Physics A Unit 1 pp. 80 to 93 Vector Addition

You will learn from this initial unit how to define a vector. You will also see how to add vectors using two different methods. Finally you will apply your knowledge of vectors to the specific scenarios of boats traveling on a river or airplanes flying with random wind patterns to specific destinations. Before taking your first physics test you should be prepared to work problems and answer questions over the following outline.

I. Geometry Review

A. Sine, Cosine and Tangent for Right Triangles

B. Law of Sines and Law of Cosines for Non-Right Triangles

II. Vector Addition

A. Parallel, Antiparallel and Perpendicular Vectors

B. Method of Components using Sine, Cosine and Tangent Functions

C. Using Law of Sines and Law of Cosines

III. Applications

A. Boats and River Currents

B. Airplanes and Wind Vectors

This unit will take eight standard class periods. Day 1 covers all of Geometry Review. Day 2 describes II.A. Day 3 investigates II.B. Day 4 covers II.C. Day 5 studies III.A. Day 6 will introduce III.B. Day 7 will be group work and practice test. Day 8 will be the first test in the Physics A course.

Lesson 4 using II.C is an optional lesson for some schools. This lesson may be dropped entirely from this unit and still leave a student adequately prepared for college physics.

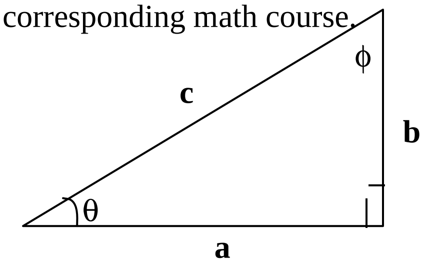
Lesson 1-01 **Geometry Review**

A right triangle has three sides and three, inside angles. One of the angles must be 90° in order to make the triangle *right*. Be sure that you know how to place your calculator in the degree mode since you will work exclusively in degrees in this course. You can do this by [MODE], [⇓], [⇓], [⇒], [ENTER]. [2nd], [MODE] returns you to home screen.

- All keystrokes in this course will be placed in [ ] if you are to strike a button. All menu choices will be in { } or by arrows as in the above keystrokes for moving the cursor through menu choices.

Today's lesson covers the parts of the right triangle and should be nothing more than review for everybody. A later lesson will cover parts of non-right triangles and is most likely new material that most students will not see until later in this year's corresponding math course.

<u>Right Triangle</u>



All right triangles have two unknown, acute angles that we shall designate as angle θ and angle ϕ. These two angles will always add to 90° for a right triangle. The three sides of the right triangle are composed of a longest side and two other sides. The longest side is opposite to the 90° angle and is usually designated as side **c**. Refer to above figure. Side c is also known as the *hypotenuse*. With the five parts labeled in the above diagram you should be able to do the following.

- Given values of any two parts find the values of the other three parts, except when the two are θ and ϕ.

The relationship among the lengths of the three sides is written as the Pythagorean Theorem. The relationship between the two angles has already been stated. Both are listed in the text box to the right. From these you can find the length of one side if you know the length of the other two sides or if you know one of the acute angles you can find the measure of the other acute angle. You will use these equations to check your answers in today's exercise. After today use them as often as possible to find values as well as to check. Both identities are pure in that they relate pure lengths or pure angles.

$$\mathbf{a}^2 + \mathbf{b}^2 = \mathbf{c}^2$$

$$\theta + \phi = 90^\circ$$

But how are lengths of sides related to angles? Suppose that you are given a side and an angle. How are those two related? What if you are given the lengths of two sides and know nothing about the angles. How can you use side lengths to find angles?

The relationship among angles and sides are shown in the three text boxes below. They are stated in the most generic terms possible. Be sure that you recognize that opposite means "length of a side across from an angle". Adjacent means "length of a side" next to an angle but not the hypotenuse.

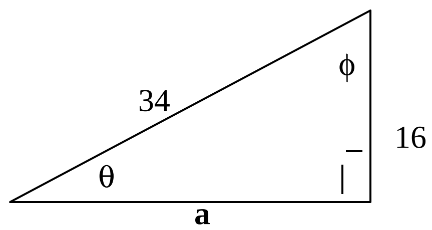
$$\sin \angle = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \angle = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \angle = \frac{\text{opposite}}{\text{adjacent}}$$

In the following exercises you are to attempt to find the remaining three parts of the given right triangles using only definitions of sine, cosine and tangent as stated at the bottom of the previous page. Check your sides using Pythagorean Theorem and check your angles to see if they sum to about 90° give or take about a half degree. Before working through the exercise two examples are shown.

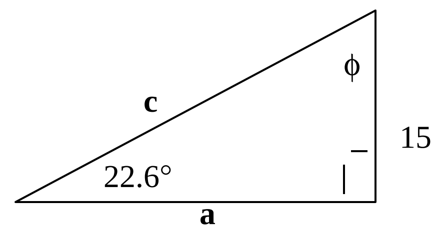
Example #1- Given two lengths



1. From the figure above the hypotenuse is 34 units long and the vertical side is 16 units long.
2. Angle θ can be found since the side opposite to θ and the hypotenuse are both known.
3. $\theta = \sin^{-1}(16/34) = 28.1°$
4. When using the calculator to get the value of an angle always start with the [2nd] key. In the above example the keystrokes are [2nd], [sin], 16, [÷], [34], [ENTER].
5. Angle φ is found in a similar manner using the cosine definition since the 16 is adjacent.
6. $\phi = \cos^{-1}(16/34) = 61.9°$
7. Length of side **a** can be found using any of the definitions and one of the found angles. Tangent is chosen just because it has not been used yet.
8. tan (28.1) = 16/ **a** or
   a = 16 / tan(28.1) = 30.0 units
9. To check angles add 28.1° and 61.9° and you get 90°.✓
10. To check the sides $\sqrt{(30^2 + 16^2)}$ and find that the hypotenuse is 34.✓

You randomly make up right triangles with two given length values until you can find the other three parts in less than 3 minutes including checking answers.

