velocity of car for first 5 hours is ___ kph.



The velocity vs. time graphs shown below have time marked in seconds and velocity marked in meters/sec.

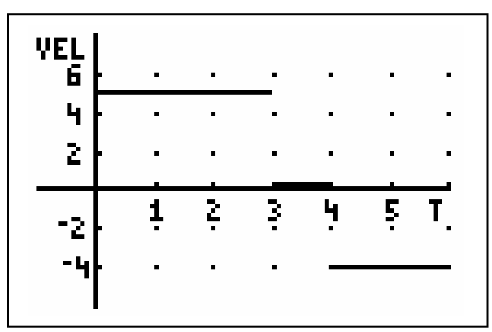
The object shown in the graph below starts at x = +5m at t = 0 seconds.

10. The distance traveled during the first two seconds is ___ m.
11. The displacement from t = 2s to t = 5s is ___ m.
12. The final position of the object at t = 6 s is x = ___ m.



The object shown in the graph below starts at a position of x = -6 m at t = 0 s.

13. The initial velocity of the object is ___ m/s.
14. The object crosses through the origin at t = ___ seconds.
15. The position of the object at t = 4 s is x = ___ m.
16. The displacement traveled during the time from t = 4s to t = 5s is ___ m.
17. Final position at t = 6s is x = ___m.



Answers to Homework

1. 40 kph to the right
2. 0 kph; at rest
3. 60 kph to left
4. At t = 3 hrs object changed from a speed of 40 kph to 0 kph.
5. At t = 4 hrs object went from rest to moving backwards at 60 kph.
6. 150 km to the right of the origin
7. 37.5 kph to the left
8. 56.25 km to the right of origin
9. 27 kph to the left
10. 8 m
11. 9 m to the left
12. 5m + 8m + -9m + 1m = 5m or the original position.
13. 5 m/s to the right
14. 1.2 seconds
15. 9m to right of origin
16. 4m to the left
17. 1m to the right of origin

Lesson 1-08 **Constant Speed (Part II)**

In the last lesson you learned that "**d**=**v**t" can be used for the constant speed motion when the **d** is for displacement and the **v** is for velocity. There is a second form of this same equation that appears to be something totally different. Displacement has been defined as distance with direction. But there is another way to define displacement. Displacement is a change in position. For an object moving along the x-axis the change in position or displacement is shown in the above box. In today's lesson we will use this new definition of displacement to modify our working equation. Then we will take advantage of the graphing calculator to visually demonstrate some problems.

Displacement = $\Delta x = x_f - x_o$ Change is <u>always</u> final - initial

The table to the right shows some initial and final positions for objects moving along the x-axis and the resulting displacement. Pause for a moment of inspection to be sure that you understand how each displacement was determined from the initial and final coordinates. After you have mastered this we can move on to modification of the equation.

x_o	x_f	**Δx**
+2.4m	+3.6m	+1.2m
+3.0m	+1.2m	-1.8m
-1.0m	+1.0m	+2.0m

Now the "d" in "**d**=**v**t" can be rewritten in terms of changes in position. The initial position can be moved to the opposite side of the equation making it a positive term. The final result is an equation that does not give how far and in what direction an object is moved. Instead the new form of the equation gives the final position in terms of time, velocity and initial position. The result is shown in the box to the right. When reading physics problems there is a very sure sign that you should use this form of "**d**=**v**t" instead of the more general form. When a problem starts listing where an object starts or where it stops instead of how far it moves. Those "where it starts" issues are dealing with the explicit information of initial position.

$$\boxed{\mathbf{x}_f = \mathbf{v}t + \mathbf{x}_o}$$

<u>Example Problem #1</u>

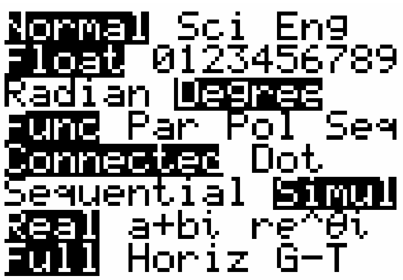
At noon a car on the interstate is at mile marker number 200. The car begins driving east at 70mph. What is the position equation for this car? Where is the car at 3 pm? How would the equation and answer change if the car was driving west instead of east?

a) $\mathbf{x}_f$ = (+70mph)t + 200mi b) $\mathbf{x}_f$ = +70mp~~h~~(3~~hrs~~) + 200mi = 410 miles c) Driving west would be represented as a negative velocity so a′) $\mathbf{x}_f$ = (**–**70mph)t + 200mi and the final answer would be that the car is at -10 miles. There are no such things as negative mile markers on the interstate. What you could conclude however is that the car is now 10 miles across the state line into the neighboring state. Note that mile markers count up when traveling in the positive directions of east and north. Mile markers count down when moving in the negative direction of south and west. I would love to know who made that choice and why?

If this is as far as you could go with position vs. time instead of displacement vs. time I would tell you that it is not worth the trouble. You could stick to "d=vt" and then add the displacement to the initial coordinate so why bother with the above extra details? It so happens that the above boxed equation is very similar to the equation of a straight line, y = mx +b. If you let the slope of the straight line represent the velocity of an object and the original position represent the y-intercept then some very nifty things can be done with a graphing calculator. This is the only reason for not skipping today's lesson.

<u>Graphing Motion on the TI 83 Calculator</u>

Mode: Be sure that you have the choices shown below. It is critical that you are in *functional* (Func) where you will graph y = f(x).



The button to the right of the [ALPHA] key will represent one of four variables depending upon your choice in mode. The button is X, T, θ, n depending upon your choice of <u>Func</u>tional, <u>Par</u>ametric, <u>Pol</u>ar or <u>Seq</u>uential. Since you are using the functional mode the button is the [X] button.

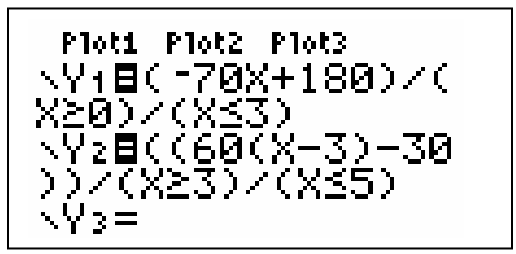
You will graph $Y = VX + X_o$. The colored variables should have different values substituted into the equation. The calculator will think that it is graph Y when the final position will be plotted on the y-axis. The calculator will think that it is graphing X when we will be using the x-axis for time.

