Mathematics

Victorian Certificate of Education Study Design

Victorian Curriculum and Assessment Authority
2005

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Latoya BARTON
The sunset (detail)
from a series of twenty-four
9.0 x 9.0 cm each, oil on board

Tarkan ERTURK
Visage (detail)
201.0 x 170.0 cm
synthetic polymer paint, on cotton duck

Liana RASCHILLA
Teapot from the *Crazy Alice* set
19.0 x 22.0 x 22.0 cm
earthenware, clear glaze. lustres

Nigel BROWN
Untitled physics (detail)
90.0 x 440.0 x 70.0 cm
composition board, steel, loudspeakers, CD player, amplifier, glass

Kate WOOLLEY
Sarah (detail)
76.0 x 101.5 cm, oil on canvas

Chris ELLIS
Tranquility (detail)
35.0 x 22.5 cm
gelatin silver photograph

Christian HART
Within without (detail)
digital film, 6 minutes

Kristian LUCAS
Me, myself, I and you (detail)
56.0 x 102.0 cm
oil on canvas

Merryn ALLEN
Japanese illusions (detail)
centre back: 74.0 cm, waist (flat): 42.0 cm
polyester cotton

Ping (Irene VINCENT)
Boxes (detail)
colour photograph

James ATKINS
Light cascades (detail)
three works, 32.0 x 32.0 x 5.0 cm each
glass, flourescent light, metal

Tim JOINER
14 seconds (detail)
digital film, 1.30 minutes

Lucy McNAMARA
Precariously (detail)
156.0 x 61.0 x 61.0 cm
painted wood, oil paint, egg shells, glue, stainless steel wire

Accredited by the Victorian Qualifications Authority
33 St Andrews Place, East Melbourne, Victoria 3002

Developed and published by the Victorian Curriculum and Assessment Authority
41 St Andrews Place, East Melbourne, Victoria 3002

This completely revised and reaccredited edition published 2005.

Edited by Ruth Learner
Cover designed by Chris Waldron of BrandHouse
Desktop published by Julie Coleman

Mathematics

ISBN 1 74010 299 1

Contents

IMPORTANT INFORMATION

Accreditation period

Units 1–4: 2006–2011

The accreditation period commences on 1 January 2006.

Other sources of information

The *VCAA Bulletin* is the only official source of changes to regulations and accredited studies. The *VCAA Bulletin*, including supplements, also regularly includes advice on VCE studies. It is the responsibility of each VCE teacher to refer to each issue of the *VCAA Bulletin*. The *VCAA Bulletin* is sent in hard copy to all VCE providers. It is available on the Victorian Curriculum and Assessment Authority's website at www.vcaa.vic.edu.au

To assist teachers in assessing school-assessed coursework in Units 3 and 4, the Victorian Curriculum and Assessment Authority publishes an assessment handbook that includes advice on the assessment tasks and performance descriptors for assessment.

The current year's *VCE and VCAL Administrative Handbook* contains essential information on assessment and other procedures.

VCE providers

Throughout this study design the term 'school' is intended to include both schools and other VCE providers.

Photocopying

VCE schools only may photocopy parts of this study design for use by teachers.

Introduction

RATIONALE

Mathematics is the study of function and pattern in number, logic, space and structure. It provides both a framework for thinking and a means of symbolic communication that is powerful, logical, concise and precise. It also provides a means by which people can understand and manage their environment. Essential mathematical activities include calculating and computing, abstracting, conjecturing, proving, applying, investigating, modelling, and problem posing and solving.

This study is designed to provide access to worthwhile and challenging mathematical learning in a way which takes into account the needs and aspirations of a wide range of students. It is also designed to promote students' awareness of the importance of mathematics in everyday life in a technological society, and confidence in making effective use of mathematical ideas, techniques and processes.

AIMS

It is an underlying principle of the Mathematics study that all students will engage in the following mathematical activities:

1. **Apply knowledge and skills:** The study of aspects of the existing body of mathematical knowledge through learning and practising mathematical algorithms, routines and techniques, and using them to find solutions to standard problems.
2. **Model, investigate and solve problems:** The application of mathematical knowledge and skills in unfamiliar situations, including situations which require investigative, modelling or problem-solving approaches.
3. **Use technology:** The effective and appropriate use of technology to produce results which support learning mathematics and its application in different contexts.

These three types of mathematical activity underpin the outcomes for each unit of Mathematics. They are intended to both guide the work of teachers and students throughout Mathematics and to promote and develop key aspects of working mathematically.

This study is designed to enable students to:

- develop mathematical knowledge and skills;
- apply mathematical knowledge to analyse, investigate, model and solve problems in a variety of situations, ranging from well-defined and familiar situations to unfamiliar and open-ended situations;
- use technology as an effective support for mathematical activity.

STRUCTURE

The study is made up of the following units:

Foundation Mathematics Units 1 and 2

General Mathematics Units 1 and 2

Mathematical Methods Units 1 and 2

Mathematical Methods (CAS*) Units 1 and 2

Further Mathematics Units 3 and 4

Mathematical Methods Units 3 and 4

Mathematical Methods (CAS) Units 3 and 4

Specialist Mathematics Units 3 and 4

Each unit deals with specific content and is designed to enable students to achieve a set of outcomes. Each outcome is described in terms of the key knowledge and skills students are required to demonstrate.

Foundation Mathematics Units 1 and 2 provide continuing mathematical development of students entering VCE who need mathematical skills to support their other VCE subjects, including VET studies, and who do not intend to undertake Unit 3 and 4 studies in VCE Mathematics in the following year. Foundation Mathematics Units 1 and 2 do not provide a basis for undertaking Unit 3 and 4 studies in Mathematics.

General Mathematics Units 1 and 2 provide courses of study for a broad range of students and may be implemented in a number of ways. Students intending to study Specialist Mathematics Units 3 and 4 should be provided with access to a rigorous implementation of General Mathematics Units 1 and 2, which emphasises mathematical structure and the justification of results through general case arguments.

Mathematical Methods Units 1 and 2 have a closely sequenced development of material, intended as preparation for Mathematical Methods Units 3 and 4. **Mathematical Methods Units 3 and 4** may be taken alone or in conjunction with either Specialist Mathematics Units 3 and 4 or Further Mathematics Units 3 and 4, and provide an appropriate background for further study in, for example, science, humanities, economics or medicine.

Mathematical Methods (CAS) Units 1 and 2 have a closely sequenced development of material, intended as preparation for Mathematical Methods (CAS) Units 3 and 4. They also provide a suitable preparation for Mathematical Methods Units 3 and 4. **Mathematical Methods (CAS) Units 3 and 4** may be taken alone or in conjunction with either Specialist Mathematics Units 3 and 4 or Further Mathematics Units 3 and 4, and provide an appropriate background for further study in, for example, science, humanities, economics or medicine.

*Computer Algebra System

Further Mathematics Units 3 and 4 are intended to be widely accessible. They provide general preparation for employment or further study, in particular, where data analysis is important. The assumed knowledge and skills for Further Mathematics Units 3 and 4 are drawn from General Mathematics Units 1 and 2. Students who have done only Mathematical Methods Units 1 and 2 or only Mathematical Methods (CAS) Units 1 and 2 will also have had access to assumed knowledge and skills to undertake Further Mathematics.

Specialist Mathematics Units 3 and 4 are normally taken in conjunction with Mathematical Methods Units 3 and 4 or Mathematical Methods (CAS) Units 3 and 4, and the areas of study extend and develop material from Mathematical Methods Units 3 and 4 and Mathematical Methods (CAS) Units 3 and 4. Specialist Mathematics Units 3 and 4 are intended for those with strong interests in mathematics and those who wish to undertake further study in mathematics and related disciplines.

The structure of VCE Mathematics is summarised below.

Units 1 and 2

Foundation Mathematics
Unit 1 Unit 2

Foundation Mathematics Units 1 and 2 are an alternative to General Mathematics Units 1 and 2, Mathematical Methods Units 1 and 2 and Mathematical Methods (CAS) Units 1 and 2. Students who take Foundation Mathematics Units 1 and 2 would not be taking Mathematical Methods Units 1 and 2 or Mathematical Methods (CAS) Units 1 and 2 in the same year, and would not proceed to study a Units 3 and 4 Mathematics in the following year. Some students may choose to take Foundation Mathematics Units 1 and 2 in conjunction with General Mathematics Units 1 and 2.

General Mathematics
Unit 1 Unit 2

General Mathematics Units 1 and 2 may be taken alone or in conjunction with Mathematical Methods Units 1 and 2 or Mathematical Methods (CAS) Units 1 and 2. They contain assumed knowledge and skills for related material in Further Mathematics Units 3 and 4. They are strongly recommended, in addition to Mathematical Methods Units 1 and 2 or Mathematical Methods (CAS) Units 1 and 2, as preparation for Specialist Mathematics Units 3 and 4.

Mathematical Methods
Unit 1 Unit 2

Mathematical Methods Units 1 and 2 may be taken alone or in conjunction with General Mathematics Units 1 and 2. They contain assumed knowledge and skills for Mathematical Methods Units 3 and 4. Students may complete Mathematical Methods Unit 1 followed by General Mathematics Unit 2. Completing General Mathematics Unit 1 followed by Mathematical Methods Unit 2 is not generally advised without additional preparatory work. Similarly, completing Mathematical Methods Unit 1 followed by Mathematical Methods (CAS) Unit 2 is not generally advised without additional preparatory work.

Mathematical Methods (CAS)
Unit 1 Unit 2

Mathematical Methods (CAS) Units 1 and 2 may be taken alone or in conjunction with General Mathematics Units 1 and 2. They contain assumed knowledge and skills for Mathematical Methods (CAS) Units 3 and 4 and also for Mathematical Methods Units 3 and 4. Students may complete Mathematical Methods (CAS) Unit 1 followed by General Mathematics Unit 2. Completing General Mathematics Unit 1 followed by Mathematical Methods (CAS) Unit 2 is not generally advised without additional preparatory work.

Units 3 and 4

Further Mathematics
Unit 3 Unit 4

Further Mathematics Units 3 and 4 may be taken alone or in conjunction with Mathematical Methods Units 3 and 4 or Mathematical Methods (CAS) Units 3 and 4. Unit 3 has a prescribed core (Data analysis) and one selected module. Unit 4 has two selected modules.

Mathematical Methods
Unit 3 Unit 4

Mathematical Methods (CAS)
Unit 3 Unit 4

Mathematical Methods Units 3 and 4 **or** Mathematical Methods (CAS) Units 3 and 4 may be taken alone or in conjunction with either Further Mathematics Units 3 and 4 or Specialist Mathematics Units 3 and 4. Students can **not** undertake study of both Mathematical Methods Units 3 and 4 and Mathematical Methods (CAS) Units 3 and 4.

Specialist Mathematics
Unit 3 Unit 4

Specialist Mathematics Units 3 and 4 are normally taken in conjunction with Mathematical Methods Units 3 and 4 or Mathematical Methods (CAS) Units 3 and 4. Mathematical Methods Units 3 and 4 and Mathematical Methods (CAS) Units 3 and 4 contain assumed knowledge and skills for Specialist Mathematics Units 3 and 4.

Some possible combinations of Mathematics units

The following table gives some possible combinations of units for students who continue with Mathematics at the Units 3 and 4 level.

Units 1 and 2	Units 3 and 4
Foundation Mathematics 1 and General Mathematics 2	Further Mathematics 3 and 4*
General Mathematics 1 and 2	Further Mathematics 3 and 4
Mathematical Methods 1 and 2 or Mathematical Methods Methods (CAS) 1 and 2	Mathematical Methods 3 and 4
Mathematical Methods (CAS) 1 and 2	Mathematical Methods (CAS) 3 and 4
Mathematical Methods 1 and 2	Mathematical Methods (CAS) 3 and 4*
General Mathematics 1 and 2 Mathematical Methods 1 and 2	Mathematical Methods 3 and 4, alone or with Specialist Mathematics 3 and 4
General Mathematics 1 and 2 Mathematical Methods (CAS) 1 and 2	Mathematical Methods (CAS) 3 and 4, alone or with Specialist Mathematics 3 and 4
General Mathematics 1 Mathematical Methods 2 or Mathematical Methods (CAS) 2	Mathematical Methods 3 and 4* or Mathematical Methods (CAS) 3 and 4*
General Mathematics 1 or 2 Mathematical Methods 1 and 2	Mathematical Methods 3 and 4, alone or with Specialist Mathematics 3 and 4*
General Mathematics 1 or 2 Mathematical Methods (CAS) 1 and 2	Mathematical Methods (CAS) 3 and 4, alone or with Specialist Mathematics 3 and 4*
Mathematical Methods 1 or Mathematical Methods (CAS) 1 General Mathematics 2	Further Mathematics 3 and 4
General Mathematics 1 and 2 Mathematical Methods 1 and 2	Further Mathematics 3 and 4 Mathematical Methods 3 and 4
General Mathematics 1 and 2 Mathematical Methods (CAS) 1 and 2	Further Mathematics 3 and 4 Mathematical Methods (CAS) 3 and 4
Mathematical Methods 1 and 2 or Mathematical Methods (CAS) 1 and 2	Further Mathematics 3 and 4
Mathematical Methods (CAS) 1 and 2	Further Mathematics 3 and 4 Mathematical Methods (CAS) 3 and 4
Mathematical Methods 1 and 2 or Mathematical Methods (CAS) 1 and 2	Further Mathematics 3 and 4 Mathematical Methods 3 and 4

* for these combinations of units, students should discuss with their school what additional study is advisable with respect to assumed knowledge and skills for the respective mathematics at Units 3 and 4 level.

In particular, students intending to study both Mathematical Methods Units 3 and 4 and Specialist Mathematics Units 3 and 4, or Mathematical Methods (CAS) Units 3 and 4 and Specialist Mathematics Units 3 and 4 should, in all but the most exceptional cases, prepare by studying both Mathematical Methods Units 1 and 2 and General Mathematics Units 1 and 2, or by studying both Mathematical Methods (CAS) Units 1 and 2 and General Mathematics Units 1 and 2. Although it is possible to prepare for Mathematical Methods Units 3 and 4 by studying only Mathematical Methods Units 1 and 2 and for Mathematical Methods (CAS) Units 3 and 4 by studying only Mathematical Methods (CAS) Units 1 and 2, a much firmer basis is obtained by also studying General Mathematics Units 1 and 2.

ENTRY

There are no prerequisites for entry to Foundation Mathematics Units 1 and 2, General Mathematics Units 1 and 2, Mathematical Methods Units 1 and 2 or Mathematical Methods (CAS) Units 1 and 2. However, students attempting Mathematical Methods or Mathematical Methods (CAS) are expected to have a sound background in number, algebra, function, and probability. Some additional preparatory work will be advisable for any student who is undertaking Mathematical Methods Unit 2 without completing Mathematical Methods Unit 1 or who is undertaking Mathematical Methods (CAS) Unit 2 without completing Mathematical Methods (CAS) Unit 1.

Units 3 and 4 of a study are designed to be taken as a sequence. Students must undertake Unit 3 of a study before entering Unit 4 of that study.

Enrolment in Specialist Mathematics Units 3 and 4 assumes a current enrolment in, or previous completion of, Mathematical Methods Units 3 and 4 or Mathematical Methods (CAS) Units 3 and 4.

Students may obtain credit towards satisfactory completion of the VCE for up to eight units of Mathematics.

Students may **not** obtain credit for more than four units of Foundation Mathematics Units 1 and 2, General Mathematics Units 1 and 2 and either Mathematical Methods Units 1 and 2 or Mathematical Methods (CAS) Units 1 and 2.

Students may obtain credit for either Mathematical Methods Units 1 and 2 or Mathematical Methods (CAS) Units 1 and 2, but **not** for both of these sequences.

Students may **not** obtain credit for both Further Mathematics Units 3 and 4 and Specialist Mathematics Units 3 and 4. In a given year, a student may **not** enrol in both Further Mathematics Units 3 and 4 and Specialist Mathematics Units 3 and 4.

Students may obtain credit for either Mathematical Methods Units 3 and 4 or Mathematical Methods (CAS) Units 3 and 4, but **not** for both of these sequences.

Students must undertake Unit 3 prior to undertaking Unit 4. Units 1 to 4 are designed to a standard equivalent to the final two years of secondary education. All VCE studies are benchmarked against comparable national and international curriculum.

DURATION

Each unit involves at least 50 hours of scheduled classroom instruction.

CHANGES TO THE STUDY DESIGN

During its period of accreditation minor changes to the study will be notified in the *VCAA Bulletin*. The *VCAA Bulletin* is the only source of changes to regulations and accredited studies and it is the responsibility of each VCE teacher to monitor changes or advice about VCE studies published in the *VCAA Bulletin*.

MONITORING FOR QUALITY

As part of ongoing monitoring and quality assurance, the Victorian Curriculum and Assessment Authority will periodically undertake an audit of Mathematics to ensure the study is being taught and assessed as accredited. The details of the audit procedures and requirements are published annually in the *VCE and VCAL Administrative Handbook*. Schools will be notified during the teaching year of schools and studies to be audited and the required material for submission.

SAFETY

This study may involve the handling of potentially hazardous substances and/or the use of potentially hazardous equipment. It is the responsibility of the school to ensure that duty of care is exercised in relation to the health and safety of all students undertaking the study.

USE OF INFORMATION AND COMMUNICATIONS TECHNOLOGY

In designing courses for this study teachers should incorporate information and communications technology where appropriate and applicable to the teaching and learning activities. The Advice for Teachers section provides specific examples of how information and communications technology can be used in this study.

Mathematics and technology

Developments in mathematics and technology have been linked throughout history. In some cases developments in mathematics have led to the emergence of new technologies, while in other cases the use of new technology has stimulated developments in mathematics and its theoretical and practical applications.

Computation, proof, modelling and problem solving are key mathematical activities, both with respect to investigation of the structure and theory of mathematics itself, and also in application to practical contexts. Over time, various technologies have been developed and used to support and further these investigations and applications, including drawing and construction tools and devices in geometry; abacuses, counting boards, pen and paper algorithms, diagrams and schema and slide rules for arithmetic and algebra; and mechanical calculators and devices for arithmetic, algebra, probability and calculus. Until the 20th century, each of these technologies required human involvement at input, operation, output and interpretation stages of the process. From the late 1930s, electro-mechanical and electronic calculators and computers which automatically carry out computations were constructed. From the 1950s, these were further developed into the modern digital calculators and computers with which we are familiar today. These can carry out various numerical, graphical and symbolic computations and manipulations, depending on the range and combination of functionalities, programs and mathematically able software they support. Such technologies have now been used in academia, industry, research, business and commerce for several decades as tools for investigation, problem solving, modelling and communication. Increasingly, from the latter part of the 1970s, they have been used in education to support teaching and learning of the mathematics curriculum.

The use of technology in the senior mathematics curriculum, and in secondary schooling mathematics assessments, in particular examinations, has evolved over the last several decades as different technologies have become more widely available, affordable and integrated into mainstream teaching and learning practice. From the late 1970s, scientific calculators have played a supporting role in mathematics courses in the senior secondary school. In more recent years, mathematically able software such as function graphers, spreadsheets, statistics analysis systems, dynamic geometry systems and computer algebra systems (CAS), on hand-held and computer platforms, have created opportunities for exploration and analysis not previously accessible to teachers and students.

The appropriate use of technology to support and develop the teaching and learning of mathematics is to be incorporated throughout each VCE mathematics unit and course, and in related assessments. This will include the use of some of the following technologies for various areas of study or topics: graphics and CAS calculators, spreadsheets, graphing and numerical analysis packages, dynamic geometry systems, statistical analysis systems, and computer algebra systems.

Teachers and students should use these technologies in the teaching and learning of new material, skills practice, standard applications and investigative work. In conjunction with this, the development of sound mental skills and by hand skills (that is, using a written algorithm or procedure) is essential to ensure that students have access to a range of methods, understand the mathematical processes implemented by technology and can make sensible choices about possible approaches and tools to use in a given situation. Students also need to develop a strong sense of the reasonableness of results, whether obtained with or without the assistance of technology, and be able to recognise equivalent forms of representation of mathematical expressions. These are complementary aspects of working mathematically, and a sound understanding of related concepts, skills and processes underpins their efficient and effective application in different theoretical and practical contexts.

Students are expected to be able to apply concepts, skills and processes, involving computation, construction, data analysis, symbolic manipulation, solving equations, graph sketching, drawing on the content from the areas of study, and the key knowledge and skills of the outcomes for each unit and course as applicable. This work should take place with a clear focus on key aspects of mathematical reasoning – formulation, solution, interpretation and communication.

The Victorian Curriculum and Assessment Authority will specify approved technology for Units 3 and 4 of the VCE Mathematics study on an annual basis.

KEY COMPETENCIES AND EMPLOYABILITY SKILLS

This study offers a number of opportunities for students to develop key competencies and employability skills. The Advice for Teachers section provides specific examples of how students can demonstrate key competencies during learning activities and assessment tasks.

LEGISLATIVE COMPLIANCE

When collecting and using information, the provisions of privacy and copyright legislation, such as the Victorian *Information Privacy Act 2000* and *Health Records Act 2001*, and the federal *Privacy Act 1988* and *Copyright Act 1968* must be met.

Assessment and reporting

SATISFACTORY COMPLETION

The award of satisfactory completion for a unit is based on a decision that the student has demonstrated achievement of the set of outcomes specified for the unit. This decision will be based on the teacher's assessment of the student's performance on assessment tasks designated for the unit. Designated assessment tasks are provided in the details for each unit. The Victorian Curriculum and Assessment Authority publishes an assessment handbook that includes advice on the assessment tasks and performance descriptors for assessment for Units 3 and 4.

Teachers must develop courses that provide opportunities for students to demonstrate achievement of outcomes. Examples of learning activities are provided in the Advice for Teachers section.

Schools will report a result for each unit to the Victorian Curriculum and Assessment Authority as S (Satisfactory) or N (Not Satisfactory).

Completion of a unit will be reported on the Statement of Results issued by the Victorian Curriculum and Assessment Authority as S (Satisfactory) or N (Not Satisfactory). Schools may report additional information on levels of achievement.

AUTHENTICATION

Work related to the outcomes will be accepted only if the teacher can attest that, to the best of their knowledge, all unacknowledged work is the student's own. Teachers need to refer to the current year's *VCE and VCAL Administrative Handbook* for authentication procedures.

LEVELS OF ACHIEVEMENT

Units 1 and 2

Procedures for the assessment of levels of achievement in Units 1 and 2 are a matter for school decision. Assessment of levels of achievement for these units will not be reported to the Victorian Curriculum and Assessment Authority. Schools may choose to report levels of achievement using grades, descriptive statements or other indicators.

Units 3 and 4

The Victorian Curriculum and Assessment Authority will supervise the assessment of all students undertaking Units 3 and 4.

In the study of Mathematics the student's level of achievement will be determined by school-assessed coursework and two end-of-year examinations. The Victorian Curriculum and Assessment Authority will report the student's level of performance on each assessment component as a grade from A+ to E or UG (ungraded). To receive a study score, students must achieve two or more graded assessments and receive S for both Units 3 and 4. The study score is reported on a scale of 0–50. It is a measure of how well the student performed in relation to all others who took the study. Teachers should refer to the current year's *VCE and VCAL Administrative Handbook* for details on graded assessment and calculation of the study score. Percentage contributions to the study score in Mathematics are as follows:

Further Mathematics

- Unit 3 school-assessed coursework: 20 per cent
- Unit 4 school-assessed coursework: 14 per cent
- Units 3 and 4 examination 1: 33 per cent
- Units 3 and 4 examination 2: 33 per cent

Mathematical Methods

- Unit 3 school-assessed coursework: 20 per cent
- Unit 4 school-assessed coursework: 14 per cent
- Units 3 and 4 examination 1: 22 per cent
- Units 3 and 4 examination 2: 44 per cent

Mathematical Methods (CAS)

- Unit 3 school-assessed coursework: 20 per cent
- Unit 4 school-assessed coursework: 14 per cent
- Units 3 and 4 examination 1: 22 per cent
- Units 3 and 4 examination 2: 44 per cent

Specialist Mathematics

- Unit 3 school-assessed coursework: 14 per cent
- Unit 4 school-assessed coursework: 20 per cent
- Units 3 and 4 examination 1: 22 per cent
- Units 3 and 4 examination 2: 44 per cent

Examination 1 for Mathematical Methods Units 3 and 4 and Mathematical Methods (CAS) Units 3 and 4 is a common technology free examination. Details of the assessment program are described in the sections on Units 3 and 4 in this study design.

Units 1 and 2:
Foundation Mathematics

Units 1 and 2: Foundation Mathematics

Foundation Mathematics provides for the continuing mathematical development of students entering VCE, who need mathematical skills to support their other VCE subjects, including VET studies, and who do not intend to undertake Unit 3 and 4 studies in VCE Mathematics in the following year. Provision of this course is intended to complement General Mathematics and Mathematical Methods. It is specifically designed for those students who are not provided for in these two courses. Students completing this course would need to undertake further mathematical study in order to attempt Further Mathematics Units 3 and 4.

In Foundation Mathematics there is a strong emphasis on using mathematics in practical contexts relating to everyday life, recreation, work and study. Students are encouraged to use appropriate technology in all areas of their study. These units will be especially useful for students undertaking VET studies.

The areas of study for Units 1 and 2 of Foundation Mathematics are 'Space, shape and design', 'Patterns and number', 'Handling data' and 'Measurement'.

At the end of Unit 1, students will be expected to have covered material equivalent to two areas of study. All areas of study will be completed over the two units. Unit 2 can be used to complement Unit 1 in development of the course material. Some courses may be based on the completion of an area of study in its entirety before proceeding to other areas of study. Other courses may consist of an ongoing treatment of all areas of study throughout Units 1 and 2. It is likely that a contextual approach will lead to the development of implementations that draw on material from all areas of study in each semester.

In developing courses based on the following areas of study, teachers should give particular attention to the opportunity for embedding content in contexts based on students' VCE (VET) and VCAL studies, work (part-time or work experience), personal or other familiar situations.

AREAS OF STUDY

1. Space, shape and design

This area of study covers the geometric properties of lines and curves, shapes and solids and their graphical and diagrammatic representations. Consideration of scale, and labelling and drawing conventions enables students to interpret domestic, industrial and commercial plans and diagrams.

This area of study will include:

- properties of two-dimensional shapes, including angles and symmetry;
- enlargement and reduction of diagrams and models;
- two-dimensional scaled plans and diagrams, and plans of three-dimensional objects, including nets and perspective diagrams;
- diagrams which incorporate scale and labelling conventions of relevant dimensions;
- plans, models and diagrams and how accurately they depict the object represented.

2. Patterns and number

This area of study covers basic number operations and the representation of patterns in number in different forms. Consideration of approximation strategies and standard calculations enable students to obtain estimates and exact values in a variety of common contexts.

This area of study will include:

- practical problems requiring basic number operations;
- place value in decimal fractions and related metric measures;
- decimals and common vulgar fractions and their use in practical contexts;
- practical problems containing decimal fractions, fractions and percentages, particularly in making decisions about money and time in familiar situations;
- simple rates in practical contexts such as average speed for a journey, wages for hours worked;
- application of approximation strategies to achieve, for example, estimates of materials to be ordered, travelling time, conversions between units;
- the application of formulas to obtain required information in specific contexts (such as the cost of a taxi fare or the capacity of a swimming pool);
- the use of simple symbolic expressions to represent patterns in number and formulas related to practical applications.

3. Handling data

This area of study covers the collection, presentation and basic analysis of data. Consideration of different forms of data representation enables students to create appropriate and effective data summaries and critically interpret common media presentations.

This area of study will include:

- the common features, conventions and basic terminology used when interpreting and preparing information in graphical or tabular form;
- interpretation and use of graphs, graphics and tables, including flow charts, timetables, maps and plans; for example, to:
  - follow the sequence of operations in a production flow chart;
  - plan a travel or delivery route;
  - plan a travel itinerary;
  - arrange furniture/equipment/stores in accordance with a floor plan;
- common methods of presenting data, including:
  - simple frequency tables;
  - simple graphs, for example bar and line graphs and pie graphs;
- use and interpretation of average (mean, median and mode) and range of a set of data in practical situations and in the media;
- application of technology such as calculators, graphics calculators, computer packages to the display of data in various forms such as bar graphs, line graphs and pie graphs.

4. Measurement

This area of study covers the use of the metric system in familiar and everyday measurement activities. Consideration of conventions and practices for degree of accuracy and the use of appropriate units enable students to make measurements relevant to a variety of common contexts.

This area of study will include:

- measurement and the metric system;
- reading, recording and analysing digital and analog instrument scales;
- workplace problems involving metric measurement with consideration of required accuracy and tolerances, rounding and approximation strategies;
- measurement applications, including:
  - using counting and estimating strategies to determine the amount of items/parts/products purchased or produced;
  - quantity calculations and estimations of required materials from plans and specifications; for example, determination of amount of carpet from a plan;
  - providing estimates of time for task completion;
  - measuring and estimating required quantities for specific activities, including cases where original quantities are increased or decreased; for example, determine ingredients for a meal for six people from a recipe for four people.

OUTCOMES

For each unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass all of the selected areas of study for each unit. For each of Unit 1 and Unit 2, the outcomes apply to the content from the areas of study selected for that unit.

Outcome 1

On completion of this unit the student should confidently and competently use mathematical concepts and skills from the areas of study.

Space, shape and design

To achieve this outcome the student will draw on knowledge and related skills outlined in area of study 1.

Key knowledge

This knowledge includes

- names and properties of common geometric shapes in two dimensions and three dimensions;
- forms of two-dimensional representations of three-dimensional objects, including nets and perspective diagrams;
- symbols and conventions for the representation of geometric objects; for example, point, line, ray, angle, diagonal, edge, curve, face and vertex;
- symbols and conventions for related measurement units.

Key skills

These skills include the ability to

- interpret plans, diagrams and their conventions;
- represent three-dimensional objects in diagrams;

- assemble three-dimensional objects from plans, instructions or kits;
- describe objects using accurate and appropriate geometric language;
- create patterns based on regular two-dimensional shapes and combinations of these shapes;
- use drawing equipment, computer drawing packages or geometry software to create and modify shapes and designs.

Patterns and number

To achieve this outcome the student will draw on knowledge and related skills outlined in area of study 2.

Key knowledge

This knowledge includes

- decimal place value basis of number scales;
- arithmetic operations and what they represent (sum, difference, product, quotient);
- equivalent forms for expressing the same quantity; for example, fractions, decimals, percentages;
- levels of accuracy required for a particular problem or context;
- relationship between quantities and related formulas.

Key skills

These skills include the ability to

- apply arithmetic operations, according to their correct order;
- decide whether answers to mathematical problems are reasonable;
- use leading digit approximation to obtain estimates of calculations;
- use a calculator for multi-step calculations;
- check results of calculations for accuracy;
- recognise the significance of place value after the decimal point;
- evaluate decimal fractions to the required number of decimal places;
- round up or round down numbers to the required number of decimal places;
- convert vulgar fractions into equivalent decimal fractions, using a calculator;
- compare and order decimal fractions;
- calculate percentages using a calculator;
- solve problems which require the application of decimal fractions; for example, the calculation of money in relation to practical contexts;
- substitute correct values into formulas and perform related calculations.

Handling data

To achieve this outcome the student will draw on knowledge and related skills outlined in area of study 3.

Key knowledge

This knowledge includes

- features of graphs, tables, maps and plans;
- key terminology used in relation to graphs, maps, sketches, plans, charts and tables;
- purposes for using different forms of data;
- conventions for correct labelling of graphs, choice of scales and units;

- data collection in a variety of contexts and for a variety of purposes; for example, keeping personal records for budgeting, keeping records for taxation purposes, gathering opinions through surveys and questionnaires for determining customer/employee satisfaction, monitoring quality control of a production process;
- key features which pertain to a range of visual presentations;
- types of data (categorical and numerical) and appropriate forms of representation;
- use and interpretation of mean and median as ‘average’, and range as spread;
- categories into which information can be sorted.

Key skills

These skills include the ability to

- present information in appropriate visual forms and distinguish between information presented in graphical and tabular formats;
- distinguish between graphs, maps, sketches, plans, charts and tables;
- use key terminology in relation to graphs, maps, sketches, plans, charts and tables;
- accurately read graphs, maps, sketches, plans, charts and tables of common objects;
- determine mean, median and range of a set of data using technology;
- transfer information from one form of representation to another (for example, table to graph and vice versa).

Measurement

To achieve this outcome the student will draw on knowledge and related skills outlined in area of study 4.

Key knowledge

This knowledge includes

- definitions of common metric units of length, area, volume, and mass;
- relative scale of metric units, for example mm, cm, m, km;
- rules for rounding to a specified degree of accuracy;
- conversion factors for representing common metric quantities in decimal form;
- procedures for estimation and calculation.

Key skills

These skills include the ability to

- identify common notation for metric measurement;
- use a variety of metric units for capacity (mL, L);
- use the metric length system with a variety of metric units (mm, cm, km);
- estimate metric weight using appropriate units;
- demonstrate an appropriate unit for measurement for particular applications;
- select appropriate standard measurement for length, width, perimeter, area, surface area, volume, height and weight;
- calculate and interpret areas, surface areas and volumes;
- use standard units and common conversions;
- calculate accurately and efficiently;
- use decimals and metric measure, for example money, weight and/or capacity to make calculations.

Outcome 2

On completion of this unit the student should be able to apply and discuss mathematical procedures to solve practical problems in familiar and new contexts, and communicate their results.

To achieve this outcome the student will draw on knowledge and related skills outlined in all the areas of study.

Key knowledge

This knowledge includes

- the use of relevant and appropriate mathematics in areas relating to their study, work, social or personal contexts;
- commonly encountered uses of mathematics in aspects of everyday life;
- commonly used methods of presenting and communicating mathematics in everyday life (for example, charts, graphs, maps, tables and plans).

Key skills

These skills include the ability to

- identify and recognise how mathematics can be used in everyday situations and contexts, making connections between mathematics and the real world;
- undertake a range of mathematical tasks, applications and processes, including measuring, counting, estimating, calculating, drawing, modelling and discussing;
- interpret results and outcomes of the use and application of mathematics in a context, including how appropriately and accurately they fit the situation;
- represent, communicate and discuss the results and outcomes of the use and application of mathematics in context.

Outcome 3

On completion of this unit the student should be able to select and use technology to apply mathematics in a range of practical contexts.

To achieve this outcome the student will draw on knowledge and related skills outlined in all the areas of study.

Key knowledge

This knowledge includes

- order of operations built into technology;
- limits of settings which are required for effective use of technology; for example, viewing windows for graphs, dimensions of lists, and related menus;
- forms of data representation using technology;
- the appropriate selection of technology for a given context.

Key skills

These skills include the ability to

- use a calculator for computation whenever necessary;
- produce tables of values, graphs, diagrams or collections of data which relate to specific contexts;
- organise and present information in a clear and useful form;
- interpret and discuss data produced by different technologies in various tabular, graphical or diagrammatic forms.

ASSESSMENT

The award of satisfactory completion for a unit is based on a decision that the student has demonstrated achievement of the set of outcomes specified for the unit. This decision will be based on the teacher's assessment of the student's overall performance on assessment tasks designated for the unit.

The key knowledge and skills listed for each outcome should be used as a guide to course design and the development of learning activities. The key knowledge and skills do not constitute a checklist and such an approach is not necessary or desirable for determining the achievement of outcomes. The elements of key knowledge and skills should not be assessed separately.

Assessment tasks must be a part of the regular teaching and learning program and must not unduly add to the workload associated with that program. They must be completed mainly in class and within a limited timeframe. Teachers should select a variety of assessment tasks for their assessment program to reflect the key knowledge and skills being assessed and to provide for different learning styles.

For this unit students are required to demonstrate achievement of three outcomes. As a set these outcomes encompass all areas of study.

Demonstration of achievement of Outcomes 1and 2 must be based on the student's performance on a selection of assessment tasks. Where teachers allow students to choose between tasks they must ensure that the tasks they set are of comparable scope and demand.