Example #2- Given a length and angle

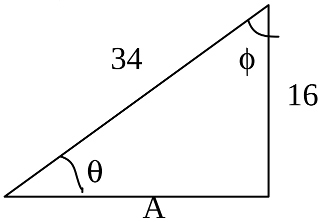


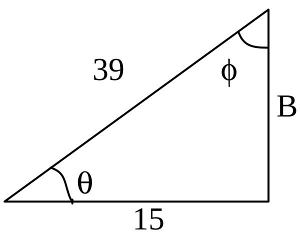
1. From the definition of sine the value of the hypotenuse is found.
   sin (22.6) = 15/ **c** or **c** = 15/sin(22.6). So **c** = 39.0 units
2. From the definition of tangent the length of side **a** can be found.
   tan (22.6) = 15/**a** or **a** = 15/tan(22.6). So **a** = 36.0 units.
3. Angle φ can be found using any definition since all three sides have been determined.
   $\phi = \cos^{-1}(15/39) = 67.38°$.
4. The angles are checked by adding.
   22.6° + 67.38° = 89.98° ✓
   This answer checks even if it is not exactly 90 since rounding error was introduced. The answer is still within a half degree of being correct and is therefore acceptable.
5. The two found sides will be used to determine if we get the original given value back from the diagram. You should get $\mathbf{b} = \sqrt{(\mathbf{c}^2 - \mathbf{a}^2)}$ when searching for an unknown side. Our value is $\sqrt{(39.0^2 - 36.0^2)} = 15$.✓

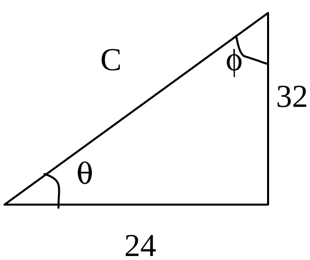
You randomly create right triangles with one known angle and one known length be it side or hypotenuse. Find remaining values using trig functions until you can get & check answers in less than 3 min.

Practice Problems for Right Triangle Geometry

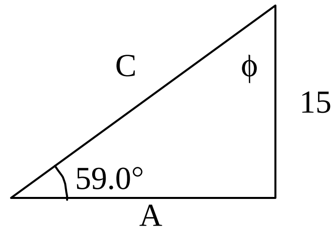
Given any two sides of a right triangle, use sine, cosine and tangent to find the interior angles and the remaining side. Use Pythagoras's Theorem to check the sides. Use $\theta + \phi = 90°$ to check the angles.

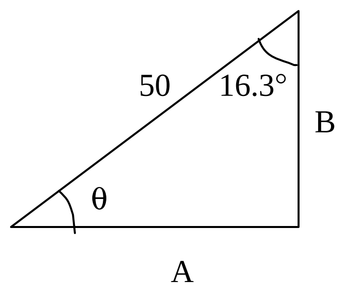


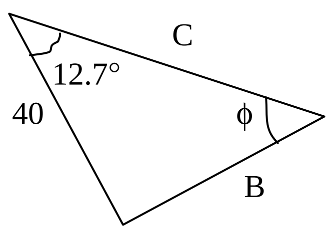




Given one of the interior angles besides 90° and one side of a right triangle, use sine, cosine and tangent to find the remaining sides and the other angle. Check the same way as before.







Be sure that you can find and check each triangle in less than 4 minutes.

Lesson 1-02 **Vectors Defined**

A *vector* is a value that has direction. All measurements made in other science courses have been *scalar* in that only values are needed. Consider 20 grams, 98.6°F and 1.00 grams/cc. These are three examples of scalar values.

Sometime direction is important. Suppose that two cars collide while one is moving at 60 mph and the other is moving at 59 mph. Is the crash critical? It depends on if the two cars meet head-on or if the faster car rear-ends the slower car. Identical numbers before the collision but two totally different results. Direction must be considered in many of the problems of physics.

A vector is a measurement that has a number and a direction. The number part of a vector is called "*magnitude*". You could say that it is the scalar part of the vector. The direction part is usually given as an angle. We will investigate angles more in tomorrow's lesson. For today, directions like forward, backwards and sideways will do.

<u>Billy Bob and Jimmy Joe</u>

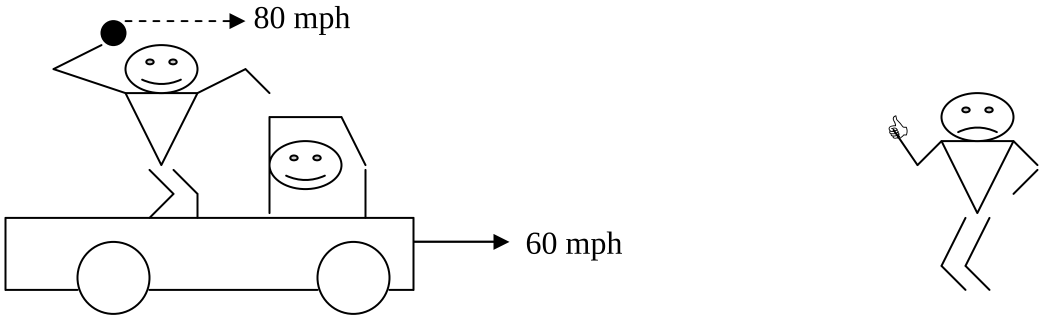
Billy Bob owns a nice pickup truck. Jimmy Joe has a bunch of rotten eggs that he needs to get rid of. Billy Bob drives his truck with Jimmy Joe in the back. They are going across town to the rival high school neighborhood for some "not so bright" action. On the way across town, Billy Bob stops at a store for a six pack of his favorite highly caffeinated beverage. It has been determined that Jimmy Joe can toss a raw egg at 80 mph on a day when the wind is calm.

<u>Scenario #1</u>

Billy Bob is driving down the street at 60 mph with Jimmy Joe in the back of the truck. Ahead is a hitchhiker (HH) looking for a ride? Jimmy Joe throws three eggs at the hitchhiker while driving along the street.

- The first throw is forward and parallel to the motion of the truck.
- The second throw is sideways perpendicular to the motion of the truck.
- The third toss is backwards, against the motion of the truck.

See the figure below:



Warning! Do not try this at home without adult supervision.

A) During the first throw that is forward, parallel to the direction of motion of the truck, the following are observed:
   1. Billy Bob and Jimmy Joe see the egg move forward at _____ mph.