<u>Example Problem</u>

A car starts at mile marker 180 at noon. The car moves at 70 mph to the west for 3 hours. At 3 pm the car drives at 60 mph to the east for two hours. Show the graph of position vs. time on the calculator.

For the first part of the trip we will type into the [y =] button the following equation: $Y_1 = -70X + 180$. But you only wish to have this graphed for the time interval $0 < t < 3$. Since the time interval is on the x-axis we can enter the equation for Y_1 as follows: $Y_1 = (-70X + 180) / (x \geq 0) / (x \leq 3)$. The latter two factors will only allow the graph to occur between 0 and 3 hours. The greater than or equals is [2nd], [MATH], [4] and less than or equals is [2nd], [MATH], [6].

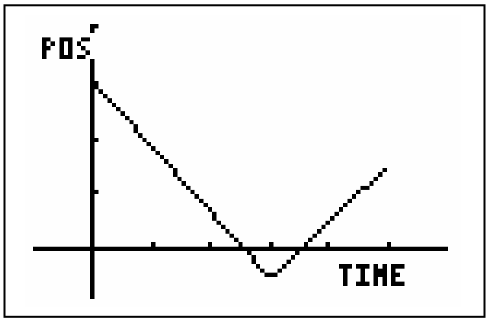
Graphing the second equation is a bit tricky. One would think that the equation would be $Y_2 = 60X – 30$ since during this interval the speed is +60mph and the initial position for the interval is at -30 miles. This is all true except that you do not want the graph to turn on until the third hour. By replacing X with (X-3) you will delay the graph turning on until 3 pm. $Y_2 = (60(X–3) – 30)$ and you want to graph only the correct segment so you again divide by the correct interval commands.



<u>Window</u>

The time interval is from $0 < t < 5$ but you want to have a border both before and after the given interval of time. Something like $-0.99 < X < 5.99$ is suggested for minimum and maximum X values. Use a scale of 1 hour for the x-axis. The y-axis is for the positions of the car that range from the initial 180 miles down to the -30 miles. Perhaps a range of $-50 < Y < 240$ with a scale of 60 would be OK. Setting the window takes some practice and should be done after the functions are entered. Some people use [2nd], [GRAPH] to give them an idea of values for the window.

The resulting graph using the described functions and window is shown below. Labeling of the axis will be explained later.



Analyzing the Graph

Once the function is graphed you can answer an infinite number of questions about the graph from the calculator.

You can use the [TRACE] button to locate the coordinates for any place on the graph(s). Try [TRACE], [1.5], [ENTER]. At 1.5 hours the position is at the 75 mile marker. Try [TRACE], [4], [ENTER]. Nothing happens because your cursor is still tracing Y_1. If you use the up-down arrow keys the cursor will hop from one function to another and you can move to Y_2 where the readout is 30 miles at the 4^{th} hour.

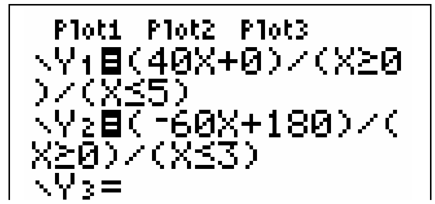
You can use [2^{nd}], [TRACE], [2] to locate where the graph crosses the x-axis. For our problem the graph crosses the axis twice. The calculator will guide you through left bound, enter, right bound, enter, enter so that it can narrow the domain of the search. The car is at the state line for the first time at 2.571 hours into the trip. The car is again at the state line at the 3.5 hour mark.

You can use [2^{nd}], [TRACE], [6] to find the slope of the graph at any point. This should give you the velocity.

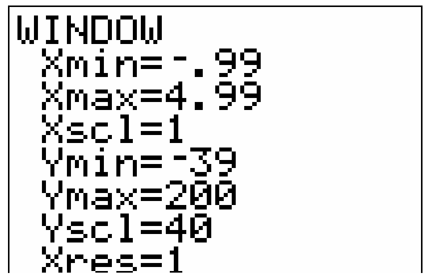
2^{nd} Example Problem

Two trains are on the same track. Train A leaves the station at mile marker 0 headed east at 40 mph. Train B is at mile marker 180 headed west at 60 mph. When and where do the trains meet?

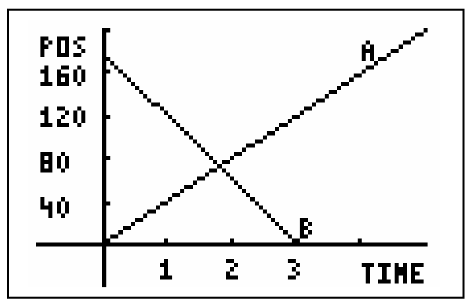
For [y =] use:



For [WINDOW] use:



You should have a graph that looks something like below:



You can see from the graph that train A and train B intersecte somewhere just before the second hour and just about at mile marker 80 miles. But where and when exactly is the key? Try [2^{nd}], [TRACE], [5] and then strike the [ENTER] key three times. The calculator is pausing in order for you to move the cursor close to the intersection. This helps the calculator in case there are multiple intersections of the functions. The trains meet at t=1.8 hours at the 72 mile marker. This method is really worth it for multiple objects in motion.

Relative Velocity

There was an easier way to get the answer that does not require use of a graphing calculator or learning all of those buttons. You still need to be familiar with the previous ideas because there are times later when that way is the best way. There is something called relative velocity. If you were the engineer of one train and looked at the other train how fast would it appear to be moving? That is what relative velocity is all about. Relative velocity is defined in the above box. The values are vector values.

$$\boxed{\mathbf{V_{REL} = V_{THEM} - V_{YOU}}}$$

Suppose that you are in the front of Train A with very good eyesight. You would see Train B to appear to head towards you at a speed of V_{REL} = -60mph – (+40mph). From your perspective, Train B appears to be headed in your direction at a speed of 100 mph. Now you can use the equation d=vt to find the time to impact. The two trains are originally separated by d=180 miles and approach each other at a relative speed of v=100 mph. The time to impact is t = 180/100 hrs or 1.8 hours. The position of the collision is found by plugging that time back into either position equation. For Train A the collision occurs at X = 40mph(1.8hrs) + 0 miles or X=72 miles. For train B the collision occurs at X= -60mph(1.8hrs) + 180 miles or X=72 miles.