Demonstration of achievement of Outcomes 1 and 2 must be based on a selection of the following tasks:

- investigations and projects; for example, a report on an application or use of mathematics such as, costing of a birthday party, budgeting for a holiday, a survey of types of television programs, or design of a car park;
- assignments, summary or review notes of mathematics that students have encountered in their work or study; for example, a written or multimedia or an oral presentation of wages calculations, materials estimation for a task, personal budgeting; and
- tests of mathematical skills developed from investigations.

For each unit demonstration of achievement of Outcome 3 must be based on the student's performance on a selection of tasks which incorporate the effective and appropriate use of technology in contexts related to topics in the selected material from the areas of study. This could, for example, include use of a CAD package in a design task, or presentation software to report on an investigation.

The achievement of this outcome should be assessed on student demonstration of the key knowledge and skills in carrying out technology-based mathematical tasks within the assessment tasks for Outcomes 1 and 2.

Advice for teachers (Foundation Mathematics Units 1 and 2)

DEVELOPING A COURSE

A course outlines the nature and sequence of teaching and learning necessary for students to demonstrate achievement of the set of outcomes for a unit. The areas of study describe the knowledge required for the demonstration of each outcome. Outcomes are introduced by summary statements and are followed by the key knowledge and skills which relate to the outcomes.

Teachers must develop courses that include appropriate learning activities to enable students to develop the knowledge and skills identified in the outcome statements in each unit.

For Units 1 and 2, teachers must select assessment tasks from the list provided. Tasks should provide a variety and the mix of tasks should reflect the fact that different types of tasks suit different knowledge and skills, and different learning styles. Tasks do not have to be lengthy to make a decision about student demonstration of achievement of an outcome.

In determining a course of study for Foundation Mathematics teachers are encouraged to embed content in contexts which are meaningful and of interest to the students. A variety of approaches could be used to achieve this. Sometimes teachers will use an extended, practical investigation that could be undertaken to develop specific mathematical knowledge and skills related to the course. At other times, teachers may introduce the key knowledge and skills associated with part of the area of study and then provide opportunities to apply the knowledge and skills in shorter projects and investigations.

Two approaches to course organisation are shown here to highlight alternative approaches that can deliver a suitable program. Both sample courses could be used in either semester to enable all outcomes to be demonstrated.

SAMPLE COURSE A

Topic 1: Cars

Weeks 1 to 3	*Cars: Buying a car*	*Areas of study*
	Investigate the costs of buying a used car, given a realistic budget and income, and using at least two different modes of paying off the purchase (cash; personal loan, credit card, finance company, etc.); take into account deposit and ability to pay off the outstanding debt. Document the two methods and report on the differences, and discuss advantages and disadvantages of each. Supporting teaching activities: • demonstrate and discuss how mathematics is embedded in everyday situations in order to be able to identify what maths to use in order to solve problems • revise fractions, decimals, percentages and related calculations, and use of calculators and spreadsheets. • model and discuss methods of reporting and communicating results of mathematical investigations.	Patterns and number Handling data
Weeks 4 and 5	*Extension and follow up applications – students negotiate a project along the lines of:*	*Areas of study*
	Prepare a budget and plan for an end of term BBQ for between 100 people and 200 students. Include a timeline of tasks to be completed. Work out a cost per student that includes a profit in order to donate it to a good cause. Investigate and report on drink driving issues such as: • standard drinks and driving on P-Plates • accidents and drink driving – research, find and report on statistics of accidents and drink driving. Investigate mobile telephone charges. Make a comparison of costs for a given number of calls or a given time of calls in a year, include cost of purchasing the phone, rental or service charge as well as call costs.	Patterns and number Handling data

Topic 2: Gardening and landscaping

Weeks 6 to 8	Students select a garden area of the school yard (or a local community venue such as a kindergarten, primary school, hall) that needs redevelopment and replanting. They design and cost for its redesign with a scale plan with specified dimensions, and include costs for edging the garden beds, new soil or bark chips and plants. Costs are researched by visiting suppliers or using the Internet. Supporting teaching activities: • demonstrate and discuss how mathematics is embedded in everyday situations in order to be able to identify what maths to use in order to solve problems • review common geometric shapes, and similarity and symmetry properties. Identify these shapes in common objects such as in containers, packaging and design • review and teach about scale drawings and plans • revise metric measurements and calculation of areas and volumes • model and discuss methods of reporting and communicating results of mathematical investigations.	*Areas of study* Space, shape and design Patterns and number Measurement
Weeks 9 and 10	*Extension and follow up applications – students choose a project along the lines of:* Plan and cost for the painting of a room in the school or their own room at home (or at the same venue as above). Include an estimate of the time needed to undertake the task. Use a scale drawing of the rooms to calculate the areas of walls to be painted. Base it on actual costs – researched by visiting hardwares or from information gathered from the Internet. Plan and cost for sowing grass in an area of the school yard (or a local community venue such as a kindergarten, primary school, hall). Design and cost a backyard with specified dimensions, including paving, laying lawn, edging garden beds and providing service area for rubbish bins or compost.	*Areas of study* Space, shape and design Patterns and number Measurement

Topic 3: Sport

Weeks 11 to 14	*Investigate your favourite sport and undertake the following types of analysis and reporting:* • Draw a scale plan of a playing field/area for the sport accompanied by a written description. • Investigate and analyse the scoring system for your sport, and generalise the scoring into an algebraic sentence and equation. • Write (and draw/illustrate) an explanation for the scoring in the game for someone who has never seen a game of your sport before. • Research and find statistics for one team in your chosen sport in at least two different games in the same competition. Compare the two results. Represent them graphically and analyse them in terms of averages (mean, median and mode).	*Areas of study* Space, shape and design Patterns and number Handling data Measurement

Supporting teaching activities:

- Demonstrate and discuss how mathematics is embedded in everyday situations in order to be able to identify what maths to use in order to solve problems.
- Teach, review and model data and statistical analysis tools, including data collection and representation (raw data, tables, graphs) and mean and median.
- Review and teach how data can be displayed graphically, e.g. by using technology, spreadsheet or graphics calculator.
- Review, teach and model how algebra can be used to generalise in familiar activities and how you can substitute into such formula to calculate results for any situation.
- Model and discuss methods of reporting and communicating results of mathematical investigations.

Weeks 15 and 16

Reflection and/or follow up:

Students prepare and present a brief individual report to the whole class on their favourite maths project/investigation for the semester, reflecting on why they liked it and what mathematics they learned and applied. The presentation can be oral, visual or written.

And/or:

Students actually undertake one of the projects they investigated, like planting the garden, holding the BBQ, etc.

Areas of study

Depends on topics chosen

SAMPLE COURSE B

In this outline, each area is studied sequentially with two weeks allotted to teaching knowledge and skills and then two weeks for investigations which incorporate this mathematics.

Space, shape and design

Weeks 1 and 2

Review common geometric shapes, and similarity and symmetry properties. (Circles, squares, triangles, cuboids, cylinders and pyramids.) Identify these shapes in common objects, in containers, packaging and design.

Identify features of buildings from perspective drawings and floor plans.

Revise metric measurements and calculation of areas and volumes.

Weeks 3 and 4

A selection of investigations, culminating in a written report, a chart or display prepared on a computer, or an oral report.

Investigations
Classroom space

In an investigation of the classroom space and the playground space in a school, the following questions are posed:

- Do students have the same amount of space in different classrooms?
- What are the key features that determine the amount of space available to students?

When agreement is reached on the meaning of space in these questions, students begin collecting appropriate data. Working in groups and sharing the tasks may be an efficient way to work. The teacher may provide some data such as class sizes, ceiling heights, a scaled plan of the school. The students will make some measurements, and in some cases they may estimate distances by pacing out the perimeters of the spaces. Students then investigate ways of maximising the space available for specific purposes. For example:

- What room dimensions are possible for a given amount of wall space? What dimensions would be recommended?

Consideration should be given to a range of factors besides simple maximisation of floor area or room volume. The findings may be presented in a written or an oral report. Some diagrams or scale drawings with appropriate use of units of measurement may be required.

Energy efficient houses

An investigation of energy efficiency in houses.

Houses with most of the windows facing north will be warmer in winter and cooler in summer than houses with south facing windows (cold in winter) or west (hot in summer).

Students are required to measure the area of the windows in their house and show the aspect of their house from each of the four compass points in sketches or scale drawings.

Calculations of the proportion of window area facing north for the whole house can be made, and all results for the class combined. Some decisions on how to group the data will be needed.

Teachers may need to collect some house plans to give to students who are unable to make their own measurements.

Parklands

How green is our local area? Use a transparent grid overlay to estimate the fraction of parkland shown on a page of a street directory. Using the scale shown in the street directory, find the areas represented by a page. Calculate the area of the parkland from the fraction.

A comparison of different suburbs can be made if class results are shared.

Centres of shapes

Finding the centre of a triangle. Mark a triangle in the long jump pit and bury a chocolate bar at the centre. Each student has one chance to dig with a small spade to find the centre of the triangle. Students can use paper triangles and fold from a vertex to the midpoint of the opposite side to find the centre. They may also use folding of other shapes to find the centre which may be defined as the ‘balance point’.

Scale and area

A spreadsheet or dynamic geometry systems can be used to investigate the changes in area of circles, triangles, and rectangles as their dimensions are doubled, halved or trebled. Geometric designs with given or calculated proportions can be produced.

Paving plans

Draw up a plan to pave a BBQ area with rectangular pavers or a mixture of square and rectangular pavers.

Patterns and number

Weeks 5 and 6

Revise simple fractions, decimals, percentages, and ratio as an extension of fractions.

Basic calculations, order or operations (calculators are used for all operations).

Test on skills and their applications in routine questions.

Weeks 7 and 8

Investigations
Booklists

Use the school booklist for Year 11 with the new price of books, to prepare a price list for the second-hand book sale. Assumptions need to be discussed, a reasonable price may be two-thirds of the original price, prices may be given to the nearest dollar to make giving change easier.

The odd cents

Collect a detailed docket from a supermarket with at least twenty items. Calculate the price if all individual items are rounded to the nearest five cents. Write out a new docket with the rounded prices, calculate the total. Compare it with the original total; express the difference as a percentage. Collect all the data from the class and determine what proportion of dockets have a lower total if the prices are rounded item by item.

Costing a party

This investigation can be extended or reduced and is suitable for group work.

Investigate the cost of an eighteenth birthday for say 50 or 100 guests at three different venues: at home, fully catered at a commercial venue or a less formal arrangement as in hiring a local hall and contracting out the catering. Assumptions need to be discussed such as quantities of food and drink per person, and provision of music. A unit cost can be calculated and compared for the six cases.

For each case, express the component costs as percentages. A spreadsheet may be used to present the information.

Handling data

Weeks 9 and 10

Review the ways in which data can be displayed graphically.

Information given can be displayed graphically using technology, spreadsheets or graphing calculators.

Convert tables to a graphical display, and present information given in a graphical display as a table or chart either in full or for a selected purpose.

Investigations
Data collection and analysis

Use data provided to show the distribution of road accidents month by month and according to the age of drivers.

Measure reaction time of class members using the 'dropped ruler method' for each member of the class, display and analyse results.

Measure body lengths such as hand-span, shoulder height, arm length and then group data to display results. Provide a picture of the average person.

Interpreting flowcharts

Set up flowcharts for tasks such as developing a photograph, colouring hair, cleaning and oiling a skateboard, mending a bicycle tyre puncture, assembling ingredients and cooking a cake.

Measurement

Weeks 13 and 14

Review measurement in metric units from previous work.

Measure temperatures in degrees Celsius.

Review scale for plans and drawings.

Calculate costs for multi-task projects.

Estimate time to complete multi-task projects.

Weeks 15 and 16

Investigation

Given the floor plan of a house and a price list for different floor finishes and coverings, set up a spreadsheet to calculate the cost for each room and the total cost. Use the spreadsheet to investigate the cost by varying the type of material used such as using carpet instead of polishing the floors.

USE OF INFORMATION AND COMMUNICATIONS TECHNOLOGY

In designing courses and developing learning activities for Foundation Mathematics, teachers should make use of applications of information and communications technology and new learning technologies, such as computer-based learning, multimedia and the World Wide Web, where appropriate and applicable to teaching and learning activities.

The World Wide Web will often be the source of mathematical information and data that forms the basis of students' research as part of data collection and investigative tasks.

In Foundation Mathematics the student should be able to select and use appropriate technology to apply mathematics to a range of practical contexts. These skills include the ability to:

- use a calculator for computation whenever necessary;
- produce tables of values, graphs, diagrams or collections of data which relate to specific contexts;
- organise and present information in a clear and useful form;
- interpret and discuss data produced by different technologies in various tabular, graphical or diagrammatic forms.

Students should also know:

- order of operations built into technology;
- limits of settings which are required for effective use of technology; for example, viewing windows for graphs, dimensions of lists, and related menus;
- forms of data representation using technology;
- the appropriate selection of technology for a given context.

KEY COMPETENCIES AND EMPLOYABILITY SKILLS

Students undertaking the following types of assessment, in addition to demonstrating their understanding and mastery of the study, typically demonstrate the following key competencies and employability skills.

Assessment task	Key competencies and employability skills
Investigations and projects	Planning and organising, problem solving, using mathematical ideas and techniques, (written) communication, self management, team work, use of information and communications technology
Assignments and analysis tasks	Planning and organising, problem solving, using mathematical ideas and techniques, use of information and communications technology
Tests	Using mathematical ideas and techniques, use of information and communications technology

LEARNING ACTIVITIES

Examples of learning activities for each unit are provided in the following sections. Extended examples are highlighted by a shaded box. The examples that make use of information and communications technology are identified by this icon ICT.

Units 1 and 2: Foundation Mathematics

AREA OF STUDY 2: Patterns and number

Outcome 1

Confidently and competently use mathematical skills from the areas of study.

Examples of learning activities

identify correct order of operations and check results with an appropriate calculator

convert metric distances (e.g. on a floor plan) from millimetres to metres

divide a given amount of money (less than $100) ten ways (or five ways) without a calculator

divide an amount of money or a metric distance (e.g. three or seven ways) and round results to two decimal places using a calculator

use a given formula to calculate the area of a triangle to a given degree of accuracy

practice rounding off the cost of supermarket items

list qualities for the ingredients of a standard recipe which has been increased (or decreased)

calculate wages due for a work period that includes overtime or above-award time

compare different cost/rental schemes for mobile phones

AREA OF STUDY 1: Shape, space and design

Outcome 2

Apply and discuss mathematical procedures to solve practical problems in familiar and new contexts, and communicate their results.

Examples of learning activities

identify geometrical shapes used in a variety of tiling patterns

develop several tiling patterns (or tesselations) using a combination of (regular) geometric shapes

devise a scaled floor plan of a classroom or room in a house

use isometric conventions to represent a three-dimensional object

contrast and evaluate several tourist maps of the central business district

compare pencil and paper procedures for drawing a regular polygon (e.g. a hexagon) with those required for a computer drawing package

estimate the amount of parkland in a given area of a town or city

AREA OF STUDY 1: Space, shape and design

Outcome 3

Select and use technology to apply mathematics to a range of practical contexts.

Examples of learning activities

- use a spreadsheet to represent a formula (e.g. area of a circle) and to calculate specific values given other known values
- use timetables (e.g. air, rail or bus) obtained from a web page to plan a journey allowing sufficient time for changeover between different modes
- devise procedures to draw different regular polygons using a computer drawing package
- investigate changes in areas of circles, triangles and rectangles as their dimensions are doubled, trebled or halved
- generate different net representations of a given three-dimensional object
- use several computer-generated perspectives of an object or location
- use a computer drawing package to create a family of related shapes or to modify a given shape

Investigation 1: Gardening and landscaping

Detailed example

SAMPLE COURSE A, TOPIC 2

Students select a garden area of the school yard (or a local community venue such as a kindergarten, primary school, local hall) that needs redevelopment and replanting.

Start with an initial class discussion of the task and decide what it involves – consider the various tasks in order for the redevelopment and replanting of a garden or a lawn. Explain that the task is to be undertaken by students preferably working in small groups. Use this as an opportunity to identify the maths skills required to complete the task, in readiness for intergrating the teaching of the necessary maths skills as they progress through the activity.

Establish with the group that the task includes the following stages (which could be drawn up as a worksheet of instructions, depending on student skill levels):

1. Choose the area to be redeveloped/planted and get teacher approval.
2. Draw a scale plan of the area with annotated dimensions.
3. Describe and name the design and shapes of the area.
4. Specify details of what work needs to be completed and what materials would be required (e.g. soil, sand, bark chips, edging, plants, lawn seeds, etc.).
5. Calculate actual lengths, areas and volumes of materials needed.
6. Determine the number of plants or lawn seeds required, taking into account the space each requires etc.
7. Visit a hardware store or use the Internet to find the prices and cost of materials and plants.
8. Use the prices to calculate the total cost of the job.
9. If practical and resources can be found or financed, undertake the task.
10. Record and report on the task, include plans, costings and photos if possible.

Supporting teaching activities:

- Demonstrate and discuss what mathematics is involved in an everyday situation such as landscaping and gardening, including drawing scale diagrams, in estimating and measuring, in calculating areas and volumes, in calculating costs, etc.
- Review common geometric shapes, and similarity and symmetry properties. Identify these shapes in common objects – in buildings, containers, packaging and design. Link to Stages 2 and 3.
- Review and teach about scale drawings and plans. Link to Stage 2.
- Revise metric system, estimating and taking measurements and calculation of areas and volumes. Link to Stages 2, 4, 5, 6 and 7.
- Review, teach and model how algebra can be used in familiar situations and how you can substitute into formula to calculate in this case areas and volumes. Link to Stages 4, 5, 6 and 8.
- Revise fractions, decimals and related calculations. Link to Stages 2, 4, 5, 6, 7 and 8.

Investigation 2: Medication dose

Detailed example

A particular medication is manufactured in solid and liquid forms. Liquid forms are used in particular for children, as either ‘drops’ or ‘elixir’, with different concentrations and dosages depending on age range and weight. There are four main age ranges:

0 – 1 ‘Drops’

1 – 5 ‘Elixir’ (24mg/1mL 100mL bottle)

Age (years)	Average weight (kg)	Dose (mL)
1 – 2	10.2 – 12.6	6 – 8
2 – 3	12.6 – 14.7	8 – 9
3 – 4	14.7 – 16.5	9 – 10
4 – 5	16.5 – 18.5	10 – 12
5	18.5 – 20.5	12 – 13

Where the weight of the child exceeds 20.5 kg, the medication dose is to be calculated at a rate of 15mg/kg of body weight.

a) Construct a line graph which could be used to predict the dose for different weights within the average weight range listed above, and use it to predict the doses for several different weights within this range.

b) Calculate the dose for a 25 kg child.

c) What would be the predicted dose for an 8 kg child, if this relationship were to be used?

d) Find a simple formula which can be used to calculate the dose for a child in this weight range. Check the predicted dose from your formula with that of your graph, for several different average weight values.

e) What would be the required dose for a person of your own body weight?

As children grow older, it is not practical to keep increasing the volume of the dose, since it will gradually become too much to comfortably swallow. For older children the concentration of the elixir is increased, and the volume of the dose adjusted accordingly.

5 – 12 ‘Elixir’ (48mg/1mL 100mL bottle)

Age (years)	Average weight (kg)	Dose (mL)
5 – 6	18.5 – 20.5	6
6 – 7	20.5 – 22.5	6 – 7
7 – 8	22.5 – 25.0	7 – 8
8 – 9	25.0 – 28.5	8 – 9
9 –10	28.5 – 32.5	9 – 10
10 – 11	32.5 – 37.0	10 – 11
11 – 12	37.0 – 41.5	11 – 12

Where the weight of the child exceeds 41.5 kg, the medication dose is to be calculated at a rate of 15mg/kg of body weight.

a) Construct a line graph which could be used to predict the dose for different weights within the average weight range listed above, and use it to predict the doses for several different weights within this range.

b) Calculate the dose for a 55 kg child.

c) Compare the doses for a child weighing 20.5 kg for both tables.

d) Find a simple formula for calculating the medication dose for a child in this weight range. Check the predicted dose from your formula with that of your graph, for several different average weight values.

e) What would be the required dose for a person of your own body weight?

12 – Adults (Tablets)

Other areas for consideration

Students could investigate dose rates associated with well-known medications, such as pain relievers, across different weight and age groups, and in different formulations.

In particular, technology could be used to create tables and graphs to display data, and to predict values from formulas.

Units 1 and 2:
General Mathematics

Units 1 and 2: General Mathematics

General Mathematics provides courses of study for a broad range of students and may be implemented in a number of ways. Some students will not study Mathematics beyond Units 1 and 2, while others will intend to study Further Mathematics Units 3 and 4. Others will also be studying Mathematics Methods Units 1 and 2 or Mathematics Methods (CAS) Units 1 and 2 and intend to study Mathematical Methods Units 3 and 4, or Mathematical Methods (CAS) Units 3 and 4 and, in some cases, Specialist Mathematics Units 3 and 4 as well. The areas of study for Unit 1 and Unit 2 of General Mathematics are 'Arithmetic', 'Data analysis and simulation', 'Algebra', 'Graphs of linear and non-linear relations', 'Decision and business mathematics' and 'Geometry and trigonometry'.

Units 1 and 2 are to be constructed to suit the range of students entering the study by selecting material from the six areas of study using the following rules:

- for each unit, material covers four or more topics selected from at least three different areas of study;
- courses intended to provide preparation for study at the Units 3 and 4 level should include selection of material from areas of study which provide a suitable background for these studies;
- selected material from an area of study provide a clear progression in key knowledge and key skills from Unit 1 to Unit 2.

The appropriate use of technology to support and develop the teaching and learning of mathematics is to be incorporated throughout the course. This will include the use of some of the following technologies for various areas of study or topics: graphics calculators, spreadsheets, graphing packages, dynamic geometry systems, statistical analysis systems, and computer algebra systems.

AREAS OF STUDY

1. Arithmetic

This area of study covers applications of arithmetic involving natural numbers, integers, rational numbers, real numbers and complex numbers, matrices and sequences and series.

Matrices

This topic will include:

- definition of a matrix;
- matrix addition, subtraction, multiplication by a scalar and multiplication of matrices;

- identity and inverse matrices and their properties;
- applications of matrices in contexts such as stock inventories, solving simultaneous linear equations, transformations of the plane or networks;
- calculator or computer applications for higher-order matrices.

Integer and rational number systems

This topic will include:

- review of properties and computation with natural numbers, integers, and rational numbers;
- forms of representation including, for example, products of powers of primes, decimal place value, equivalence of decimal and fractional forms (terminating and infinite recurring decimals for rational numbers);
- geometric representation of natural numbers, integers and rational numbers (by similarity) on a number line;
- application of integer arithmetic and rational number arithmetic to:
  - divisibility and modular arithmetic, clocks, calendars and time zones;
  - computation in practical situations involving small and large numbers using scientific notation;
  - ratio, proportion and percentage applications such as map scales, dilution factors, medicine doses, gear ratios, currency exchange rates, measurement conversion rates and other rates of change.

Real and complex number systems

This topic will include:

- definition and properties of real and complex number systems;
- irrational numbers and complex numbers as roots to quadratic equations;
- the golden ratio φ, its geometric interpretation, and rational approximations to its exact value;
- geometric representation of irrational numbers that are roots to quadratic equations on the real number line and representation of complex numbers on an argand diagram;
- operations with irrational numbers of the form $a + b\sqrt{n}$ where a, b are rational numbers and n is a positive integer which is not a perfect square;
- operations with complex numbers of the form $a + bi$ where a, b are rational numbers and $i^2 = -1$.

Sequences and series

This topic will include:

- sequences and series as maps between the natural numbers and the real numbers, and the use of technology to generate sequences and series and their graphs;
- sequences generated by recursion: arithmetic ($t_{n+1} = t_n + d$), geometric ($t_{n+1} = rt_n$) and fixed point iteration (for example $t_1 = 2$, $t_{n+1} = t_n^2$; $t_1 = 0.5$, $t_{n+1} = 0.8t_n(1 - t_n)$);
- practical applications of sequences and series, such as financial arithmetic, population modelling and musical scales.

2. Data analysis and simulation

This area of study covers the display, summary, and interpretation of univariate and bivariate data, and the design, construction and evaluation of probability simulation models.

Univariate data

This topic will include:

- categorical data and numerical data;
- data displays and their interpretation: frequency tables and bar charts for categorical data; dot plots, stemplots, frequency tables and histograms (including relative frequency, percentage frequency and cumulative frequency) for numerical data;
- summary of numerical data using measures of central tendency and spread: mean, median and mode, range, interquartile range (IQR), variance and standard deviation;
- five-number summary for a set of data {minimum, Q1, Q2 = median, Q3, maximum} and its graphical representation by boxplot.

Bivariate data

This topic relates to work in the Linear graphs and modelling topic from the **Graphs of linear and non-linear relations** area of study and will include:

- scatterplots;
- informal interpretation of patterns and features of scatterplots;
- correlation and regression:
  - use and interpretation of the quadrant, q, correlation coefficient;
  - fitting a line to data with an appropriate linear association for a dependent variable with respect to a given independent variable, by eye and using the two mean method, determining the equation of this line, and using this equation for prediction. Informal consideration of closeness of fit (how close the data points are to the fitted line).

Simulation

This topic will include:

- random experiments, events and event spaces;
- probability as an expression of long run proportion;
- stages in using a simulation in a mathematical model: formulation, solution, interpretation, validation, improvement of model;
- types of simulations:
  - bernoulli and markov trials;
  - simple queueing problems;
  - multi-event problems (for example, traffic lights, games, lotto, card collecting).

3. Algebra

This area of study includes linear and non-linear relations and equations and algebra and logic.

Linear relations and equations

This topic will include:

- substitution and transposition in linear relations, such as temperature conversion;
- the solution of linear equations, including literal linear equations;
- developing formulas from word descriptions, substitution of values into formulas;

- the construction of tables of values from a given formula using technology;
- linear relations defined recursively and simple applications;
- the algebraic and graphical solution of simultaneous linear equations in two variables;
- solution of worded problems involving a linear equation or simultaneous linear equations in two variables.

Non-linear relations and equations

This topic will include:

- substitution and transposition in non-linear relations, such as volume mensuration formulas, radiation intensity and distance and simple logarithmic scales;
- developing formulas from word descriptions, substitution of values into formulas;
- the construction of tables of values from a given formula by use of calculator, computer algebra system or spreadsheet;
- the solution of non-linear equations using algebra, tables, graphs and simple numerical approaches such as bisection, secants or simple iteration (including the use of continued fractions to approximate the irrational roots of a quadratic equation) to determine a root for an equation over an interval in which it is known to exist;
- solution of word problems using non-linear equations;
- the solution of simultaneous equations arising from the intersection of a line with a parabola, circle or rectangular hyperbola, and their solution using algebra, graphs, tables and simple iteration.

Algebra and logic

This topic will include:

- propositions, connectives and truth tables;
- tautologies, validity and proof patterns;
- the application of proof to number patterns and algebra;
- electronic gates and circuits;
- laws and properties of boolean algebra;
- boolean algebra and its application to circuit simplification.

4. Graphs of linear and non-linear relations

This area of study covers the sketching and interpretation of linear and non-linear graphs, modelling with linear and non-linear graphs, variation and a numerical and graphical approach to rectilinear motion.

Linear graphs and modelling

This topic will include:

- determining gradients, intercepts and the equations of straight lines from graphs;
- plotting and sketching straight lines given an equation;
- determining points of intersection of straight line graphs by graphical and algebraic methods;
- simple applications of linear modelling, such as fitting a line of best fit by eye to data values, identifying the equation of best fit, use of this line for prediction, informal discussion of closeness of fit;
- the construction and interpretation of line segment graphs;
- graphs of linear inequalities.

Sketching and interpreting linear and non-linear graphs

This topic will include:

- sketching relations in the cartesian plane from descriptions, equations or formulas and identifying their key features;
- sketching relations in the cartesian plane from rules and tables of values;
- polar coordinates and polar graphs;
- interpreting graphical representations of practical data;
- graphical representation of circles, ellipses, parabolas and hyperbolas; sketch graphs, including focus-directrix properties;
- sketching graphs by addition of ordinates; identifying asymptotes;
- sketching the graph of reciprocal and square relations from the graph of a simple relation.

Variation

This topic will include:

- numerical, graphical and algebraic approaches to direct, inverse and joint variation;
- transformation of data to establish relationships between variables, for example, x^2, $\frac{1}{x}$ to linear;
- modelling of given non-linear data using the relationships $y = kx^2 + c$, $y = \frac{k}{x} + c$, where k is a positive real number;
- modelling of data using the logarithmic function $y = a\log_{10}(x) + b$, where a is a positive real number.

Kinematics

This topic will include:

- diagrammatic and graphical representation of empirical position-time data for a single particle in rectilinear motion, including examples with variable velocity (data may be obtained by a student moving along a 100 metre tape according to a given set of instructions, data logging or previous experimental data);
- graphical modelling and numerical analysis of position-time and velocity-time relationships based on continuous hybrid functions formed by straight line segments, including consideration of average velocity and distance traveled over an interval;
- modelling and analysis of rectilinear motion under constant acceleration, including use of constant acceleration formulas: $v = u + at$, $v^2 = u^2 + 2as$, $s = \frac{1}{2}(u + v)t$ and $s = ut + \frac{1}{2}at^2$;
- qualitative graphical analysis of the relationship between position-time, velocity-time and acceleration time graphs for simple cases of rectilinear motion involving variable acceleration;
- numerical approximation to instantaneous rate of change of a function f at time $t = a$ by evaluation of the central difference $\frac{f(a+h) - f(a-h)}{2h}$ for small values of h using technology; and its application to approximate evaluation of instantaneous velocity and instantaneous acceleration in simple cases of rectilinear motion involving variable velocity and variable acceleration;
- approximation of velocity-time relationships by step functions; and its application to approximate evaluation of distance travelled in simple cases of rectilinear motion involving variable velocity and variable acceleration, as a sum of areas of rectangles, using technology.

5. Decision and business mathematics

This area of study covers definitions and applications of undirected graphs, linear programming and financial arithmetic.

Networks

This topic will include:

- description of networks in terms of faces (regions), vertices and edges;
- faces (regions) and the application of euler's formula;
- traversibility of a network by considering the order of vertices in the network and rules for following a path: eulerian paths and circuits and applications, and hamiltonian paths and circuits and applications;
- applications of networks to simple distance or time minimisation problems;
- trees and minimum spanning trees and applications.

Linear programming

This topic will include:

- graphs of relations from linear equations and linear inequalities;
- the solution of simultaneous linear equations by algebraic, numerical and graphical methods;
- graphical approaches to solving simple optimisation problems using linear programming.

Financial arithmetic

This topic will include:

- cash flow in common savings and credit accounts including interest calculations;
- applications of simple interest and compound interest formulas;
- comparison of purchase options, including cash, credit card, bank loan, time payments (hire purchase) and store cards;
- appreciation and depreciation of assets, including investment of money, capital gains of physical assets, and depreciation of assets by inflation.

6. Geometry and trigonometry

This area of study includes shape and measurement, coordinate geometry, trigonometry, vectors and geometry in two dimensions and three dimensions.

Shape and measurement

This topic will include:

- mensuration (angle, length, boundary, area, surface area and volume);
- pythagoras theorem in two dimensions and simple examples in three dimensions;
- similarity and symmetry in two dimensions and applications to maps, art, tessellations, plans;
- similarity in three dimensions and application to scale models;
- tests for similarity and symmetry.

Geometry in two and three dimensions

This topic will include:

- angle sum of a triangle and of polygons;
- straight edge and compass constructions such as:
  - construction of a line parallel to a given line;
  - bisecting a given angle;

- perpendicularly bisecting a line segment;
- construction of a perpendicular to a given line from a point not on the line;
- construction of exact angles of 60°, 30°, 45°;

- construction and investigation of various regular and star polygons;
- construction and investigation of polyhedra, platonic solids;
- geometry in art and design, tessellations, patterns, perspective;
- theorems relating to angles in a circle, such as:
  - the angle subtended at the circumference is half the angle subtended at the centre by the same arc;
  - angles in the same segment of a circle are equal (and converse);
  - opposite angles in a cyclic quadrilateral are supplementary;
  - the alternate segment theorem;
- theorems on intersecting chords where the chords intersect inside or outside the circle (as secants), as well as the limiting case, where one of the lines is a tangent.

Coordinate geometry

This topic will include:

- pythagoras theorem and its application to finding the distance between two points;
- calculation of coordinates of the mid-point of a line segment;
- gradients of parallel and perpendicular lines;
- finding equations of straight lines (including vertical lines) from given information;
- cutting a line segment internally and externally in a given ratio;
- application of coordinate geometry: for example, design, orienteering, navigation and geometrical proofs.

Vectors

This topic will include:

- concept of the position vector of a point in the cartesian plane;
- the representation of plane vectors as ordered pairs (a, b);
- plane vectors as directed line segments;
- the representation of a vector (a, b) in the form $a\underset{\sim}{i} + b\underset{\sim}{j}$ where $\underset{\sim}{i}$ and $\underset{\sim}{j}$ are the standard orthogonal unit vectors;
- the magnitude of a plane vector (a, b) and its calculation;
- addition of plane vectors, using components or the parallelogram rule;
- simple vector algebra (addition, subtraction, multiplication by a scalar);
- applications of vectors; for example, geometric proofs, orienteering, navigation, and statics.

Trigonometric ratios and their applications

This topic will include:

- right-angled triangles and solutions to problems involving right-angled triangles using sine, cosine and tangent;
- the relationships $\sin^2(\theta) + \cos^2(\theta) = 1$, $\cos(\theta) = \sin(90^\circ - (\theta))$ and $\sin(\theta) = \cos(90^\circ - (\theta))$;
- two-dimensional applications including angles of depression and elevation;
- exact values of sine, cosine and tangent for 30°, 45° and 60°;
- solution of triangles by the sine and cosine rules;

- areas of triangles, including the formula $A = \sqrt{s(s-a)(s-b)(s-c)}$;
- circle mensuration: radian measure, arc length, areas of sectors and segments;
- applications, for example, navigation and surveying in simple contexts.

OUTCOMES

For each unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass all of the selected areas of study for each unit.

Outcome 1

On completion of this unit the students should be able to define and explain key concepts in relation to the topics from the selected areas of study, and apply a range of related mathematical routines and procedures.

Arithmetic

To achieve this outcome the student will draw on knowledge and related skills outlined in area of study 1.

Key knowledge

This knowledge includes

- representations of number in various structures and contexts, such as matrices, number systems and sequences;
- knowledge of special forms such as identity, inverse, conjugate, limit value;
- operations on number and algorithms for computation in a variety of contexts;
- strategies for approximation and estimation of the results of computation.

Key skills

These skills include the ability to

- define and represent number in various structures and contexts such as integer, rational, real and complex number systems, ordered sets of numbers such as sequences and series and matrices;
- identify and determine special forms such as identity, inverse, conjugate, reciprocal, limit value;
- perform computations and apply algorithms in various structures and contexts and interpret results;
- form estimates for the results of computations and calculate approximate results to a specified accuracy.

Data analysis and simulation

To achieve this outcome the student will draw on knowledge and related skills outlined in area of study 2.

Key knowledge

This knowledge includes

- representations of different data types;
- key statistics and their interpretation in various contexts;
- the notion of randomness;
- uses of simulation models.

Key skills

These skills include the ability to

- represent different types of data using a variety of graphs and/or diagrams;
- represent events and event spaces using graphs, lists and tables and/or diagrams;
- generate random numbers or a random sequence of events;
- estimate likely frequency of outcomes involving multiple events from simulation results.

Algebra

To achieve this outcome the student will draw on knowledge and related skills outlined in area of study 3.

Key knowledge

This knowledge includes

- symbols and formulas used to express linear or non-linear algebraic relations;
- laws and rules for symbolic manipulation of these expressions in various structures;
- types of problems in various contexts where algebra can be used to express relationships and obtain solutions.