   2. The hitchhiker sees the egg moving towards him at _____ mph.

B) During the second throw the egg is tossed sideways or perpendicular to the direction of motion of the truck. When the egg velocity is perpendicular to the truck velocity the following are observed.
   1. Billy Bob and Jimmy Joe see the egg move sideways at _____ mph

   2. The hitchhiker sees the egg moving at _____ mph at an angle of _____° from the forward direction.

C) During the third throw that is backwards, antiparallel to the direction of motion of the truck, the following are observed:
   1. Billy Bob and Jimmy Joe see the egg move backwards at _____ mph.

   2. The hitchhiker sees the egg moving towards him at _____ mph.

From the above scenarios you should have learned several things. Fill in the blanks below.

1. A vector that describes speed and direction is called ___________.
2. When two vectors are parallel they can be combined using __________.
3. When two vectors are antiparallel they can be combined using ________.
4. When two vectors are perpendicular they can be combined using ___________ to get the magnitude and using ___________ to find the direction.

<u>Scenario #2</u>

Due to the large consumption of caffeinated beverages, Billy Bob needs to find a restroom. He increases his truck's forward speed to 80 mph with Jimmy Joe still in the back of the pickup truck. A second hitchhiker is noticed between Billy Bob's truck and the nearest restroom. Jimmy Joe makes three new attempts to egg a hitchhiker.

D) During the first throw that is forward, parallel to the direction of motion of the truck, the following are observed:
   1. Billy Bob and Jimmy Joe see the egg move forward at _____ mph.

   2. The hitchhiker sees the egg moving towards him at _____ mph.

E) During the second throw the egg is tossed sideways or perpendicular to the direction of motion of the truck. When the egg velocity is perpendicular to the truck velocity the following are observed.
   1. Billy Bob and Jimmy Joe see the egg move sideways at _____ mph

   2. The hitchhiker sees the egg moving at _____ mph at an angle of _____° from the forward direction.

F) During the third throw that is backwards, antiparallel to the direction of motion of the truck, the following are observed:
   1. Billy Bob and Jimmy Joe see the egg move backwards at _____ mph.

   2. The hitchhiker sees the egg moving towards him at _____ mph.

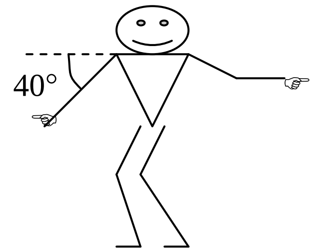
Bring to class tomorrow three pens, pencils, crayons, etc that write in different colors such as blue, red and green.

Lesson 1-03 **Vector Addition**

http://id.mind.net/~zona/mstm/physics/mechanics/vectors/componentAddition/componentAddition2.htm

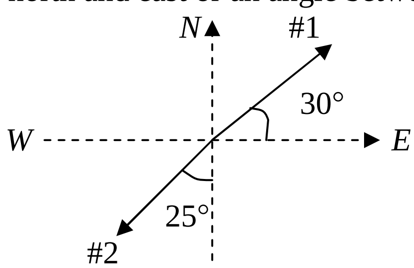
In the previous lesson you learned how to combine vectors that are parallel, antiparallel and perpendicular. But what if the vectors are not at nice angles? How do you combine vectors that are different by 70° or 135°? There are three different approaches to solving problems of this nature. In today's lesson you will learn how to combine coplanar vectors using the *method of components*.

Directions of Angles



Angles in this course are described in two different planes. Vertical angles are measured from a line that runs parallel to the ground. Consider the figure standing in the diagram to the right. The figure's left hand points in the horizontal direction. The right hand points at 40° below the horizontal. If something points at 40° below the horizontal then it also is pointing at 50° from the vertical direction.

For the hand that is pointing parallel to the ground we know that it is not pointing up or down but merely sideways. We still have not completely described the direction. Is the hand pointing north or south or at some angle between north and east or an angle between south and west? How do we describe those directions? Consider the two angles shown in the figure to the right. The angles are shown as you would see them if sitting in a tree looking down on the angles from above. The coordinate axes are all drawn parallel to the ground and pointing along cardinal points of the compass.



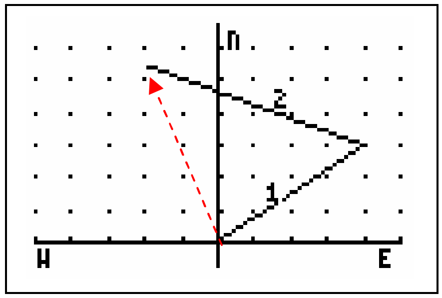
The first vector is shown pointing at an angle that is "30° from the east axis. But which way from the east axis is the vector drawn? Is the vector drawn 30° north of the east axis or is the angle drawn 30° south of the east axis? This angle is read as "30° North of East". The second direction (East in this case) is the reference axis that the angle is drawn to. The first direction is the way the angle is drawn off the reference axis. Note that "30° North of East" is the same direction as "60° East of North". You can use the complimentary angle and reverse the axis directions to report the same heading. The second angle shown in the figure is 25° West of the South axis or "25° W of S". Do you know where 42° N of W would be drawn? If not go find out before going to next section.

Head to Tail Method

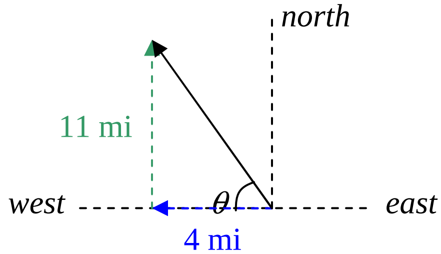
When adding to vectors the two can be drawn head-to-tail. The resultant sum of the two vectors is equivalent to the single arrow that goes from the tail of the first vector to the head of the second vector. Lengths of arrows are drawn to scale in order to represent the magnitude of each vector. A protractor is used to get the direction of each vector correct. The problem with this method is that there is a great deal of human error involved and the errors can be additive to create an answer that is only approximately close. Consider the example problem on next page.
http://hyperphysics.phy-astr.gsu.edu/hbase/vect.html

Example Problem

Billy Bob lost his driving privileges for a week because of a minor egg throwing incident. He had to walk to his friend's house from his own home. Billy Bob first walks 10 miles at 37° North of East. He then walks 13 miles at 22.6° North of West. How far is Billy Bob from home? You could draw these vectors with 1 cm = 1 mile and use a compass to get the paths in the correct directions. At the end of vector 1 you start to draw the 13 cm of length for vector 2. Again use a compass to get direction correct. Your result would be similar to the smaller diagram in the text box to the right. For the figure shown each dot is separated by a distance of 2 cm (representing 2 miles) from each other dot. You can draw a line from the origin to the end of vector 2. Using your ruler to measure the distance of this line will tell you how far Billy Bob is from home? You can also use a protractor to measure the angle.