When two objects are in relative motion you can find a meeting time in position in one of three ways: 1) Write the position equations and use algebra to analyze four equations with four unknowns. 2) Write the position equations and graph them to locate the intersection of the functions. 3) Use d=vt and the relative velocity equation.

Homework: reproduce the graph on page 3 of this unit.

Lesson 1-09 Acceleration Defined

After today's lesson you should be able to define acceleration verbally, graphically, numerically and algebraically. Acceleration like velocity is a rate. Velocity is how fast displacement changes with time. Acceleration is how fast velocity changes with time.

Acceleration: Rate of change of velocity with time

Symbol: **a** (a vector quantity indicated by bold type)

Units/ Dimensions: m/s^2 or m/s/s or (m/s each second).

Two Types of Acceleration

Acceleration is defined to be change in velocity divided by change in time. This definition is complicated by the fact that velocity has two parts and if either part changes then there is acceleration taking place. Recall that velocity is a measure of speed and direction. If an object changes speed then it accelerates. You accelerate a car by using the gas pedal or the brake pedal. Either of these will change the speed of the car and cause it to accelerate.

Direction is also a part of velocity. When an object changes direction it also accelerates. You accelerate a car by turning the steering wheel too. Moving around a circle at constant speed avoids the first kind of acceleration (changing speed) but is a perfect example of the second type of acceleration (changing direction or curving). The acceleration due to curving has a special name- centripetal acceleration. This kind of acceleration will be studied in another unit. In this unit we will focus on the acceleration due to changing speed. For this unit test be prepared to recognize the existence of both types of acceleration but concentrate on the applications of changing speed only.

Since acceleration is the rate of change of velocity with time it is easy to spot acceleration in a position vs. time graph. At the top of page 3 on the right-hand side are shown three position vs. time graphs. Only one of the three graphs has a constant slope and a constant velocity. The other two graphs show a changing slope and therefore exhibit acceleration. Since graphs A and C have changing slope they are examples of accelerated objects on a position vs. time graph.

Acceleration can be defined mathematically by a simple algebraic equation. Although the equation is applied to both types of acceleration it is almost exclusively used for changing speed types of acceleration only.

$$\mathbf{a} = \mathbf{\Delta v} / \Delta t$$

The equation is shown in the text box to the right. Realize that the deltas represent change and are mathematically defined as "final minus initial". The direction of the acceleration vector is the same as the direction of the change in velocity. The numerator has units of m/s and the denominator has units of seconds so that the resulting unit of acceleration is m/s^2. Go ahead and say "meters per second squared" and go ahead and write "m/s^2" but do not think about acceleration in either term. Always think about acceleration in this unit as "meters per second each second".

Consider the example of a car accelerating from rest at a rate of 3 m/s^2 for 5 seconds before traveling at a constant speed. This means that the car is changing speed by 3m/s each second for the first five seconds of travel. Speeds at different times are shown below for the first 6 seconds of travel.

Velocity	0 m/s	3 m/s	6 m/s	9 m/s	12 m/s	15 m/s	15 m/s
Time	0 sec	1 sec	2 sec	3 sec	4 sec	5 sec	6 sec
Total distance							

As long as you can add you can continue to write down new speeds for any interval of time. Consider a motorcycle that accelerates at 4 m/s^2 for 6 seconds from rest. Fill in the table below and determine the final speed of the motorcycle.

Velocity	0 m/s						
Time	0 sec	1 sec	2 sec	3 sec	4 sec	5 sec	6 sec
Total distance							

What about distance traveled? How can a person find the distance moved by an object? Is there a simple method that will enable you to fill in the third row in the above tables? Yes. Can you use d=vt since the object is changing speed? (Sort of) You have to use the average velocity instead of beginning or end velocity.

Return to the car that accelerated from rest at 3 m/s^2 for 5 seconds. During the first second it had an average speed of (0+3)/2 or 1.5 m/s. Using d=vt we can find that the car moved d = $(1.5m/\cancel{s})(1\ \cancel{sec})$ = 1.5 m. During the first two seconds the car had an average speed of (0+6)/2 or 3 m/s. The car moved for the first 2 seconds a total distance of d= $(3m/\cancel{s})(2\cancel{sec})$= 6 m. For the first 5 seconds the car moved at an average speed of 7.5 m/s. The total distance covered during the first 5 seconds is d = $(7.5m/\cancel{s})(5\cancel{sec})$ = 37.5 m. Beware that you can use the d = $(v_{AVE})*t$ as long as there is constant acceleration. The total distance traveled by the car for 6 seconds is 52.5 m and d = $(v_{AVE})*t$ fails since the acceleration changed from 3 m/s^2 to 0 m/s^2. Watch out for piece wise continuous stuff. Take this time to fill in the remainder of both charts on the previous page.

<u>Gravity and Free Fall</u>

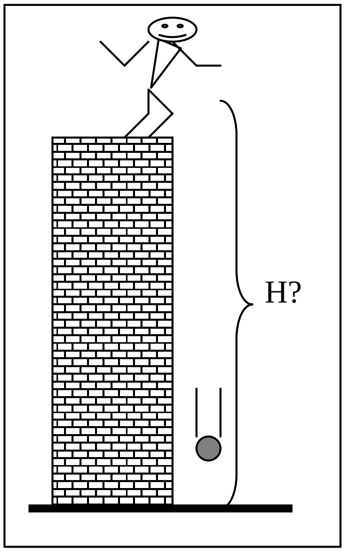
When an object is moving vertically against gravity (either up or down) but is under the influence nothing else besides the attraction to Earth it is said, “To be in free fall”. If an object is moving vertically with the aid of rockets or falling with the aid of parachutes or air resistance then you do not have free-fall conditions. In this course we will neglect that affect of air resistance. For objects in free fall the rate of acceleration is -9.8 m/s^2. We will round this to -10 m/s^2 in order to approximate things in our head. Acceleration due to gravity is so common that it has its own symbol. The value of g is shown to the right.