Key skills

These skills include the ability to

- use symbols and formulas to construct expressions involving linear or non-linear algebraic relations;
- use algebraic laws and rules to manipulate these expressions correctly in various structures;
- write algebraic expressions and use them to solve a variety of problems in different contexts.

Graphs of linear and non-linear relations

To achieve this outcome the student will draw on knowledge and related skills outlined in area of study 4.

Key knowledge

This knowledge includes

- the graphical forms of linear and/or non-linear relations and their key features;
- methods for fitting curves to a set of points;
- contexts for which particular types of relations can be used to model and solve problems;
- definition of functions and relations and their key properties.

Key skills

These skills include the ability to

- find equations which represent particular relations or functions and use these relations to find values for the variables involved;
- represent these functions and relations using suitable graphs, and clearly identify key features of these graphs;
- use linear and/or non-linear functions and relations to model data in a variety of different situations, and solve particular problems related to those situations.

Decision and business mathematics

To achieve this outcome the student will draw on knowledge and related skills outlined in area of study 5.

Key knowledge

This knowledge includes

- key terms and definitions used in graphical and/or algebraic formulation of optimisation and decision-making problems;
- procedures and algorithms for solving optimisation and decision-making problems;
- contexts in which relevant optimisation and decision problems occur.

Key skills

These skills include the ability to

- formulate and construct diagrams, graphs, and/or algebraic representations in application contexts;
- apply procedures, algorithms and carry out computations to solve problems in application contexts;
- interpret solutions in contexts.

Geometry and trigonometry

To achieve this outcome the student will draw on knowledge and related skills outlined in area of study 6.

Key knowledge

This knowledge includes

- key geometric and/or trigonometric terms, properties, relationships and theorems as they apply to related structures, objects, constructions and operations;
- algorithms and computations for solving geometric and/or trigonometric problems;
- standard application contexts for geometric structures, objects, constructions and manipulations.

Key skills

These skills include the ability to

- define key geometric and/or trigonometric terms, relationships, objects and constructions;
- apply key concepts and theorems to solve geometric and/or trigonometric problems;
- use geometric and/or trigonometric structures, objects, properties and constructions, operations or manipulations to model a variety of application contexts.

Outcome 2

On completion of this unit the student should be able to apply mathematical processes in non-routine contexts, and analyse and discuss these applications of mathematics in at least three areas of study.

To achieve this outcome the student will draw on knowledge and related skills outlined in at least three areas of study.

Key knowledge

This knowledge includes

- the application of mathematical content from one or more areas of study in a given context for investigation;
- specific and general formulations of concepts used to derive results for analysis within a given context for investigation;
- the role of examples, counter-examples and general cases in developing mathematical analysis;
- inferences from analysis and their use to draw valid conclusions related to a given context for investigation.

Key skills

These skills include the ability to

- specify the relevance of key mathematical content from one or more areas of study to the investigation of various questions in a given context;
- develop mathematical formulations of specific and general cases used to derive results for analysis within a given context for investigation;
- make inferences from analysis and use these to draw valid conclusions related to a given context for investigation;
- communicate conclusions using both mathematical expression and everyday language; in particular, the interpretation of mathematics with respect to the context for investigation.

Outcome 3

On completion of this unit the student should be able to use technology to produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches in at least three areas of study.

To achieve this outcome the student will draw on knowledge and related skills outlined in at least three areas of study.

Key knowledge

This knowledge includes

- exact and approximate technological specification of mathematical information such as numerical data, graphical forms and the solutions of equations;
- domain and range requirements for the technological specification of graphs of functions and relations;
- the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions or equations;
- the similarities and differences between formal mathematical expressions and their representation in various technology applications;
- the appropriate selection of a technology application in a variety of mathematical contexts.

Key skills

These skills include the ability to

- distinguish between exact and approximate presentations of mathematical results, and interpret these results to a specified degree of accuracy;
- produce results using technology which identify examples or counter-examples for propositions;
- produce tables of values, families of graphs or collections of other results which support general analysis in problem-solving, investigative or modelling contexts;

- use appropriate domain and range technological specifications which illustrate key features of graphs of functions and relations;
- identify the relation between numerical, graphical and symbolic forms of information about functions and equations, and the corresponding features of those functions or equations;
- specify the similarities and differences between formal mathematical expressions and their representation in various technology applications;
- make appropriate selections for technology applications in a variety of mathematical contexts, and provide a rationale for these selections;
- relate the results from a particular application to a mathematical task (investigative, problem solving or modelling).

ASSESSMENT

The award of satisfactory completion for a unit is based on a decision that the student has demonstrated achievement of the set of outcomes specified for the unit. This decision will be based on the teacher's assessment of the student's overall performance on assessment tasks designated for the unit.

The key knowledge and skills listed for each outcome should be used as a guide to course design and the development of learning activities. The key knowledge and skills do not constitute a checklist and such an approach is not necessary or desirable for determining the achievement of outcomes. The elements of key knowledge and skills should not be assessed separately.

Assessment tasks must be a part of the regular teaching and learning program and must not unduly add to the workload associated with that program. They must be completed mainly in class and within a limited timeframe. Teachers should select a variety of assessment tasks for their assessment program to reflect the key knowledge and skills being assessed and to provide for different learning styles.

For this unit students are required to demonstrate achievement of three outcomes. As a set these outcomes encompass all areas of study.

Demonstration of achievement of Outcomes 1 and 2 must be based on the student's performance on a selection of assessment tasks. Where teachers allow students to choose between tasks they must ensure that the tasks they set are of comparable scope and demand.

Demonstration of achievement of Outcome 1 must be based on a selection of the following tasks:

- assignments;
- tests;
- summary or review notes.

Demonstration of achievement of Outcome 2 must be based on a selection of the following tasks:

- projects;
- short written responses;
- problem-solving tasks;
- modelling tasks.

These tasks may also have relevance to the assessment of Outcome 1.

For each unit, demonstration of achievement of Outcome 3 must be based on the student's performance on a selection of tasks completed in demonstrating achievement of Outcomes 1 and 2, which incorporate the effective and appropriate use of technology in contexts related to topics in the selected material from the areas of study.

Advice for teachers (General Mathematics Units 1 and 2)

DEVELOPING A COURSE

A course outlines the nature and sequence of teaching and learning necessary for students to demonstrate achievement of the set of outcomes for a unit. The areas of study describe the learning context and the knowledge required for the demonstration of each outcome. Outcomes are introduced by summary statements and are followed by the key knowledge and skills which relate to the outcomes.

Teachers must develop courses that include appropriate learning activities to enable students to develop the knowledge and skills identified in the outcome statements in each unit.

For Units 1 and 2, teachers must select assessment tasks from the list provided. Tasks should provide a variety and the mix of tasks should reflect the fact that different types of tasks suit different knowledge and skills, and different learning styles. Tasks do not have to be lengthy to make a decision about student demonstration of achievement of an outcome.

SAMPLE COURSES

Following are four sample course outlines which meet the needs and interests of different groups of students. In constructing courses, teachers may well choose to incorporate more areas of study and/or topics studied in a unit than are illustrated in the sample courses.

Sample course 1: A General Mathematics course with a financial/business orientation

Unit 1		Unit 2	
Area of study	Topic	Area of study	Topic
Arithmetic	*Integer and rational number systems*	Decision and business mathematics	*Financial arithmetic*
Data analysis and simulation	*Univariate data*	Data analysis and simulation	*Bivariate data*
Graphs of linear and non-linear relations	*Linear graphs and modelling*	Decision and business mathematics	*Linear programming*
Algebra	*Linear relations and equations*	Arithmetic	*Matrices*
Data analysis and simulation	*Simulation*	Another topic selected as appropriate if time permits	
4 areas of study, 5 topics		3 areas of study (possibly 4), 4 topics (possibly 5)	

For students with an interest in art or design, the topics selected from the Arithmetic area of study could be replaced by topics selected from the Geometry and trigonometry area of study, and/or included as an additional topic.

Sample course 2: A General Mathematics course as preparation for Further Mathematics Units 3 and 4 with, for example, 'Networks and decision mathematics', 'Geometry and trigonometry' and 'Matrices' as the three selected modules.

Unit 1		Unit 2	
Area of study	Topic	Area of study	Topic
Data analysis and simulation	*Univariate data*	Geometry and trigonometry	*Trigonometric ratios and their applications*
Data analysis and simulation	*Bivariate data*	Geometry and trigonometry	*Shape and measurement*
Graphs of linear and non-linear relations	*Linear graphs and modelling*	Geometry and trigonometry	*Geometry in two and three dimensions*
Algebra	*Linear relations and equations*	Decision and business mathematics	*Networks*
Decision and business mathematics	*Financial arithmetic*	Arithmetic	*Matrices*
4 areas of study, 5 topics		3 areas of study, 5 topics	

If, for example, the 'Graphs and relations' module was to be selected instead of the 'Networks and decision mathematics' module in the intended Further Mathematics course, two of the 'Geometry and trigonometry' topics in Unit 2 could be replaced by the topics Linear programming and Variation.

Sample course 3: A General Mathematics course, which, in conjunction with Mathematical Methods or Mathematical Methods (CAS) Units 1 and 2, would provide a possible preparation for Mathematical Methods or Mathematical Methods (CAS) Units 3 and 4, respectively.

Unit 1		Unit 2	
Area of study	Topic	Area of study	Topic
Data analysis and simulation	*Univariate data*	Data analysis and simulation	*Simulation*
Geometry and trigonometry	*Coordinate geometry*	Geometry and trigonometry	*Shape and measurement*
Graphs of linear and non-linear relations	*Linear graphs and modelling*	Geometry and trigonometry	*Trigonometric ratios and their applications*
Algebra	*Linear relations and equations*	Algebra	*Non-linear relations and equations*
Graphs of linear and non-linear relations	*Variation*	Arithmetic	*Sequences and series*
4 areas of study, 5 topics		4 areas of study, 5 topics	

There are alternative topics which could be selected for either Unit 1 or Unit 2, for example, Matrices, Real and complex number systems or Sketching and interpreting linear and non-linear graphs.

Sample course 4: A General Mathematics course, which, in conjunction with Mathematical Methods Units 1 and 2 or Mathematical Methods (CAS) Units 1 and 2, prepares students for Specialist Mathematics Units 3 and 4.

Unit 1		Unit 2	
Area of study	Topic	Area of study	Topic
Arithmetic	*Real and complex number systems*	Graphs of linear and non-linear relations	*Sketching and interpreting linear and non-linear graphs*
Geometry and trigonometry	*Coordinate geometry*	Geometry and trigonometry	*Geometry in two and three dimensions*
Geometry and trigonometry	*Trigonometric ratios and their applications*	Geometry and trigonometry	*Vectors*
Algebra	*Non-linear relations and equations*	Graphs of linear and non-linear relations	*Kinematics*
Arithmetic	*Matrices*	Algebra	*Algebra and logic*
3 areas of study, 5 topics		3 areas of study, 5 topics	

The emphasis in this course is to provide opportunities for access to a rigorous exploration of aspects of mathematical structure and proof. Teachers may wish to include some ‘Data analysis and simulation’ topics or select them as alternative topics to some of those listed above.

Sample teaching sequence

The following teaching sequence for Sample course 2 is based on the assumption of 18 effective teaching weeks in a semester. Teachers will need to vary this sequence based on their selection of areas of study and topics. The amount of time taken to cover a particular topic, and the sequence of topic coverage, will vary depending on teaching approaches and the mathematics background of the group of students.

Unit 1		Unit 2	
Topic	Weeks	Topic	Weeks
Financial arithmetic	3	Shape and measurement	4
Univariate data	4	Geometry in two and three dimensions	4
Linear graphs and modelling	4	Trigonometric ratios and their applications	4
Linear relations and equations	3	Networks	3
Bivariate data	4	Matrices	3

USE OF INFORMATION AND COMMUNICATIONS TECHNOLOGY

In designing courses and developing learning activities for General Mathematics Units 1 and 2, teachers should make use of applications of information and communications technology and new learning technologies, such as computer-based learning, multimedia and the World Wide Web, where applicable to teaching and learning activities.

KEY COMPETENCIES AND EMPLOYABILITY SKILLS

Students undertaking the following types of assessment, in addition to demonstrating their understanding and mastery of the study, typically demonstrate the following key competencies and employability skills.

Assessment task	Key competencies and employability skills
Assignments	Use of information and communications technology
Tests	Self management, use of information and communciations technology
Summary or review notes	Self management
Projects	Communication, team work, self management, planning and organisation, use of information and communciations technology, initiative and enterprise
Short written responses	Communication, problem solving
Problem-solving tasks	Communication, problem solving, team work, use of information and communciations technology
Modelling tasks	Problem solving, planning and organisation, use of information and communciations technology

LEARNING ACTIVITIES

Examples of learning activities for each unit are provided in the following sections. The examples that make use of information and communications technology are identified by this icon.

Units 1 and 2: General Mathematics

Outcome 1	Examples of learning activities
Define and explain key concepts in relation to the topics from the selected areas of study, and apply a range of related mathematical routines and procedures.	skills practice on standard mathematical routines through work on an appropriate selection of exercises (e.g. computations and applications associated with different number systems) construction of summary or review notes related to a topic or area of study (e.g. the use of coordinate geometry and/or vectors in situations where information can be readily specified in coordinate form) assignments structured around the development of samples cases of standard applications of mathematical skills and procedures in readily recognisable situations (e.g. the use of sample proportions to provide information about the population from which the sample was drawn, such as polling of voting intentions) exercises such as the identification of key statistics related to interpretation of samples of data from a particular context (e.g. rental prices in different suburbs) in univariate statistics, or patterns in data sets which suggest suitable relationships to select for transformation of data to linearity in variation (where the information used may have been produced by relevant technology)

Outcome 2	Examples of learning activities
Apply mathematical processes in non-routine contexts, and analyse and discuss these applications of mathematics in at least three areas of study.	investigative projects; e.g. exploring the construction of different possible lines of good fit, and their closeness of fit, for economic data, or the application of matrix algebra in simple price, stock and cost situations, or the logistic function as an example of a sequence generated by fixed point iteration collections of related problem-solving tasks, e.g. determining optimal values in network applications (minimising cost of cable connections), or the use of trigonometry in surveying tasks (constructing a new series of walks in a national park, orienteering) modelling tasks, e.g. the use of linear programming techniques in running a small manufacturing business a set of applications questions requiring analysis and extended response related to a particular context, e.g. the use of geometry in architecture and design, a report on item response analysis for a collection of multiple-choice questions (this would involve a clear and detailed explanation as to why other alternatives did not provide an appropriate response to the question asked, for example, why is a particular simulation suitable for a given set of conditions, and related probabilities) presentation on research into a particular application of mathematics, such as the use of boolean algebra in electrical circuit theory, devising a simple calculator (this could involve the use of presentation information technologies)

Outcome 3

Use technology to produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches in at least three of the areas of study.

Examples of learning activities

The use of technology should be developed as an integral part of the range of learning activities for Outcomes 1 and 2. In particular, these can include:

investigations based on the production of collections of diagrams which support the development of geometric results or theorems (e.g. using CAD or a dynamic geometry system) and their possible application in various contexts or the exploration of graphical images produced by using non-cartesian coordinate based specification of relations (e.g. using a computer algebra system)

activities based on the application of a variety of different techniques to a particular type of task, e.g. different technologies can be used to implement various graphical, tabular, numeric and analytic approaches associated with solving appropriate equations involving both linear and non-linear relations

'workshop' type sessions where, for example, spreadsheets are used to explore various finance-related situations (such as the performance of share market investment portfolios) and/or the behaviour of different types of sequences and series

APPROACHES TO COURSE DEVELOPMENT

The following are examples of approaches to course development, including how the use of various technologies may be integrated with the development of key knowledge and skills, and approaches to assessment for selected topics from the sample courses.

Linearity, functions, graphs and equations

There are two main situations where linear functions are used to model trends in a set of data: the first is when the data exhibits essentially linear behaviour (that is, constant difference between consecutive values of the dependent or 'output' variable), the second is when a scatterplot of the data exhibits some linearity, and where a reasonable 'line of best fit' is possible. In the latter context, a line of best fit is obtained by sight or, alternatively, by using various statistical fitting methods. In each case the coordinates of two key points are used to determine a gradient for the line, and the coordinates of a point which is required to lie on the line are used to determine the value of the y axis intercept.

If the coordinates of two points are:

$$(x_1, y_1) \text{ and } (x_2, y_2)$$

then the line which contains these two points has the equation:

$$y = mx + c$$

and gradient:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

The value of the y axis intercept can then be evaluated as:

$$c = y_1 - mx_1$$

This can be readily done by hand if the coordinates have integer values, to obtain an equation where m and c will be rational numbers. Given:

$$m = \frac{a}{b}$$

a is interpreted as the 'rise' and *b* as the 'run' of the gradient. This and the value of *c* can be used to both sketch the graph of the function and interpret such sketches. The gradient *m*, is interpreted in modelling contexts as being representative of the constant rate of change in the variable *y* with respect to the variable *x*. If the values of the coordinates involved are decimal values, a calculator is generally used to find *m* and *c*. An alternative method for finding the values of *m* and *c* is to solve the system of simultaneous equations:

$y_1 = mx_1 + c$ and $y_2 = mx_2 + c$

for *m* and *c*. This approach can be generalised to quadratic and other polynomial functions. The use of technology to solve the system of equations (using matrices or built-in equation solver functions) may or may not be appropriate, depending on the nature of the coordinates. Where the process is to be repeated several times, and analysis is intended to focus on the change in key features of the graph (with respect to variation of coordinate values), a graphics calculator, CAS or spreadsheet can be a useful support for developing student understanding of the behaviour of linear functions in this form.

Various dynamic geometry software packages are capable of plotting pairs of points and the line segments joining them on grids and coordinate axes. The values of rise, run, gradient, angle to the horizontal and *y* axis intercept can be calculated and automatically re-calculated as the position of a point is varied, enabling students to see how these quantities change with changes in position of a point. Alternatively, function graphers and computer algebra systems can be used to define a general linear function in terms of the parameters *m* and *c* and an investigation of the graphs of these functions and key features such as coordinates of axis intercepts undertaken by systematic variation of these parameters. Whichever approach or combination of approaches is taken, students need to develop a sound understanding of the relation between the gradient and *y* axis intercepts of a linear function, the graph of this function, and modelling interpretations of these parameters to support work on finding trend lines of best fit for data represented in a 'linear looking' scatterplot.

Correlation values provide a measure, on a scale from –1 to 1, of the degree of linear association between data relating two sets of variables. Some correlation measures are relatively robust (changes in the location of a point or points has little effect on the correlation, such as quadrant, or *q*, correlation) while others are very sensitive (such as *Pearson's correlation*, the *r* correlation measure produced by calculators). A qualitative analysis of a scatterplot can provide the basis for estimating an approximate correlation value. Graphics calculators, spreadsheets and statistical software are very useful for developing conceptual understanding of the behaviour of correlation in terms of the distribution of points on a scatterplot. A useful activity is to start with a set of 20 points, originally all lying on a straight line (e.g. with a perfect positive correlation of 1), and then by arbitrarily varying the location of one point, explore the change in correlation. This can be extended to a fixed subset of points from the original collection (to what extent does it matter from where these points are selected?) and finally rearrangements of all points in the original set. Students could be asked to obtain arrangements with a particular correlation value, for example a correlation of around –0.25.

Students could investigate suitable application contexts where the work on correlation is combined with work on regression analysis. This could include comparison of different approaches to finding lines of 'best fit' and their usefulness as predictors in different situations. Data can be provided for suitable contexts, or students could be asked to *hypothesise* situations in which they believe a certain correlation value is likely to occur (e.g. a slight negative correlation). This analysis will then involve discussing the reasons which led to the proposed hypothesis, and hence the relationship between correlation and cause. An investigative assessment task based on this type of activity could cover two areas of study, through the Linear graphs and modelling topic from the 'Graphs of linear and non-linear relations' area of study and the topic Bivariate data from the 'Data analysis and simulation' area of study. If this activity involves the use of technology, it will substantively address key knowledge and skills for Outcomes 2 and 3. Another application task from a third area of study, for example an assignment on modelling investment strategies in the Financial arithmetic topic from the 'Decision and business mathematics' area of study could, in conjunction with the previous task, provide the basis for assessing student achievement of Outcomes 2 and 3. Similar combinations of tasks can be organised for other selections of topics and areas of study. The assessment of student achievement of Outcome 1 could be based on a collection of approximately four tests or assignments related to topics from the selected areas of study as well as summary or review notes where appropriate. This may include two or three standard 'facts and skills' tests that incorporate some or all of multiple-choice, short-answer or extended-response items and one or two exercise type assignments. For example, there could be tests on the topics Univariate data, Bivariate data, and Linear graphs and modelling, with an assignment on Linear relations and equations. This assignment could be based around exploration of analytical, graphical and numerical approaches to solving equations and simultaneous equations. Graphics calculators, spreadsheets, function graphers and computer algebra systems could all be used, in particular when comparing exact solutions to approximate values for solutions, and observing the effect of variation in gradient and intercept on the location of the point of intersection of the lines involved, when the lines do in fact intersect. For example, the solution of the equation:

$ax + b = c$

can be seen as a special case of the simultaneous linear equation system:

$y = ax + b$ and $y = mx + c$

where m has the value 0. Hence a single equation is interpreted as the intersection of a horizontal line (gradient, $m = 0$) with the line $y = ax + b$. It may be appropriate for students to produce a summary or a review of key definitions and worked examples for typical problems.

Practical computation using matrices

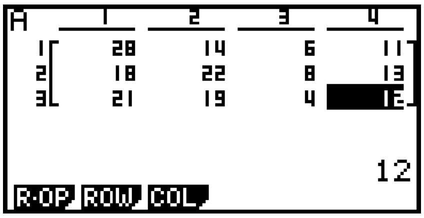
Consider three stores A-Mart, B-Mart, and C-Mart, all of which stock Item 1, Item 2, Item 3 and Item 4 (these could be soap, shampoo or any common product in the store stock inventory). To analyse the state of the market, the sales of these items are monitored over a period of several weeks. Matrices are rectangular arrays of data which can be used to support this analysis. The figures for the first week of sales are given in the following table:

Table 1 – Week 1 Sales

Store	Item 1	Item 2	Item 3	Item 4
A-Mart	28	14	6	11
B-Mart	18	22	8	13
C-Mart	21	19	4	12

This information can be stored as a 3×4 matrix, which can later be manipulated using matrix algebra to provide other sorts of information. We will call this matrix A.

This data can be stored as a calculator or computer array. While calculations for problems involving matrices of small order can be readily manipulated by hand, the relevant computations quickly become very time consuming for higher order matrices. In this example a graphics calculator has been used; however, spreadsheets and computer algebra systems also have efficient functionality for dealing with matrix algebra more generally.

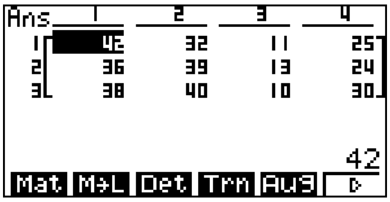


The sale data for Week 2 is shown in the table below:

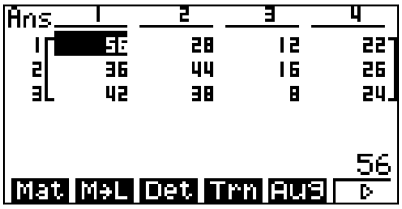
Table 2 – Week 2 Sales

Store	Item 1	Item 2	Item 3	Item 4
A-Mart	14	18	5	14
B-Mart	18	17	5	11
C-Mart	17	21	6	18

This data can similarly be stored in matrix *B*. The total sales (for each store and item) across both weeks can be calculated as *A* + *B* to obtain the following matrix:



If the Week 1 sales were doubled, then 2 A can be evaluated to obtain:



Consider price List 1 of recommended retail prices (RRP) for each of the items as given in the table below:

Table 3 – List 1 RRP

Item	Price
1	1.78
2	0.93
3	5.80
4	2.33

This can be represented as a 4×1 cost matrix C. Matrix multiplication is defined in such a way that it automatically computes the value for the total sales across all items for each store in Week 1. The answer matrix $T = A \times C$ will then need to be a 3×1 matrix, listing the total value of all sales for each store. Hence:

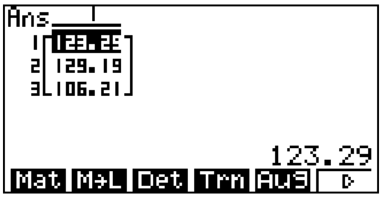
$$T = \begin{bmatrix} 28 & 14 & 6 & 11 \\ 18 & 22 & 8 & 13 \\ 21 & 19 & 4 & 12 \end{bmatrix} \times \begin{bmatrix} 1.78 \\ 0.93 \\ 5.80 \\ 2.33 \end{bmatrix}$$

To evaluate the first value for T which represents the total sales for the A-Mart store

$28 \times 1.78 + 14 \times 0.93 + 6 \times 5.80 + 11 \times 2.33 = 123.29$ is calculated. This gives the first value in the matrix T:

$$\begin{bmatrix} A - Mart : total \\ B - Mart : total \\ C - Mart : total \end{bmatrix} = \begin{bmatrix} 123.29 \\ \\ \end{bmatrix}$$

Computing $T = A \times C$ gives:



If the RRP remains the same for Week 2 the corresponding total sales matrix can also be computed.

The combined sales for Week 1 and Week 2, for this common RRP, can be evaluated in two different ways:

$(A + B) \times C$ or $(A \times C) + (B \times C)$

which exemplifies the distributive property of multiplication over addition for these matrices. There are many possible variations on this theme. Scalar multiples of the appropriate matrices can be used to represent price discounts and corresponding percentage sale increases. The differences between old and new sales can be computed. Alternatively, List 1 RRP could apply to Week 1 sales and List 2 RRP (shown in Table 4 below) applies to Week 2 sales.

Table 4 – List 2 RRP

Item	Price
1	1.58
2	1.11
3	5.50
4	2.49

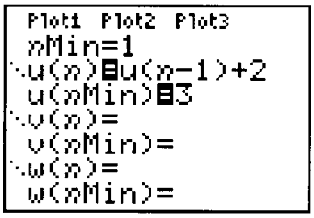
The sales performance for each of the stores with respect to the different RRP Lists (List 1 and List 2) can also be obtained by matrix multiplication. The sales for both weeks can be compared using a 4×2 matrix which combines both RRP Lists. These can be obtained by computing the matrix product:

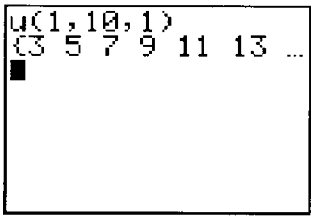
$$\begin{bmatrix} 28 & 14 & 6 & 11 \\ 18 & 22 & 8 & 13 \\ 21 & 19 & 4 & 12 \end{bmatrix} \times \begin{bmatrix} 1.78 & 1.58 \\ 0.93 & 1.11 \\ 5.80 & 5.50 \\ 2.33 & 2.49 \end{bmatrix}$$

A range of different scenarios lead students through a practical introduction to key elements of matrix algebra, while combining experience of by hand manipulation with a broader exploration of key elements of matrix algebra with the assistance of technology. Variations on such an approach can be used to relate learning activities to each of the three outcomes.

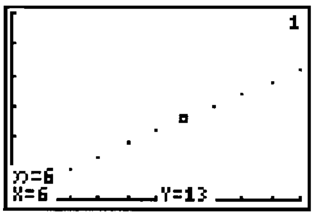
Generating sequences using technology

Technology can be used to generate sequences recursively and to plot corresponding graphs in activities that draw on key knowledge and skills from all three outcomes. For example, the arithmetic sequence where the first term is $a = 3$ and each term is obtained by an addition of constant difference $d = 2$ to the previous terms, can be generated recursively, where $u(1) = a$ and $u(n) = u(n-1) + d$, using a graphics calculator as shown below:

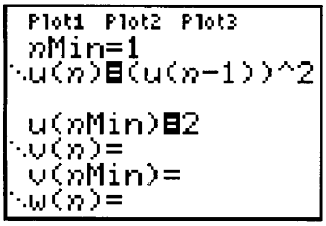




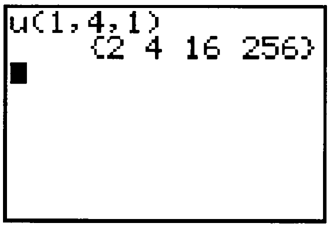
The built-in calculator function $u(m, n, s)$ starts at the m^{th} term and lists n values of the sequence in steps of s. The following graph also shows the specific point corresponding to the 6^{th} term in the sequence, (6, 13):



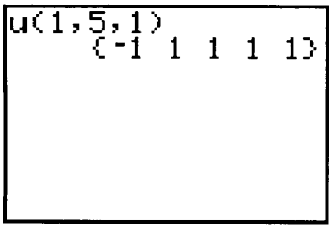
This approach can also be used to generate other sequences by fixed point iteration such as $u(1) = a$, $u(n) = (u(n-1))^2$ for different initial values of a:



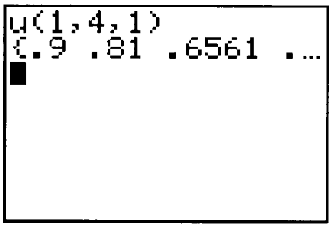
$a = 2$



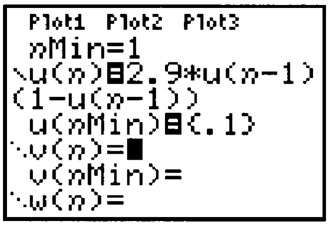
$a = -1$

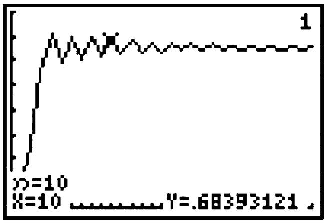


$a = 0.9$



The logistic function, where $u(1) = a$, and $u(n) = ku(n-1)(1 - u(n-1))$ has applications to population modelling and can be used to illustrate sensitivity in terms of the initial condition and defining parameter. The following graph shows the case where $a = 0.1$ and $k = 2.9$:





An assessment task related to this material could be an investigation on the nature of the graph produced for different values of these for the logistic function, leading to the construction of web-diagrams and consideration of convergent, divergent, oscillating and chaotic behaviour.

APPROACHES TO ASSESSMENT

In General Mathematics Units 1 and 2 teachers must select assessment tasks from those designated for these units. Assessment tasks are likely to assess key knowledge and skills related to more than one outcome; however, certain types of task will be especially suited to providing students with the opportunity to demonstrate key knowledge and skills related to achievement of a particular outcome or outcomes. There should be a clear link between the nature of learning activities and corresponding elements of assessment tasks.

A summary of a possible assessment program for Sample course 2 is given below

Unit 1		Unit 2	
Univariate data	Test	Trigonometric ratios and their applications	Test
Bivariate data	Test, investigative task	Shape and measurement	Test, design problem task
Linear graphs and modelling	Test	Geometry in two and three dimensions	Assignment
Linear relations and equations	Assignment	Networks	Modelling task
Financial arithmetic	Review and summary of sample problems	Matrices	Assignment

Other combinations of tasks are possible, both in terms of the intended student group and the particular implementation of General Mathematics. When tests and assignments are used, teachers may decide to specify or construct components of these tasks where graphics calculators, CAS or other technologies are not to be used (or not able to be used), while requiring student use of technology on other components of the assessment. Summary or review notes constructed from various learning activities, or possibly in preparation for each test, could be used to assess key knowledge and skills related to Outcome 1. These could include definitions of key concepts, statements of important results or formulas, and sample solutions to routine problems. Where these also incorporate samples of work which technology is used to produce (for example, tables of values, graphs or solution of equations) this would provide feedback on student achievement on related key knowledge and skills for Outcome 3.

In work on practical applications of 'Geometry and trigonometry' this could, for example, include a flow diagram for identification of types of shapes and their properties, along with procedures for identifying what information is given or to be found with respect to these shapes (right-angled, non-right-angled triangles, included or non-included angles) and where decisions are to be made about the order in which information is to be obtained in the solution process. A worked collection of 'typical problem' cases could be related to key knowledge and skills for Outcome 1, while an assignment based on application of this to unseen examples as part of usual practice activity could be related to key knowledge and skills for Outcome 2.

Alternatively, an assignment based on a collection of questions requiring progressively more complex and detailed analysis could be used to provide feedback on key knowledge and skills for Outcomes 2 and 3. For example, an assignment could be structured around a collection of contexts involving variation. This may initially involve analysis of which type of variation matches up with a range of tables of values, and subsequently lead to investigating which of several possible models best fits a limited collection of empirical data points derived from experiment (such as the law of allometry in science). Graphics calculators, spreadsheets, statistical software or computer algebra systems can all be used to explore readily the behaviour of the distribution in terms of these parameters through rules, tables and graphs. This will provide students with the opportunity to demonstrate understanding of key knowledge and skills related to Outcome 3.

Units 1 and 2: Mathematical Methods

Unit 1: Mathematical Methods

Mathematical Methods Units 1 and 2 are designed as preparation for Mathematical Methods Units 3 and 4. The areas of study for Unit 1 are ‘Functions and graphs’, ‘Algebra’, ‘Rates of change and calculus’ and ‘Probability’. At the end of Unit 1, students will be expected to have covered the material outlined in each area of study given below, with the exception of ‘Algebra’ which should be seen as extending across Units 1 and 2. This material should be presented so that there is a balanced and progressive development of skills and knowledge from each of the four areas of study with connections among and across the areas of study being developed consistently throughout both Unit 1 and Unit 2.

Students are expected to be able to apply techniques, routines and processes involving arithmetic, algebraic manipulation, equation solving, graph sketching, differentiation and integration with and without the use of technology, as applicable. Students should have facility with relevant mental and by hand approaches in simple cases.

The appropriate use of technology to support and develop the teaching and learning of mathematics is to be incorporated throughout the unit. Students are encouraged to use graphics calculators, spreadsheets, statistical software, graphing packages or computer algebra systems as applicable across the areas of study, both in the learning of new material and the application of this material in a variety of different contexts.

Familiarity with determining the equation of a straight line from combinations of sufficient information about points on the line or the gradient of the line and familiarity with pythagoras theorem and its application to finding the distance between two points is assumed. Students should also be familiar with quadratic and exponential functions, algebra and graphs, and basic concepts of probability.

AREAS OF STUDY

1. Functions and graphs

This area of study covers the graphical representation of functions of a single real variable and the study of key features of graphs of functions such as axis intercepts, domain (including maximal domain) and range of a function, asymptotic behaviour and symmetry.

This area of study will include:

- distance between two points in the cartesian plane, coordinates of the midpoint of a line segment, and gradients of parallel and perpendicular lines;
- use of the notation $y = f(x)$ for describing the rule of a function and evaluation of $f(a)$, where a is a real number or a symbolic expression;

- graphs and their use to express and interpret relations and inverse relations;
- graphs of power functions $y = x^n$ for $n \in N$ and $n = -1, -2, \frac{1}{2}$ and transformations of these to the form $y = a(x + b)^n + c$ where a, b and $c \in R$;
- sketch graphs of linear, quadratic and cubic polynomial functions (with approximate location of stationary points) for rules given in factorised form and in expanded form (including the use of simple transformations);
- sketch graphs of quartic polynomial functions with rule given in factorised form (with approximate location of stationary points);
- the 'vertical line test' and its use to determine whether a relation is a function, including its application to circles with equations of the form $(x - a)^2 + (y - b)^2 = r^2$ as examples of relations that are not functions.

2. Algebra

This area of study supports material in the 'Functions and graphs', 'Rates of change and calculus' and 'Probability' areas of study and this material is to be distributed between Unit 1 and Unit 2. In Unit 1 the focus is on the algebra of polynomial functions to degree 4. Content introduced in Unit 1 may be revised and further developed in Unit 2.