The vector sum is known as the "resultant" and is shown as the red arrow in the above diagram. By inspection of the diagram we can get a rough idea of how far Billy is from home. Notice that the head of the red arrow is two dots to the left of the origin. Since each dot stands for 2 miles we see that Billy is 4 miles west of home. We also see that the end of path 2 is 5 and ½ dots above the origin. Again using the scale of two miles per dot this works out to 11 miles north of home. We can take these two pieces of information and combine them since they are perpendicular. What we seek is the length of the hypotenuse, shown in black. Using Pythagorean Theorem you will find that Billy is 11.7 miles from home. Using the tangent function you will notice that the arrow points at Tan θ = 11/4 or at an angle of θ = 70° North of West.



Drawing vectors in the head to tail method is messy, awkward and prone to much error unless you wish to waste a great deal of time. There is a better way. It is known as the method of components. In the method of components you do not have to draw things to scale. Just slap the numbers down. All vectors are drawn as rays from the origin. As long as you do not change the magnitude of the vectors and as long as you keep them pointing in the same direction you can draw them in different places without changing the problem. The steps are outlined below with two examples to follow.

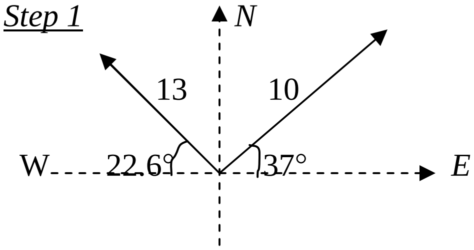
1. Draw and label each vector to be added as a ray pointing outward from the origin. The label should include magnitude and direction.
2. Find the x and the y part of each vector using adj = hyp∗cos∠ and using opp = hyp∗sin ∠.
3. Add all of the x parts to get total x and all of the y parts to get total y. Watch for negative signs here.
4. Use the total x and total y to draw <u>head to tail</u> vectors that will form the sides of a right triangle just as was done above.
5. Solve for the hypotenuse and angle of the right triangle. The hypotenuse is the magnitude of the resultant. The angle adjacent to the origin will give you the heading of the resultant

Two examples of vector addition are shown below. This is the most critical lesson of the entire unit. You should be able to add three coplanar vectors in 5 minutes or less.

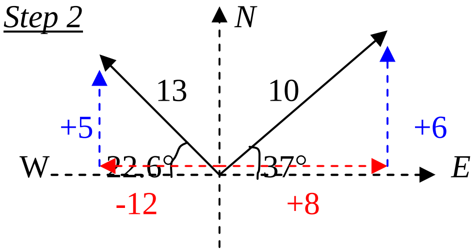
Example #1

Return to Billy Bob's walk and add the vectors using the method of components.

Step 1



Step 2



Vector #1: x part = +10cos(37°)
y part = +10sin(37°)

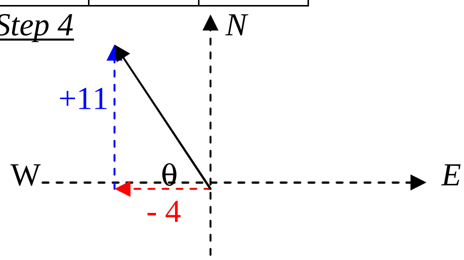
Vector #2: x part = –13cos(22.6°)
y part = +13sin(22.6°)

The positive and negative signs are inserted based on up and right as positive down and left as negative.

Step 3

vector	X part	Y part
#1	+8	+6
#2	-12	+5
total	-4	+11

Step 4



Step 5 (Substitute absolute values here!)

Magnitude?: $\sqrt{(11^2 + 4^2)}$ = 11.7 mi

Direction?: tan θ = 11/4 so θ = 70°NofW
Billy Bob is 11.7 miles at 70°NofW from home.

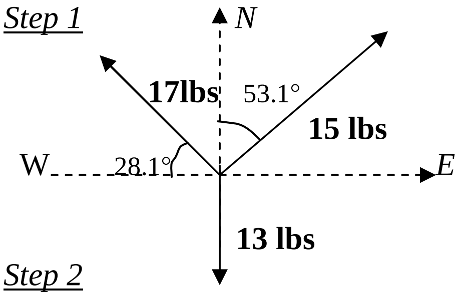
Example #2

Three students are fighting over the last scholarship to Enormous State Univ. Each pulls on the paper with a force as follows:

1) 15 lbs @ 53.1° East of North
2) 17 lbs at 28.1° North of West
3) 13 lbs due South

Which way does the scholarship move?

Step 1



Step 2

Vector #1: x part = +15sin(53.1°) = +12
y part = +15cos(53.1°) = +9

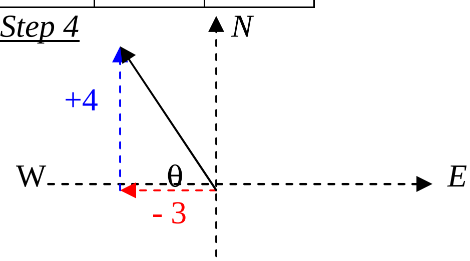
Vector #2: x part = –17cos(28.1°) = -15
y part = +17sin(28.1°) = +8

Vector #3: x part = 0
y part = –13 lbs

Step 3

vector	X part	Y part
#1	+12	+9
#2	- 15	+8
#3	0	-13
Total	-3	+4

Step 4



Step 5 (Substitute absolute values here!)