$\boxed{\mathbf{g} \cong -10\ m/s^2}$

We finish with two vertical examples.

<u>Example #1</u>

A ball is released from rest from the roof of a building and strikes the ground 4 seconds later. How high was the ball above the ground at release?



The ball passes through speeds as shown in the table below:

Time (sec)	0	1	2	3	4
Velocity(m/s)	0	-10	-20	-30	-40

The average velocity is (0+ -40m/s)/2 or -20 m/s. The ball traveled at an average velocity of -20 m/s for 4 seconds. Using $d = (v_{AVE})*t = (-20\ m/\cancel{s})(4\cancel{sec}) = -80$ m

<u>Example #2</u>

A ball is tossed upward at 30 m/s. It rises to maximum height before falling back to the original toss point where it is once again caught. How high does the ball rise? How long is it in the air?

There are two little notes that make this problem easier than it seems.

- At maximum height the velocity is always zero for an instant.
- When an object rises and then returns to its starting point it will have the same speed but in the opposite direction.

A velocity-time table is shown below for the first 6 seconds of motion.

t (sec)	0	1	2	3	4	5	6
v(m/s)	30	20	10	0	-10	-20	-30

You can see that the ball rises for 3 seconds before beginning to fall again.

While rising, the ball has an average speed of 15m/s for 3 seconds.

The ball rises to height of $d = (v_{AVE})*t = 45$ m above the tossing point.

At t=6 seconds the ball once again has its original speed but in the opposite direction. The time of flight for the ball is 6 seconds.

You have seen from this lesson that acceleration is a rate of change of velocity with time. We are focusing upon the specific idea of changing in speed with time. Since speed changes you have to use the equation d = (average speed)(time) where average speed is easily found by averaging the beginning and end values as long as the acceleration is uniform.

The method of analyzing the motion for times, speeds, heights and so forth is known as the study of *kinematics*. In today's lesson you used a very numerical approach where tables of numbers were required. In the next two days you will learn an algebraic approach that is faster and more direct. Why did we take this numerical approach if it is slower and less direct? You need to understand some of the concepts that are emphasized in this lesson before moving on to the next two lessons.

Homework

1. A truck starts from rest accelerating at a uniform rate of 2 m/s^2 for 8 seconds. How far did the truck move during the 8 seconds? What was the final speed of the truck?
2. A car is moving at 24 m/s when it decelerates at -3 m/s^2 (It is losing 3 m/s each second). How long does it take for the car to stop? How far does it travel while braking before coming to a rest?
3. A ball is tossed upward at an initial speed that is not known. It is noticed that the ball rises for a total of 2.5 seconds. What was the initial speed of the ball when it was tossed? How high did the ball rise?
4. A ball is dropped from the roof of a building and strikes the ground in 5 seconds. How high is the building? How fast is the ball moving when it hits the ground?

Answers

1. 64 m ; 16 m/s
2. 8 seconds; 96 m
3. 25 m/s; 31.3m
4. 125 m; 50 m/s

Lesson 1-10 **Four Kinematics Equations**

When objects uniformly accelerate there are four equations of motion that can be used to calculate any two unknowns. The five variables are displacement (**d**), initial velocity ($\mathbf{v_o}$), final velocity (**v**), acceleration (**a**) and time (t). Note the vector nature of all except time. When given any three of the five variables you can always calculate the two unknowns using the equations shown below. Each equation is missing one variable.

1) $d = v_o t + \frac{1}{2} a t^2$	2) $v = v_o + at$	3) $v^2 = v_o^2 + 2ad$	4) $d = \frac{1}{2}(v + v_o)t$
missing v	missing d	missing t	missing a

The following steps can be used to solve the problems:

I) Read the problem to find the three givens. List them in a vertical column. It is also recommended to draw a picture when ever possible.
   A. If an object "starts from rest" then $v_o = 0$.
   B. If an object "comes to rest" then $v_f = 0$.
   C. If an object is thrown upward and reaches maximum height then v=0 at that point.
   D. If an object returns to the initial position then displacement is zero,
   E. If an object is in vertical motion under the influence of only gravity then it will accelerate at -9.8 m/s^2. We will use g = -10 m/s^2 for this course.*
   F. If an object hits the ground **do not** use $v_f = 0$. In most cases the problem is looking into the maximum speed just before impact.*
   G. For objects that are thrown vertically upward under the influence of gravity alone the following assumption is allowed. When an object returns to initial height it will have the final speed equal to the initial speed but opposite direction. $v = -v_o$

II) List the unknowns in the same column with question marks.

III) To determine the value of one of the unknowns select an equation that is missing the other unknown. If v_o is one of the unknowns then it must be found first since it is in all four equations.

IV) Substitute values and solve for the unknown. Record your unknown in the original column in a different color of ink. This way when you go back to study the problem you will see two different colors, givens and founds.

V) Repeat Steps III & IV for the other unknown. Try to avoid using a calculated number to find a calculated number. If you do and the first answer is incorrect then you will suffer from double jeopardy by getting the second answer also incorrect.

VI) Use your two answers plus one of the givens in one of the unused equations to calculate one of the originally given values. If you get back that original value then you know that your answers are correct.

*For objects in vertical motion there are actually three different kinematics problems. The first problem involves the object being in contact with something like a hand while being tossed upwards. There is the vertical free-flight motion that takes places after the thrown object breaks contact with the thrower. Finally there is the last kinematics problem where the thrown object is in contact with the ground. We will almost always focus on the middle stage. The initial velocity of the thrown object is the speed it had when breaking contact with the hand. Worry about the flight of the object not necessarily how it became a projectile.

Reading Exercise

Read each of the following problems and apply steps I & II from the previous outline. Beside the question marks write the number of the equation that you would use to solve for that particular unknown. Leave several lines of space in between each problem. Tonight's homework will be to do the math part of the problem to get final answers.