This area of study will include:

- substitution into formulas and rearrangement of formulas;
- identification of key features of polynomials, including coefficients and degree;
- expansion of linear, quadratic and cubic polynomial expressions from factors;
- factorisation: quadratic trinomials, the remainder and factor theorems and factorisation of a cubic polynomial expression with at least one linear factor of the form $ax + b$ where a and b are integers;
- quadratic equations: obtaining rational solutions or approximations to solutions by systematic trial and error, by graphing, by simple iteration; obtaining rational and irrational solutions by completion of the square and by the quadratic formula;
- use and interpretation of the discriminant of the quadratic formula to identify the number of real solutions to a quadratic equation;
- application of completion of the square method to finding maximum or minimum values of quadratic functions;
- cubic equations and their solution by any of the following methods: graphical (including cases which do not have three solutions); numerical (systematic trial and error) and analytical, for example, factorisation of cubic polynomials where there is at least one integer solution;
- the connection between factors of $f(x)$, solutions of the equation $f(x) = 0$ and the horizontal axis intercepts of the graph of the function f;
- solution of simultaneous equations: two linear equations and one linear equation and one quadratic equation by numerical, graphical and analytical methods;
- development of polynomial models, for example, by the use of finite difference tables or solution of a system of simultaneous linear equations obtained from values of a function, or a simple combination of values of a function;
- index laws and logarithm laws, including their application to the solution of simple exponential equations.

3. Rates of change and calculus

This area of study covers constant and average rates of change and an informal treatment of instantaneous rate of change of a function in familiar contexts including graphical and numerical approaches to the measurement of constant, average and instantaneous rates of change.

This area of study will include:

- concepts of rates of change:
  - practical examples of instantaneous rates of change; for example, speedometer readings, revolution counters;
  - practical examples of average rates of change; for example, average speed on a bush walk, average slope of a hill from bottom to top;
- rate of change of a linear function: use of gradient as a measure of rate of change;
- graphs and the interpretation of rates of change; for example, where the rate of change is positive, negative, or zero;
- average rate of change: use of the gradient of a chord of a graph to describe average rate of change of $y = f(x)$ with respect to x, over a given interval;
- instantaneous rates of change:
  - as given by the gradient of the graph of a function at a given point;
  - linear functions and constant rate of change;
  - simple hybrid functions as examples of continuous functions without defined gradient at some point of their domain;
  - quadratic and cubic polynomial functions as examples of variable rates of change;
- relating the gradient function to features of the original function;
- applying rates of change in motion graphs:
  - construction and interpretation of displacement–time and velocity–time graphs;
  - informal treatment of the relationship between displacement-time and velocity–time graphs;
- the measurement of rates of change of polynomial functions by finding successive numerical approximations to the gradient of a polynomial function at a point by taking another point very close to it on either side, of the graph of the function and finding the gradient of the line joining the two points, and then repeating this procedure, leading to informal treatment of the gradient of the tangent as a limiting value of the gradient of a chord.

4. Probability

This area of study covers introductory probability theory, including the concept of events, probability and representation of event spaces using various forms such as lists, grids, venn diagrams, karnaugh maps, tables and tree diagrams. Impossible, certain, complementary, mutually exclusive, conditional and independent events involving one, two or three events (as applicable) including rules for computation of probabilities for compound events.

This area of study will include:

- random experiments, events and event spaces;
- probability as an expression of long run proportion;
- simulation using simple generators such as coins, dice, spinners, random number tables and technology;
- display and interpretation of results of simulations;
- probability of simple and compound events;
- lists, grids, venn diagrams, karnaugh maps and tree diagrams;

- the addition rule for probabilities;
- conditional probability, $\Pr(A|B) = \dfrac{\Pr(A \cap B)}{\Pr(B)}$;
- independence, and the multiplication rule for independent events, $\Pr(A \cap B) = \Pr(A).\Pr(B)$ when A and B are independent events.

OUTCOMES

For this unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass all of the areas of study for the unit.

Outcome 1

On completion of this unit the student should be able to define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.

To achieve this outcome the student will draw on knowledge and related skills outlined in all the areas of study.

Key knowledge

This knowledge includes

- the definition of a function, the concepts of domain, and range and notation for specification of the domain, range and rule of a function;
- equation and features of a straight line, including gradient and axis intercepts, midpoint of a line segment and parallel and perpendicular lines;
- key features and properties of power and polynomial functions and their graphs;
- the effect of transformations (dilation from the coordinate axes; reflection in the coordinate axes and translation) and simple combinations of these transformations on the graphs of linear and power functions;
- equations of circles at the origin and translations of these;
- factorisation patterns, the quadratic formula and discriminant, the remainder and factor theorems and the null factor law;
- index (exponent) laws and logarithm laws;
- average and instantaneous rates of change and their interpretation with respect to the graphs of functions;
- probability as long run proportion;
- forms of representation of an event space;
- probabilities for a given event space are non-negative and the sum of these probabilities is one;
- rules for computing probabilities of compound events from simple events, in particular cases involving mutual exclusiveness, independence and conditional probability.

Key skills

These skills include the ability to

- determine by hand the length of a line segment and the coordinates of its midpoint, the equation of a straight line given two points or one point gradient; and the gradient and equation of lines parallel and perpendicular to a given line through some other point;
- specify the rule, domain and range of a relation and identify a relation that is also a function;

- substitute integer, simple rational and exact irrational values into formulas, including the rules of functions and relations, and evaluate these by hand;
- rearrange simple algebraic equations and inequalities by hand;
- expand and factorise linear and simple quadratic expressions with integer coefficients (including difference of perfect squares and perfect squares) by hand;
- express $ax^2 + bx + c$ in completed square form by hand, where a, b and c are integers;
- express a cubic polynomial, with integer coefficients, as the product of a linear factor $(x – a)$, where a is an integer, and a quadratic factor, by hand;
- use a variety of analytic, graphical and numerical approaches, including the factor theorem, to determine and verify solutions to equations over a specified interval;
- apply index (exponent) laws and logarithm laws to manipulate and simplify expressions and solve equations involving these terms, by hand in simple cases;
- set up and solve systems of simultaneous linear equations involving up to four unknowns, by hand for a system of two equations in two unknowns;
- sketch by hand graphs of linear, quadratic and cubic polynomial functions, and quartic polynomial functions in factored form (approximate location of stationary points only for cubic and quartic functions);
- sketch by hand graphs of power functions $y = x^n$ for $n = –2, –1, \frac{1}{2}, 1, 2, 3$ and 4, and simple transformations of these, and sketch by hand circles of a given centre and radius;
- draw graphs of polynomial functions to degree 4 (*approximate* location of stationary points only for cubic and quartic functions), power functions and circles of a given centre and radius;
- describe the effect of transformations (dilation from the coordinate axes; reflection in the coordinate axes and translation), and simple combinations of these transformations, on the graphs of linear and power functions;
- use finite difference tables to obtain polynomial models for sets of data where there are constant first, second or third differences;
- use graphical, numerical and algebraic approaches to find an approximate value or the exact value (as appropriate) for the gradient of a secant or tangent to a curve at a given point;
- describe the notion of randomness and its relation to events;
- define and calculate probability as an expression of long run proportion;
- calculate probabilities for compound events by hand in simple cases;
- use rules for computing the probabilities of compound events from simple events, including the concepts of mutual exclusiveness, independence and conditional probability.

Outcome 2

On completion of this unit the student should be able to apply mathematical processes in non-routine contexts, and analyse and discuss these applications of mathematics.

To achieve this outcome the student will draw on knowledge and related skills outlined in one or more areas of study.

Key knowledge

This knowledge includes

- key mathematical content from one or more areas of study relating to a given context for investigation;
- specific and general formulations of concepts used to derive results for analysis within a given context for investigation;

- the role of examples, counter-examples and general cases in developing mathematical analysis;
- inferences from analysis and their use to draw valid conclusions related to a given context for investigation.

Key skills

These skills include the ability to

- specify the relevance of key mathematical content from one or more areas of study to the investigation of various questions in a given context;
- develop mathematical formulations of specific and general cases used to derive results for analysis within a given context for investigation;
- use a variety of techniques to verify results;
- make inferences from analysis and use these to draw valid conclusions related to a given context for investigation;
- communicate conclusions using both mathematical expression and everyday language, in particular, the interpretation of mathematics with respect to the context for investigation.

Outcome 3

On completion of this unit the student should be able to use technology to produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.

To achieve this outcome the student will draw on knowledge and related skills outlined in all the areas of study.

Key knowledge

This knowledge includes

- exact and approximate specification of mathematical information such as numerical data, graphical forms and the solutions of equations produced by the use of technology;
- domain and range requirements for the technology-based specification of graphs of functions and relations;
- the relation between numerical, graphical and symbolic forms of information about functions and equations, and the corresponding features of those functions or equations;
- the similarities and differences between formal mathematical expressions and their representation in various technology applications;
- the appropriate selection of a technology application in a variety of mathematical contexts.

Key skills

These skills include the ability to

- distinguish between exact and approximate presentations of mathematical results, and interpret these results to a specified degree of accuracy;
- produce results using technology which identify examples or counter-examples for propositions;
- produce tables of values, families of graphs or collections of other results which support general analysis in problem-solving, investigative or modelling contexts;
- use appropriate domain and range technological specifications which illustrate key features of graphs of functions and relations;
- identify the relation between numerical, graphical and symbolic forms of information about functions and equations, and the corresponding features of those functions or equations;

- specify the similarities and differences between formal mathematical expressions and their representation in various technology applications;
- make appropriate selections for technology applications in a variety of mathematical contexts, and provide a rationale for these selections;
- relate the results from a particular application to a mathematical task (investigative, problem solving or modelling).

ASSESSMENT

The award of satisfactory completion for a unit is based on a decision that the student has demonstrated achievement of the set of outcomes specified for the unit. This decision will be based on the teacher's assessment of the student's overall performance on assessment tasks designated for the unit.

The key knowledge and skills listed for each outcome should be used as a guide to course design and the development of learning activities. The key knowledge and skills do not constitute a checklist and such an approach is not necessary or desirable for determining the achievement of outcomes. The elements of key knowledge and skills should not be assessed separately.

Assessment tasks must be a part of the regular teaching and learning program and must not unduly add to the workload associated with that program. They must be completed mainly in class and within a limited timeframe. Teachers should select a variety of assessment tasks for their assessment program to reflect the key knowledge and skills being assessed and to provide for different learning styles.

For this unit students are required to demonstrate achievement of three outcomes. As a set these outcomes encompass all areas of study.

Demonstration of achievement of Outcomes 1 and 2 must be based on the student's performance on a selection of assessment tasks. Where teachers allow students to choose between tasks, they must ensure that the tasks they set are of comparable scope and demand.

Demonstration of achievement of Outcome 1 must be based on a selection of the following tasks:

- assignments;
- tests;
- summary or review notes.

Demonstration of achievement of Outcome 2 must be based on a selection of the following tasks:

- projects;
- short written responses;
- problem-solving tasks;
- modelling tasks.

Demonstration of achievement of Outcome 3 must be based on the student's performance on a selection of tasks completed in demonstrating achievement of Outcomes 1 and 2, which incorporate the effective and appropriate use of technology in contexts related to the content of the areas of study.

Unit 2: Mathematical Methods

The areas of study for Unit 2 are 'Functions and graphs', 'Algebra', 'Rates of change and calculus', and 'Probability'. At the end of Unit 2, students will be expected to have covered the material outlined in each area of study. Material from the 'Functions and graphs', 'Algebra', 'Rates of change and calculus', and 'Probability' areas of study should be organised so that there is a clear progression of knowledge and skills from Unit 1 to Unit 2 in each area of study.

Students are expected to be able to apply techniques, routines and processes involving rational and real arithmetic, algebraic manipulation, equation solving, graph sketching, differentiation and integration with and without the use of technology, as applicable. Students should be familiar with relevant mental and by hand approaches in simple cases.

The appropriate use of technology to support and develop the teaching and learning of mathematics, and in related assessments, is to be incorporated throughout the unit. Students are encouraged to use graphics calculators, spreadsheets, statistical software, graphing packages or computer algebra systems as applicable across the areas of study both in the learning of new material and the application of this material in a variety of different contexts.

AREAS OF STUDY

1. Functions and graphs

This area of study covers graphical representation of functions of a single real variable and the study of key features of graphs of functions such as axis intercepts, domain (including maximal domain) and range of a function, asymptotic behaviour, periodicity and symmetry.

This area of study will include:

- revision of trigonometric ratios and their applications to right-angled triangles, including exact values for sine, cosine and tangent of 30°, 45° and 60°;
- radians: unit circle definition, conversion between radians and degrees;
- the unit circle and definition of the sine, cosine and tangent functions as functions of a real variable, including exact values for $\frac{n\pi}{6}$ and $\frac{n\pi}{4}$, $n \in Z$
- the relationships $\sin^2(x) + \cos^2(x) = 1$ and $\tan(x) = \frac{\sin(x)}{\cos(x)}$
- symmetry properties, complementary relations and periodicity properties for the sine, cosine and tangent functions;

- graphs of circular functions of the form $y = af(bx) + c$, where f is the sine, cosine or tangent function and a, b and $c \in R$;
- simple applications of circular functions of the above form to model tidal heights, sound waves, bio-rhythms, ovulation cycles, temperature fluctuations during a day and the interpretation of period, amplitude and mean value in these contexts and their relationship to the parameters a, b and c;
- solution of simple equations of the form $f(x) = k$, $k \in R$ using both exact and approximate values, where f is the sine, cosine, or tangent function, on a given domain;
- graphs of $y = Aa^{kx} + C$ for simple cases of a, k, A and $C \in R$;
- the graph of $y = \log_a(x)$ as the graph of the inverse function of $y = a^x$, including the relationships $a^{\log_a(x)} = x$ and $\log_a(a^x) = x$;
- simple applications of exponential functions of the above form to model growth and decay in populations and the physical world, appreciation and depreciation of value in finance; the interpretation of initial value, rate of growth or decay and long run value in these contexts and their relationship to the parameters A, k and C.

2. Algebra

This area of study provides an opportunity for the revision and further development of content prescribed in Unit 1, as well as the study of related algebra material introduced in the other areas of study in Unit 2, including circular functions, exponential functions and logarithmic functions. The content as described in the ‘Algebra’ area of study in Unit 1 is to be distributed across Units 1 and 2.

3. Rates of change and calculus

This area of study covers first principles approach to differentiation, formal differentiation and anti-differentiation of polynomial functions up to degree 3 and simple power functions and related applications, including graph sketching.

This area of study will include:

- informal treatment of the concept of a limit for polynomial expression and simple rational expressions;
- the derivative as the gradient of the graph at a point and its representation by a gradient function;
- notation for derivatives: $D_x(f)$, $\frac{dy}{dx}$, $f'(x)$, $\frac{d}{dx}(f(x))$;
- first principles approach to find the gradient function for linear functions, simple quadratic functions and $f(x) = x^3$;
- first principles, graphical or numerical approaches to justify rules for finding the gradient functions of other polynomials;
- derivatives of simple polynomial functions;
- applications of differentiation:
  - finding rates of change;
  - determining maximum or minimum points for quadratic and cubic functions graphically and analytically, including interval endpoint maximum and minimum values, and their application to simple maximum/minimum problems;
  - using turning points to assist in sketching graphs of simple polynomials;
- anti-differentiation as the reverse process of differentiation:
  - developing rules for anti-derivatives of simple polynomials;
  - identifying families of curves with the same gradient function.

4. Probability

This area of study covers introductory counting principles and techniques and their application to probability, the law of total probability in the case of two events, and the application of conditional probability.

This area of study will include:

- addition and multiplication principles for counting;
- combinations: concept of a selection and computation of ${}^{n}C_{r}$ and pascal's triangle;
- applications of counting techniques to probability;
- the law of total probability for two events, $\Pr(A) = \Pr(A|B)\Pr(B) + \Pr(A|B')\Pr(B')$;
- calculation of probabilities for combinations of successive non-independent events using conditional probability.

OUTCOMES

For this unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass all of the areas of study for the unit.

Outcome 1

On completion of this unit the student should be able to define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.

To achieve this outcome the student will draw on knowledge and related skills outlined in all the areas of study.

Key knowledge

This knowledge includes

- exact values of sine, cosine and tangent for 0, $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ (and their equivalents in degrees) and integer multiples of these;
- key features and properties of the circular functions sine, cosine, and tangent and their graphs;
- the effect of transformations (dilation from the coordinate axes; reflection in the coordinate axes and vertical translation) and simple combinations of these transformations on the graphs of sine, cosine, tangent and exponential functions;
- characteristics of data which suggest the use of circular functions or exponential functions as an appropriate model for a given context;
- exponential and logarithm functions and properties of their graphs;
- the relationship between an exponential function to a given base and the logarithmic function to the same base as inverse functions;
- the derivative function f' as the rate of change or gradient function of a given function f;
- informal concept of a limit and the limit definition of the derivative function;
- the sign of the gradient at and near a point and its interpretation in terms of key features of the graph of simple polynomial functions;
- rules for finding derivatives and anti-derivatives of simple power functions and polynomials functions;
- counting techniques and their rules of application;
- representation of conditional probabilities using tree diagrams.

Key skills

These skills include the ability to

- sketch by hand graphs of the sine, cosine and exponential functions, and simple transformations of these, and sketch by hand the graphs of $\log_a(x)$ and the tangent function;
- draw graphs of circular, exponential and simple logarithmic functions over a given domain, and identify and discuss key features of these graphs;
- describe the effect of transformations (dilation from the coordinate axes; reflection in the coordinate axes and vertical translation) and simple combinations of these transformations on the graphs of the sine, cosine, tangent and exponential functions;
- solve simple equations over a specified interval related to these functions using graphical, numerical and analytical approaches;
- recognise characteristics of data in tabular or graphical form which suggest the selection of one of these types of functions as an appropriate model for the data, and obtain a corresponding function as a model for the data;
- evaluate limits for polynomial expressions and simple rational expressions;
- use a variety of approaches (numerical, graphical, first principles or by rule) to find the value of the derivative of a function at a given point;
- find by hand the derivative function and an anti-derivative function for a simple power function, or a polynomial function up to degree four;
- use derivatives to assist in the sketching of graphs of simple polynomial functions and to solve simple maximum/minimum optimisation problems;
- find a family of anti-derivative functions for a given polynomial function;
- apply counting principles and manipulations to solve problems in contexts where repetition may or may not be allowed, and order may or may not be important, including cases involving simple restrictions;
- calculate probabilities for compound events by hand in simple cases;
- use tree diagrams to calculate combinations of probabilities involving non-independent events.

Outcome 2

On completion of this unit the student should be able to apply mathematical processes in non-routine contexts, and analyse and discuss these applications of mathematics.

To achieve this outcome the student will draw on knowledge and related skills outlined in one or more areas of study.

Key knowledge

This knowledge includes

- key mathematical content from one or more areas of study relating to a given context for investigation;
- specific and general formulations of concepts used to derive results for analysis within a given context for investigation;
- the role of examples, counter-examples and general cases in developing mathematical analysis;
- inferences from analysis and their use to draw valid conclusions related to a given context for investigation.

Key skills

These skills include the ability to

- specify the relevance of key mathematical content from one or more areas of study to the investigation of various question in a given context;
- give mathematical formulations of specific and general cases used to derive results for analysis within a given context for investigation;
- use a variety of techniques to verify results;
- make inferences from analysis and use these to draw valid conclusions related to a given context for investigation;
- communicate conclusions using both mathematical expression and everyday language, in particular interpretation of mathematics with respect to the context for investigation.

Outcome 3

On the completion of this unit the student should be able to use technology to produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.

To achieve this outcome the student will draw on knowledge and related skills outlined in all the areas of study.

Key knowledge

This knowledge includes

- exact and approximate specification of mathematical information such as numerical data, graphical forms and the solutions of equations produced by the use of technology;
- domain and range requirements for the technology-based specification of graphs of functions and relations;
- the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions or equations;
- the similarities and differences between formal mathematical expressions and their representation in various technology applications;
- the appropriate selection of a technology application in a variety of mathematical contexts.

Key skills

These skills include the ability to

- distinguish between exact and approximate presentations of mathematical results, and interpret these results to a specified degree of accuracy;
- produce results using technology which identify examples or counter-examples for propositions;
- produce tables of values, families of graphs or collections of other results which support general analysis in problem-solving, investigative or modelling contexts;
- use appropriate domain and range technological specifications which illustrate key features of graphs of functions and relations;
- identify the relation between numerical, graphical and symbolic forms of information about functions and equations, and the corresponding features of those functions or equations;
- specify the similarities and differences between formal mathematical expressions and their representation in various technology applications;
- make appropriate selections for technology applications in a variety of mathematical contexts, and provide a rationale for these selections;
- relate the results from a particular application to mathematical task (investigative, problem solving or modelling).

ASSESSMENT

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Assessment tasks must be a part of the regular teaching and learning program and must not unduly add to the workload associated with that program. They must be completed mainly in class and within a limited timeframe. Teachers should select a variety of assessment tasks for their assessment program to reflect the key knowledge and skills being assessed and to provide for different learning styles.

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Demonstration of achievement of Outcome 1 must be based on a selection of the following tasks:

- assignments;
- tests;
- summary or review notes.

Demonstration of achievement of Outcome 2 must be based on a selection of the following tasks:

- projects;
- short written responses;
- problem-solving tasks;
- modelling tasks.

Demonstration of achievement of Outcome 3 must be based on the student's performance on a selection of tasks completed in demonstrating achievement of Outcomes 1 and 2, which incorporate the effective and appropriate use of technology in contexts related to the content of the areas of study.

Advice for teachers (Units 1 and 2: Mathematical Methods)

DEVELOPING A COURSE

A course outlines the nature and sequence of teaching and learning necessary for students to demonstrate achievement of the set of outcomes for a unit. The areas of study describe the learning context and the knowledge required for the demonstration of each outcome. Outcomes are introduced by summary statements and are followed by the key knowledge and skills which relate to the outcomes.

In particular, it should be noted that the key skills for Outcome 1 include routines that students are expected to be able to carry out by hand without the use of technology. In some instances these will only require writing down of a result obtained by application of a general rule or pattern, for example differentiation of a polynomial function, in other instances several steps of manipulation may be required, for example re-expression of the rule of a cubic polynomial function in terms of a specified linear factor $(x - a)$ by equating coefficients. Students should be conversant with such mental and by hand routines.

Teachers must develop courses that include appropriate learning activities to enable students to develop the knowledge and skills identified in the outcome statements in each unit.

For Units 1 and 2, teachers must select assessment tasks from the list provided. Tasks should provide a variety and the mix of tasks should reflect the fact that different types of tasks suit the assessment of different knowledge and skills. For example, analysis of the behaviour of a function, such as the location of its minimum value over a given interval in terms of changes in the values of its coefficients, could be assessed by a short and focused investigation, and incorporate the required use of technology, while the development of by hand skills could be assessed either by a test, an assignment, or a component of an investigation. Tasks do not have to be lengthy to make a decision about student demonstration of achievement of an outcome.

The following sample course outline is based on the assumption of 17 teaching weeks per semester, allowing time for review and consolidation and assessment to be incorporated. For several topics an outline of a sample task has been included.

SAMPLE TEACHING SEQUENCE

Unit 1

Topic	Time
• **Algebra** associated with linear, quadratic and cubic equations, including midpoint and length of line; completion of the square; use of quadratic formula; solution of simultaneous equations between linear and quadratic equations; remainder and factor theorems; polynomial re-expression and division; and finding polynomial models for data by finite difference tables.	4 weeks
• **Functions and graphs** based on $f(x) = x^n$, where $n \in \{-2, -1, 1, \frac{1}{2}, 1, 2, 3, 4\}$ including equations of lines parallel and perpendicular to the graph of a given linear function; sketches of graphs for expanded and factorised forms of quadratic and cubic functions; sketching from factorised form of quartic functions; the vertical line test for the graph of a function; domain and range; determining axis intercepts by hand and by technology; asymptotes; stationary points for $n = 3$ or 4 by technology; and transformation from the graph of $y = f(x)$ to the graph of $y = af(bx + c) + d$. **Sample task 1: Investigation of families of curves with the rule $f(x) = x^n$, $n \in$ Q.** Sketch and comment on features, similarities, differences and restrictions for graphs of (i) $f(x) = x^n$, $n \in \mathbb{Z}^+$ (ii) $f(x) = x^n$, $n \in \mathbb{Z}^-$ (iii) $f(x) = x^n$, $n = \frac{p}{q}$ where $p \neq q$ and $p, q \in \mathbb{Z}^+$ (v) $f(x) = x^n$, $n = \frac{p}{q}$ where $p \in \mathbb{Z}^-$ and $q \in \mathbb{Z}^+$.	4 weeks
• **Inverse and other relations** Graphical representation of an inverse relation for any relation; algebraic determination of the rule of an inverse relation for quadratic functions and hyperbola; horizontal line test for f^{-1} to exist; equation of circle in the form $(y - h)^2 + (x - k)^2 = r^2$.	2 weeks
• **Rates of change and calculus** Rates of change in practical examples such as filling a jug from tap, a leaking balloon or speed of travel; average and approximate rates of change as determined by gradient of secant connecting two points on a curve; instantaneous rate of change represented by gradient of tangent at a point; construction and interpretation of distance-time, and velocity-time graphs. **Sample task 2: Analysis of travel graphs** Distance/speed/acceleration and displacement/velocity/acceleration. Significance of: gradient, points of intersection and area under curve for the motion of two or more vehicles. Students can be presented with a distance/displacement-time graph involving two or more hybrid functions. Interpretation of these graphs and development of related speed/velocity graphs and acceleration graphs. Similarly, consideration of speed/velocity – time graph; interpretation and sketching the corresponding distance/displacement and acceleration graphs (different components of this task could be designed to be tackled with and without the use of technology).	3 weeks
• **Probability 1 – introductory probability** Long run proportion, addition and multiplication rules, independent events, mutually exclusive events; venn diagrams and karnaugh maps; and conditional probability (for example, informal consideration of simple markov chains represented by tree diagrams such as several days of weather or sets of tennis).	3 weeks

Unit 2

Topic	Time

- **Probability 2 – counting principles and applications** — 3 weeks

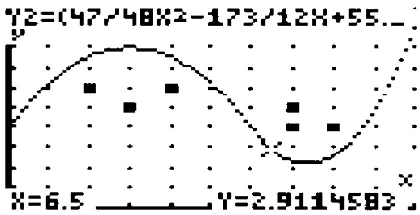
  Counting principles, ${}^nC_r$ and pascal's triangle, law of total probability for two events and calculation of probabilities for successive non-independent events using conditional probability.

- **Differentiation and anti-differentiation of polynomial functions** — 5 weeks

  Limits, differentiation by first principles, differentiation by rule, applications of differentiation, graphs of gradient function, informal treatment of continuity and differentiability, anti-differentiation as reverse of differentiation; and rules for anti-differentiation.

  Sample task 3: Modelling a path around obstacles using hybrid function polynomial functions

  Use of a 10 × 10 grid, with two randomly placed obstacles (using 3 or more grid points) and a *start* and *finish* point on the left and right side of the grid.

  Construction of a hybrid function of no more than two polynomial functions to create a smooth path from start to finish around the obstacles.

- **Circular functions** — 5 weeks

  Trigonometric ratios for right-angled triangles, radians, unit circle and definition of sine, cosine and tangent functions, exact values, symmetry properties, period and amplitude; $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$; $\sin^2(x) + \cos^2(x) = 1$; transformation from the graph of $y = f(x)$ to the graph of $y = af(bx + c) + d$;

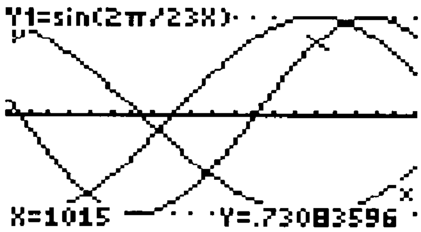
  applications of circular functions in modelling and solutions to $f(x) = B$ using exact and approximate values.

  Sample task 4: Investigation of the pseudo-science of bio-rhythms

  Simple bio-rhythms are based on the assumption that a person's physical, emotional and intellectual state of 'well being' can be modelled by a combination of circular functions

  such as: $P(t) = \sin\left(\dfrac{2\pi t}{23}\right)$, $E(t) = \sin\left(\dfrac{2\pi t}{28}\right)$ *and* $I(t) = \sin\left(\dfrac{2\pi t}{33}\right)$

respectively, where t = the number of days since birth:



Investigation of biorhythms in relation to significant dates and when any two or all three equations would be peaking, neutral or at the bottom of a trough, either separately or in combination with each other.

- **Exponential and logarithmic functions** 4 weeks

  Graphs of $y = Aa^{kx} + B$, graphs of $y = \log_a(x)$ as the inverse of $y = a^x$ and applications of exponential functions in modelling.

DETAILED EXAMPLES OF COURSE DEVELOPMENT

An introduction to polynomial functions

Polynomial functions provide one of the simplest types of functions for modelling data in a wide range of situations. They can be readily evaluated by combinations of arithmetic operations, are everywhere continuous and differentiable and their behaviour is relatively simple to describe graphically. They are used to approximate more complex functions such as circular functions, to construct curves through points such as by a computer-assisted drawing package and to develop quantitative models for sets of data such as the bending of a beam under load or for trend data. This data may have been obtained from various sources including experiments or records, possibly from other VCE studies, journals, magazines, websites or newspapers. This type of source material can be used to develop an integrated approach to key concepts skills and processes related to the study of polynomial functions. An important property of polynomial functions is that for any set of n distinct points (which do not all lie on a straight line), there is a polynomial function of degree $n – 1$ whose graph will pass exactly through these n points. If there are n points:

x_1	x_2	x_3	x_4	...	x_n
y_1	y_2	y_3	y_4	...	y_n

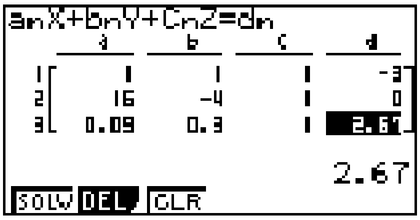
and p is a polynomial function of degree $n – 1$ with rule $p(x)$, then the set of simultaneous linear equations:

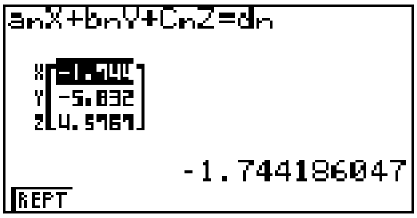
$$\{y_1 = p(x_1), y_2 = p(x_2) \ldots y_n = p(x_n)\}$$

can be solved to determine the coefficients of the powers of p. In the case of a quadratic function, q, the coordinates of three such points would be required to determine the coefficients a, b and c of $q(x) = ax^2 + bx + c$. If the coordinates of the three points are (1, –3), (–4, 0) and (0.30, 2.67), this would result in the corresponding set of simultaneous linear equations:

$$-3 = a + b + c \qquad 0 = 16a - 4b + c \qquad 2.67 = 0.09a + 0.30b + c$$

Such a system of equations can be solved by hand; however, the equation-solving facilities of many graphics calculators (or the use of their matrix algebra functions), spreadsheets and computer algebra systems will easily carry out this process for both integer and non-integer values of the coordinates involved. The following graphics calculator screen dumps provide an illustrative output (note that the coefficients *a*, *b* and *c* of the original rule become the values to be determined (unknowns) of the system of equations, which are represented by the variables *X*, *Y* and *Z* respectively in the calculator format).





In this case the function *q* would have the particular rule $q(x) = –1.74\,x^2 + 5.83x + 4.58$, where the coefficients are correct to two decimal places. A suitable domain of interpretation would need to be specified with respect to the context under consideration.

This approach also extends to situations where more complex systems of equations are required to be solved. While students should be able to apply the relevant processes by hand in straightforward cases, modelling real data using this method will require the appropriate support of technology to obtain solutions to this type of problem efficiently. A graphics calculator, spreadsheet or computer algebra system can be used to find these coefficients, draw the corresponding graph and perform related analysis of key features (e.g. find *x* axis intercepts, maximum/minimum values); however, an algebraic approach is required to establish general results. Solving the general equation for $q'(x) = 0$ gives a stationary point when $2ax + b = 0$ or $x = -\frac{b}{2a}$. The corresponding maximum or minimum value (depending on the sign of *a*) can then be found by evaluating $q\left(-\frac{b}{2a}\right)$. Knowledge of this general result facilitates the analysis of maximum and minimum values of data modelled by quadratic functions.

The solutions of a quadratic equation of the form $q(x) = k$ can be found by reference to the quadratic formula, which is based on the completion of the square process. Any quadratic function with rule $q(x) = ax^2 + bx + c$ can be re-expressed in the completed square form $q(x) = A(x + B)^2 + C$ where $A = a$, which can also be used to sketch its graph as a sequence of transformations of the graph of $y = x^2$.

For the case of cubic polynomial functions there is no simple formula like the quadratic formula since not all graphs of cubic polynomial fucntions are transformations of the graph of the basic function with rule $y = x^3$. Thus, while: $c(x) = ax^3 + bx^2 + cx + d$ and $c(x) = (x – k)\,(lx^2 + mx + n) + r$ are general forms of the rule for a cubic polynomial function, this is not the case for the form: $c(x) = A(x + B)^3 + C$.

The second form is found by re-expressing $c(x)$ in terms of $(x – k)$ and leads to the remainder theorem, $c(k) = r$ and its corollary the factor theorem: if $c(k) = 0$ then $(x – k)$ is a factor of $c(x)$. This re-expression can be carried out by polynomial division, or by equating coefficients of the powers of *x* in the cubic polynomial, for example: $x^3 – 3x^2 + 6x – 7 = x^2\,(x – 2) –x\,(x – 2) + 4(x – 2) + 1 = (x – 2)\,(x^2 – x + 4) + 1$.

In many exercises students deal with cases where $r = 0$ and the quadratic expression $(lx^2 + mx + n)$ has simple integer or rational factors. They should be able to identify intervals within which a root to the equation $c(x) = 0$ must lie, and could also seek to identify whether such a root is rational. For example, if it is assumed that the particular cubic polynomial function with rule $c(x) = 2x^3 + 3x^2 – 22x – 33$ has at least one linear factor $(ux + v)$ which leads to a rational root then: $c(x) = 2x^3 + 3x^2 – 22x – 33 = (ux + v)\,(lx^2 + mx + n)$ where *u* and *v* are integers such that $ul = 2$ and $vn = 33$. Hence *u* would a factor of 2 and *v* a factor –33, so it is only necessary to test whether $c\left(\frac{-v}{u}\right) = 0$ for the possible rational combinations of $p = \pm 1, \pm 2$ with $q = \pm 1, \pm 3, \pm 11, \pm 33$. Inspection reveals that $c\left(\frac{-3}{2}\right) = 0$, hence $(2x + 3)$ is a linear factor of $c(x)$. Re-expression gives $x^2 - 11 = \left(x+\sqrt{11}\right)\left(x-\sqrt{11}\right)$ as the other factor. While approximate numerical values of key features of cubic polynomial functions can be identified using technology (for example, coordinates of axial intercepts and stationary points), description of the general behaviour of these functions requires an analytic approach. The nature and existence of stationary points can be obtained from examination of the quadratic equation related to the derivative function. This can be well supported by the use of numerical approximations to the gradient of the tangent on particular graphs using technology.

An approach to developing the general rule for the derivative of x^n

Technology can be used to develop a numerical approach for conjecturing a limit value for the gradient of a function at a specific x value. If the function with the rule $f(x) = x^2$ is considered, then both the following expressions can be evaluated for various values of x:

right secant gradient = $\frac{f(x+h)-f(x)}{h}$ and left secant gradient = $\frac{f(x)-f(x-h)}{h}$

For example, if $x = 2$, a numerical approximation for $f'(2)$ can be obtained by considering a table of values for these expressions where h varies from say 0.1 to 0.0001. The values in the table below can be seen to strongly suggest convergence to a limiting value of 4 for $f'(2)$. Initially, this can be done by hand as a collection of individual calculations to develop student familiarity with the process. For the particular function with the rule $f(x) = x^2$ when $x = 2$, these expressions simplify to:

right secant gradient = $\frac{(2+h)^2-4}{h}$ and left secant gradient = $\frac{4-(2-h)^2}{h}$

h	Left secant gradient	Right secant gradient
0.1	3.9	4.1
0.01	3.99	4.01
0.001	3.999	4.001
0.0001	3.9999	4.0001

When a sufficient range of x values has been systematically investigated, the rule for $f'(x)$ can be conjectured as likely to be $f'(x) = 2x$. A similar approach can be taken for other functions of the form $f(x) = x^n$. Graphics calculators or spreadsheets are an effective technology for carrying out systematic numerical analysis of this kind. The following spreadsheet illustrates how this might be done for values of the right secant gradient. The rows are to be read from left to right to see the limit process emerge. The values of h can be changed arbitrarily to smaller values if required. Note that these provide an underestimate of the gradient when $x < 0$, and an overestimate of the gradient when $x > 0$. A similar spreadsheet could be produced for values of the left secant gradient.