Magnitude?: $\sqrt{(3^2 + 4^2)}$ = 5 lbs

Direction?: tanθ = 4/3 so θ = 53.1°NofW
The scholarship moves in a direction of 53.1° North of West

Exercise Problems for Class and Home:

1. Jimmy Joe is lost in the swamp. He has walked the following distances from his cabin:
   a) 1442 yards @ 56.3° N of E
   b) 1887 yards @ 58.0° W of N
   c) 700 yards due south

   How far is Jimmy Joe from his cabin?
2. Three dogs at night are pulling on a bone. Each dog exerts the following forces on the bone.
   a) 26.0 pounds @ 67.4° N of E
   b) 34.5 pounds @ 60.5° E of S
   c) 16.0 pounds due west

   What is the net force acting on the bone and in which direction will it move? (Joy to the world!)
3. Crazy Carl is trapped while hang-gliding in a tropical storm just above the LSU campus. Before he can safely land he is blown along the following paths.
   a) 25.0 miles @ 53.1° N of E
   b) 60.4 miles @ 24.4° N of W
   c) 54.0 miles due south

   How far and in what direction does he land from the LSU Stadium?
4. The Three Stooges are trying to move a crate.
   a) Larry pushes with a force of 228 pounds @ 52.1° N of E.
   b) Curly pushes with a force of 260 pounds due west.
   c) Moe pushes with a force of 130 pounds at 67.4° E of S.

   What is the net force on the crate? Which way does the crate move?

<u>Answers?</u>

1. 1700 yards @ 62° N of W
2. 25 pounds @ 16.3° N of E
3. 41 miles @ 12.8° S of W
4. 130 pounds due north

There are more practice problems on the following page. Be sure that you can add three coplanar vectors in 5 minutes or less.

More practice on next page!

More Vector Addition Practice Problem

1. A bird flies from its nest in the old, oak tree taking the following paths:
   a) 21.6 meters @ 56.3° N of E
   b) 12.2 meters @ 55.0° N of W
   c) 16.0 meters due south
   Give a resultant vector to go straight
   from the nest to the bird.
2. On a leisurely flight a pilot is watching the ground rather than her instruments. Without paying attention she has flown along the following route:
   a) 51 miles @ 65° N of E
   b) 25 miles @ 53.1 ° E of S
   c) 28 miles due west
   What vector would carry the pilot directly <u>back</u> to her starting point?
3. In an effort to locate a great fishing spot in the Gulf a group leaves port along the following paths.
   a) 22 miles due south
   b) 9.4 miles @ 32.0° W of N
   c) 16.6 miles @ 32.7° N of E
   After the third leg of the journey they run out of gasoline. What vector would take a second boat directly from the same port to the stranded group?
4. Ninety-nine red balloons are released into the air and float along the following route:
   a) 8.1 miles @ 60.3° N of E
   b) 10.8 miles @ 33.7° N of W
   c) 8.0 miles due south
   Where are the balloons now?
5. While on an ocean cruise Sting places a message in a bottle. The bottle floats along the following path before washing ashore on a small island:
   a) 17.2 km @ 54.5° E of N
   b) 17 km @ 28.1° N of W
   c) 20 km @ 53.1° E of S
   How far was Sting from the island when he dropped the “message in a bottle”?

<u>Answers</u>

1. 13 meters @ 67.4° N of E

2. 34 miles @ 66.5° S of W

3. 10.3 miles @ 29° S of E

4. 7.1 miles @ 45° N of W

5. 16.2 km @ 21.8° S of W

Lesson 1-04 **Vector Addition II**

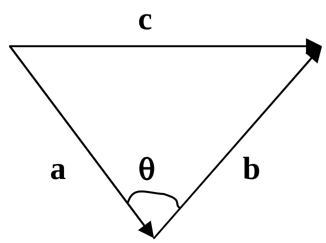
There is an alternative approach to adding vectors that starts out like the head to tail method but uses two rules for geometry that you most likely do not know. This technique is faster than the previous method if there are only two vectors involved. If there are three or more vectors the previous method is recommended.

Geometry Review

When drawing two vectors head-to-tail you are most probably not going to get a right triangle. The following two laws are valid for all triangles.

The Law of Cosines

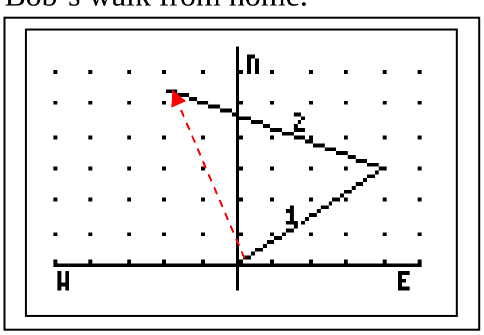
The Law of Cosines is really the general form of the Pythagorean Theorem. It describes the relation among all three sides of a triangle with one particular angle. The angle in the formula is the angle opposite to side c. See the figure below:



Suppose that you draw vectors **a** & **b** in a head-to-tail manner as shown above. The vector **c** is the resultant or sum of vectors **a** and **b**. Written another way **a** + **b** = **c**. How are the lengths of the vectors related? The Law of Cosines is one of the two ways to describe a relationship among the vectors.

$$c^2 = a^2 + b^2 - 2ab\cos(\theta)$$

This equation could have been used in Billy Bob's walk from home.



Vector 1 is 10 miles at 37° N of E. We assume that to be vector **a**. Vector 2 is 13 miles at 22.6° N of W. We assume that to be vector **b**. The angle between vectors **a** and **b** is the sum of the two angles above, 37° + 22.6° = 59.6° = θ. Now we can use the Law of Cosines to determine the length of side **c**.

$|c| = \sqrt{[10^2 + 13^2 - 2(10(13)\cos(59.6)]}$ or the length of side c is 11.72. This is in good agreement with previous two methods. The Law of Cosines gives to you the magnitude of the sum but you still need to know the direction. There is a second formula that will help.

The Law of Sines

This law relates the sin of any angle to its opposite side length. You can write the lengths of the sides as the numerator or as the denominator.

$$\frac{\text{Sin A}}{a} = \frac{\text{Sin B}}{b} = \frac{\text{Sin C}}{c}$$

We can now do a ratio using side c and side b.

Sin (B)/ 13 = Sin (59.6)/11.72 and find Sin (B) = 0.957 so angle B = 73.1°.

The angle between vector **a** and the east axis is 37°. The angle between vector **a** and vector **c** is 73.1°. The angle between vector c and the west axis is 180° – 37° – 73.1° = 69.9° N of West.

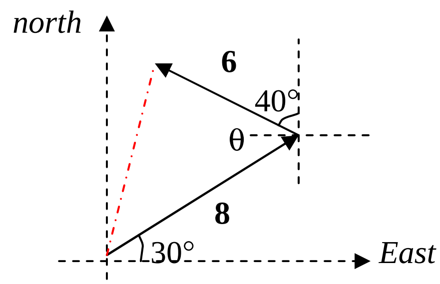
This method is better for those who like geometry and the previous method is better for those who like algebra.