1. A car uniformly accelerates from rest to a final speed of 88 mph (39.2 m/s) in 4 seconds. What is the acceleration rate of the car? How far did it travel?
2. A car traveling at 27 m/s uniformly accelerates to rest over a distance of 60.8m. What is the deceleration rate? How long did it take to stop?
3. A car traveling at 10 m/s uniformly accelerates at $4 m/s^2$ over a distance of 28.8 meters. How long did the acceleration last? What is the final speed of the car?
4. A motorcycle uniformly accelerates at 6 m/s/s over a distance of 58.5 meters. The distance is covered in 3.8 seconds. What is the final speed? What was initial speed?
5. A person stands at the edge of a bridge over the river. A rock is released from rest and falls vertically for 2.5 seconds before hitting the water below. Approximately how high is the bridge above the water? What was impact speed of the rock?
6. A ball is thrown straight up at an initial speed of 24 m/s. How long did it take to reach maximum height? How high did it rise before falling again?
7. A child throws a small ball vertically upward. The ball is caught at the initial height 1.8 seconds after being thrown. What was the net displacement for the ball? What was the initial speed of the ball? What was the final speed of the ball? How high did it rise? (This last question is a different problem from the rest.)
8. A car uniformly accelerates from rest at $3 m/s^2$ for 8 seconds. At the end of the 8 seconds the car uniformly brakes at $-5 m/s^2$ for 4 seconds. What total distance is traveled by the car during the 12 seconds of motion? This is a two part problem.

Problem #	d (m)	v_0 (m/s)	v (m/s)	a (m/s^2)	t (sec)
1					
2					
3					
4					
5					
6					
7					
8a					
8b					

Lesson 1-11 **Four Kinematics Equations**

Work last night's homework problems in class. Emphasize the mathematical pitfalls.

Lesson 1-12 **Graphing Accelerated Motion**

When initial conditions are given, you can use the graphing calculator to analyze an object's motion. The first of the four equations will be used in a modified form.

1) $\mathbf{d} = v_o t + ½ at^2$

The variable "**d**" is actually an object's displacement or the vector difference in the final and initial coordinates. In other words, $\mathbf{d} = x_f - x_o$ for objects moving horizontally and $\mathbf{d} = y_f - y_o$ for objects moving vertically. The graphing calcu-lator is better suited for showing actual coordinates rather than displacement.

2) $x_f = x_o + v_o t + ½ at^2$

Suppose for example that a car is moving at 15 m/s to the right when it is 20 m to the left of an intersection. The car uniformly decelerates at 4 m/s^2. What is the car's speed when it arrives at the intersection? We could graph the following equation:

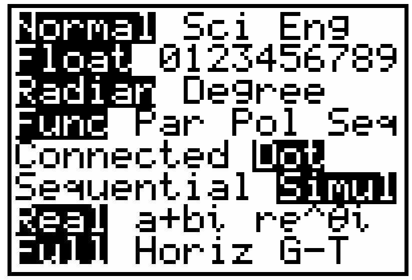
$x = -20 + 15t - ½(4)t^2$

As another example consider a person standing at the top edge of a 80 m high cliff. A ball is tossed upward into the air at 30 m/s. How high does it rise? How long is it in the air before it strikes the ground if it just misses the edge of the building on the return trip? What will be the impact speed with the ground? We could graph the following equation:

$y = +80 + 30t + ½ (-10) t^2$

In either case the numerical values for initial position, velocity and acceleration are substituted into the general form of equation (2) be it for x or y position.
In today's lesson we will learn to avoid the algebra by using the calculator.

Graphing Calculator Method
First you will need to place the calculator into the correct mode. The key selection will be the <u>func</u>tion choice.

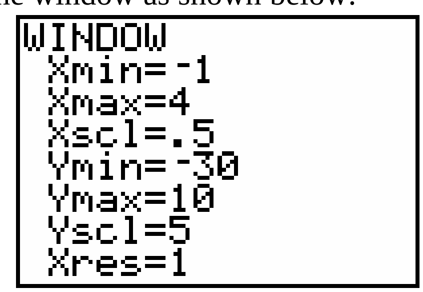


Be sure that all stat plots are turned off.
[2nd] → [y =] → [4] → [ENTER]
You don't want the calculator to plot numbers from any lists while you are trying to get it to plot equations.

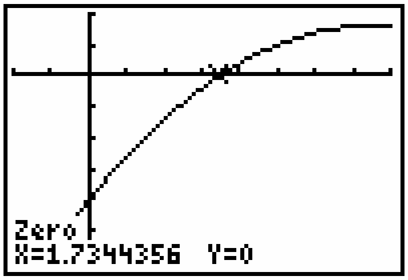
We can now graph equations but there is one drawback. The function mode will graph your position (x or y) on the y-axis and the time will be plotted on the x-axis. Keep in mind that the "x" in the equation is time while the "y" is position. Go to [y=] and enter the equation of motion for the car from the left column as shown below.

$Y_1 = -20 + 15X - 2X^2$

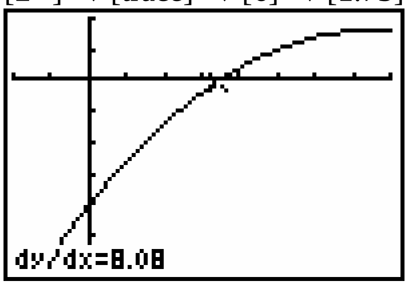
Set the window as shown below:



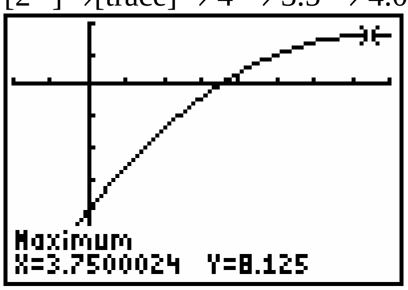
Now hit graph and look at the car's motion. Since y represents the car's position then you can [trace] until the cursor reaches a value of y=0. The cursor will not exactly land on y=0 but you can take advantage of the [2nd] → [trace] → [2] → 1.5 → [enter] → [2] → [enter].



The car reaches the intersection in 1.73 seconds. We can also find the speed of the car at the instant it reaches the intersection by finding the slope of the graph at that point.
[2nd] → [trace] → [6] → [1.73] → [enter].