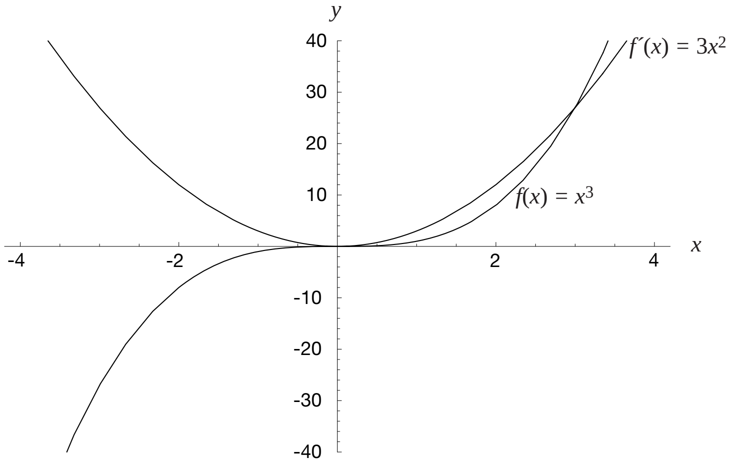
Numerical evaluation of derivative approximations by spreadsheet

		value of h as h tends to zero			
		0.1	0.01	0.001	0.0001
secant formula		$((x + 0.1)^2 - x^2)/0.1$	$((x + 0.01)^2 - x^2)/0.01$	$((x + 0.001)^2 - x^2)/0.001$	$((x + 0.0001)^2 - x^2)/0.0001$
x values	–7	–13.9	–13.99	–13.999	–13.9999
	–6	–11.9	–11.99	–11.999	–11.9999
	–5	–9.9	–9.99	–9.999	–9.9999
	–4	–7.9	–7.99	–7.999	–7.9999
	–3	–5.9	–5.99	–5.999	–5.9999
	–2	–3.9	–3.99	–3.999	–3.9999
	–1	–1.9	–1.99	–1.999	–1.9999
	0	0.1	0.01	0.001	0.0001
	1	2.1	2.01	2.001	2.0001
	2	4.1	4.01	4.001	4.0001
	3	6.1	6.01	6.001	6.0001
	4	8.1	8.01	8.001	8.0001
	5	10.1	10.01	10.001	10.0001
	6	12.1	12.01	12.001	12.0001
	7	14.1	14.01	14.001	14.0001

An advantage of this approach is that it enhances the ability to deal effectively with an important concept, that the derivative is defined when two limits have an identical value (the limiting values as h tends to 0 for both the left secant gradient and the right secant gradient), that is the gradient of the unique tangent at x. An informal notion of differentiability arises naturally from such considerations. This can then lead to the development of a first principles approach where one of these limits is applied to simple power function such as $f(x) = 1$, $f(x) = x$, $f(x) = x^2$ and $f(x) = x^3$. A general case argument could be used to summarise the emerging pattern. Alternatively, a computer algebra system could be useful in developing these ideas further, where students have previously worked through the cases for $n = 1$, 2 and 3 by hand.

The graph of a particular function, say $f(x)= x^3$, and its derivative graph can be plotted for each case, and consistency checked, for several x values:

```
Plot [{f[x], f´[x]},{x, -3, 3}, AxesLabel → {x, y}]
```



If the more general case $f(x) = x^n$ is considered, then it is possible to use technology with symbolic manipulation functionality to produce the following table of results:

Table [{n, f[x], Expand $\left[\dfrac{\text{f[x+h]-f[x]}}{\text{h}}\right]$ Limit $\left[\dfrac{\text{f[x+h]-f[x]}}{\text{h}}\right]$, h → 0}, {n, 0, 10}]

0	1	0	0
1	x	1	1
2	x^2	h = 2x	2x
3	x^3	$h^2 + 3hx + 3x^2$	$3x^2$
4	x^4	$h^3 + 4h^2x + 6hx^2 + 4x^3$	$4x^3$
5	x^5	$h^4 + 5h^3x + 10h^2x^2 + 10hx^3 + 5x^4$	$5x^4$
6	x^6	$h^5 + 6h^4x + 15h^3x^2 + 20h^2x^3 + 15hx^4 + 6x^5$	$6x^5$
7	x^7	$h^6 + 7h^5x + 21h^4x^2 + 35h^3x^3 + 35h^2x^4 + 21hx^5 + 7x^6$	$7x^6$
8	x^8	$h^7 + 8h^6x + 28h^5x^2 + 56h^4x^3 + 70h^3x^4 + 56h^2x^5 + 28hx^6 + 8x^7$	$8x^7$

Which can then be used to lead into the general case result for x^n based on expansion of $(x + h)^n$.

A combination of numerical, graphical and analytical approaches could be used to extend this work to polynomial functions. The following could then be used as an analysis task to apply these ideas to the graphs of cubic polynomial functions:

- sketch the graph of the function f with the rule $f(x) = x^3 – 4x + 7$, identifying important features;
- consider the function g with the rule $g(x)= x^3 – 4x + d$. Find the value of d for which g has only one x axis intercept and three x axis intercepts respectively;
- identify any values of d for which g has a stationary point of inflection;
- consider the function h, with the rule $h(x) = x^3 + bx^2 + 4x + 3$. Explore the nature of the graph of h in terms of different values of b.

USE OF INFORMATION AND COMMUNICATIONS TECHNOLOGY

In designing courses and developing learning activities for Mathematical Methods Units 1 and 2, teachers should make use of applications of information and communications technology and new learning technologies, such as computer-based learning, multimedia and the World Wide Web, where applicable to teaching and learning activities.

KEY COMPETENCIES AND EMPLOYABILITY SKILLS

Students undertaking the following types of assessment, in addition to demonstrating their understanding and mastery of the study, typically demonstrate the following key competencies and employability skills.

Assessment task	Key competencies and employability skills
Assignments	Learning, use of information and communications technology
Tests	Self management, use of information and communications technology
Summary or review notes	Self management
Projects	Communciation, team work, self management, planning and organisation, use of information and communications technology, initiative and enterprise
Short written responses	Communication, problem solving
Problem-solving tasks	Communication, problem solving, team work, use of information and communications technology
Modelling tasks	Problem solving, planning and organisation, use of information and communications technology

LEARNING ACTIVITIES

Examples of learning activities for each unit are provided in the following sections. The examples that make use of information and communications technology are identified by this icon ICT.

Units 1 and 2: Mathematical Methods

Outcome 1

Define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.

Examples of learning activities

skills practice on standard mathematical routines through an appropriate selection of exercises (e.g. difference tables)

construction of summary or review notes related to a topic or area of study (e.g. quadratic and cubic factorisation techniques)

assignments structured around the development of samples cases of standard applications of mathematical skills and procedures in readily recognisable situations

exercises such as the identification of key features of graphs of functions and relations on graphs produced using technology (e.g. location of axes intercepts and any stationary points, interval(s) over which a function is increasing or decreasing) and special features of particular functions and relations (e.g. asymptotic behaviour, periodicity, location of centres of circles)

Outcome 2

Apply mathematical processes in non-routine contexts, and analyse and discuss these applications of mathematics.

Examples of learning activities

investigative projects, such as exploring the properties of graphs of families of related functions or relations such as the rate at which a container fills, given a constant rate of flow

collections of problem-solving tasks, such as determining optimal values for problems related to application contexts such as maximising or minimising area or volume

modelling tasks, such as using functions to model experimental data from other areas of study, or make predictions based on modelling of trend data such as circular functions applied to periodic patterns in weather

a set of applications questions requiring analysis and extended response on a particular context such as seating arrangements with various restrictions

a report on item response analysis for a collection of multiple-choice questions (this would involve a clear and detailed explanation as to why other alternatives did not provide an appropriate response to the question asked, that is, did not account for all the information provided)

presentation on research into a particular application of mathematics (this could involve the use of presentation information technologies, e.g. an historical treatment of the development of key ideas of calculus)

Outcome 3

Use technology to produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.

Examples of learning activities

The use of technology should be developed as an integral part of the range of learning activities for Outcomes 1 and 2. In particular, these can include:

investigations based on the production of collections of tables of values or graphs for families of functions, and analysis of the behaviour of this type of function in terms of key defining parameters such as the effect of the parameters a, b, c and d on graphs of functions with rules of the form $f(x) = a(x - b)(x - c)(x - d)$; summary of the effects of variation of these parameters

activities based on the application of a variety of different techniques to a particular type of task, e.g. use different technologies to implement various graphical, tabular, numerical and analytical approaches associated with solving appropriate equations of the form: $f(x) = 0$ or $f(x) = k$ or $f(x) = g(x)$

'workshop' type sessions where, for example, students explore the relationship between a function and its derivative function using analytical, graphical and numerical approaches

APPROACHES TO ASSESSMENT

A sample summary of an assessment program for Units 1 and 2 follows:

Unit 1	
Algebra	Test
Functions and relations	Investigative task
Rates of change	Rates problem-solving task
Probability 1	Test
Unit 2	
Probability 2	Multiple-choice analysis
Differentiation and anti-differentiation of polynomial functions	Modelling problem
Circular functions	Periodic motion modelling task
Exponential and logarithmic functions	Test

Other tests and assignments could be utilised throughout the course in order to monitor student progress. Teachers should consider where technology can most effectively be used in assessment, in particular what tasks, or parts of tasks, are most suited to assessment of mental, by hand and technology assisted approaches. Summary or review notes constructed in preparation for prescribed tests could be part of the required material for assessment of Outcome 1. These could include definitions of key concepts, statements of important results or formulas, and sample routine problems. Alternatively, an assignment based on a collection of questions requiring progressively more complex and detailed analysis could be used to assess student achievement of aspects of Outcomes 2 and 3.

DETAILED EXAMPLES

Algebra – some sample Unit 1 by hand questions

1. Determine the mid-point and length of the line segment from (–6, 3) to (2, 5).
2. Solve $x + 2y = 8$ and $2x - y = -11$ simultaneously for x and y.
3. Express $x^2 + 4x - 6$ in completed square form and hence state the exact solutions for $x^2 + 4x - 6 = 0$.
4. Find the remainder when $2x^4 - 3x^3 - 6x^2 + 4x - 5$ is divided by $x + 1$.
5. Show that $(x - 2)$ is a factor of $p(x) = x^3 - 3x - 2$ and hence determine all linear factors of $p(x)$ over R.
6. a) Use finite differences to determine the degree of the polynomial function which has been used to generate this set of data:

x	2	3	4	5	6	7	8
y	–4	–2	4	14	28	46	68

   b) Hence work out the rule of the polynomial function.

Functions and graphs – a sample Unit 1 investigation

The purpose of this task is to provide a context which can be use to select and carry out function fitting processes to trend data and analyse these trends.

Summary of key concepts and processes

Concisely but clearly define the terms *function*, *model* and *trend*.

Outline the processes that can be used to fit a function to a set of data as a trend curve and discuss the strengths and limitations of each approach.

Application of these processes to a selection of data from a variety of different contexts

Obtain a set of data for each of the following situations: a linear looking trend, a quadratic looking trend and a cubic looking trend, and draw a corresponding graph of the data (include one set of data trend which could well be modelled by a function with a negative coefficient of its highest power).

For each set of data find a function of good fit. Use a different approach in each case. Plot the function of good fit on the original graph and discuss how well the function describes the pattern or trend of the original data. Comment on how well you believe your fitted function will perform as a predictor of 'future values'.

Analysis of a given context to predict possible future trends using several different approaches

Provide an article to students on current data on market share value, and the graphical projections of several different institutions for future directions. Have students draw a graph for the known data.

Fit a linear, quadratic, cubic and quartic trend function to this data. Decide which type of trend function matches up most closely to the forecast of each institution. Comment on which forecast you believe is most likely to be accurate.

Development of quantitative and qualitative criteria for assessing ‘goodness of fit’, and consideration of the appropriateness of a particular approach in a given context

Consider your fitted functions from the previous section of work. With respect to the known data, give a qualitative analysis that shows how you would select the function which provides the ‘best’ fit, and identify this function.

Construct a numerical measure that shows how you would select the function which provides the ‘best’ fit, and identify this function.

Comment on the selection of a function of best fit in both cases.

Possible approaches to finding a function of good fit include: relating key features of the function to relevant information in the data (for example, a clear maximum value in a symmetric ‘rise and fall’ situation to the vertex of a parabola, linking x axis intercepts to ‘natural’ zero values form the context); difference equations (where an essentially constant 1st, 2nd or 3rd difference is identified for consecutive data points), selecting key points according to some rationale and using simultaneous equations to determine the coefficients of an appropriate polynomial or using built-in least square fitting algorithms. A graphics calculator or other suitable technology can be used to assist students in exploring different possibilities through the capability to recalculate rules and re-draw graphs where selected points or other data are varied.

Rates of change and calculus: a sample Unit 1 problem-solving task

Students are asked to determine the time it takes to fill a variety of shaped containers with water from a tap that pours at a constant rate of 0.1L/s, graph the corresponding depth of water versus time graph and present their findings in a brief written report. For example, the containers could be:

Container	Description
A	Rectangular prism 140 cm long, 60 cm wide and 40 cm high.
B	Cylinder 40 cm deep and has a radius of 40 cm.
C	Trapezoidal prism 140 cm long, 50 cm wide at the base, 70 cm wide at the top and 40 cm high.
D	Inverted truncated rectangular prism (a simple common bath shape) that is 40 cm deep, 150 cm long at the top and 130 cm long at the base, 50 cm wide at the base and 70 cm wide at the top.
E	The dimensions of a bath and determining a reasonable flow rate from your tap (use a measuring jug and determine the volume over 10 seconds).

Probability: a sample Unit 2 multiple-choice items response analysis question

Students analyse a small collection of probability related multiple-choice questions. This involves determining the correct response and identifying possible reasons that would lead to the selection of at least two of the other incorrect responses for each question.

For example:

Amy plays tennis. Amy gets her first service in play 60% of the time. If she gets her first service in play she has a 80% chance of winning the point. If Amy has a second serve she gets that in play 90% of the time but she only has a 50% chance of winning the point. For example, the following is a conditional probability question that can be effectively tackled using a tree diagram.

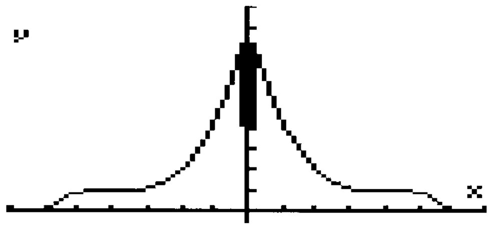
The probability that Amy wins the first point of a match is:

A. 0.48 B. 0.93 C. 0.18 D. 1.3 E. 0.64

Rates of change and calculus: A sample Unit 2 differentiation and anti-differentiation analysis question

Consider the situation where an 18 m high spire is to be built with a vertical cross section in the form of the graphs of two cubic polynomial functions:

$f(x) = 0.25(x + 4)^3 + 2$, $x \in [-6, 0]$ and $g(x) = -0.25(x - 4)^3 + 2$, $x \in (0, 6]$ as illustrated below:



If the graph is not given, students could be asked to draw the graph of the hybrid function.

In the first part of the analysis question, students could be asked to find the average rate of change in height between several pairs of points, for example, between $x = 0$ and $x = 6$.

In the second part of the analysis question, they could then be asked to determine the gradient, correct to a specified accuracy of the sides of the spire at several points, for example when $x = 0$, 2 and 6.

In the third part of the analysis question, students could be asked to find an approximation to the cross-sectional area using a series of rectangles, for example left and right rectangles of constant base lengths of 0.5 m.

In the final part of the analysis question, students could be asked to find the anti-derivative function, F, of f, and compare computed values of this to the area approximation obtained earlier, for example $F(0) - F(-6)$ to the rectangle approximations from $x = -6$ to $x = 0$.

Units 1 and 2: Mathematical Methods (CAS)

Unit 1: Mathematical Methods (CAS)

Mathematical Methods (CAS) Units 1 and 2 are designed as preparation for Mathematical Methods (CAS) Units 3 and 4. They also provide a suitable preparation for Mathematical Methods Units 3 and 4. The areas of study for Unit 1 are 'Functions and graphs', 'Algebra', 'Rates of change and calculus' and 'Probability'. At the end of Unit 1, students will be expected to have covered the material outlined in each area of study given below, with the exception of 'Algebra' which should be seen as extending across Units 1 and 2. This material should be presented so that there is a balanced and progressive development of skills and knowledge from each of the four areas of study with connections among and across the areas of study being developed consistently throughout both Units 1 and 2.

Students are expected to be able to apply techniques, routines and processes involving rational and real arithmetic, algebraic manipulation, equation solving, graph sketching, differentiation and integration with and without the use of technology, as applicable. Students should be familiar with relevant mental and by hand approaches in simple cases.

The appropriate use of computer algebra system (CAS) technology to support and develop the teaching and learning of mathematics, and in related assessments, is to be incorporated throughout the unit. Other technologies such as spreadsheets, dynamic geometry or statistical analysis software may also be used, as appropriate, for various topics from within the areas of study for the course.

Familiarity with determining the equation of a straight line from combinations of sufficient information about points on the line or the gradient of the line and familiarity with pythagoras theorem and its application to finding the distance between two points is assumed. Students should also be familiar with quadratic and exponential functions, algebra and graphs, and basic concepts of probability.

AREAS OF STUDY

1. Functions and graphs

This area of study covers the graphical representation of functions of a single real variable and the study of key features of graphs of functions such as axis intercepts, domain (including maximal domain) and range of a function, asymptotic behaviour and symmetry.

This area of study will include:

- distance between two points in the cartesian plane, coordinates of the midpoint of a line segment, and gradients of parallel and perpendicular lines;
- use of the notation $y = f(x)$ for describing the rule of a function and evaluation of $f(a)$, where a is a real number or a symbolic expression;

- graphs of power functions $y = x^n$ for $n \in N$ and $n = -1, -2, \frac{1}{2}$ and transformations of these to the form $y = a(x + b)^n + c$ where a, b and $c \in R$;
- graphs of polynomial functions to degree 4;
- qualitative interpretation of features of graphs, and families of graphs, including an informal consideration of rates of change, including:
  - graphs of functions that have been obtained empirically, such as heart rate data during an exercise sequence, economic trend data;
  - relations as models for data, such as the weight-height index $I = \frac{w}{h^2}$ kg/m^2;
- 'the vertical line test' and its use to determine whether a relation is a function;
- graphs of relations including those specified by conditions or constraints, such as arcs of circles or a region defined by an inequality, for example 'the set of all points whose distance from a fixed point (a, b) is less than a given value';
- graphs of inverse relations.

2. Algebra

This area of study supports material in the 'Functions and graphs', 'Rates of change and calculus' and 'Probability' areas of study and this material is to be distributed between Units 1 and 2. In Unit 1 the focus is on the algebra of polynomial functions to degree 4. Content introduced in Unit 1 may be revised and further developed in Unit 2.

This area of study will include:

- use of symbolic notation to develop algebraic expressions and represent functions, relations and equations;
- substitution into, manipulation, expansion and factorisation of algebraic expressions, including the remainder and factor theorems;
- recognition of equivalent expressions and simplification of algebraic expressions involving functions and relations, including use of exponent laws and logarithm laws;
- determination of rules of simple functions and relations from given information, including polynomial functions to degree 4 and of transformations (dilation, reflection and translation) of the square root and circle relations;
- solution of polynomial equations to degree 4, analytically, numerically and graphically;
- use of inverse functions to solve equations;
- solution of equations of the form $Af(bx) + c = k$, where A, b, c and $k \in R$, and f is sine, cosine, tangent or a^x using exact or approximate values on a given domain, and interpretation of these equations;
- the connection between factors of $f(x)$, solutions of the equation $f(x) = 0$ and the horizontal axis intercepts of the graph of the function f;
- solution and interpretation of simultaneous equations involving two functions by numerical, graphical and analytical methods;
- use of parameters to represent a family of functions and general solutions of equations involving these functions;
- development of polynomial models, for example by the use of finite difference tables or solution of a system of simultaneous linear equations obtained from values of a function, or a simple combination of values of a function;
- index laws and logarithm laws, including their application to the solution of simple exponential equations;

- application of matrices to transformations of points in the plane (dilation from the coordinate axes; reflection in the coordinate axes and the line $y = x$, and translation from the coordinate axes), and the solution of systems of simultaneous linear equations in up to four unknowns.

3. Rates of change and calculus

This area of study covers constant and average rates of change and an informal treatment of instantaneous rate of change of a function in familiar contexts, including graphical and numerical approaches to the measurement of constant, average and instantaneous rates of change.

This area of study will include:

- average and instantaneous rates of change:
  - rate of change of a linear function, use of gradient as a measure of rate of change;
  - average rate of change, use of the gradient of a chord of a graph to describe average rate of change of $y = f(x)$ with respect to x, over a given interval, practical examples of average rates of change such as average speed on a bush walk and average slope of a hill from bottom to top;
  - the measurement of rates of change of polynomials functions; achieved by finding successive numerical approximations to the gradient of a polynomial function at a point by taking another point very close to it on either side of the graph of the function, and finding the gradient of the line joining the two points, and then repeating this procedure, leading to an informal treatment of the gradient of the tangent as a limiting value of the gradient of a chord;
- tangents to curves and local linearity of differentiable curves, use of gradient of a tangent at a point on the curve to describe instantaneous rate of change of $y = f(x)$ with respect to x, practical examples of instantaneous rates of change, such as speedometer readings and revolution counters and other applications of rates of change:
  - graphs of functions and interpretation of the rate of change of a function, where the rate of change is positive, negative, or zero, relationship of the gradient function to features of the original function;
  - motion graphs, construction and interpretation of displacement–time and velocity–time graphs and informal treatment of the relationship between displacement–time and velocity–time graphs;
  - use of rates of change and corresponding graphs in other contexts, such as describing the rate of change of heartbeat during an exercise sequence or the rate of change of the height of water in a container that is being filled at a constant rate.

4. Probability

This area of study covers introductory probability theory, including the concept of events, probability and representation of event spaces using various forms such as lists, grids, venn diagrams, karnaugh maps, tables and tree diagrams. Impossible, certain, complementary, mutually exclusive, conditional and independent events involving one, two or three events (as applicable), including rules for computation of probabilities for compound events.

This area of study will include:

- random experiments, events and event spaces;
- probability as an expression of long run proportion;
- simulation using simple generators such as coins, dice, spinners, random number tables and technology;
- display and interpretation of results of simulations;

- probability of simple and compound events;
- lists, grids, venn diagrams, karnaugh maps and tree diagrams;
- the addition rule for probabilities;
- conditional probability, $\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$;
- independence, and the multiplication rule for independent events, $\Pr(A \cap B) = \Pr(A).\Pr(B)$ when A and B are independent events.

OUTCOMES

For this unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass all of the areas of study for the unit.

Outcome 1

On completion of this unit the student should be able to define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.

To achieve this outcome the student will draw on knowledge and related skills outlined in all the areas of study.

Key knowledge

This knowledge includes

- the definition of a function, the concepts of domain and range, and notation for specification of the domain, range and rule of a function;
- equation and features of a straight line, including gradient and axis intercepts, midpoint of a line segment and parallel and perpendicular lines;
- key features and properties of power and polynomial functions and their graphs;
- the effect of transformations (dilation from the coordinate axes; reflection in the coordinate axes and the line $y = x$, and translation) and simple combinations of these transformations, on the graphs of linear and power functions;
- equations of circles at the origin and translations of these;
- factorisation patterns, the quadratic formula and discriminant, the remainder and factor theorems and the null factor law;
- matrix representation of systems of simultaneous linear equations involving up to four unknowns, matrix specification of linear transformations of the plane;
- operations of matrix addition and multiplication;
- index (exponent) laws and logarithm laws;
- average and instantaneous rates of change and their interpretation with respect to the graphs of functions;
- probability as long run proportion;
- forms of representation of an event space;
- probabilities for a given event space are non-negative and the sum of these probabilities is one;
- rules for computing probabilities of compound events from simple events, in particular, cases involving mutual exclusiveness, independence and conditional probability.

Key skills

These skills include the ability to

- determine by hand the length of a line segment and the coordinates of its midpoint; the equation of a straight line given two points or one point and gradient; and the gradient and equation of lines parallel and perpendicular to a given line through some other point;
- specify the rule, domain and range of a relation and identify a relation which is also a function;
- substitute integer, simple rational and exact irrational values into formulas, including the rules of functions and relations and evaluate these by hand;
- rearrange simple algebraic equations and inequalities by hand;
- expand and factorise linear and simple quadratic expressions with integer coefficients (including difference of perfect squares and perfect squares) by hand;
- express $ax^2 + bx + c$ in completed square form where a, b and c are integers by hand;
- express a cubic polynomial, with integer coefficients, as the product of a linear factor $(x – a)$, where a is an integer, and a quadratic factor, by hand;
- use a variety of analytical, graphical and numerical approaches, including the factor theorem, to determine and verify solutions to equations over a specified interval;
- apply index (exponent) laws and logarithm laws to manipulate and simplify expressions and solve simple equations involving these terms, by hand in simple cases;
- set up and solve systems of simultaneous linear equations involving up to four unknowns, by hand for a system of two equations in two unknowns;
- sketch by hand graphs of linear, quadratic and cubic polynomial functions, and quartic polynomial functions in factored form (approximate location of stationary points only for cubic and quartic functions);
- sketch by hand graphs of power functions $y = x^n$, $n = –2, –1, \frac{1}{2}, 1, 2, 3$ and 4, simple transformations of these and sketch by hand circles of a given centre and radius;
- draw graphs of polynomial functions to degree 4 (*approximate* location of stationary points only for cubic and quartic functions), power functions and circles of a given centre and radius;
- describe the effect of transformations (dilation from the coordinate axes; reflection in the coordinate axes and the line $y = x$, and translation), and simple combinations of these transformations, on the graphs of linear functions and power functions;
- use finite difference tables to obtain polynomial models for sets of data where there are constant first, second or third differences;
- apply matrix addition and multiplication, by hand in simple cases, to the computation of images of point under transformation in the cartesian plane;
- use graphical, numerical and analytical approaches to find an approximate value or the exact value (as appropriate) for the gradient of a secant or tangent to a curve at a given point;
- describe the notion of randomness and its relation to events;
- define and calculate probability as an expression of long run proportion;
- calculate probabilities for compound events by hand in simple cases;
- use rules for computing the probabilities of compound events from simple events, including the concepts of mutual exclusiveness, independence and conditional probability.

Outcome 2

On completion of this unit the student should be able to apply mathematical processes in non-routine contexts, and analyse and discuss these applications of mathematics.

To achieve this outcome the student will draw on knowledge and related skills outlined in one or more areas of study.

Key knowledge

This knowledge includes

- key mathematical content from one or more areas of study relating to a given context for investigation;
- specific and general formulations of concepts used to derive results for analysis within a given context for investigation;
- the role of examples, counter-examples and general cases in developing mathematical analysis;
- inferences from analysis and their use to draw valid conclusions related to a given context for investigation.

Key skills

These skills include the ability to

- specify the relevance of key mathematical content from one or more areas of study to the investigation of various questions in a given context;
- develop mathematical formulations of specific and general cases used to derive results for analysis within a given context for investigation;
- use a variety of techniques to verify results;
- make inferences from analysis and use these to draw valid conclusions related to a given context for investigation;
- communicate conclusions using both mathematical expression and everyday language, in particular, the interpretation of mathematics with respect to the context for investigation.

Outcome 3

On completion of this unit the student should be able to select and use a computer algebra system and other technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.

To achieve this outcome the student will draw on knowledge and related skills outlined in all the areas of study.

Key knowledge

This knowledge includes

- exact and approximate specification of mathematical information such as numerical data, graphical forms and general or specific forms of solutions of equations produced by use of a computer algebra system;
- domain and range requirements for a computer algebra system's specification of graphs of functions and relations;
- the role of parameters in specifying general forms of functions and equations;
- the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions or equations;

- the similarities and differences between formal mathematical expressions and their computer algebra system representation;
- the appropriate selection of a technology application, in particular, computer algebra systems, in a variety of mathematical contexts.

Key skills

These skills include the ability to

- distinguish between exact and approximate presentations of mathematical results produced by a computer algebra system, and interpret these results to a specified degree of accuracy;
- produce results using a computer algebra system which identify examples or counter-examples for propositions;
- produce tables of values, families of graphs or collections of other results using a computer algebra system, which support general analysis in problem-solving, investigative or modelling contexts;
- use appropriate domain and range specifications to illustrate key features of graphs of functions and relations;
- identify the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions or equations;
- specify the similarities and differences between formal mathematical expressions and their computer algebra system representation, in particular, equivalent forms of symbolic expressions;
- make appropriate selections for a computer algebra system and other technology applications in a variety of mathematical contexts, and provide a rationale for these selections;
- relate the results from a particular application to the nature of a particular mathematical task (investigative, problem solving or modelling) and verify these results;
- specify the process used to develop a solution to a problem using a computer algebra system, and communicate the key stages of mathematical reasoning (formulation, solution, interpretation) used in this process.

ASSESSMENT

The award of satisfactory completion for a unit is based on a decision that the student has demonstrated achievement of the set of outcomes specified for the unit. This decision will be based on the teacher's assessment of the student's overall performance on assessment tasks designated for the unit.

The key knowledge and skills listed for each outcome should be used as a guide to course design and the development of learning activities. The key knowledge and skills do not constitute a checklist and such an approach is not necessary or desirable for determining the achievement of outcomes. The elements of key knowledge and skills should not be assessed separately.

Assessment tasks must be a part of the regular teaching and learning program and must not unduly add to the workload associated with that program. They must be completed mainly in class and within a limited timeframe. Teachers should select a variety of assessment tasks for their assessment program to reflect the key knowledge and skills being assessed and to provide for different learning styles.

For this unit students are required to demonstrate achievement of three outcomes. As a set these outcomes encompass all areas of study.

Demonstration of achievement of Outcomes 1 and 2 must be based on the student's performance on a selection of assessment tasks. Where teachers allow students to choose between tasks they must ensure that the tasks they set are of comparable scope and demand.

Demonstration of achievement of Outcome 1 must be based on a selection of the following tasks:

- assignments;
- tests;
- summary or review notes.

Demonstration of achievement of Outcome 2 must be based on a selection of the following tasks:

- projects;
- short written responses;
- problem-solving tasks;
- modelling tasks.

Demonstration of achievement of Outcome 3 must be based on the student’s performance on a selection of tasks completed in demonstrating achievement of Outcomes 1 and 2, which incorporate the effective and appropriate use of computer algebra system technology in contexts related to the content of the areas of study.

Unit 2: Mathematical Methods (CAS)

The areas of study for Unit 2 are ‘Functions and graphs’, ‘Algebra’, ‘Rates of change and calculus’, and ‘Probability’. At the end of Unit 2, students will be expected to have covered the material outlined in each area of study. Material from the ‘Functions and graphs’, ‘Algebra’, ‘Rates of change and calculus’, and ‘Probability’ areas of study should be organised so that there is a clear progression of skills and knowledge from Unit 1 to Unit 2 in each area of study.

Students are expected to be able to apply techniques, routines and processes involving rational and real arithmetic, algebraic manipulation, equation solving, graph sketching, differentiation and integration with and without the use of technology, as applicable. Students should be familiar with relevant mental and by hand approaches in simple cases.

The appropriate use of computer algebra system (CAS) technology to support and develop the teaching and learning of mathematics, and in related assessments, is to be incorporated throughout the unit. Other technologies such as spreadsheets, dynamic geometry or statistical analysis software may also be used, as appropriate, for various topics from within the areas of study for the course.

AREAS OF STUDY

1. Functions and graphs

This area of study covers graphical representation of functions of a single real variable and the study of key features of graphs of functions such as axis intercepts, domain (including maximal domain) and range of a function, asymptotic behaviour, periodicity and symmetry.

This area of study will include:

- revision of trigonometric ratios and their applications to right-angled triangles, including exact values for sine, cosine and tangent of 30°, 45° and 60°;
- radians: unit circle definition, conversion between radians and degrees;
- the unit circle and definition of the sine, cosine and tangent functions as functions of a real variable, including exact values for $\frac{n\pi}{6}$ and $\frac{n\pi}{4}$, $n \in Z$;
- the relationships $\sin^2(x) + \cos^2(x) = 1$ and $\tan(x) = \frac{\sin(x)}{\cos(x)}$;
- symmetry properties, complementary relations and periodicity properties for the sine, cosine and tangent functions;

- graphs of circular functions of the form $y = af(bx) + c$, where f is the sine, cosine or tangent function, and a, b and $c \in R$;
- simple applications of circular functions of the above form to model tidal heights, sound waves, bio-rhythms, ovulation cycles, temperature fluctuations during a day and the interpretation of period, amplitude and mean value in these contexts and their relationship to the parameters a, b and c;
- graphs of $y = Aa^{kx} + C$, where $a \in R^+$, for simple cases of a, A, k and $C \in R$;
- the graph of $y = \log_a(x)$ as the graph of the inverse function of $y = a^x$, including the relationships $a^{\log_a(x)} = x$ and $\log_a(a^x) = x$;
- simple applications of exponential functions of the above form to model growth and decay in populations and the physical world, appreciation and depreciation of value in finance; the interpretation of initial value, rate of growth or decay and long run value in these contexts and their relationship to the parameters A, k and C.

2. Algebra

This area of study provides an opportunity for the revision and further development of content prescribed in Unit 1, as well as the study of related algebra material introduced in the other areas of study in Unit 2 including circular functions, exponential functions and logarithmic functions. The content as described in the ‘Algebra’ area of study in Unit 1 is to be distributed across Units 1 and 2.

3. Rates of change and calculus

This area of study covers first principles approach to differentiation, formal differentiation and anti-differentiation of polynomial functions and power functions and related applications, including graph sketching.

This area of study will include:

- the derivative as the gradient of the graph of a function at a point and its representation by a gradient function;
- notation for derivatives: $D_x(f)$, $\frac{dy}{dx}$, $f'(x)$ $\frac{d}{dx}(f(x))$;
- graphical and numerical approaches to approximating the value of the gradient function for simple polynomial functions and power functions at a given point in the domain of the function;
- first principles approach to finding the gradient function for $f(x) = x^n$, $n \in Z$ and simple polynomial functions;
- derivatives of simple power functions and polynomial functions;
- applications of differentiation:
  - finding rates of change;
  - determining maximum or minimum values of functions, including interval endpoint maximum and minimum values and their application to simple maximum/minimum problems;
  - use of the gradient function, to assist in sketching graphs of simple polynomials, in particular, the identification of stationary points;
- anti-differentiation as the reverse process of differentiation and identification of families of curves with the same gradient function;
- application of anti-differentiation to problems involving straight-line motion, including calculation of distance travelled.

4. Probability

This area of study covers introductory counting principles and techniques and their application to probability, the law of total probability in the case of two events, and the application of transition matrices to conditional probabilities.

This area of study will include:

- addition and multiplication principles for counting;
- combinations: concept of a selection and computation of ${}^{n}C$, and for pascal's triangle;
- applications of counting techniques to probability;
- the law of total probability for two events $\Pr(A) = \Pr(A|B)\Pr(B) + \Pr(A|B')\Pr(B')$;
- use of 2×2 transition matrices to calculate probabilities of two state markov chains (consideration of steady state not required).