Once you get the hang of it, this method has some very definite advantages. If you must stick with a single method then the method of components is more versatile. Learn it first. You lose all advantage of speed using this method if there are three or more vectors. If there are three vectors and you choose this method then you have to add the first two vectors to find resultant of (a&b). Then you add vector 3 to the resultant of (a&b). In other words you work the problem twice. In this course you will be expected to use this method to combine only two vectors at a time.

Final warning- The Law of Sines sometimes has 2 solutions, 1 solution or no solutions. In this course that will not be a problem but you will encounter the mystery when you get to this topic in your pre-calculus or advanced math course. Beware! ☠

Example #2

Walt walks 8 km at 30° N of E. He then walks 6 km at 40° W of N. How far and in what direction should Walt walk to go straight home?



First step is to draw the vectors in a head-to-tail manner. See the diagram to the right. The given angles are also included in the diagram.

Second Step is to determine the angle between the first two vectors. Between **8** and the vertical dashed line is an angle of 60° Between the **6** and the vertical dashed line is an angle of 40°. This leaves a total of 80° between the vector of **6** and vector of **8**.

Third Step is to use the law of cosines to determine how far Walt is from home.

$$c = \sqrt{[\,8^2 + 6^2 - 2(8)(6)\cos(80)]} = 9.13 \text{ miles from home.}$$

Fourth Step is to find the angle between vector **8** and **resultant** (red dashed line) using the Law of Sines. Sin (∠) / 6 = Sin (80) / 9.13 or Sin (∠) = 0.6473. The angle is 40.3°. So the resultant is 70.3° North of East. Walt needs to walk 9.13 miles in a direction of 70.3° South of West to go straight home. You are welcome to verify this using the method of components.

Adding Two Displacement Vectors

In each of the following situations something is displaced in two steps from its starting point. In each instance, add the vectors using the head-to-tail method to determine net displacement. A complete answer will have both magnitude and direction from origin. In case of an open response problem on the test you will be expected to (1) sketch these vectors head-to-tail in order to form two sides of a triangle, (2) use the Law of Cosines to get the total displacement magnitude and (3) use Law of Sines for final direction.

1. A dog sits under the only shade tree in its territory. Suddenly the dog desires to dig a previously buried bone. From the tree the dog goes 13 meters at 22.6° north of east. Not finding the bone there the dog then goes 17 meters at 61.9° north of west. Digging at the second location yields a bone. To go directly from the tree to the bone the dog should have gone what distance in what direction?
   Answer: 20.4m @ 78.7° N of E

2. A honey bee leaves the hive in search of a clover patch. The bee first goes 23.3 yards @ 59.0° south of east. Next the bee flies 14.6 yards at 74.1° north of west where nice clover is found. In order for the rest of the bees to go straight from the hive to the clover the bees should go how far and in what direction?
   Answer: 10.0 yds @ 37° S of E

3. A goose and a crow sit in a very tall tree. They decide to meet in another spot later in the day. The goose flies 60 km @ 30° north of west. The goose then goes 15.6 km @ 39.8° north of east. The crow being smarter decides to fly directly to the meeting point. How far away is the meeting point, as the crow flies? What direction?
   Answer: 56.5 km @ 45° NW

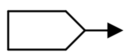
4. A plane leaves the airport at noon. The initial leg of the trip is 65 miles @ 67.4° south of west. The plane then turns to a heading of 61.9° north of west for a distance of 68 miles. In order to return to the initial airport the plane should go how far and in what direction?
   Answer: 57 miles due east

5. A hiker leaves camp for a morning stroll. Her first hike is 12 km due south. After snacking on a few wild berries she goes 10 km at 36.9° north of west. How far and in what direction should she go in order to hike straight back to camp?
   Answer: 10 km @ 36.9° N of E

We now turn our attention to several application situations that arise in physics. The first deals with boats traveling on rivers. The vector sum of the boat motor speed and the river current speed gives the resulting speed of the boat as seen by a person on shore. In the second application we look at airplanes in cross winds and so forth.

Boats & Rivers

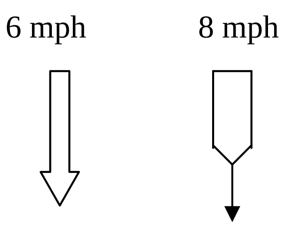
When a boat travels across the surface of a body of water there are three factors that determine the resultant velocity vector, 1) prevailing winds, 2) river currents and 3) motor inputs. In this lesson we will ignore wind influence and focus on the latter two. River currents always flow down stream as indicated with ⇓. The motor speed will always be along the direction that the boat is pointed as shown below:



We wish to focus upon four unique combinations of river current and motor input.

Boat Pointed Downstream

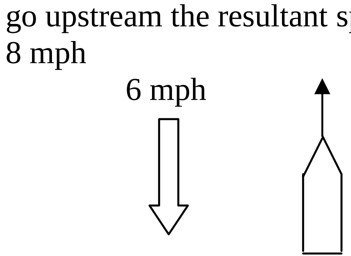
In the figure below the river current is 6 mph. The boat would travel in still water at a speed of 8mph only.



The parallel velocity vectors will give the boat a resultant speed of _____ mph.

Boat Pointed Upstream

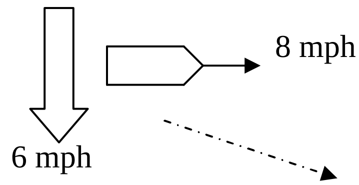
If the boat in the previous example wishes to go upstream the resultant speed is different.



The resultant speed due to anti-parallel vectors is _____ mph.

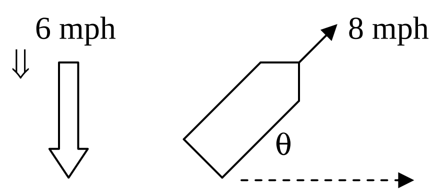
Boat Pointed Across Stream

In this particular orientation the motor is pushing perpendicular to the current flow.