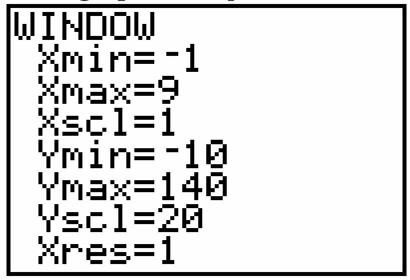
The speed of the car is +8.1 m/s as it gets to the intersection. Notice that after selection of the slope, dy/dx , I actually told the calculator exactly what time value was needed to find the slope. If you continue to follow the graph you will notice that the slope peaks some where between 3.5 and 4.0 seconds. We can use
[2nd]→[trace]→ 4 → 3.5 → 4.0 →[enter].



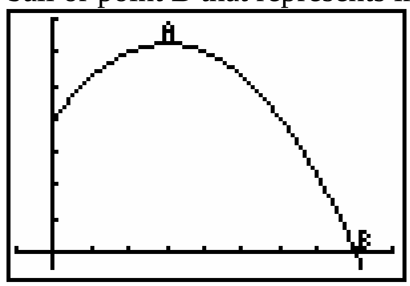
At 3.75 seconds into braking the car reached a maximum distance of 28.125 meters before coming to rest.

As you can see the graphical analysis involves 1) entering the proper equation into y=, 2) setting a proper window and 3) using the 2nd trace to analyze the graph.
Consider another example, the ball toss.

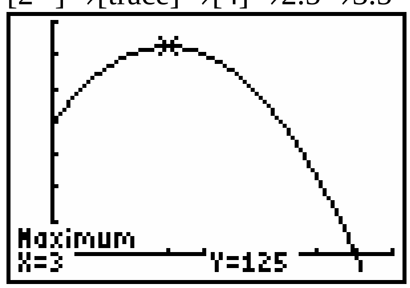
Let $Y_1 = (80 + 30X - 5X^2) / (X \geq 0)$ and the following window:
Now graph the equation.



Now you can trace until you are at point A that represents the maximum height of the ball or point B that represents impact.



The maximum value is around 3 seconds.
[2nd]→[trace]→[4]→2.5→3.5→[enter]^2



You can see that the ball rises to exactly 125m above the ground, 45m above the building in 3.0 seconds. With a slope of zero the ball is temporarily at rest.
[2nd]→[trace]→2→7→9→[enter]^2
The ball hits the ground 8 seconds after being tossed.
[2nd]→[trace]→6→8→[enter] will give the slope of the graph at 8 seconds. The impact speed of the ball is 50 m/s down.

You can graph in parametric mode as an alternative. This eliminates the problem of having the Y axis represent X and the X axis represent t. There is less confusion about variables.

Parametric Mode

In order to analyze motion in parametric mode go to the [MODE] button and change functional to parametric. Now you can enter equations as shown below:

$$X = X_o + V_oT + ½ a T^2 \qquad\qquad Y = Y_o + V_oT + ½ a T^2$$

The button to the right of the [ALPHA] button is now working as the [T] key. If you are working in a horizontal motion problem assign a constant value for Y. For example, if the object is moving horizontally fix the Y part of the Window to go -1 < Y < 3. Then for the part of the parametric equations under the [y =] assign $Y_1 = 1.5$. This will make the graph run horizontally near the middle of the window. If the problem is a vertical motion problem then assign the X value to be $X_1 = 1.5$ and use a similar range.

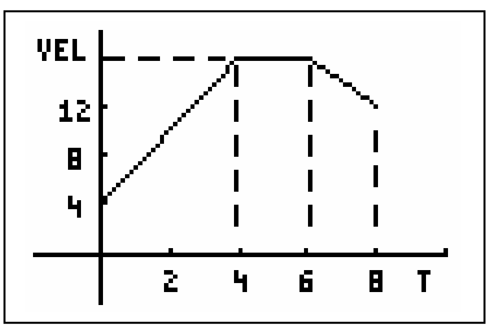
The parametric mode avoids the confusion of X being on the y-axis and t being on the x-axis but it too has some drawbacks. The graph may run over itself if the object in motion stops and backs up. Also, the [2^{nd}], [TRACE] menu has been reduced since the slopes for two dimensions are needed.

For most students using the functional mode for one dimension and the parametric mode for two dimensions is advisable.

Reading Velocity vs. Time Graphs

Although velocity vs. time graphs was mentioned at the beginning of the unit there are some concepts that need to be repeated and emphasized. Now that objects can accelerate the lines for velocity will have the possibility of a non-zero slope. When this takes place the slope will represent the acceleration of object. The area under a curve is still the displacement but it will now take the form of a trapezoid. An example is shown below:

Consider the velocity vs. time graph of an object shown below. The horizontal axis is measured in seconds and the vertical axis is measured in m/s.



The slopes are different for the three different time intervals. From 0 < t < 4 seconds the acceleration (slope) is rise/run = +12 m/s / 4 sec = + 3m/s^2. The slope from 4 < t < 6 seconds is zero and the object moves at constant speed. The acceleration for the third interval is rise/run = -4m/s / 2 sec = -2 m/s^2.

How far did the object move during each interval? The area of a trapezoid is average height times width.

For the first trapezoid the area is {(4m/~~s~~+16m/~~s~~)/2}∗ 4 ~~sec~~ = 40m.

For the middle area you see a rectangle that id height∗width or 16m/s(2sec) = 32 m. For the first 6 seconds the object has traveled a total of 72m.

During the last interval the object is still moving forward even though the speed is decreasing. Again, a trapezoid is encountered.

Area = {(16m/s+12m/s)/2}∗2 sec = 28m.

The total distance moved during the 8 seconds is 100 m.

This is the end of the second unit. Expect a test in about two days.

Shown below is a practice test.