OUTCOMES

For this unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass all of the areas of study for the unit.

Outcome 1

On completion of this unit the student should be able to define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.

To achieve this outcome the student will draw on knowledge and related skills outlined in all the areas of study.

Key knowledge

This knowledge includes

- exact values of sine, cosine and tangent for 0, $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ (and their equivalents in degrees) and integer multiples of these;
- key features and properties of the circular functions sine, cosine, and tangent and their graphs;
- the effect of transformations (dilation from the coordinate axes; reflection in the coordinate axes and vertical translation) and simple combinations of these transformations on the graphs of sine, cosine, tangent and exponential functions;
- characteristics of data which suggest the use of circular functions or exponential functions as an appropriate type of model for a given context;
- exponential and logarithmic functions and properties of their graphs;
- the relationship between an exponential function to a given base and the logarithmic function to the same base as inverse functions;
- the derivative function f' as the rate of change or gradient function of a given function f;
- informal concept of a limit and the limit definition of the derivative function;
- the sign of the gradient at and near a point and its interpretation in terms of key features of the graph of simple polynomial functions;
- rules for finding derivatives and anti-derivatives of simple power functions and polynomial functions;
- counting techniques and their rules of application;
- representation of conditional probabilities using transition matrices.

Key skills

These skills include the ability to

- sketch by hand graphs of the sine, cosine and exponential functions, and simple transformations of these, sketch by hand graphs of $\log_a(x)$ and the tangent function;
- draw graphs of circular, exponential and simple logarithmic functions over a given domain and identify and discuss key features and properties of these graphs;
- describe the effect of transformations (dilation from the coordinate axes; reflection in the coordinate axes and vertical translation) and simple combinations of these transformations on the graphs of the sine, cosine, tangent and exponential functions;
- solve simple equations over a specified interval related to these functions using graphical, numerical and analytical approaches;
- recognise characteristics of data in tabular or graphical form which suggest the selection of one of these types of functions as an appropriate model for the data, and obtain a corresponding function as a model for the data;
- evaluate limits for polynomial expressions and simple rational expressions;
- use a variety of approaches (numerical, graphical, first principles or by rule) to find the value of the derivative of a function at a given point;
- find by hand the derivative function and an antiderivative function for a simple power function or a polynomial function up to degree four;
- use derivatives to assist in the sketching of graphs of simple polynomial functions and to solve simple maximum/minimum optimisation problems;
- find a family of anti-derivative functions for a given power or polynomial function;
- apply counting principles and manipulations to solve problems in contexts where repetition may or may not be allowed, and order may or may not be important, including cases involving simple restrictions;
- calculate probabilities for compound events, by hand in simple cases;
- calculate matrix products involving 2×2 transition matrices, by hand in simple cases;
- use transition matrices to calculate combinations of probabilities involving non-independent events.

Outcome 2

On the completion of this unit the student should be able to apply mathematical processes in non-routine contexts, and analyse and discuss these applications of mathematics.

To achieve this outcome the student will draw on knowledge and related skills outlined in one or more areas of study.

Key knowledge

This knowledge includes

- key mathematical content from one or more areas of study relating to a given context for investigation;
- specific and general formulations of concepts used to derive results for analysis within a given context for investigation;
- the role of examples, counter-examples and general cases in developing mathematical analysis;
- inferences from analysis and their use to draw valid conclusions related to a given context for investigation.

Key skills

These skills include the ability to

- specify the relevance of key mathematical content from one or more areas of study to the investigation of various question in a given context;
- give mathematical formulations of specific and general cases used to derive results for analysis within a given context for investigation;
- use a variety of techniques to verify results;
- make inferences from analysis and use these to draw valid conclusions related to a given context for investigation;
- communicate conclusions using both mathematical expression and everyday language, in particular, interpretation of mathematics with respect to the context for investigation.

Outcome 3

On completion of each unit the student should be able to select and use a computer algebra system and other technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.

To achieve this outcome the student will draw on knowledge and related skills outlined in the areas of study.

Key knowledge

This knowledge includes

- exact and approximate specification of mathematical information such as numerical data, graphical forms and general or specific forms of solutions of equations produced by use of a computer algebra system;
- domain and range requirements for a computer algebra system's specification of graphs of functions and relations;
- the role of parameters in specifying general forms of functions and equations;
- the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions or equations;
- the similarities and differences between formal mathematical expressions and their computer algebra system representation;
- the appropriate selection of a technology application, in particular, computer algebra systems, in a variety of mathematical contexts.

Key skills

Theses skills include the ability to

- distinguish between exact and approximate presentations of mathematical results produced by a computer algebra system, and interpret these results to a specified degree of accuracy;
- produce results using a computer algebra system which identify examples or counter-examples for propositions;
- produce tables of values, families of graphs or collections of other results using a computer algebra system, which support general analysis in problem-solving, investigative or modelling contexts;
- use appropriate domain and range specifications to illustrate key features of graphs of functions and relations;
- identify the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions or equations;

- specify the similarities and differences between formal mathematical expressions and their computer algebra system representation, in particular, equivalent forms of symbolic expressions;
- make appropriate selections for a computer algebra system and other technology applications in a variety of mathematical contexts, and provide a rationale for these selections;
- relate the results from a particular application to the nature of a particular mathematical task (investigative, problem solving or modelling) and verify these results;
- specify the process used to develop a solution to a problem using a computer algebra system, and communicate the key stages of mathematical reasoning (formulation, solution, interpretation) used in this process.

ASSESSMENT

The award of satisfactory completion for a unit is based on a decision that the student has demonstrated achievement of the set of outcomes specified for the unit. This decision will be based on the teacher's assessment of the student's overall performance on assessment tasks designated for the unit.

The key knowledge and skills listed for each outcome should be used as a guide to course design and the development of learning activities. The key knowledge and skills do not constitute a checklist and such an approach is not necessary or desirable for determining the achievement of outcomes. The elements of key knowledge and skills should not be assessed separately.

Assessment tasks must be a part of the regular teaching and learning program and must not unduly add to the workload associated with that program. They must be completed mainly in class and within a limited timeframe. Teachers should select a variety of assessment tasks for their assessment program to reflect the key knowledge and skills being assessed and to provide for different learning styles.

For this unit students are required to demonstrate achievement of three outcomes. As a set these outcomes encompass all areas of study.

Demonstration of achievement of Outcomes 1 and 2 must be based on the student's performance on a selection of assessment tasks. Where teachers allow students to choose between tasks they must ensure that the tasks they set are of comparable scope and demand.

Demonstration of achievement of Outcome 1 must be based on a selection of the following tasks:

- assignments;
- tests;
- summary or review notes.

Demonstration of achievement of Outcome 2 must be based on a selection of the following tasks:

- projects;
- short written responses;
- problem-solving tasks;
- modelling tasks.

Demonstration of achievement of Outcome 3 must be based on the student's performance on a selection of tasks completed in demonstrating achievement of Outcomes 1 and 2, which incorporate the effective and appropriate use of computer algebra system technology in contexts related to the content of the areas of study.

Advice for teachers (Units 1 and 2: Mathematical Methods (CAS))

DEVELOPING A COURSE

A course outlines the nature and sequence of teaching and learning necessary for students to demonstrate achievement of the set of outcomes for a unit. The areas of study describe the knowledge required for the demonstration of each outcome. Outcomes are introduced by summary statements and are followed by the key knowledge and skills which relate to the outcomes.

In particular, it should be noted that the key skills for Outcome 1 include routines that students are expected to be able to carry out by hand without the use of technology. In some instances these will only require writing down of a result obtained by application of a general rule or pattern, for example differentiation of a polynomial function, in other instances several steps of manipulation may be required, for example re-expression of the rule of a cubic polynomial function in terms of a specified linear factor $(x – a)$ by equating coefficients. Students should be conversant with such mental and by hand routines.

For Units 1 and 2, teachers must select assessment tasks from the lists provided. Tasks should provide a variety and the mix of tasks should reflect the fact that different types of tasks suit the assessment of different knowledge and skills. For example, analysis of the behaviour of a function, such as the location of its minimum value over a given interval in terms of changes in the values of its coefficients, could be assessed by a short and focused investigation, and incorporate the required use of technology, while the development of by hand skills could be assessed either by a test, an assignment or a component of an investigation. Tasks do not have to be lengthy to make a decision about student demonstration of achievement of an outcome.

Sample teaching sequence

The following sample course outline is based on the assumption of 17 teaching weeks per semester, allowing time for review and assessment. Teachers are likely to draw on material from across the areas of study in covering content related to different topics in various ways, and should adjust the time allocated accordingly.

Topics and possible assessment

Topic	Weeks	Assessment
Unit 1		
Polynomial and power functions	6	Test (week 5) Project – fitting curves to data (week 6)
Functions and relations	4	'Solving equations' assignment (week 3) Test (week 4)
Rates of change	4	Test (week 3) 'Rates' problem solving/modelling task (week 4)
Probability 1 (Introductory probability)	3	Short written response – simulation task (week 4) Summary notes (week 3)
Unit 2		
Probability 2 (Counting principles, transition matrices and applications to probability)	3	Short written response – transition matrices (week 1) Test (week 3)
Exponential and logarithmic functions	4	Growth and decay assignment (week 3) Test (week 4)
Circular functions	5	Test (week 4) Project – modelling with circular functions (week 5)
Differentiation and anti-differentiation of polynomial functions	5	Test (week 4) Maximum/minimimum optimisation problem-solving task or straight line motion assignment (week 5)

Description of topics and related CAS functionality

Topic	Approximate time	CAS functionality
Unit 1		
Polynominal and power functions	**6 weeks**	
Use of the notation $y = f(x)$ for describing the rule of a function and evaluation of $f(a)$, where a is a real number or a symbolic expression. Graphs of power functions $f(x) = x^n$ for $n \in N$ and $n = -1, -2, \frac{1}{2}$ (where $x \geq 0$ when $n = \frac{1}{2}$) and transformations* of these to the form: $f(x) = a(x + b)^n + c$ where a, b and $c \in R$. Use of parameters to represent a family of power functions and general solutions of equations involving these functions.	2 weeks	Definition of $f(x)$ and evaluation of $f(a)$, where a is numeric or symbolic. Production of table of values for $f(a)$, where a is numeric or symbolic, over a given interval. Drawing the graph of $f(x)$ over a given domain. Production of a family of graphs using a parameter in the definition of $f(x)$. Solution of $a(x + b)^n + c = k$ where k is a real number.

Unit 1 (continued)		
Topic	**Approximate time**	**CAS functionality**
Polynominal and power functions	**6 weeks**	
Application of matrices to transformations of points in the plane (dilation, reflection and translation)* and the solution of systems of simultaneous linear equations# in up to four unknowns.	1 week	Specification of matrices and evaluation of sum, difference, product and inverse solution of matrix equations of the form $AX = B$. Solution of a system of simultaneous linear equations.
Graphs of polynomial functions to degree 4. Substitution into, and expansion and factorisation of, algebraic expressions, including the remainder and factor theorems. Solution of polynomial equations to degree 4, analytically, numerically and graphically. Connection between factors of $f(x)$, solutions of the equation $f(x) = 0$ and the horizontal axis intercepts of the graph of the function f.	2 weeks	Simplification of algebraic expressions and polynomial division. Expansion and factorisation of polynomial algebraic expressions. Solution of polynomial equations numerically, analytically and graphically.
Development of polynomial models for sets of data, for example, by the use of finite difference tables or solution of a system of simultaneous linear equations# obtained from values of a function, or a simple combination of values of a function.	1 week	Evaluation of difference between consecutive values in a table or list. Solution of system of simultaneous equations: $\{f(x_1) = y_1 , f(x_2) = y_2, \dots , f(x_n) = y_n\}$.
Functions and relations	4 weeks	
Qualitative interpretation of features of graphs, and families of graphs including an informal consideration of rates of change. Use of symbolic notation to develop algebraic expressions and represent functions, relations and equations. Calculation of coordinates of the midpoint of a line segment, and gradients of parallel and perpendicular lines. Graphs of relations specified by conditions or constraints, and the graphs of inverse relations.	2 weeks	Drawing graphs of relations that are not functions, including inequalities. Determination of the rule of an inverse function or relation. Drawing the graph of an inverse function or relation.
Determination of rules of simple functions and relations from given information, and of transformations (dilation, reflection and translation) of the square root and circle relations. Use of inverse functions to solve equations, including consideration of equations involving functions that are not one-to-one. Solution and interpretation of simultaneous equations involving two functions by analytical, numerical and graphical methods.	2 weeks	Substitution of values into algebraic expressions and simplification of these expressions. Analytical solution of equations using inverse functions. Analytical and numerical solution of $f(x) = g(x)$. Identification of points of intersection of corresponding curves.

Unit 1 (continued)		
Topic	**Approximate time**	**CAS functionality**
Rates of change	**4 weeks**	
Rate of change of a linear function, use of gradient as a measure of rate of change; average rate of change, use of the gradient of a chord of a graph to describe average rate of change of $y = f(x)$ with respect to x, over a given interval. The measurement of rates of change of polynomials functions by finding successive numerical approximations to the gradient of a polynomial function at a point, leading to an informal treatment of the gradient of the tangent as a limiting value of the gradient of a chord (formed on either side at a given point).	1 week	Evaluation of gradient of chord as: $\dfrac{f(x+h)-f(x)}{h}$ and as: $\dfrac{f(x)-f(x-h)}{h}$ Production of tables of values for gradient of chord for different x and h. Evaluation of the limiting value of $\dfrac{f(x+h)-f(x)}{h}$ as $h \to 0$. Evaluation of $f'(x)$ as a limit expression.
Tangents to curves, use of gradient of a tangent at a point on the curve to describe instantaneous rate of change of $y = f(x)$ with respect to x. Graphs of functions and interpretation of the rate of change of the function, where the rate of change is positive, negative, or zero, relationship of the gradient function of a function to features of the original function.	2 weeks	Determination of $f'(x)$. Drawing the graph of $f(x)$ and $f'(x)$ for a given function over a specified domain.
Motion graphs, construction and interpretation of displacement–time and velocity–time graphs and informal treatment of the relationship between displacement–time and velocity–time graphs. Use of rates of change and corresponding graphs in other contexts.	1 week	Drawing graphs of $x(t)$ and $x'(t)$.
Probability (introduction)	**3 weeks**	
Random experiments, events and event spaces. Probability as an expression of long run proportion. Simulation using simple generators such as coins, dice, spinners, random number tables and computers. Display and interpretation of results of simulations.	½ week	Generation of random numbers and tables of values of random numbers.
Probability of simple and compound events. Lists, grids, venn diagrams, karnaugh maps and tree diagrams.	1 week	
The addition rule for probabilities. Conditional probability and independence, law of total probability, the multiplication rule for independent events. Probabilities involving sequences of repeated experiments for independent or conditional events.	1½ weeks	Evaluation of probabilities for sequences of repeated trials.

Unit 2		
Topic	**Approximate time**	**CAS functionality**
Probability 2 (applications of probability)	**3 weeks**	
Counting principles, combinations and the relationship of combinations to pascal's triangle. Applications to probability.	1½ weeks	Expansion of expressions of the form $(x + a)^n$.
Use of transition matrices to calculate probabilities of combinations of successive non-independent events (two-state markov chains).	1½ weeks	Evaluation of matrix powers T_n and application to computation of probabilities: $T_{n+1} = T^n \times S_0$.
Exponential and logarithmic functions	**4 weeks**	
Recognition of equivalent expressions and simplification of algebraic expressions involving functions and relations, including use of exponent and logarithm laws.	1 week	Manipulation and simplification of algebraic expressions involving exponential or logarithmic terms.
Graphs of $y = Aa^{kx} + B$, where $a \in R^+$, $a \neq 1$, for simple cases of A and $B \in R$, the graph of $y = \log_a x$ as the inverse of $y = a^x$, for simple cases of a where $a \in R^+$, $a \neq 1$, informal discussion of the inverse relationship.	1 week	Transformation of $f(x) = a^x$, production of families of graphs of exponential functions based on variation of parameters used to define rule, determination of rules for inverse functions.
Solution of equations of the form $Af(kx) + B = C$, where A, B, C and $k \in R$, and $f(x) = a^x$ for $a \in R^+$, $a \neq 1$, using exact or approximate values and interpretation of these equations.	1 week	Analytic, numeric and graphical solution of equations involving exponential functions, including general cases in terms of parameters used to specify these equations.
Illustration of the application of exponential functions of the form $y = Aa^{kx} + B$, for simple cases of A, k and $B \in R$, in data modelling such as growth and decay models in populations and the physical world, appreciation and depreciation of value in finance and the interpretation of initial, value, rate of growth or decay and long run value in these contexts and their relationship to parameters A, k and B.	1 week	Generation of sequences of values with constant ratio between successive terms.
Circular functions	**5 weeks**	
Revision of trigonometric ratios and their applications to right-angled triangles, including exact values for sine, cosine and tangent of 30°, 45° and 60°; radians: unit circle definition, conversion between radians and degrees; the unit circle and definition of sine, cosine and tangent functions as functions of a real variable, including exact values for $\frac{n\pi}{6}$ and $\frac{n\pi}{4}$, $n \in Z$.	1 week	Computation of exact values for circular functions where the independent variable is $\frac{n\pi}{6}$ and $\frac{n\pi}{4}$, $n \in Z$ and approximate values for all $x \in R$.

Unit 2 (continued)		
Topic	**Approximate time**	**CAS functionality**
The relationships $\sin^2(x) + \cos^2(x) = 1$, $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$, symmetry properties, complementary relations and periodicity properties for sine, cosine and tangent functions, symmetry properties of sine, cosine and tangent over the interval $[0, 2\pi]$.	1 week	Use of symmetry, complementarity and periodicity to identify families of values of x for which $f(x) = k$, $k \in R$. Manipulation of expressions involving circular functions to identify equivalent expressions using specified relationships and formulas.
Graphs of circular functions of the form $y = af(bx) + c$ where f is sine, cosine or tangent, and a, b and $c \in R$.	1 week	Transformation of $f(x) = \sin(x)$, $f(x) = \cos(x)$ and $f(x) = \tan(x)$ to produce families of graphs of circular functions based on variation of parameters used to define rules.
Solution of equations of the form $af(bx) + c = k$, where a, b, c and $k \in R$, and f is sine, cosine or tangent using exact or approximate values over one period and interpretation of these equations.	1 week	Analytic, numeric and graphical solution of equations involving sine, cosine and tangent, including general cases in terms of parameters used to specify these equations.
Illustration of the application of circular functions of the form $y = af(xb) + c$, where f is sine or cosine for simple values of a, b and $c \in R$, in data modelling such as tidal heights, sound waves, bio-rhythms, ovulation cycles, temperature fluctuations during a day and the interpretation of period and amplitude in these contexts and their relationship to the parameters a, b and c.	1 week	Draw graphs of transformed circular functions of sine and cosine.
Differentiation and anti-differentiation of power and polynomial functions	**5 weeks**	
The derivative as the gradient of the graph at a point and its representation by a gradient function; notation for derivatives: $D_x(f)$, $\dfrac{dy}{dx}, f'(x), \dfrac{d}{dx}(f(x))$. Graphical and numerical approaches to approximating the value of the gradient function for simple polynomial and power functions at a given point in the domain of the function. First principles approach to finding the gradient function for $f(x) = x^n$, $n \in Z$ and simple polynomial functions. Derivatives of simple power functions and polynomial functions	1 week	Evaluation of numerical approximations to the gradient function at $x = a$ by: $\dfrac{f(b) - f(a)}{b - a}$ for b close to a, or by: $\dfrac{f(x+h) - f(x)}{h}$ for small values of h. Determination of derivatives from first principles using limits. Finding $f'(x)$, evaluation of $f'(a)$.

Unit 2 (continued)		
Topic	**Approximate time**	**CAS functionality**
Applications of differentiation – finding rates of change – determining maximum or minimum values of functions, including interval endpoint maximum and minimum values and their application to simple maximum/minimum problems – use of the gradient function to assist in sketching graphs of simple polynomials, in particular, the identification of stationary points.	2 weeks	Drawing graphs of f and f'. Solution of $f'(x) = k$, in particular where $k = 0$. Simplification of a set of algebraic expressions in several variables to a function of a single variable. Identification of maximum and minimum values of f over its domain.
Anti-differentiation as the reverse process of differentiation and identification of families of curves with the same gradient function.	1 week	Determination of an anti-derivative for a given function. Specification of a general family of anti-derivatives for a given function, determination of a specific anti-derivative for a given function satisfying suitable boundary conditions.
Application of anti-differentiation to problems involving straight line motion, including calculation of distance travelled (possible use of modulus function).	1 week	Evaluation of expressions for distance in terms of differences in position (which may involve turning points).

COURSE DEVELOPMENT

An introduction to polynomial functions

Polynomial functions provide one of the simplest types of functions for modelling in a wide range of situations and for introducing the mathematical processes which a computer algebra system can apply (its *functionality*).

When a computer algebra system is used to *define* a polynomial function f, or more specifically its rule $f(x)$ as a formal symbolic object, this can then be used as a unifying construct that underpins many aspects of the course. At the simplest level, a computer algebra system can be used to carry out processes such as *evaluate* $f(x)$ when numerical and/or algebraic expressions are *substituted* for the variable x; construct a *table* for a specified sequence of values of x; *plot* a graph of $f(x)$ over a given interval; *solve* equations such as $f(x) = x$, *differentiate* $f(x)$ and *integrate* $f(x)$.

Computer algebra system functionality can be combined so that several processes are carried out together, for example to solve $f'(x) = 0$ for x, to produce a table of expressions for $f(x + h) – f(x)$ and $f(x) – f(x – h)$ where $f(x) = x^n$ for a sequence of values of n; or to produce a collection of graphs that illustrate the effect of varying a parameter used in the definition of a function, such as for $f(x) = x(x – 2)(x – k)$ where k takes integer values of from –4 to 4. Many of these processes apply equally well to other types of functions.

Computer algebra system functionality can be used in transformations of graphs, algebraic re-expression of mathematical expressions in equivalent forms, addition of ordinates, solving simultaneous equations, rates of change and differentiation from first principles and evaluation of powers of matrices.

Polynomial functions are readily evaluated by combinations of arithmetic operations, are everywhere continuous and differentiable and their behaviour is relatively simple to describe graphically. They are used to approximate more complex functions such as circular functions, to construct curves through points such as by a computer-assisted drawing package and to develop quantitative models for sets of data such as the bending of a beam under load or for trend data and coding of electronic communications. Relevant data may have been obtained from various sources, including experiments or records, possibly from other VCE studies, journals, magazines, websites or newspapers. This type of source material can be used to develop an integrated approach to key concepts, skills and processes related to the study of polynomial functions.

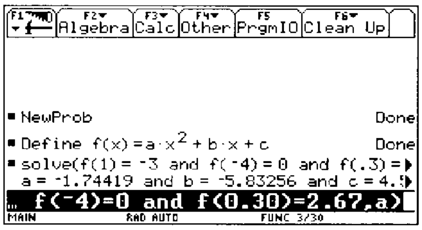
An important property of polynomial functions is that for any set of *n* distinct points (which do not all lie on a straight line), there is a polynomial function of degree $n - 1$ whose graph will pass exactly through these *n* points. If there are *n* points:

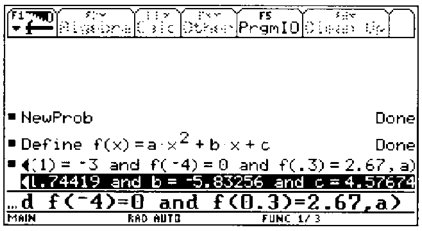
x_1	x_2	x_3	x_4	...	x_n
y_1	y_2	y_3	y_4	...	y_n

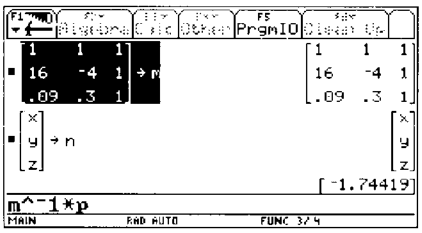
and *p* is a polynomial function of degree $n - 1$ with rule $p(x)$, then the set of simultaneous linear equations: $\{y_1 = p(x_1), y_2 = p(x_2) \ldots y_n = p(x_n)\}$ can be solved to determine the coefficients of the powers of *p*. In the case of a quadratic function, *q*, the coordinates of three such points would be required to determine the coefficients *a*, *b* and *c* of $q(x) = ax^2 + bx + c$. If the coordinates of the three points are $(1, -3)$, $(-4, 0)$ and $(0.30, 2.67)$.

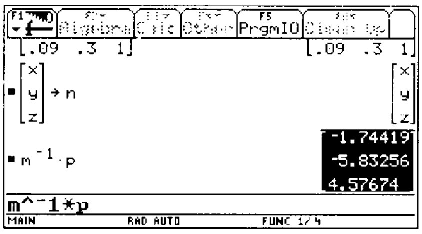
This results in the set of simultaneous linear equations: $\{q(1) = -3, q(-4) = 0, q(0.30) = 2.67\}$ or equivalently $\{-3 = a + b + c, 0 = 16a - 4b + c, 2.67 = 0.09a + 0.30b + c\}$.

Such a system of equations can be solved by hand; however, the equation-solving facilities of many computer algebra systems (or the use of their matrix algebra functions) will easily carry out this process for both integer and non-integer values of the coordinates involved. The following screen dumps provide an illustrative output (note that the coefficients *a*, *b* and *c* of the original rule become the values to be determined (unknowns) of the system of equations, which are represented by the variables *x*, *y* and *z* respectively in the calculator format).









In this case the function q would have the particular rule $q(x) = -1.74\,x^2 + 5.83x + 4.58$, where the coefficients are correct to two decimal places. A suitable domain of interpretation would need to be specified with respect to the context under consideration.

This approach also extends to situations where more complex systems of equations are required to be solved. While students should be able to apply the relevant processes by hand in straightforward cases, modelling real data using this method will require the appropriate support of technology to obtain solutions to this type of problem efficiently. A computer algebra system can be used to find these coefficients, draw the corresponding graph and perform related analysis of key features (such as axis intercepts, maximum/minimum values); however, an algebraic approach is required to establish general results. Solving the general equation for $q'(x) = 0$ gives a stationary point when $2ax + b = 0$ or $x = -\dfrac{b}{2a}$. The corresponding maximum or minimum value (depending on the sign of a) can then be found by evaluating $q\left(-\dfrac{b}{2a}\right)$. Knowledge of this general result facilitates the analysis of maximum and minimum values of data modelled by quadratic functions.

The solutions of a quadratic equation of the form $q(x) = k$ can be found by reference to the quadratic formula, which is based on the completion of the square process. Any quadratic function with rule $q(x) = ax^2 + bx + c$ can be re-expressed in the completed square form $q(x) = A(x + B)^2 + C$ where $A = a$, which can also be used to sketch its graph as a sequence of transformations of the graph of $y = x^2$.

For the case of cubic polynomial functions there is no simple formula like the quadratic formula (although there is a complicated general formula which involves the use of complex numbers) since not all graphs of cubic polynomial functions are transformations of the graph of the basic function with rule $y = x^3$. Thus, while: $c(x) = ax^3 + bx^2 + cx + d$ and $c(x) = (x - k)$ are general forms of the rule for a cubic polynomial function, this is not the case for the form: $c(x) = A(x + B)^3 + C$

The second form is found by re-expressing $c(x)$ in terms of $(x - k)$ and leads to the remainder theorem, $c(k) = r$ and its corollary the factor theorem: if $c(k) = 0$ then $(x - k)$ is a factor of $c(k)$. This re-expression can be carried out by polynomial division, or by equating coefficients of the powers of x in the cubic polynomial, for example: $x^3 - 3x^2 + 6x - 7 = x^2\,(x - 2) - x\,(x - 2) + 4(x - 2) + 1 = (x - 2)\,(x^2 - x + 4) + 1$

In many exercises, students deal with cases where $r = 0$ and the quadratic expression $(lx^2 + mx + n)$ has simple integer or rational factors. They should be able to identify intervals within which a root to the equation c(x) = 0 must lie, and could also seek to identify whether such a root is rational. For example if it is assumed that the particular cubic polynomial function with rule $c(x) = 2x^3 + 3x^2 - 22x - 33$ has at least one linear factor $(ux + v)$ which leads to a rational root then: $c(x) = 2x^3 + 3x^2 - 22x - 33 = (ux + v)$ where u and v are integers such that $ul = 2$ and $vn = -33$. Hence u would a factor of 2 and v a factor of –33, so it is only necesary to test whether $c\left(\dfrac{-v}{u}\right) = 0$ for the possible rational combinations of $p = \pm1, \pm2$ with q = ±1, ±3, ±11, ±33. Inspection reveals that $c\left(\dfrac{-3}{2}\right) = 0$, hence $(2x + 3)$ is a linear factor of $c(x)$.

Re-expression gives $x^2 - 11 = (x + \sqrt{11})(x - \sqrt{11})$ as the other factor. While approximate numerical values of key features of cubic polynomial functions can be identified using technology (such as coordinates of axis intercepts and stationary points), description of the general behaviour of these functions requires an analytic approach. The nature and existence of stationary points can be obtained from examination of the quadratic equation related to the derivative function. Students could be asked to investigate under what conditions a cubic polynomial function will have rational x axis intercepts and rational x values for its stationary points.

An approach to developing the general rule for the derivative of x^n

A computer algebra system can be readily used to develop a numerical approach for conjecturing a limit value for the gradient of a function at a specific x value. If the function with the rule $f(x) = x^2$ is considered, then both the following expressions can be evaluated for various values of x:

right secant gradient = $\dfrac{f(x+h)-f(x)}{h}$ and left secant gradient = $\dfrac{f(x)-f(x-h)}{h}$

For example, if $x = 2$, a numerical approximation for $f'(2)$ can be obtained by considering a table of values for these expressions where h varies from say 0.1 to 0.0001. The values in the table below can be seen to strongly suggest convergence to a limiting value of 4 for $f'(2)$. Initially this can be done by hand as a collection of individual calculations to develop student familiarity with the process. For the particular function with the rule $f(x) = x^2$ when $x = 2$ these expressions simplify to:

right secant gradient = $\dfrac{(2+h)^2-4}{h}$ and left secant gradient = $\dfrac{4-(2-h)^2}{h}$

h	Left secant gradient	Right secant gradient
0.1	3.9	4.1
0.01	3.99	4.01
0.001	3.999	4.001
0.0001	3.9999	4.0001

When a sufficient range of x values has been systematically investigated the rule for $f'(x)$ can be conjectured as likely to be $f'(x) = 2x$. A similar approach can be taken for other functions of the form $f(x) = x^n$. An advantage of this approach is that it enhances the ability to deal effectively with an important concept, that the derivative is defined when two limits (the limiting values as h tends to 0 of both the left secant gradient and the right secant gradient) have an identical value, that is the gradient of the unique tangent at a given in the domain of the function. An informal notion of differentiability arises naturally from such considerations.

This can then lead to the development of a first principles approach where one of these limits is applied to simple power functions such as $f(x) = 1$, $f(x) = x$, $f(x) = x^2$ and $f(x) = x^3$. Computer algebra systems can be used to develop these ideas further where students have previously worked through the cases for $n = 1$, 2 and 3 by hand.

If the more general case $f(x) = x^n$ is considered, then it is possible for computer algebra system functionality to produce summaries of related computations such as:

$$\text{Table }\left[\left\{n,\ f[x],\ \text{Expand}\left[\frac{f[x+h]-f[x]}{h}\right], \text{Limit}\left[\frac{f[x+h]-f[x]}{h}\right], h \to 0\right\}, \{n, 0, 10\}\right]$$

0	1	0	0
1	x	1	1
2	x^2	$h + 2x$	$2x$
3	x^3	$h^2 + 3hx + 3x^2$	$3x^2$
4	x^4	$h^3 + 4h^2x + 6hx^2 + 4x^3$	$4x^3$
5	x^5	$h^4 + 5h^3x + 10h^2x^2 + 10hx^3 + 5x^4$	$5x^4$
6	x^6	$h^5 + 6h^4x + 15h^3x^2 + 20h^2x^3 + 15hx^4 + 6x^5$	$6x^5$
7	x^7	$h^6 + 7h^5x + 21h^4x^2 + 35h^3x^3 + 35h^2x^4 + 21hx^5 + 7x^6$	$7x^6$
8	x^8	$h^7 + 8h^6x + 28h^5x^2 + 56h^4x^3 + 70h^3x^4 + 56h^2x^5 + 28hx^6 + 8x^7$	$8x^7$

These can then be used to lead into the general case result for x^n based on expansion of $(x + h)^n$.

A combination of numerical, graphical and analytical approaches could be used to extend this work to polynomial functions, where again students can tackle simple first principles linear and quadratic function examples by hand. The following could then be used as an analysis task to apply these ideas to the graphs of cubic polynomial functions:

- sketch the graph of the function f with the rule $f(x) = x^3 – 4x + 7$, identifying important features
- consider the function g with the rule $g(x)= x^3 – 4x + d$; find the value of d for which g has only one x axis intercept and three x axis intercepts respectively
- identify any values of d for which g has a stationary point of inflection
- consider the function h, with the rule $h(x) = x^3 + bx^2 + 4x + 3$. Explore the nature of the graph of h in terms of different values of b.

USE OF INFORMATION AND COMMUNICATIONS TECHNOLOGY

In designing courses and developing learning activities for Mathematical Methods (CAS) Units 1 and 2, teachers should make use of applications of information and communications technology and new learning technologies, such as computer-based learning, multimedia and the World Wide Web, where applicable to teaching and learning activities.

KEY COMPETENCIES AND EMPLOYABILITY SKILLS

Students undertaking the following types of assessment, in addition to demonstrating their understanding and mastery of the study, typically demonstrate the following key competencies and employability skills.

Assessment task	Key competencies and employability skills
Assignments	Learning, use of information and communications technology
Tests	Self management, use of information and communications technology
Summary or review notes	Self management
Projects	Communciation, team work, self management, planning and organisation, use of information and communications technology, initiative and enterprise
Short written responses	Communication, problem solving
Problem-solving tasks	Communication, problem solving, team work, use of information and communications technology
Modelling tasks	Problem solving, planning and organisation, use of information and communications technology

LEARNING ACTIVITIES

Examples of learning activities for each unit are provided in the following sections. The examples that make use of information and communications technology are identified by this icon ICT.

Units 1 and 2: Mathematical Methods (CAS)

Outcome 1

Define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.

Examples of learning activities

skills practice on standard mathematical routines through work on an appropriate selection of exercises (e.g. difference tables, transition matrices)

construction of summary or review notes related to a topic or area of study (e.g. quadratic and cubic factorisation techniques, conditional probability and independence)

assignments structured around the development of samples cases of standard applications of mathematical skills and procedures in readily recognisable situations

exercises such as the identification of key features of graphs of functions and relations on graphs produced using technology (e.g. location of axes intercepts and any stationary points, interval(s) over which a function is increasing or decreasing) and special features of particular functions and relations (e.g. asymptotic behaviour, periodicity, location of centres of circles)

Outcome 2

Apply mathematical processes in non-routine contexts, and analyse and discuss these applications of mathematics.

Examples of learning activities

investigative projects, such as exploring the properties of graphs of families of related functions or relations (e.g. the rate at which a container fills, given a constant rate of flow)

collections of problem-solving tasks, such as determining optimal values for problems related to application contexts (e.g. maximising or minimising area or volume)

modelling tasks, such as using functions to model experimental data from other areas of study, or make predictions based on modelling of trend data (e.g. circular functions applied to periodic patterns in weather, long run probabilities in two-state markov chains)

a set of applications questions requiring analysis and extended response on a particular context (e.g. seating arrangements with various restrictions)

a report on item response analysis for a collection of multiple-choice questions (this would involve a clear and detailed explanation as to why other alternatives did not provide an appropriate response to the question asked, that is did not account for all the information provided)

presentation on research into a particular application of mathematics (this could involve the use of presentation information technologies, an historical treatment of the development of key ideas of calculus)

Outcome 3

Select and use a computer algebra system and other technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.