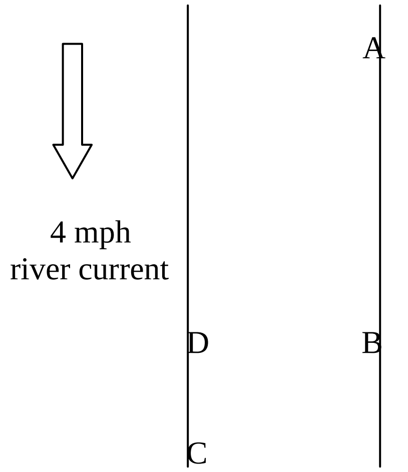
As a result of the perpendicular vectors the boat is pointed to the right while drifting along the dashed arrow path. To use distance = rate * time with this type of motion requires caution. Do not use a horizontal distance with the vertical speed or a total distance along the hypotenuse with a horizontal speed. It is advised to draw a velocity triangle. Then draw a congruent displacement triangle. The resultant speed of the above boat is ______mph. If the distance across a river is 0.75 miles then this boat would cross the river in a time of _______ minutes.

Boat Angled Upstream to Move Perpendicular to Current



Although the boat is pointed upstream at an angle, the vertical part of 8 mph will exactly cancel with the river current. As a result the boat moves sideways due to the horizontal part of the 8 mph. For the above situation the boat must be angled at θ =____ degrees. The resulting speed of the boat is ______ mph to the right.

Billy Bob is going fishing. There is a good fishing hole exactly 6 miles downstream from the boat ramp but on the other side of the river. The river current on this particular day flows at 4 mph. Billy's trolling motor can move the boat at 7 mph. Key points on the river are shown in the figure below:



Pt A is the boat ramp.
Pt B is 6 miles below boat ramp.
Pt D is a fishing hole exactly 1.0 mile across from Pt. B.
Pt C is on the same side of the river as point D but downstream.

A to B Trip

Billy gets in his boat and goes directly from point A to point B. Write in the blanks below Billy's speed and time to go from A to B.

V_{AB} = ______ mph t_{AB} = ______min

B to C Trip

At point B he points his boat straight across the current at the fishing spot. While putting bait on his line and not paying much attention to what is going on he does not look up until arriving at point C. What is Billy's velocity from B to C? How long does it take him to go from B to C? How far below point D is point C?

V_{BC} = _____ mph @ _______° below BD line

t_{BC} = _______ minutes

d_{CD} = ______ miles

C to D Trip

Realizing his mistake there is nothing to do now but turn the boat upstream and go from C to D. What is the resulting speed? How long does it take to go from C to D?

V_{CD} = _______ mph

t_{CD} = _______ minutes

After fishing for one hour and catching three catfish, two turtles and an old tire Billy is ready to go home.

D to B Trip

To correct for the mistake made crossing the river last time Billy must point is boat at what angle above the DB line? What is his resulting speed? How long will it take him to cross?

θ = ______° above DB line

V_{DB} = _______ mph

t_{DB} = _______ minutes

B to A Trip

After arriving at point B Billy turns the boat upstream and heads back to the ramp. What is his time to travel from B to A?

t_{BA} = ___________ minutes

What was the total time of Billy's fishing trip in hours and minutes?

4 hours and 3 min, do your values agree?
Yes or **No** (circle one)

http://hyperphysics.phy-astr.gsu.edu/hbase/HFrame.html (mechanics, velocity/acceleration, boat in current)

Lesson 1-06 **Vector Applications Part II**

Airplanes, Crosswinds, Tailwinds & Headwinds

A pilot has to fly her airplane 500 miles due north from Baton Rouge to St. Louis every day. On a calm day she can make the trip in 5 hours at maximum airspeed. (After her first couple of months on the job she will get a faster plane!) Assume that the plane always travels at maximum speed and analyze the plane's motion on the following days:

1. On a calm day the plane can travel at a maximum speed of ______ mph north.

2. On the second day on the job the wind is blowing at 50 mph from the south. The plane will have a net speed of

   ______mph north. The trip takes

   ______ hrs & ______ minutes.

3. On the third day on the job the wind now blows at 50 mph from the north. Repeat the calculations from part (2).

   Net velocity = _________mph north

   Time of flight = ______hrs ____min

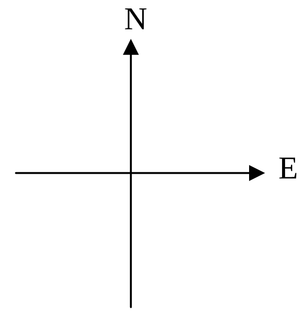
4. On the fourth day on the job the wind is blowing from the west at 50 mph. The pilot ignores the crosswind and aims the plane due north since it was not a problem on previous flights! The plane's resultant speed for this part of the flight is ____ mph. After 5 hrs of flight she is astonished to be somewhere besides St. Louis. The plane will be _____ miles east of St. Louis after 5 hours of flight. If she immediately banks the plane into a new heading of due west the total time of flight into St. Louis is ______ hrs.

5. To avoid the problem in the previous step the pilot should have directed the plane at a heading of _____° WofN.

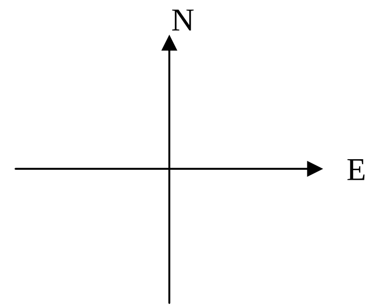
   As a result the plane would have a speed of _____ mph north.

   The time of flight would be _____ hrs.

6. On the next day's flight the wind blows at 50 mph at 53.1° N of E. At what heading should the plane be directed so that it goes due north? What will be the resultant speed of the plane? How long will the flight last?



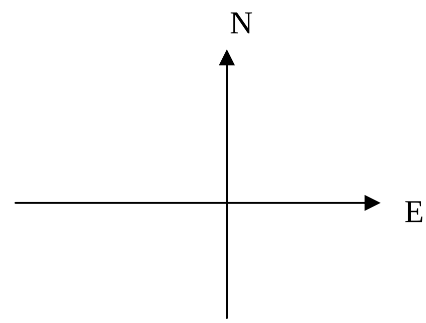
7. On the next day's flight the wind velocity is 50 mph at 37 ° South of East. What heading should the plane be directed into so that it flies due north? What will be the resulting speed of the plane? How long will the flight last?