Problem I A

A ball is rolling down a hill. After crossing a line on the hill it has the following measured speeds at two second intervals:

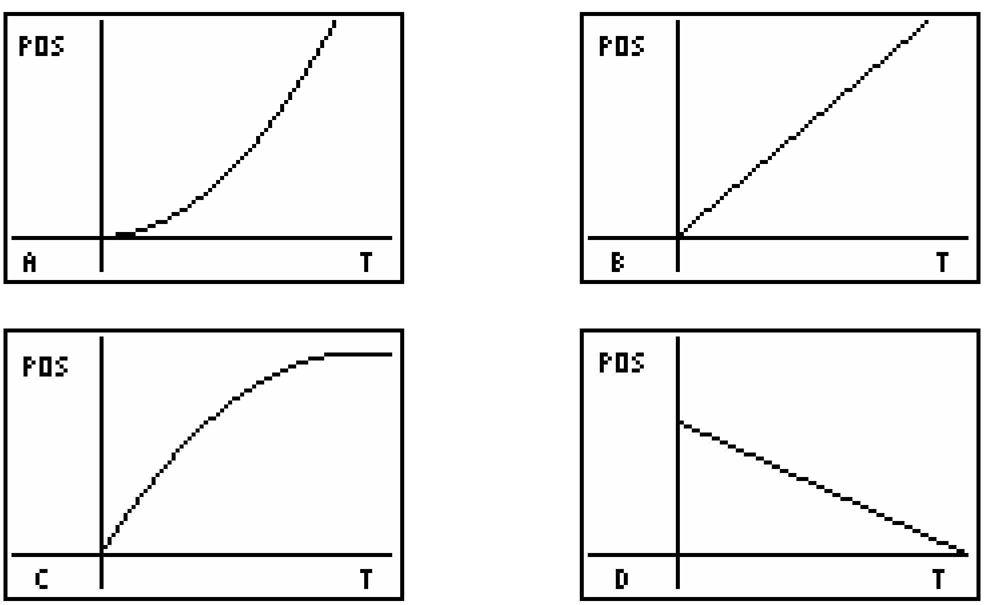
V_1 = 8 m/s	V_2 = 14 m/s	V_3 = 20 m/s	V_4= 26 m/s
t_1= 0 sec	t_2= 2 sec	t_3= 4 sec	t_4 = 6 sec

1. The average acceleration for the ball is about ___ m/s^2.
   a) 2 b) 3 c) 4 d) 5 e) 6
2. The speed of the ball at t_5 will be ___ m/s.
   a) 29 b) 32 c) 35 d) 38 e) 41
3. The distance traveled from t_1 to t_2 is ___ m.
   a) 16 b) 19 c) 22 d) 25 e) 28
4. The average velocity of the ball from t_2 to t_3 is ___ m/s.
   a) 14 b) 15 c) 16 d) 17 e) 18

Problem II

A car is traveling down a long, straight road when it uniformly brakes to rest at a rate of –3m/s^2 over a distance of 121.5 meters. Use this information to respond to items 7-10.

5. The braking time lasts for ___ seconds.
   a) 5 b) 6 c) 7 d) 8 e) 9
6. The speed of the car just before brakes were applied is ___ m/s.
   a) 21 b) 24 c) 27 d) 30 e) 33
7. During the last three second of braking before coming to rest the car travels a distance of ___ m.
   a) 13.5 b) 24.3 c) 35.1 d) 46.6 e) 57.6
8. A graph of position versus time for the motion of the car would look like which of the following?



Problem III A

A ball is thrown vertically upward at 36 m/s from the edge of a 127.4 m high cliff. Respond to items 11 – 15 about the ball while it is in the air. Ignore any effects of air resistance.

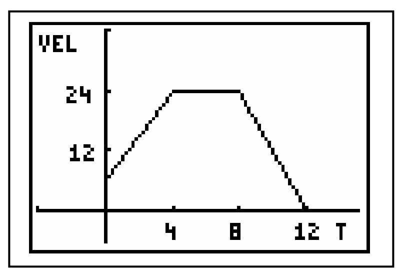
9. The ball reaches a maximum height of ___ meters above the base of the cliff.
   a) 115 b) 157 c) 192 d) 231 289
10. The ball will reach maximum height in ___ seconds after being thrown.
   a) 1.9 b) 2.8 c) 3.6 d) 4.4 e) 5.3
11. The ball will remain in the air for ___ seconds after being thrown.
   a) 6.2 b) 7.6 c) 8.7 d) 9.8 e) 11.2
12. The ball will strike the earth at the base of the cliff at a speed of ___ m/s.
   a) 48 b) 56 c) 62 d) 68 e) 76

Problem IVA

A grasshopper on steroids jumps straight up into the air with an initial speed of 18 m/s. Assume that the insect is unable to fly.

13. The grasshopper will remain in the air for ___ seconds.
   a) 3.0 b) 3.6 c) 4.2 d) 4.8 e) 5.4
14. The grasshopper reaches a maximum height of ___ meters above the ground.
   a) 16.2 b) 22.1 c) 28.8 d) 36.5 e) 42.4
15. Upon landing the grasshopper has a total displacement of ___ meters.
   a) 0 b) 32.4 c) 44.1 d) 57.6 e) 72.9
16. At one second into the flight the grasshopper's speed is ___ m/s.
   a) 0 b) 5 c) 8 d) 11 e) 14
17. If the grasshopper doubles its jumping speed at lift-off then its maximum height will
   a) double b) triple c) quadruple d) half e) none of these

Use the velocity-time graph to answer the remaining items.



18. The acceleration of the object at t=2 seconds is ___ m/s^2.
   A) 3 B) 4.5 C) 6 D) 9.5 E) 12
19. The distance traveled during the first 4 seconds is ___ meters.
   A) 36 B) 48 C) 52 D) 60 E) 96
20. The acceleration at t=10 seconds is -____ m/s^2.
   A) 2 B) 4 C) 6 D) 8 E) 10
21. The distance traveled during the interval from t=4 seconds to t = 8 seconds is ___ m.
   A) 36 B) 48 C) 52 D) 60 E) 96
22. The average velocity for the object from t=4 seconds to t=12 seconds is ____ m/s.
   A) 6 B) 12 C) 18 D) 24 E) 30

Solutions to practice test 102 **Use $g = -10\ m/s^2$**

Problem set #1

1. For any interval shown the velocity is increasing by 6 m/s so $\Delta v = +6m/s$. The time intervals are $\Delta t = 2$ seconds. Since $a \equiv \Delta v/\Delta t$, so a= +6m/s / 2sec. **B**
2. $V_5 = V_4 + \Delta V$ or $V_5 = 26m/s + 6m/s$. The speed is changing by +6 m/s each second so do 26m/s + 6m/s =32m/s. **B**
3. The average speed during this interval is (8+14)/2 or 11 m/s. Using d=vt where v is average velocity of 11m/s and time is 2 second, d=(11m/s)2sec or 22 m/s. **C**
4. Average velocity is found by averaging beginning and end speeds. (14+20)/2 or 17 m/s for v_{AVE}. **D**