Examples of learning activities

the use of computer algebra systems should be developed as an integral part of the range of learning activities for Outcomes 1 and 2; in particular, these can include:

investigations based on the production of collections of tables of values or graphs for families of functions, and analysis of the behaviour of this type of function in terms of key defining parameters such as the effect of the parameters a, b, c and d on graphs of functions with rules of the form $f(x) = a(x - b)(x - c)(x - d)$; summary of the effects of variation of these parameters

activities based on the application of a variety of different techniques to a particular type of task, e.g. use a computer algebra system to implement various graphical, tabular, numerical and analytical approaches associated with solving appropriate equations of the form: $f(x) = 0$ or $f(x) = k$ or $f(x) = g(x)$

'workshop' type sessions where, for example, students explore the relationship between a function and its derivative function using analytical, graphical and numerical approaches

APPROACHES TO ASSESSMENT

In Units 1 and 2 of the study teachers must select assessment tasks from those designated for each unit.

When tests are used teachers may decide to specify or construct components of a test where CAS calculators are not to be used (or not able to be used), while allowing or expecting students use on other components of the test. Summary or review notes constructed in preparation for each test could be part of the required material for assessment of Outcome 1. These could include definitions of key concepts, statements of important results or formulas, and sample routine problems. Where these also incorporate samples of work where technology is used to produce, for example, tables of values, graphs or solve equations, this would provide feedback on student achievement of related aspects of Outcome 3. In the work on Probability this could, for example, include a flow diagram for identification of types of problems involving counting principles, where decisions are to be made about whether repetition is allowed or not, whether some elements of a set are to be grouped or not, and whether all elements are distinguishable or not. The development and use of this flow diagram could be related to assessing student achievement of Outcome 1, while an assignment based on application of this to unseen examples could be related to assessing student achievement of Outcome 2.

Alternatively, an assignment based on a collection of questions requiring progressively more complex and detailed analysis could be used to assess student achievement of aspects of Outcomes 2 and 3. For example, an assignment could be structured around a context incorporating transition matrices. This may involve analysis of variations on a theme related to sampling problems, or investigating the behaviour of different initial probabilities values and teachers may wish to extend this to the 'steady state' for the transition matrix, $T^{n+1} S \approx T^n S$. Such a task would provide students with the opportunity to demonstrate key knowledge and skills related to Outcome 3.

Units 3 and 4: Further Mathematics

Units 3 and 4: Further Mathematics

Further Mathematics consists of a compulsory core area of study ‘Data analysis’ and then a selection of three from six modules in the ‘Applications’ area of study. Unit 3 comprises the ‘Data analysis’ area of study which incorporates a statistical application task, and one of the selected modules from the ‘Applications’ area of study. Unit 4 comprises the two other selected modules from the ‘Applications’ area of study.

Assumed knowledge and skills for the ‘Data analysis’ area of study are contained in the topics: Univariate data, Bivariate data, Linear graphs and modelling, and Linear relations and equations from General Mathematics Units 1 and 2.

The appropriate use of technology to support and develop the teaching and learning of mathematics is to be incorporated throughout the units. This will include the use of some of the following technologies for various areas of study or topics: graphics calculators, spreadsheets, graphing packages, statistical analysis systems, dynamic geometry systems, and computer algebra systems. In particular, students are encouraged to use graphics calculators, spreadsheets or statistical software in ‘Data analysis’, dynamic geometry systems in ‘Geometry and trigonometry’ and graphics calculators, graphing packages or computer algebra systems in the remaining areas of study, both in the learning of new material and the application of this material in a variety of different contexts.

AREAS OF STUDY

There are two areas of study:

1. Data analysis – core material
2. Applications – module material:
   Module 1: Number patterns
   Module 2: Geometry and trigonometry
   Module 3: Graphs and relations
   Module 4: Business-related mathematics
   Module 5: Networks and decision mathematics
   Module 6: Matrices

1. Data analysis

This area of study covers the presentation, summary, description and analysis of univariate and bivariate sample data. This area of study includes:

Displaying, summarising and describing univariate data, including:

- types of data: numerical and categorical; discrete and continuous;
- review of methods for displaying data: dot plots, stemplots (including the use of split stems), barcharts (including barcharts segmented appropriately using percentages) and frequency histograms), description in terms of shape, including symmetry and skewness (positive and negative), centre and spread and possible outliers where appropriate;
- review of summary sample statistics: mode and range; mean and standard deviation; and median and interquartile range (IQR) for describing the centre and spread of a univariate numerical data set, and conditions for their use;
- the use and interpretation of boxplots (including boxplots with outliers, and the '$Q_1 - 1.5 \times$ IQR and $Q_3 + 1.5 \times$ IQR' criterion for identifying possible outliers);
- modelling bell shaped distributions by the normal distribution; the 68-95-99.7% rule and its use in giving meaning to the standard deviation and determining appropriate percentages and intervals, including standard z scores and their application to comparing data values from different distributions;
- random numbers and their use to draw simple random samples from a population; display, appropriately summarise and describe these samples.

Displaying, summarising and describing relationships in bivariate data, including:

- identification of dependent (response) and independent (explanatory) variable (where appropriate);
- back-to-back stemplots and their use in displaying a relationship between a numerical variable (for example, height) and a two-valued categorical variable (for example, gender), and parallel boxplots to display a relationship between a numerical variable and a two or more level categorical variable (for example, age group);
- the relationship between a numerical variable and a categorical variable using summary statistics;
- tables and/or associated barcharts segmented appropriately using percentages, and their use in displaying and describing the relationship between two categorical variables (for example, voting intention and gender);
- scatterplots and their use in displaying and describing, in terms of direction (positive, negative), form (linear, non-linear) and strength (strong, moderate, weak), the association between two numerical variables (for example, height and weight);
- approximation of pearson's product-moment correlation coefficient, r, from a scatterplot and use of technology with bivariate statistics to calculate this correlation coefficient;
- use and interpretation of r;
- correlation and causation;
- calculation of the coefficient of determination, r^2, and interpretation of this coefficient in terms of explained variation.

Introduction to regression, including:

- independent and dependent variables, fitting lines to bivariate numerical data, by eye, the three median line (as a graphical technique) and the least squares methods, interpretation of slope and intercepts, and use of lines to make predictions; extrapolation and interpolation; residual analysis to check quality of fit (residual value defined as *actual value – predicted value*);

- estimation of the equation of an appropriate line of best fit from a scatterplot, use of the formulas $b = r\frac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$; and use of technology with bivariate statistics to determine the coefficients of the corresponding equation, $y = a + bx$, of the least squares regression line;
- transformation of some forms of non-linear data to linearity by transforming one of the axes scales using a square, log or reciprocal transformation.

Displaying, summarising and describing time series data, including:

- qualitative analysis of time series; recognition of trend, seasonal, cyclic and random patterns;
- seasonal adjustments; seasonal effects and indices, deseasonalisation of the data using yearly averages;
- median smoothing (as a graphical technique) and smoothing using a moving average with consideration of the number of terms required and centring where required;
- fitting a trend line to data by eye, by three median fit (as a graphical technique) or by the least squares method;
- forecasting using trend lines (with the data deseasonalised where necessary).

2. Applications

Students must undertake one of the following modules in Unit 3 and two of the following modules in Unit 4. This area of study covers the application of number, geometry and trigonometry, graphs and relations, business-related mathematics, networks and decision mathematics and matrices in a variety of practical contexts. This area of study includes:

Module 1: Number patterns

This module covers arithmetic and geometric sequences, first order linear difference equations, fibonacci and related sequences and the solution of related equations numerically or graphically (algebraic approaches may be used where applicable but are not required).

Arithmetic and geometric sequences, including:

- the recognition of arithmetic sequences and the evaluation of terms and the sum of a finite number of terms and applications;
- the recognition of geometric sequences, the evaluation of terms and the sum of a finite number of terms; applications, including growth models;
- infinite geometric sequences and the sum of an infinite geometric sequence;
- contrasting arithmetic and geometric sequences through the use of graphs involving a discrete variable.

Difference equations, including:

- generation of the terms of a sequence from a difference equation, graphical representation of such a sequence and interpretation of the graph of the sequence;
- arithmetic and geometric sequences as specific cases of first-order linear difference equations;
- other first-order linear difference equations used to model change;
- setting up and using difference equations to represent practical situations such as growth models in various contexts (numerical and graphical solution of related equations);
- fibonacci and related sequences and applications (numerical and graphical solution of related equations).

Module 2: Geometry and trigonometry

This module covers the application of geometric and trigonometric knowledge and techniques to various two-dimensional and three-dimensional practical spatial problems.

Familiarity with the trigonometric ratios sine, cosine and tangent, similarity and congruence, pythagoras theorem, basic properties of triangles and applications to regular polygons, corresponding, alternate and co-interior angles and angle properties of regular polygons is assumed.

Geometry, including:

- pythagoras theorem in two and three dimensions and the use and applications of similarity;
- calculation of surface area and volume of regular and composite solids;
- application of the effect of changing linear dimensions (that is, if the linear scale factor is k, then the area scale factor is k^2 and the volume scale factor is k^3).

Trigonometry, including:

- the solution of right-angled triangles using trigonometric ratios;
- the solution of triangles using the sine and cosine rules;
- evaluation of areas of non-right-angled triangles using the formulas $A = \frac{1}{2}ab\sin(C)$ and $A = \sqrt{s(s-a)(s-b)(s-c)}$.

Applications, including:

- specification of location (distance and direction) in two dimensions using three figure bearings;
- calculation of angles and distances in a vertical plane (that is, finding or using angles of elevation and depression);
- interpretation and use of a contour map to calculate distances and the average slope between two points;
- calculation of unknown angles and distances given triangulation measurements.

Module 3: Graphs and relations

This module covers the graphical representation and analysis of linear and non-linear relations as models for various practical contexts as well as graphical and algebraic approaches to solving equations and inequalities.

Familiarity with plotting and sketching straight lines and methods of determining the equation of a straight line given the coordinates of two points on the line, or gradient and one point on the line, is assumed.

Construction and interpretation of graphs, including:

- the construction and interpretation of straight-line graphs, line segment graphs and step graphs to represent practical and everyday situations (which could include, for example, conversion graphs, income tax schedules, and postal charges);
- graphical and algebraic solution of simultaneous linear equations in two unknowns; applications which could include, for example, break-even analysis, where cost and revenue functions are linear;
- the interpretation of given non-linear graphs that represent practical and familiar situations, including significance of intercepts, slope, maximum/minimum points and average rate of change; for example, distance-time graphs, tidal heights, pulse rates at different levels of exercise;
- non-linear graphs and their construction from tables of data; interpolation and extrapolation to predict values; estimation of maximum/minimum values and location; reading coordinates of points of intersection for applications such as break-even analysis; interpretation of slope;

- graphical representation of relations of the form $y = kx^n$ for $n = -2, -1, 1, 2, 3$; obtaining a linear graph by plotting y against x^n; applications to determining the constant of proportionality and testing the appropriateness of a particular model for a given set of data, for example, braking distances, volumes, light intensity.

Linear programming, including:

- the transfer from a description of an optimisation problem to its mathematical formulation, including the introduction of variables, constraints and an objective function;
- systems of linear inequalities and their graphs;
- the use of graphical methods to solve simple linear programming problems with two decision variables, such as blending and manufacturing problems.

Module 4: Business-related mathematics

This module covers the application of numerical computations and graphical techniques to the formulation and solution of various problems in business and financial contexts, including the solution of related equations.

Financial transactions and asset value, including:

- financial computations involving multiple transactions (such as bank account balances), percentage changes and charges (such as discounting, capital gains, stamp duty, GST);
- graphical and tabular representation and calculation of the value of money for a series of consumer price changes (inflation);
- the use and comparison of flat rate, reducing balance and unit cost methods of calculating depreciation.

Loans and investments, including:

- use and comparison of simple and compound interest in investment and loan applications without periodic payments;
- annuity investments involving a series of regular and equal deposits; consideration of the effects of initial and periodic deposit values, frequency of deposits, interest rate, and length of investment;
- the ordinary perpetuity as a series of regular payments from an investment that continues indefinitely;
- reducing balance loans as particular applications of annuities, with applications including housing loans, time payment plans (hire purchase) and credit/store cards; consideration of the effects of varying the repayment amount, the frequency of repayments, and the interest rate on the total repayment time and total interest paid;
- determination of effective interest rates from nominal interest rates.

Module 5: Networks and decision mathematics

This module covers the use of undirected and directed graphs (networks) to the modelling of situations involving the spatial representation of relationships and the optimisation of various measures such as coverage, flow, time and allocation.

Undirected graphs and networks, including:

- graphical and matrix representation of graphs (networks);
- planar graphs and applications of euler’s formula;
- eulerian and hamiltonian paths and their applications;
- shortest path(s) between two given points in a network by informal methods;
- trees and minimum spanning trees and applications to minimum connector problems by use of prim’s algorithm.

Directed graphs and networks, including:

- graphical and matrix representation of directed graphs (networks), including application to dominance and reachability;
- critical path analysis, including network construction (with activity as edge), location of the critical path by forward and backward scanning;
- network flow; ‘minimum cut-maximum flow’ theorem;
- assignment problems, for example, representation by bipartite graph; optimal allocation including use of the hungarian algorithm.

In this module it is expected that students will have developed a working knowledge of the following terms: graph, subgraph, vertex (= node), edge, degree of a vertex, loop, isolated vertex, connected graph, circuit, tree, spanning tree, complete graph, simple graph, bipartite graph, digraph, reachable, hamiltonian and eulerian paths (lines) and circuits, network, cut-set, capacity, planar graph, adjacency matrix (loop = 1 edge) critical path, dummy activity, earliest and latest start and finish times, ‘crashing’ (as a means of shortening the critical path) and float (slack) in non-critical activities. Matrix arithmetic is not required but may be used.

Module 6: Matrices

This module covers the matrix representation of discrete data in rectangular arrays, and the application of matrix arithmetic to the analysis of problems in practical situations. Technology is to be used to carry out computations as applicable.

Matrix representation and its application, including:

- matrix representation of data from a variety of situations in a rectangular array whose order is defined by the number of rows and columns in the array;
- application of matrix arithmetic (sum, difference, scalar multiple, product) to solving practical problems, for example stock inventories, total value of sales, mark-ups and discounts;
- application of simultaneous linear equations in practical situations, and their solution using the inverse matrix method.

Transition matrices, including:

- application of simple transition matrices up to 4×4 to analyse practical situations such as consumer preferences in shopping, including an informal consideration of steady state (no noticeable change from one state to the next).

In this module it is expected that students will have developed a working knowledge of the following terms: matrix, vector (as a row matrix or column matrix), scalar, row, column, order, inverse, state, initial state, transition matrix and steady-state (state vectors should be represented as column vectors). State vectors should be represented as column vectors. Students should use technology to carry out matrix arithmetic as applicable. Matrix multiplication should be developed from the notion of a sum of products of weighted values. Generally, matrices of low order, with up to five rows or columns would be used. However, for modelling tasks and problems in coursework, and related assessment, higher order matrices could be considered.

UNIT 3 OUTCOMES

For this unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass ‘Data analysis’ area of study and one module from the ‘Applications’ area of study.

Outcome 1

On completion of this unit the student should be able to define and explain key terms and concepts as specified in the content from the areas of study, and use this knowledge to apply related mathematical procedures to solve routine application problems.

To achieve this outcome the student will draw on knowledge and related skills outlined in area of study 1.

Data analysis

Key knowledge

This knowledge includes

- the standard statistical terms and techniques used to display, summarise and describe univariate data for both categorical and numerical data;
- the concept of sample and population and the use of random numbers as a means of selecting a simple random sample of data from a population;
- the standard terms and techniques used to display and describe associations in bivariate data for both categorical and numerical data;
- the technique of regression as a means of modelling the relationship between two numerical variables with a straight line;
- the role of residual analysis and the coefficient of determination in making decisions about the appropriateness of a particular regression model;
- the concept of data linearisation through transformation;
- the terms used to describe standard patterns in time series in qualitative terms, the role of smoothing and deseasonalisation in helping to identify these patterns, and some simple techniques for quantifying these patterns and enabling forecasting of future values;
- the assumptions and/or limitations that underlie the applications of statistical techniques.

Key skills

These skills include the ability to

- use a range of standard statistical techniques and terms to display, summarise, describe and interpret patterns in both univariate and bivariate data, and outline the assumptions and/or limitations relating to the application of these skills;
- obtain a simple random sample from a given population using a table of random numbers or an alternative random number generator;
- use the technique of linear regression to model a relationship between two numerical variables and use residual analysis in conjunction with the coefficient of determination (where appropriate) to test the suitability of this model;
- interpret the parameters in a linear regression equation in relation to the situation being modelled;
- use one of the listed data transformations (square, log, reciprocal), where appropriate, to linearise a set of bivariate data as a means of improving the fit of a regression model;

- display a time series (which may have been smoothed where necessary) graphically and use this to identify and describe any possible trend patterns and predict future values;
- use a range of simple techniques to describe features such as seasonality and trend in time series.

The description of the key knowledge and key skills required to be demonstrated for the selected ‘Applications’ module are contained in the details of Unit 4.

Outcome 2

On completion of this unit the student should be able to use mathematical concepts and skills developed in the ‘Data analysis’ area of study to analyse a practical and extended situation, and interpret and discuss the outcomes of this analysis in relation to key features of that situation.

To achieve this outcome the student will draw on knowledge and related skills outlined in area of study 1.

Key knowledge

To achieve this outcome the student should demonstrate knowledge of the statistical concepts and techniques that apply in a given practical situation and the assumptions and/or limitations that underlie their application.

Key skills

These skills include the ability to

- use a range of appropriate statistical techniques to analyse a practical situation and to draw valid conclusions from analysis;
- present the various mathematical outcomes of analysis in a logical and organised manner;
- draw together the results of analysis in relevant conclusions and communicate these in a coherent manner.

Outcome 3

On completion of this unit the student should be able to select and appropriately use technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches in the area of study ‘Data analysis’ and the selected module from the ‘Applications’ area of study.

To achieve this outcome the student will draw on knowledge and related skills outlined in area of study 1 and the selected module from area of study 2.

Key knowledge

This knowledge includes

- the operating characteristics and capabilities of a graphics calculator and/or other technologies (for example, spreadsheets, geometry software, statistical software or computer algebra systems) in application to appropriate contexts related to the areas of study;
- the appropriate selection of a technology application in a variety of mathematical contexts.

Key skills

These skills include the ability to

- make appropriate selections for technology applications in a variety of contexts, and provide a rationale for these selections;
- set up parameters for a technology to produce required results;
- use technology to produce results which are relevant to a given task;
- relate the results produced by a particular technology to the nature of a particular mathematical task (investigative, problem solving or modelling).

UNIT 4 OUTCOMES

For this unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass the two selected modules from the 'Applications' area of study.

Outcome 1

On completion of this unit the student should be able to define and explain key terms and concepts as specified in the content from the 'Applications' area of study, and use this knowledge to apply related mathematical procedures to solve routine application problems.

Module 1: Number patterns

To achieve this outcome the student will draw on knowledge and related skills outlined in area of study 2.

Key knowledge

This knowledge includes

- the properties of arithmetic and geometric sequences and the sum of terms of a sequence, key terms, related formulas and the form of graphical representations;
- the properties of sequences derived from first-order linear difference equations, related formulas and the form of graphical representations;
- typical contexts for the application of sequences, the sum of terms of a sequence, difference equations and the conditions under which various formulas are relevant;
- fibonacci and related sequences and their applications.

Key skills

These skills include the ability to

- calculate terms and sums of terms for arithmetic and geometric sequences, and sequences generated by first-order difference equations;
- draw and compare corresponding graphs of these sequences;
- use a variety of approaches to solve difference equations and problems related to the use of these sequences in typical application contexts;
- test whether a given sequence is arithmetic, geometric, derived from a first-order linear difference equation or none of these;
- solve problems involving fibonacci and related sequences.

Module 2: Geometry and trigonometry

To achieve this outcome the student will draw on knowledge and related skills outlined in area of study 2.

Key knowledge

This knowledge includes

- the concepts of similarity, scale and the effect of re-scaling by a linear factor on area, surface area and volume;
- pythagoras theorem, trigonometric ratios sine, cosine and tangent, the sine and cosine rules, and area of triangle formulas;
- basic geometric and trigonometric concepts associated with orienteering, navigation and surveying.

Key skills

These skills include the ability to

- represent practical problems in two-dimensional and three-dimensional geometric form;
- apply pythagoras theorem, scale and similarity to solve problems in two dimensions and three dimensions;
- use trigonometric relationships to solve problems in two dimensions and three dimensions;
- apply geometry and trigonometry in circumstances related to orienteering, navigation and surveying.

Module 3: Graphs and relations

To achieve this outcome the student will draw on knowledge and related skills outlined in area of study 2.

Key knowledge

This knowledge includes

- key features of straight line, line segment and step graphs and the form of related tables of values;
- the concept of break-even analysis and its relation to graphic and tabular representation of relations;
- non-linear relations, constant of proportionality and key features;
- linear inequalities, systems of linear inequalities and their properties;
- the role of variables, constraints and objective functions in linear programming optimisation.

Key skills

These skills include the ability to

- use graphs and tables of linear and non-linear relations to interpret information in a variety of application contexts;
- identify key properties of graphs and relate these to specific features in a given context;
- determine the nature of a relationship, and the corresponding constant of proportionality, as applicable to model a given set of data;
- set up, solve and interpret equations in a given context;
- formulate optimisation problems where linear programming is an appropriate technique;
- use graphical techniques to solve these types of problems, and interpret the results.

Module 4: Business-related mathematics

To achieve this outcome the student will draw on knowledge and related skills outlined in area of study 2.

Key knowledge

This knowledge includes

- financial transactions as they apply to account debits and credits and cash flow, percentage changes and applications;
- terms, concepts and definitions associated with the value of money in investment and loan situations, including an understanding of the effects of inflation;
- formulas related to these application contexts;
- methods of calculating values and periods of time in relation to these application contexts.

Key skills

These skills include the ability to

- calculate values associated with business-related mathematics contexts;
- produce graphs and tables for business-related problems;
- find the time interval, or number of periods required, for a particular value to be reached for annuities, interest or depreciation and repayment;
- compare different types of rates, depreciations and loans.

Module 5: Networks and decision mathematics

To achieve this outcome the student will draw on knowledge and related skills outlined in area of study 2.

Key knowledge

This knowledge includes

- the key elements of a graph or network (vertex, edge, face, loop, degree of a vertex, weight, direction);
- different forms of representation of graphs (edge and vertex sets, network diagrams, matrices);
- types of graphs (simple, planar, connected, complete, tree, bi-partite, directed graph);
- subgraphs of graphs (spanning trees, eulerian and hamiltonian paths and circuits);
- key theorems and algorithms (network flow, minimum spanning tree, assignment, allocation, critical path).

Key skills

These skills include the ability to

- draw and modify graphs according to given specifications;
- model applications in a variety of contexts using various representations of graphs as networks;
- identify particular characteristics of graphs, carry out constructions on graphs and calculate various measures related to graphs;
- use algorithms such as forward and backward scanning, ‘minimum cut-maximum flow’, prim’s and hungarian algorithms, and ‘crashing’ as applicable, to optimise associated characteristics in given contexts.

Module 6: Matrices

To achieve this outcome the student will draw on knowledge and related skills described in area of study 2.

Key knowledge

This knowledge includes

- terms, symbols, notations and conventions for the representation of matrices (including vectors as row matrices or column matrices) and matrix operations;
- matrix form of simultaneous linear equations and matrix inverse;
- initial state, transition matrices and steady state.

Key skills

These skills include the ability to

- represent data from situations using scalars, vectors (as row matrices or column matrices) and matrices;
- use technology to calculate scalar multiples, sums and differences, products and inverses of matrices, including simple combinations of these operations as applicable;
- solve systems of simultaneous linear equations using the matrix inverse method;
- model situations using transitions matrices, and identify and/or calculate initial, transition and steady states in practical situations using technology.

Outcome 2

On completion of this unit the student should be able to apply mathematical processes in contexts related to the ‘Applications’ area of study, and analyse and discuss these applications of mathematics.

To achieve this outcome the student will draw on knowledge and related skills outlined in area of study 2.

Key knowledge

This knowledge includes

- key mathematical content relating to a given application context;
- specific and general formulations of concepts used to derive results for analysis within a given context;
- the role of examples, counter-examples and general cases in developing analysis;
- how results of analysis can be drawn together to make valid conclusions related to a given application context.

Key skills

These skills include the ability to

- specify what key mathematical content is relevant to a given application context;
- formulate specific and general cases which can be used to derive results for analysis within a given application context;
- draw together results of analysis to make valid conclusions related to a given application;
- communicate these conclusions using both mathematical language and everyday language, in particular, interpreting mathematics with respect to features of the application context.

Outcome 3

On the completion of this unit the student should be able to select and appropriately use technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches related to the selected modules for this unit from the 'Applications' area of study.

To achieve this outcome the student will draw on knowledge and related skills outlined in area of study 2.

Key knowledge

This knowledge includes

- the operating characteristics and capabilities of technology in application to appropriate contexts related to the area of study;
- examples of appropriate selection of a given technology application in a variety of mathematical contexts.

Key skills

These skills include the ability to

- make appropriate selections for technology applications in a variety of mathematical contexts, and provide a rationale for these selections;
- set up parameters for a technology to produce required results;
- use technology to produce results which are relevant to a given task;
- explain clearly how the results produced by a particular technology relate to nature of a particular mathematical task (investigative, problem solving or modelling).

ASSESSMENT

The award of satisfactory completion for a unit is based on a decision that the student has demonstrated achievement of the set of outcomes specified for the unit. This decision will be based on the teacher's assessment of the student's overall performance on assessment tasks designated for the unit. The Victorian Curriculum and Assessment Authority publishes an assessment handbook that includes advice on the assessment tasks and performance descriptors for assessment.

The key knowledge and skills listed for each outcome should be used as a guide to course design and the development of learning activities. The key knowledge and skills do not constitute a checklist and such an approach is not necessary or desirable for determining the achievement of outcomes. The elements of key knowledge and skills should not be assessed separately.

Assessment of levels of achievement

The student's level of achievement in Units 3 and 4 will be determined by school-assessed coursework and two end-of-year examinations.

Contribution to final assessment

School-assessed coursework for Unit 3 will contribute 20 per cent and for Unit 4 will contribute 14 per cent to the study score.

The level of achievement for Units 3 and 4 is also assessed by two end-of-year examinations, which will contribute 66 per cent to the study score.

School-assessed coursework

Teachers will provide to the Victorian Curriculum and Assessment Authority a score representing an assessment of the student's level of achievement.

The score must be based on the teacher's rating of performance of each student on the tasks set out in the following table and in accordance with an assessment handbook published by the Victorian Curriculum and Assessment Authority. The assessment handbook also includes advice on the assessment tasks and performance descriptors for assessment.

Assessment tasks must be a part of the regular teaching and learning program and must not unduly add to the workload associated with that program. They must be completed mainly in class and within a limited timeframe. Where optional assessment tasks are used, teachers must ensure that they are comparable in scope and demand. Teachers should select a variety of assessment tasks for their program to reflect the key knowledge and skills being assessed and to provide for different learning styles.

Outcomes	Marks allocated*	Assessment tasks
Unit 3		A data analysis *application task* using contexts for investigation from a suitable data set selected by the teacher. The task has three components of increasing complexity: • display and organisation of univariate and bivariate data • consideration of general features of the data • analysis of the data such as regression analysis, the use of transformations to linearity, deseasonalisation, smoothing, or analysis of time series. **and** An *analysis task* for the first selected module, from one of the four types of analysis task described for Unit 4. The outcomes are to be assessed across the *application task* and the *analysis task*.
Outcome 1 Define and explain key terms and concepts as specified in the content from the areas of study, and use this knowledge to apply related mathematical procedures to solve routine application problems.	30	15 *The application task* 15 *Analysis task*
Outcome 2 Use mathematical concepts and skills developed in the 'Data analysis' area of study to analyse a practical and extended situation, and interpret and discuss the outcomes of this analysis in relation to key features of that situation.	20	20 *The application task*
Outcome 3 Select and appropriately use technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches in the area of study 'Data analysis' and the selected module from the 'Applications' area of study.	10	5 *The application task* 5 *Analysis task*
Total marks	**60**	

*School-assessed coursework for Unit 3 contributes 20 per cent to the study score.

Outcomes	Marks allocated*	Assessment tasks
Unit 4		Two *analysis tasks*, with the three outcomes assessed across the tasks. Each *analysis task* is a short item of 2–4 hours duration over 1–2 days selected from: • an assignment where students have the opportunity to work on a broader range of problems in a given context; **or** • a short and focused investigation, challenging problem or modelling task; **or** • a set of application questions requiring extended response analysis in relation to a particular topic or topics; **or** • item response analysis for a collection of multiple-choice questions, including analysis of item distractors and their relationship to conceptual, process or reasoning error. The two *analysis tasks* are to be of a different type.
Outcome 1 Define and explain key terms and concepts as specified in the content from the 'Applications' area of study, and use this knowledge to apply related mathematical procedures to solve routine application problems.	15	7 *Analysis task 1* 8 *Analysis task 2*
Outcome 2 Apply mathematical processes in contexts related to the 'Applications' area of study, and analyse and discuss these applications of mathematics.	15	8 *Analysis task 1* 7 *Analysis task 2*
Outcome 3 Select and appropriately use technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches related to the selected modules for this unit from the 'Applications' area of study.	10	5 *Analysis task 1* 5 *Analysis task 2*
Total marks	**40**	

*School-assessed coursework for Unit 4 contributes 14 per cent to the study score.

End-of-year examinations

The student's level of achievement for Units 3 and 4 will also be assessed by two examinations based on tasks related to Outcomes 1 to 3 in each unit.

Examination 1

Description

The task is designed to assess students' knowledge of mathematical concepts, their skills in carrying out mathematical algorithms and their ability to apply concepts and skills in standard ways.

The task will consist of multiple-choice questions drawn from the 'Data analysis' and 'Applications' areas of study.

The task will be set by an examination panel using criteria published annually by the Victorian Curriculum and Assessment Authority.

Format

Students are required to respond to multiple-choice questions covering the core and three selected modules. Student access to an approved graphics calculator or CAS will be assumed by the setting panel.

Conditions

The task will be completed under the following conditions:

- Duration: one and a half hours.
- Date: end-of-year, on a date to be published by the Victorian Curriculum and Assessment Authority.
- Victorian Curriculum and Assessment Authority examination rules will apply. Details of these rules are published annually in the *VCE and VCAL Administrative Handbook*.
- An approved graphics calculator or CAS and **one** bound reference, text (which may be annotated) or lecture pad, may be brought into the examination.
- The examination will be marked by a panel appointed by the Victorian Curriculum and Assessment Authority.

Contribution to final assessment

The examination will contribute 33 per cent to the study score.

Examination 2

Description of task

Students are required to respond to four sets of extended-answer questions, equally weighted from the Core, ‘Data analysis’, and the three selected ‘Applications’ modules. Student access to an approved graphics calculator or CAS will be assumed by the setting panel.

The task is designed to assess students’ ability to understand and communicate mathematical ideas, and to interpret, analyse and solve both routine and non-routine problems.

Students should attempt all the extended-answer questions, involving multi-stage solutions of increasing complexity.

The task will be set by an examination panel using criteria published annually by the Victorian Curriculum and Assessment Authority.

Conditions

The task will be completed under the following conditions:

- Duration: one and a half hours.
- Date: end-of-year, on a date to be published annually by the Victorian Curriculum and Assessment Authority.
- Victorian Curriculum and Assessment Authority examination rules will apply. Details of these rules are published annually in the *VCE and VCAL Administrative Handbook*.
- An approved graphics calculator or CAS and **one** bound reference, text (which may be annotated) or lecture pad, may be brought into the examination.
- The examination will be marked by a panel appointed by the Victorian Curriculum and Assessment Authority.

Contribution to final assessment

The examination will contribute 33 per cent to the study score.

Advice for teachers (Units 3 and 4: Further Mathematics)

DEVELOPING A COURSE

A course outlines the nature and sequence of teaching and learning necessary for students to demonstrate achievement of the set of outcomes for a unit. The areas of study describe the learning context and the knowledge required for the demonstration of each outcome. Outcomes are introduced by summary statements and are followed by the key knowledge and skills which relate to the outcomes.

Teachers must develop courses that include appropriate learning activities to enable students to develop the knowledge and skills identified in the outcome statements in each unit.

In Units 3 and 4, assessment is structured. For school-assessed coursework the assessment tasks are prescribed. The contribution that each task makes to the total score for school-assessed coursework is also stipulated.

In Unit 3, the 'Data analysis' area of study is to be undertaken by all students along with one module from the 'Applications' area of study in Unit 3. Unit 4 comprises the study of two of the remaining five modules in the 'Applications' area of study. Any implementation of General Mathematics Units 1 and 2, which is intended as a preparation for Further Mathematics Units 3 and 4, must be designed to cover the assumed material for the 'Data analysis' area of study.

SAMPLE COURSE SEQUENCES

The following represents a sample 27-week sequence for a Further Mathematics Units 3 and 4 course.

Core – Data analysis

Week

1–3 Data presentation and analysis, including the use of technology to summarise and display results

4–5 Correlation and regression techniques, including appropriate use of technology

6–7 Transformations to linearity and residual analysis

8–10 Time series data and seasonality, smoothing, trendlines and forecasting

11–12 Data analysis – Application task

Applications (3 Modules covered in any order)

Number patterns and applications

Week

1 Arithmetic sequences and applications
2 Geometric sequences and applications
3 First-order difference equations, including arithmetic and geometric sequences and others of the form $f(n + 1) = af(n) + b$ or $t_{n+1} = rt_n + d$
4 Applications of first-order difference equations
5 Fibonacci and related sequences and applications.

The analysis task, which is the school-assessed component for this module, could be based on exploring related number patterns.

Geometry and trigonometry

Week

1 Pythagoras theorem in two and three dimensions
2 Surface area and volume of solids; similarity, including effect of changing dimensions
3 Basic trigonometry and applications
4 Trigonometry with non-right-angled triangles
5 Applications (distances and directions, triangulation, contour maps)

The analysis task, which is the school-assessed component for this module, could involve a survey or calculation of distances that cannot be measured directly; investigation of models in terms of changes in scale and their effect on area and volume.

Graphs and relations

Week

1 Straight-line graphs, line segments and step graphs
2 Simultaneous linear equations in two unknowns, interpretation of non-linear graphs
3 Construction of non-linear graphs, $y = kx^n$ where $n = –2, –1, 1, 2, 3$
4 Plot y versus x^n and applications
5 Linear programming and optimisation

The Analysis task, which is the school-assessed component for this module could include modelling data, break-even analysis and optimisation.

Business-related mathematics

Week

1 Transactions and balances (percentage changes, fees and charges, maintaining account balances)
2 Compound growth and decay modelled using the Consumer Price Index (CPI); depreciation methods
3 Simple and compound interest
4 Annuity investments and applications
5 Reducing balance loans and applications

The analysis task, which is the school-assessed component for this module, could include comparing bank loans, time payment plans, CPI, number of periods to pay out a loan and other related considerations.

Networks

Week

1 Undirected graphs, planar graphs, euler’s formula, application of adjacency matrices
2 Eulerian paths, hamiltonian paths and circuits and applications
3 Shortest paths between two points, trees and minimum spanning trees
4 Directed graphs and applications including use of adjacency matrices in dominance and reachability applications, network flow, scheduling, allocation
5 Critical path analysis and applications

The analysis task, which is the school-assessed component for this module, could include travelling salesperson problems, flow in gas pipe lines, and electric powerline placement.

Matrices

Week

1–2 Matrix representation and arithmetic, use of technology and applications
Matrix multiplication; technological approaches to matrix solutions of applications
3 Inverse matrices; solution of simultaneous equations and applications
4 Matrix powers; 2×2 transition matrices
5 Transition matrices of higher order and development of steady states

The analysis task, which is the school-assessed component for this module, could include formulation and matrix solutions of problems involving simultaneous equations, and the determination of transition and possible equilibrium states in practical examples of markov chains.