8. Repeat step #6 if the wind velocity is 65 mph @ 67.4° North of East.

   ________° W of N   v = ________ mph

   __________ hrs and _____ minutes



Partial Answers

2. a) 150 mph   b) 3hrs & 20 minutes

3. a) 50 mph   b) 10 hrs even

4. a) 112 mph   b) 250 miles east   c) 10 hrs even

5. a) 30°WofN   b) 86.8 mph   c) 5 hrs & 46 minutes

6. a) $\sin\theta = (30/100)$ b) v = 135.4 mph   c) 3 hrs and 42 minutes

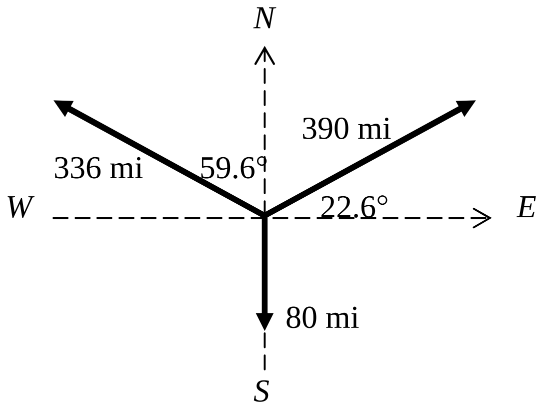
7. a) 23.6°   b) 61.7 mph   c) 8 hrs & 7 minutes

8. c) 3 hrs and 11 minutes

This concludes the first unit of physics. Test within 48 hours or as soon as possible after that. Tomorrow expect small group work on a practice test. Bring your calculators and be prepared to work for the entire period.
http://hyperphysics.phy-astr.gsu.edu/hbase/HFrame.html (mechanics, velocity/acceleration, airplane and wind)

Vector Addition <u>Practice Test</u> 1A

Add the following vectors to get a sum.
Vector #1: 390 mi @ 22.6° N of East
Vector #2: 336 mi @ 59.6° W of North
Vector #3: 80 mi due South

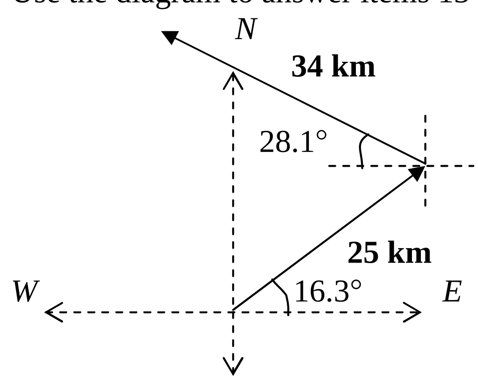


1. North-south part of vector #2 is ___mi.
   A) 150 B) 170 C) 290 D) 360
2. The east-west part of vector #1 is ___mi.
   A) 150 B) 170 C) 290 D) 360
3. The total north-south displacement for all three vectors is ___ mi.
   A) 240 B) 290 C) 360 D) 400
4. The total east-west displacement for all three vectors is ___ mi.
   A) 70 B) 290 C) 360 D) 650
5. The total displacement from beginning of the first vector to the end of the last is ___ miles.
   A) 70 B) 240 C) 250 D) 310
6. The heading from the beginning of the first vector to the end of the third vector is ___° N of East.
   A) 16.2 B) 28.1 C) 53.1 D) 73.7

The motor of a boat has only a single speed of 20 mph in still water. On this particular day the boat is traveling on a river that has a current of 15 mph.

7. To travel three miles <u>downstream</u> the boat would take ___ minutes.
   A) 5.1 B) 9.0 C) 12.0 D) 36.0
8. To travel two miles <u>upstream</u> the boat would take ___ minutes.
   A) 3.4 B) 6.0 C) 8.0 D) 24.0
9. The boat is now pointed directly ⊥ to the current. To travel 1.25 miles across the river takes ___ minutes.
   A) 3.0 B) 3.75 C) 5.0 D) 15.0
10. Upon reaching the opposite side in item #9 the boat has drifted ___ miles downstream.
    A) 0.75 B) 0.94 C) 1.25 D) 3.75
11. To make the boat actually move ⊥ to the river current the boat should be pointed ___° upstream from the intended direction of travel.
    A) 22.6 B) 36.9 C) 48.6 D) 53.1
12. The resulting time to cross the 1.25 mile wide river is ___ minutes.
    A) 3.7 B) 5.7 C) 7.7 D) 9.7

A bird leaves home and flies 25 km at 16.3° N of E. The bird then turns and flies 34 km @ 28.1° n of W. The parts of the bird's trip are shown as solid arrows in the figure below. Use the diagram to answer items 13 – 16.



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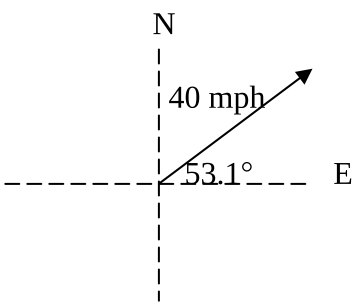
13. The acute angle between the two vectors of the bird's flight is ___°.
    A) 44.4 B) 78.2 C) 102 D) 136
14. The net displacement for the bird has a magnitude of ___ km.
    A) 23.8 B) 37.9 C) 46.2 D) 54.8
15. The bird is at a heading of ___° N of W from its original starting point.
    A) 40 B) 43.3 C) 56.7 D) 75.5
16. The bird flies a third flight and ends up exactly due west of home. The third flight was ___ km due south.
    A) 16 B) 23 C) 30 D) 42

An airplane can travel at a speed of 150 mph on a calm day. On this particular day the wind is blowing at 40 mph at 53.1° North of East. The pilot needs to fly due east for a distance of 240 miles.

Wind Velocity



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17. In order to fly due east the pilot must cancel a crosswind of ___ mph.
    A) 16 B) 24 C) 32 D) 40
18. To accomplish the goal in item #17 the plane should be placed on a heading of ___° South of East.
    A) 8 B) 10 C) 12 D) 14
19. The resulting speed of the plane would be ___ mph.
    A) 122.5 B) 146.5 C) 170.5 D) 174
20. The time required to travel 240 miles due east is ___ hours.
    A) 1.1 B) 1.4 C) 1.6 D) 2.0

Answers

1. B
2. D
3. A
4. A
5. C
6. D
7. A
8. D
9. B
10. B
11. C
12. B
13. A
14. A
15. D
16. B
17. C
18. C
19. C
20. B