Problem set #2

5. You must find answer to item #6 before getting answer to #5.
6. Using $v = v_o + at$; $0 = v_o^2 + 2(-3)121.5$ or $0 = v_o^2 + -279$ or $v_o^2 = +279$; $v_o = 27m/s$. The answer to #6 is **C**. Now we can get time using $v = v_o + at$. Subbing in values, 0 = 27 + (-3)t. The total braking time is 9 sec. **E** for #5.
7. During the last three seconds of braking you have $a=-3m/s^2$, $v_f = 0$ and t = 3 seconds. Using $v = v_o + at$ gives $0 = v_o + (-3m/s^2)(3seconds)$ so the speed at the beginning of the last 3 seconds is +9m/s. Now you can use $d= (v_o + v)t/2$ to find distance traveled. d= (9+0)3/2 or d=13.5m. **A**
8. Graphs B&D are for objects moving at constant speed but our car is slowing down. Correct answer is either A or B. Graph A starts from rest and has increasing speed since the slope goes from zero value to positive value. By elimination only graph B fits. Also note that Graph B has a beginning positive slope (i.e. initial velocity) and a zero value for final slope (at rest). **C**

Problem set #3

There are two distinct problems in this set. From toss to maximum height is one problem (A to B). From toss to impact with ground is another problem (A to C). Items 9 & 10 cover A to B. Items 11 and 12 cover the A to C path.

9. For the path of A to B we have values d=?; v_o= 36 m/s; v_f = 0m/s; $a=-10m/s^2$ and we also don't know time. You get max height using $v^2 = v_o^2 + 2ad$. Subbing in values, 0 = 1296 – 20d or 20d = 1296 or d = 64.8m above the toss point. Height above cliff base is 64.8m plus 127.4m =192.2m. **C**
10. Using $v = v_o + at$ gives 0=36+-10t or a time of 3.6 seconds. **C**
11. Find answer to #12 first then do #11.
12. For the path of A to C we have givens d=-127.4m; v_o=+36m/s; v_f =?;t=?; $a=-10m/s^2$. Use these values to answer item #12 first. $v_f^2 = v_o^2 + 2ad$. Subbing values $v_f^2=[36^2 +2(-10)(-127.4)] =3844$ or $v_f = \pm 62$ m/s. Note here that when you take a square root of a number there are always ± solutions. For our case the ball is going down when it hits the ground so v_f = -62m/s. The speed part of velocity is merely the number, 62m/s. Answer to #12 is **C.** Now we can use $v_f = v_o + at$ to get time of flight for the ball. -62=+36 +-10t or -98=-10t so time is t=9.8 seconds. **D**

Problem set #4

13. We will work this problem from jump to maximum height. d=?; v_o=18m/s; v=0m/s; $a=-10m/s^2$; t=?. The rise time is found using $v=v_o + at$ or 0=18+-10t. The rise time is 1.8 seconds. Since the grasshopper falls back to the same place the falling time is also 1.8 seconds giving a total time of 3.6s. **B**

14. Using $v^2 = v_o^2 + 2ad$; Subbing values gives $0 = (18^2) + 2(-10)d$ or d=16.2. **A**
15. Grasshopper is back where it started. **A**
16. Acceleration takes away 10m/s each second for a rising object. 18m/s -10m/s =8m/s. Or if you don't wish to think about it, $v = v_o + at$. Subbing values gives v=18 +(-10)(1) = 8 **C**
17. Go back to problem #14 and replace 18 with 36. You will see that max height is 4 times as high or quadrupled. **C**
18. The time, t=2seconds, tells us to consider the first line interval. Acceleration is the slope of this interval so you can do rise (+18m/s) over run (4sec). Alternately, you could use $v_f = v_o + at$ and solve for "a". You have 24 = 6 + a(4). Either way $a=4.5m/s^2$. **B**
19. The distance traveled is the area underneath the velocity segment. For trapezoids, Use base times average height. Area = 4sec(24+6)/2. Alternately, if you got stuck use the last kinematics equation, $d=(v_o+v_f)t/2$. Subbing values gives d=(6+24)4/2. Either way the answer is 60m. **D**
20. Use same approach as in #18. The slope is found with a rise = -24 m/s and a run of 4 seconds. Slope = -6 m/s. Alternately, $v = v_o + at$. Subbing values for that entire segment, gives 0=24+a(4) or a=-6m/s/s.
21. You have constant speed for the segment in this question since you have a zero slope. With constant speed, d=vt will do nicely. d=vt=24(4)=96m. Note had you not realized that d=vt would work here you can still get the correct answer from 1st kinematics equation. $d= v_ot + ½ at^2$. Subbing in values gives $d= 24(4) + ½ (0)4^2$ =96m. The answer is **E**.
22. To get average velocity, divide the total displacement by total time. For the 4-8sec interval d=96m. For the 8-12sec interval, $d= v_ot + ½ at^2$ or $d=24(4)+1/2(-6)4^2$ or 48 m. You could have noticed that the area of this part is half of the area of the 4-8 second part and merely done ½ of 96m. Either way the total displacement is 96m+48m=144m. The displacement of 144 m occurred in 8 seconds. The average velocity is 144m/8sec or 18m/s. **C**

<u>Test 102 A & B Objectives</u>

1. Given corresponding velocity and time lists determine kinematics values such as displacement, average velocity, acceleration, etc. See Problem IA.
2. Consider an object moving horizontally. Given any three out of the five variables d, v_o, v_f, a and t use the four kinematics equations to calculate correct values for the other two. See Problem IIA
3. Repeat objective 2 except for objects moving vertically under the influence of gravity alone. See Problem III&IVA.
4. Given graphs of position vs. time identify which graphs are for objects moving with increasing, decreasing or constant speed. Identify initial velocity from initial slope of the graph. Problem IIA8.
5. Given a graph of velocity vs. time be able to identify acceleration from slope and displacement from area under curve.

Test format will include multiple choice questions for concepts, open response problems for analytical skills and perhaps a written paragraph or two for comparing and contrasting *motion with constant velocity and constant acceleration.*