USE OF INFORMATION AND COMMUNICATIONS TECHNOLOGY

In designing courses and developing learning activities for Further Mathematics, teachers should make use of applications of information and communications technology and new learning technologies, such as graphic calculators, computer-based learning, multimedia and the World Wide Web, where applicable to teaching and learning activities.

Statistical analysis and presentation software such as Minitab and FX-Stat can be used for data analysis. Dynamic geometry software such as Geometer’s Sketchpad and Cabri Geometry can be used in the Geometry and trigonometry module.

KEY COMPETENCIES AND EMPLOYABILITY SKILLS

Students undertaking the following types of assessment, in addition to demonstrating their understanding and mastery of the content of the study, typically demonstrate the following key compentencies and employability skills.

Assessment task	Key competencies and employability skills
Application task	Planning and organising, solving problems, using mathematical ideas and techniques, (written) communication, use of information and communications technology
Analysis task	Planning and organising, solving problems, using mathematical ideas and techniques, use of information and communications technology

LEARNING ACTIVITIES

Examples of learning activities for each unit are provided in the following sections. Extended examples are highlighted by a shaded box. The examples that make use of information and communications technology are identified by this icon.

Unit 3: Further Mathematics

Outcome 1

Define and explain key terms and concepts as specified in the content from the areas of study, and use this knowledge to apply related mathematical procedures to solve routine application problems.

Examples of learning activities

skills practice on standard mathematical routines through appropriate exercises; this should include development of the ability to identify problems as either being in a standard form, or readily transformable to a standard form, the efficient and accurate application of the relevant mathematical routine (e.g. the application of calculators or spreadsheets to obtain summary statistics for data sets which are presented as unordered and unformatted lists – this would also link to Outcome 3), and systematic checking of both the reasonableness and accuracy of results (does the mean or median value make sense in terms of an appropriate graph of the data); the effective use of technology, such as relating bivariate correlation and regression summary statistics to an estimated scatterplot line of best fit ‘by eye’, and subsequently drawing an accurate graph (to a specified degree of accuracy) would also provide a link for this type of work to Outcome 3

error identification and analysis exercises; work through a range of ‘worked solutions’ and identify and rectify missing steps in working or errors in working (e.g. selection and use of random numbers to develop samples from a population)

construction of summary or review notes related to a topic or area of study (e.g. the conditions under which particular axes transformations could be used to transform data sets to linearity)

presentation of a range of typical problems associated with an area of study and worked solutions (e.g. case studies involving deseasonalisation of data)

assignments structured around the development of standard applications of mathematical skills and procedures in readily recognisable situations (e.g. summarising and describing time series data – this can be checked by the use of graphics calculators, spreadsheets, or statistical software and hence linked to Outcome 3)

These examples have been linked to the ‘Data analysis’ area of study; teachers will readily identify similar examples for selected modules.

Outcome 2

Use mathematical concepts and skills developed in the 'Data analysis' area of study to analyse a practical and extended situation, and interpret and discuss the outcomes of this analysis in relation to key features of that situation.

Examples of learning activities

investigative projects (e.g. use of univariate summary statistics to compare the distribution of rental property prices in different suburbs)

problem-solving tasks (e.g. comparing 'by eye', median and mean-based approaches to finding lines of best fit in context)

modelling tasks (e.g. which basketball team has the 'best statistics', and how are these statistics related to performance in final series)

Unit 4: Further Mathematics

Outcome 2

Apply mathematical processes in contexts related to the 'Applications' area of study, and analyse and discuss these applications of mathematics.

Examples of learning activities

a set of applications questions requiring analysis and extended response related to a particular context (e.g. different investment strategies)

a report on item response analysis for a collection of multiple-choice questions (e.g. different rules for difference equations)

presentation on research into a particular application of mathematics (e.g. critical path analysis in scheduling problems)

Outcome 3

Select and appropriately use technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches related to the selected modules for this unit from the 'Applications' area of study.

Examples of learning activities

ICT technology should be used where appropriate and applicable to enhance the teaching and learning of mathematics throughout the course; it is assumed that all students will have access to graphics calculators in Further Mathematics Units 3 and 4; while students may be introduced to different statistical measures using various routines which are initially evaluated by longhand for small data sets (in particular, where the construction of the measure as a model of certain data characteristics is being discussed), they should quickly transfer to the use of graphics calculators to explore features of data sets, including various representations of the data, and the analysis of these features based on relevant summary statistics obtained using the graphics calculator; graphics calculators offer many opportunities to explore mathematical ideas in ways which will enhance a students understanding of their underlying concepts, e.g. graphing regions of the plane specifications by inequalities

ICT teachers should develop courses that encourage the appropriate use of a variety of other technology, such as spreadsheets or various statistical analysis systems, which provide platforms for developing a variety of ways for presenting data analysis, in particular where analysis is required on larger data sets (e.g. tables in Module 4) where the underlying behaviour of the relevant variable(s), or any relationships between variables, is not evident (Module 3); dynamic geometry systems can be used in exploring geometric concepts and their applications

EXTENDED LEARNING ACTIVITIES

A data analysis application

Core–Data analysis

The following task is designed to cover core content. Some decisions need to be made on selection of particular content for emphasis. For example, if a task involves seasonality and students have to deseasonalise their data, then transforming data to improve the model may not be included. Consequently, teachers need to ensure that sufficient attention is given to balancing potential rigour and depth, with an appropriate time allocation for the task.

In the completion of the application task students must use the appropriate technology to analyse the data. The following sample application task ‘Exchange rates and tourism’ is designed to cover all three outcomes for the core.

Detailed example

EXCHANGE RATES AND TOURISM

In local and overseas money markets, the value of the Australian dollar fluctuates, in part, according to the international view of the health of the economy. Overall, however, a rising value of the Australian dollar reduces the competitiveness of Australia as a destination for travel. In contrast a fall in the value of our dollar leads hopefully to making Australia a relatively cheaper destination (relevant data sets should be provided to students).

The following investigation will look at international arrivals and examine the validity of predicting trends. It will also examine the possible relationship between the number of tourists arriving and the exchange rate.

International arrivals for holidays in Australia are highly seasonal. The data provided is the deseasonalised monthly arrivals to Australia for holidays.

1. (a) Choose two consecutive years of arrival data at random, starting between January 1995 and December 2002. You may commence the data at any month. Explain your method of selection.

   (b) For your selected period, plot the data and discuss any features you notice about arrivals over this time.

   (c) Determine the equation of the least squares regression line which can be used to predict the number of arrivals at any time in your chosen period. Draw your regression line on the graph. Interpret the gradient and intercept.

2. (a) Perform a residual analysis, including a residual plot, based on your regression line fitted in 1(c), and comment whether the linear model is suitable.

   (b) Based on the analysis of residuals in 2(a) and the scatterplot, apply either a logarithmic or square transformation to your data, justifying your choice. Determine if the straight line resulting from this transformation is more appropriate than the one derived directly in 1(c).

   (c) Use your preferred model to predict the number of arrivals both one month and one year beyond your sample. Compare the predictions with the real data, using percentage error to determine the accuracy of your predictions.

3. Construct a scatterplot for the sample period of 1(a) with deseasonalised holiday arrival numbers as the dependent variable plotted against the Australian-US dollar exchange rate as the independent variable. Find and interpret the values of the product moment correlation (pearson’s *r*) and the coefficient of determination. Comment on your findings. Should consideration be given to the time delay between the exchange rate when flights are booked and the time of arrival?

Sample analysis tasks

Appropriate analysis tasks for each of the modules will require 160–200 minutes to be completed and provide a significant coverage of the associated key knowledge and skills.

The following sample activities are intended to provide suggestions of possible approaches only and require further development to meet individual school needs. They provide examples of different approaches including assignments, challenging problems, modelling investigations, sets of analysis questions and item response analysis of multiple-choice questions.

Module 1 – Number patterns and applications

The following example is a modelling task that requires students to demonstrate an understanding of different types of sequences, their representation as difference equations and their application to practical contexts. The task requires students to extensively apply appropriate technology (such as a graphic calculator in sequence mode) to model the data in different formats.

The task could be modifed to be more of a challenging problem by not providing information about the types of sequences to be used for each swimmer.

Detailed example

TRAINING FOR THE BIG EVENT

Alex, Bree and Con begin a training program in preparation for a major swimming carnival in a year's time. The three athletes plan their programs so that each will follow a sequence to increase their weekly training to 250 laps in 52 weeks.

Alex is currently swimming 80 laps per week. He decides to increase his weekly distance by a fixed amount each week.

Bree is currently swimming 25 laps per week. She decides to use a geometric progression to build up to 250 laps per week.

Con is currently swimming 7.5 laps per week. He decides to increase his weekly distance by using a sequence of the form $t_n = at_{n-1} + b$ with b representing an additional 3 laps added to a proportional increase on the previous week's distance.

Determine the arithmetic sequence that Alex has chosen. Write down a different equation that would generate this sequence.

Determine the geometric sequence that Bree has chosen (r accurate to 4 decimal places). Write down a different equation that would generate this sequence.

Express the sequence used by Con (r accurate to 3 decimal places) as a difference equation.

For each of the swimmers, determine the number of laps they are completing in weeks 10, 26 and 40.

Determine the total number of laps Alex and Bree complete in the 52 weeks of training. Describe how the total distance covered by Con compares to the other two athletes. A graph of the sequences will help answer this question.

Con wishes to reach the target number of laps in 40 weeks, again using a sequence of the form $t_n = at_{n-1} + b$. Find appropriate values of a and b.

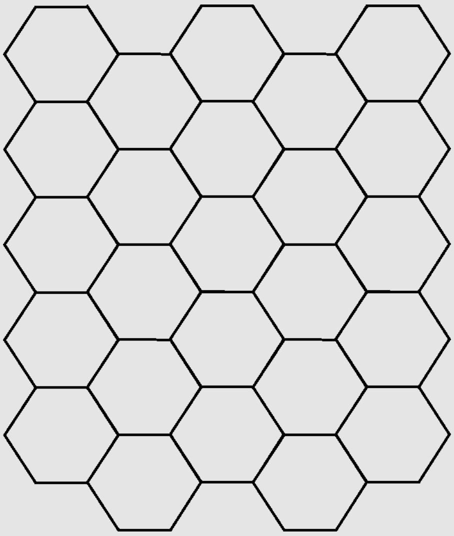
Module 2 – Geometry and trigonometry

The following task is an example of using a set of application questions requiring extended response analysis to form a task.

Detailed example

BUILDING WALLS

Aristotle valued the aesthetic appearance of the Greek city wall. He was disturbed by the earlier polygonal walls, which were made of irregular shapes. He convinced Philip, King of Macedonia, that the walls of Miea should be constructed with regular hexagonal blocks as shown below:



On one of the hexagonal faces, mark two adjacent corners A and B of the face of the block, and mark O at the centre of this hexagonal face. The length of each edge of each hexagon is 1.5 metres. Mark in the point C midway between A and B.

1. (a) Draw a scale diagram of this face, and show:
   (i) all three of the longest diagonals
   (ii) the magnitudes of the angles on this hexagonal face.

   (b) Show that the length OA is 1.5 metres, explaining your reasoning.

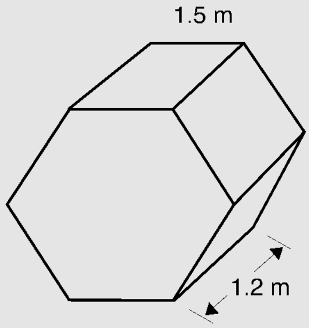
   (c) Find the length OC in metres to three decimal places.

   (d) If the wall is ten blocks high, find the height of the wall in metres.

   (e) Find the area of the triangle OAB. Give the answer in square metres to three decimal places.

   (f) Find the area of one of the hexagonal faces, giving your answer in square metres accurate to one decimal place.

   (g) Each hexagonal block is 1.2 metres deep as shown in the following diagram:



   Find the volume of one hexagonal block, giving your answer accurate to one decimal place.

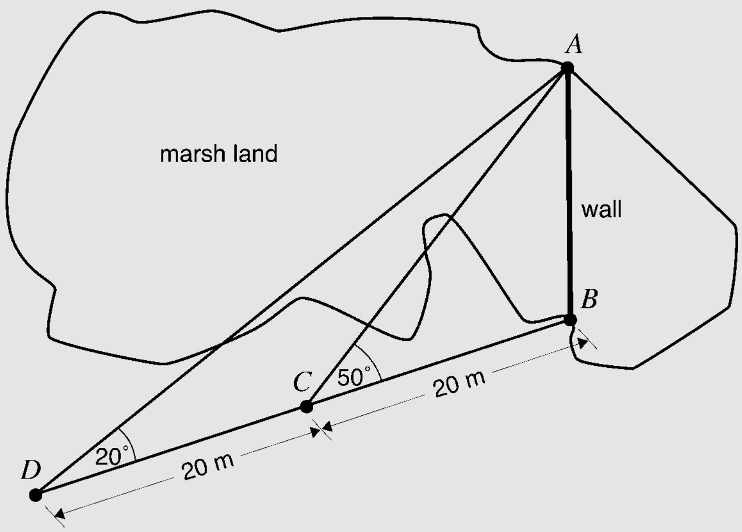
   (g) Using a spreadsheet or otherwise, complete the table shown by finding the area of the face and volume for blocks with side lengths as shown in the table:

Side length (am)	Area of face	Volume of block
50		
60		
70		
80		
90		
100		
110		
120		
130		
140		

continued

Detailed example (continued)

2. Aristotle wanted to see a scale model of a section of the wall before it was built. The scale he chose was 1:25.
   (a) What would be the length of an edge of a hexagonal face of a block for the model? Give your answer in centimetres.
   (b) What is the ratio of the area of the block in the model to that of an actual block in the wall? Explain your result.
   (c) Referring to part (b), discuss the ratio of the volume of a block to that of its model.
3. A part of the wall is to cross marshlands. Aristotle wanted to find out the length of this part of the wall but did not want to get his sandals muddy. To overcome this problem, he made the measurements shown on the diagram below.



   (a) Find the distance AC in metres accurate to two decimal places.
   (b) Find the distance AB which is the length of the wall to be constructed across the marshland. Give your answer to the nearest metre.
4. To form an impressive city entrance, a tower in the form of a hexagonal pyramid is proposed at the point marked D in the diagram above.
   (a) If the wall shown in the diagram runs directly north to south, find the compass bearing of the tower from point A.
   (b) The horizontal cross sections of the tower are to be regular hexagons. The hexagonal base of the tower is to have sides of length 11 metres. An observation tower is to be attached to the tower 18 metres above the ground. At this height, the hexagonal cross-section has sides of length five metres. Find the height of the tower in metres.
   (c) In order to construct the tower, Aristotle needs to find the angle each triangular face makes with the hexagonal base. Find this angle.

This task lends itself very well to using a dynamic geometry system.

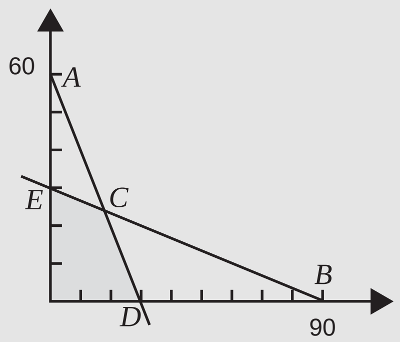
Module 3 – Graphs and relations

The following task is an example of using a set of application questions requiring extended response analysis to form a task.

Detailed example

BLENDING PROFITS

1. In the diagram below:
   (a) Find the equations of the lines AD and EB.
   (b) Find the coordinates of C, the intersection point of these two lines.
   (c) Using your function grapher, and appropriate window settings, check your result. Explain the steps that you followed in achieving this.
   (d) The shaded area shown is the solution region for a set of four simultaneous linear inequalities. Find these four inequalities.



McKeown's vineyard sells two different types of red wines, Type X and Type Y.

2. (a) The profit on the sales of Type X wine is $2.10 per litre, and on Type Y wine is $1.50 per litre. If $P is the overall profit (in thousands of dollars), when x thousand litres of Type X wine and y thousand litres of Type Y are sold, develop a formula for P in terms of x and y and explain your reasoning.
   (b) (i) If $63,000 profit is made in a given year, sketch a straight line that represents this profit model.
   (ii) If 12,000 litres of Type X wine was sold, find the number of litres of Type Y sold, and show this on your graph.
   (iii) Find another possible set of values for the different types of wines sold that correspond to this profit. Show this on your graph.

3. Type X is a blend in which half the wine is made from Cabernet grapes, and half is made from Shiraz grapes. Type Y is a blend in which one sixth is Cabernet wine, one-third is Shiraz wine and the remainder is made from Mataro grapes.

   In a typical year, a total of 15 thousand litres of Cabernet wine, 20 thousand litres of Shiraz wine and 22.5 thousand litres of Mataro wine are available for blending into Type X and Type Y wines.

   (a) If x thousand litres of Type X wine are produced in a typical year, and y thousand litres of Type Y are also produced in that year, develop a set of constraints for x and y as linear inequalities.
   (b) Sketch these inequalities on one set of axes, and shade the feasible region.
   (c) Find the coordinates of the intersections of all lines and label these on your diagram.
   (d) Determine the quantities of Type X and Type Y wines that should be produced to obtain the maximum profit using the model from Question 2.
   (e) What is this maximum profit?
   (f) How many thousands of litres of each of the original wines, Cabernet, Shiraz and Mataro are needed to make this total quantity of Type X and Type Y wine?

Detailed example (continued)

4. In one poor season, the Mataro grapes produced by the vineyard are only enough to make 14,000 litres of Mataro wine for blending, but the other grapes are unaffected and are enough to produce Cabernet and Shiraz wine in the same quantities as in a typical year.
   (a) Draw a diagram showing the appropriate linear inequalities and the feasible region.
   (b) Assuming that the profits on both types of wines, X and Y, are unchanged:
      (i) what is the maximum possible profit in this case to the nearest $100?
      (ii) give the number of litres of each wine that should be produced for a maximum profit.

5. McKeown's wines find that the price that they can command for wine X is inversely proportional to the amount of wine X produced. In a given year, when 18,000 litres of wine X was produced, they found that the wine commanded a price of $16 per litre.
   (a) Find a relationship linking the price per litre of Wine X to the number of thousand litres produced.
   (b) Sketch this relationship on a set of axes.
   (c) Discuss the practical limitations of this model.

   The use of graphics calculator, function grapher or computer algebra system to draw graphs and solve equations would relate to Outcome 3.

Module 4 – Business-related mathematics

The following example demonstrates an assignment style approach to an analysis task. A more investigative approach could be developed by asking students to consider a purchase of their own choice or to compare calculations with actual property value data.

This example relies on extensive use of financial (TVM) software.

Detailed example

INVESTING IN YOUR HOME

Sarah and Michael purchased a house in January 1986 for $145 000.

The property has increased in value by 6% pa over the twenty years to 2006. Determine the value of the house in January 2006.

Sarah and Michael bought the house using a 20% deposit and monthly repayments based on the reducing balance at the end of each month. The interest rate for the first five years of the loan (1986–1991) was 12% pa. Calculate the monthly repayments for this period and the total amount paid on the loan.

For the remainder of the loan, Sarah and Michael used a series of fixed interest rates as follows.

Jan 1991 – Jan 1996: 10.5% pa
Jan 1996 – Jan 2001: 8.25% pa
Jan 2001 – Jan 2006: 5.95% pa

Repayments were adjusted during each period according to the given interest rates.

Determine the monthly repayments for each of these 5-year periods.

Calculate the total amount that Sarah and Michael repay over the life of the loan.

Based on these calculations comment on the financial outcome of the house purchase.

Sarah and Michael considered the option of maintaining the monthly repayments at the initial amount throughout the life of the loan. Calculate how this would affect the final term of the loan and the total amount repaid.

continued

Detailed example (continued)

Inflation rates during the period 1986–2006 showed a decline similar to home loan interest rates.
In 5-year periods the annual inflation rates are summarised as:

Jan 1986 – Jan 1991: 8.0% pa
Jan 1991 – Jan 1996: 6.25% pa
Jan 1996 – Jan 2001: 5.5% pa
Jan 2001 – Jan 2006: 4.0% pa

Use these rates to calculate the 2006 value of Sarah and Michael's initial deposit after taking inflation into account (as a percentage of the 2006 value of the house).

Consider the alternative for Sarah and Michael of not purchasing the house in 1986, but investing their money (including equivalent monthly deposits) in a saving account paying quarterly compound interest based on the quarterly minimum balance. Savings account interest rates during the twenty-year period were 1.5% lower than home loan interest rates. Calculate the final balance of the account.

An item response approach

Alternatively, teachers could use an item response analysis task for a collection of multiple-choice questions, including analysis of item distractors and their relationship to conceptual, process or reasoning error.

Such a task could be structured in two parts.
In the first part, students tackle a collection of multiple-choice items similar to questions from past examination papers, correct these and then carry out analysis of selected responses to questions that have been answered incorrectly. In the second part of the task, students provide a detailed analysis of each alternative for a small number of related items.

Students could also be asked further questions related to a given multiple-choice item, for example:

Interest on Kris' bank account is paid yearly on 30 June and is calculated on the minimum monthly balance. The interest rate is 6% pa. For the year 2003–04, the complete statement for Kris' account, before adding interest, is shown:

Date	Credit	Debit	Balance
30 June 2003	Interest $37.00		$2137.00
13 April 2004		$1025.00	$1112.00

Assuming no other deposits or further withdrawals were made after 13 April 2004, the total interest in dollars to be credited to this account on 30 June 2004 is given by the expression:

A. $0.005 \times (2137 \times 9 + 1112 \times 3)$

B. $0.005 \times (2137 \times 10 + 1112 \times 2)$

C. $0.005 \times (2000 + 2137 \times 8 + 1112 \times 3)$

D. $0.06 \times (2137 \times 9 + 1112 \times 3)$

E. $0.06 \times (2137 \times 10 + 1112 \times 2)$

a. What is the minimum balance in June 2003?

b. For how many months is the minimum balance $2137?

c. What is the minimum balance in April?

d. What error of reasoning would lead a student to select alternative D?

e. Explain how the correct alternative can be determined.

f. Show how each of the alternatives, and information in the table, would need to be modified if the interest rate was 2% pa.

Module 5 – Networks and decision mathematics

The following sample task is presented as a series of related analysis questions. It could easily be modified to be a challenging problem. For example, students could be asked to formulate their own precedence relationships for the critical path analysis and investigate various crashing options.

Detailed example

A ROUND OF GOLF

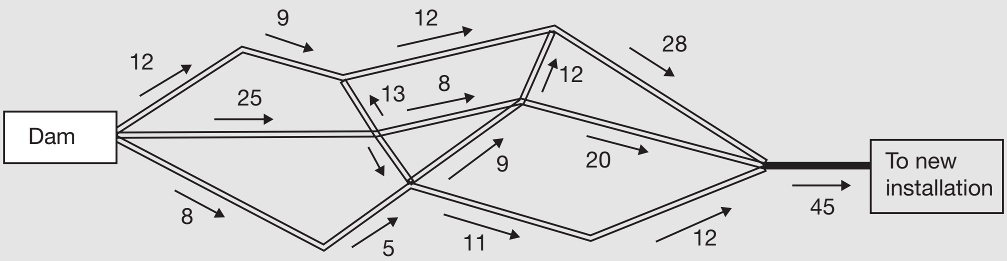
The Eagle's Nest Golf Club aims to extend its irrigation system to a new part of the course.

The following table shows the activities that have to be undertaken, the time required for each activity and any precedence relationships.

Activity number	Activity	Time (Days)	Prerequisites
1	Conduct topographical survey	6	–
2	Draw up plans	4	1
3	Arrange finances	5	–
4	Purchase materials	8	2, 3
5	Employ contractors	5	2, 3
6	Make alternative arrangements for golfers	10	–
7	Excavate	14	5, 6
8	Lay pipes	8	4, 7
9	Test and trouble shoot	2	8
10	Reseed fairways	3	8

Draw a network diagram showing the critical path. Find shortest time for completion of the project, the critical path and slack of any activities.

The new watering system will be added to the end of an existing set of water pipes as shown. The maximum flow of each of the pipes in the system is shown (in litres per second).

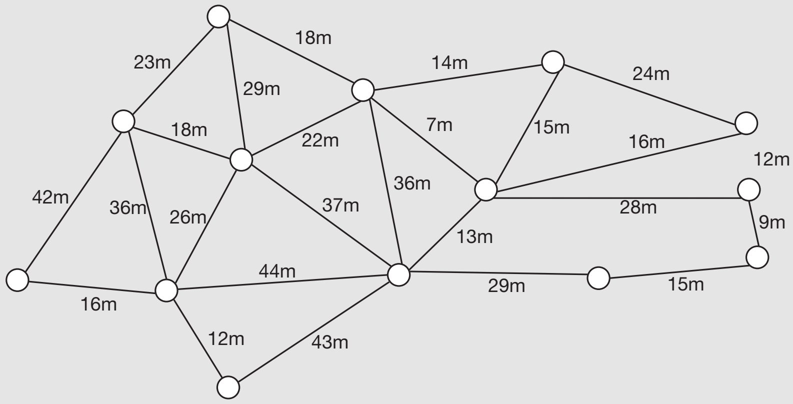


To function effectively, the new installation will require a water delivery flow rate of 35 litres per second. To achieve this overall flow rate, the engineer suggests the upgrade of one pipe to a higher flow rate. Describe one way of doing this, including the amount by which the flow rate of the pipe needs to be increased.

continued

Detailed example (continued)

The water will be dispersed through the new installation using a spanning tree network of pipes connecting sprinkler heads. Possible connections between sprinkler heads and the length of pipe required for each connection are shown.



Determine the minimum length of pipe required to create the network.

Module 6 – Matrices

The following example illustrates an assignment style approach to the analysis task. Students apply routine practices to solve the given problems. The task could easily be modified to an open-ended modelling task. For example, in the final section relating to transition matrices, students could be asked to find a transition matrix that leads to the No Extras model becoming the largest seller.

Detailed example

RUNNING HOT

1. The Speedy Shoe Company produces three models of sport shoes: the Extreme, the Sports and the No Extras models.

   Production of a pair of each of the models requires the following inputs:

   Extreme: 6g fabric, 5g rubber, 2g of plastic
   Sports: 5g fabric, 5g rubber, 1g plastic
   No Extras: 4g fabric, 3g rubber, 3g plastic

   (a) $\begin{bmatrix}S\\C\\P\end{bmatrix}$ is the column matrix for the amount of fabric, rubber and plastic used.

   $\begin{bmatrix}E\\S\\N\end{bmatrix}$ is the column matrix that shows the number of Extreme, Sports and No Extras models that are produced.

   Write the matrix equation to be determined $\begin{bmatrix}S\\C\\P\end{bmatrix}$

   (b) The company receives an order for 16 pairs of Extreme, 11 pairs of Sports and 18 pairs of No Extras models. Determine the inputs required to complete the order.

   (c) The company is advised by the supplier that only 176g fabric, 152g rubber and 67g of plastic is available. Determine how many of each model can be produced by fully utilising all of these inputs.

continued

Detailed example (continued)

2. In the first six months of 2006, the Speedy Shoe Company sold 3200 pairs of shoes. The Extreme model accounted for 60% of sales, the Sports 25% of sales and the No Extras 15% of sales.

   (a) How many pairs of each model were sold in the six-month period?

   (b) Market research by the company has determined that the total market for a six-month period is 10,000 shoes. The sales for each model in the Speedy range and all other brands can be shown as a column matrix $\boldsymbol{P}_0 = \begin{bmatrix} e \\ s \\ n \\ r \end{bmatrix}$

   where e = proportion of Extreme shoes sold
   s = proportion of Speedy shoes sold
   n = proportion of No Extras shoes sold
   r = proportion of other shoes sold

   Determine the elements of $\boldsymbol{P}_0$.

   (c) Surveyed consumers have indicated their buying intentions for the next six months as:

	Extreme	Speedy	No Extras	Other
Extreme	47%	33%	5%	19%
Speedy	25%	54%	32%	8%
No Extras	12%	9%	48%	6%
Other	16%	4%	15%	67%

   That is, 47% of consumers who purchased Extreme shoes in the first half of 2006 will buy another pair in the second half of the year. Twenty-five per cent will purchase a pair of Speedys, 12% will purchase the No Extras model and 16% will change to another brand.

   (d) Enter this information into a 4 × 4 transition matrix, $\boldsymbol{T}$.

   (e) Find the proportions of sales for the second half of 2006, $\boldsymbol{P}_1$ by performing the matrix multiplication, $\boldsymbol{P}_1 = \boldsymbol{T}\,\boldsymbol{P}_0$

   Assume the total shoe sales remains at 10,000 pairs for each six-month period, and consumer buying decisions follow the same pattern as shown above.

   (f) Determine the proportion of total sales and the number of each of the speedy models sold in the second half of 2009.

   (g) Can the Speedy Shoe company expect the sales of each model to continually fluctuate or establish an equilibrium? If an equilibrium is to be achieved, give proportions of each model to the nearest percentage.

   (h) Based on these projections, which model will be the biggest seller in the Speedy range?

Units 3 and 4: Mathematical Methods

Units 3 and 4: Mathematical Methods

Mathematical Methods Units 3 and 4 consists of the following areas of study: 'Functions and graphs', 'Calculus', 'Algebra' and 'Probability' which must be covered in progression from Unit 3 to Unit 4, with an appropriate selection of content for each of Unit 3 and Unit 4. Assumed knowledge and skills for Mathematical Methods Units 3 and 4 are contained in Mathematical Methods Units 1 and 2, and these will be drawn on, as applicable in the development of related content from the areas of study, and key knowledge and skills for the outcomes of Mathematical Methods Units 3 and 4.

In Unit 3, a study of Mathematical Methods would typically include a selection of content from the areas of study 'Functions and graphs', 'Algebra' and applications of derivatives and differentiation to identifying and analysing key features of the functions described in these areas of study and their graphs. In Unit 4, this selection would typically consist of remaining content from the areas of study: 'Functions and graphs', 'Calculus', 'Algebra' and the study of random variables and discrete and continuous probability distributions and their application. For Unit 4, the content from the 'Calculus' area of study would be likely to include the treatment of anti-differentiation, integration, the relation between integration and the area of regions specified by lines or curves described by the rules of functions, and simple applications of this content.

The selection of content from the areas of study should be constructed so that there is a development in the complexity and sophistication of problem types and mathematical processes used (modelling, transformations, graph sketching and equation solving) in application to contexts related to these areas of study. There should be a clear progression of knowledge and skills from Unit 3 to Unit 4 in each area of study.

Students are expected to be able to apply techniques, routines and processes involving rational and real arithmetic, algebraic manipulation, equation solving, graph sketching, differentiation and integration with and without the use of technology, as applicable. Students should be familiar with relevant mental and by hand approaches in simple cases.

The appropriate use of technology to support and develop the teaching and learning of mathematics, and in related assessments, is to be incorporated throughout the units. This will include the use of some of the following technologies for various areas of study or topics: graphics calculators, spreadsheets, graphing packages, statistical analysis systems, and computer algebra systems. Students are encouraged to use graphics calculators, spreadsheets, statistical software, graphing packages or computer algebra systems as applicable across the areas of study.

AREAS OF STUDY

1. Functions and graphs

This area of study covers the behaviour of functions of a single real variable including key features of their graphs such as axis intercepts, turning points and stationary points of inflection, domain (including maximal domain) and range, asymptotic behaviour and symmetry. The behaviour of these functions is to be linked to applications in practical situations.

This area of study will include:

- graphs and identification of key features of graphs of the following functions:
  - power functions, $y = x^n$, for $n \in Q$;
  - exponential functions, $y = a^x$;
  - logarithmic functions, $y = \log_e(x)$ and $y = \log_{10}(x)$, the relationship $a = e^k$ where $k = \log_e(a)$;
  - circular functions, $y = \sin(x)$, $y = \cos(x)$ and $y = \tan(x)$;
  - modulus function, $y = |x|$ where $|x| = x$ when $x \geq 0$ and $|x| = -x$ when $x < 0$;
- transformation from $y = f(x)$ to $y = Af(n(x + b)) + c$, where A, n, b and $c \in R$, and f is one of the functions specified above and the relation between the graph of the original function and the graph of the transformed function such as:

  $y = \dfrac{0.4}{(x+3)^2}$ $\quad$ $y = -10\sin(\pi(x - 0.25)) - 5$ $\quad$ $y = 3(x + 7)^5 - 2$
- graphs of polynomial functions;
- graphs of sum, difference, product and composite functions of f and g (given $r_g \subseteq d_f$), where f and g are functions of the types specified above such as:

  $y = \sin(x) + 2x$ $\quad$ $y = |\cos(2x)|$ $\quad$ $y = x^2e^{kx}, k \in R$

  $y = \dfrac{a}{x^2+1}, a \in R$ $\quad$ $y = (x^2 - 2)^n, n \in N$;
- graphical and numerical solution of equations;
- graphs of inverse functions derived from graphs of original functions;
- recognition of the general form of possible models for data presented in graphical or tabular form, using polynomial, power, circular, exponential and logarithmic functions.

2. Algebra

This area of study covers the algebra of functions, including composition of functions, inverse functions and the solution of equations. This area of study includes the identification of appropriate solution processes for solving equations, and systems of simultaneous equations, presented in various forms. It covers recognition of equations and systems of equations that are solvable using inverse operations or factorisation, and the use of graphical and numerical approaches for problems involving equations where exact value solutions are not required or which are not solvable by other methods. This should support work in the other areas of study.

This area of study will include:

- review of factorisation of polynomials (including the remainder and factor theorems) and its use in curve sketching, equation solving and determining the nature of stationary points;
- exponential and logarithm laws (including the change of base relationship);
- solution of simple exponential and logarithmic equations; for example, $e^{2x} - 4e^x + 3 = 0$;
- solution of simple equations involving circular functions of the form $f(ax) = k$ where f is the sine, cosine or tangent function and k is a real constant;
- one-to-one and many-to-one functions, conditions for the existence of inverse functions;

- finding inverses of functions such as
  $y = A(x + b)^2 + B$ $\quad$ $y = \dfrac{A}{x+b} + B$
  $y = Ae^{kx} + B$ $\quad$ $y = A\log_e(ax + b) + B$

3. Calculus

This area of study covers graphical treatment of limits, continuity and differentiability (including local linearity) of functions of a single real variable and differentiation, anti-differentiation and integration of these functions. This material is to be linked to applications in practical situations.

This area of study will include:

- deducing the graph of the gradient function, including its domain, from the graph of a function;
- rules for derivatives of x^n, (for $n \in Q$), e^x, $\log_e(x)$, $\sin(x)$, $\cos(x)$, $\tan(x)$, and linear combinations of these functions (formal derivation not required);
- product, chain and quotient rules for differentiation; applications involving simple manipulations; for example, derivatives of
  $\sqrt{4-x^2}$ $\quad$ $x^3\sin(2x)$ $\quad$ $e^{\cos(x)}$ $\quad$ $\dfrac{\log_e(x)}{x}$ $\quad$ $\tan(kx)$;
- applications of differentiation to curve sketching, stationary points (turning points, and stationary points of inflection), equations of tangents and normals, maximum/minimum problems, and rates of change, including simple cases of related rates of change, and numerical evaluation of derivatives;
- approximation of the coordinates of stationary points, approximation of intervals where the rate of change is positive or is negative, estimation of the y value for a given x value and approximation of the rate of change at any point;
- the relationship $f(x + h) \approx f(x) + hf'(x)$ for a small value of h, its use to estimate a value of $f(x + h)$ close to a known value of $f(x)$, and its geometric interpretation;
- the relation between the graph of an anti-derivative function and the graph of the original function;
- informal approximation to areas under curves by left rectangles and right rectangles;
- informal treatment of the fundamental theorem of calculus;
- definite and indefinite integrals of x^n and $(ax + b)^n$ where $n \in Q$, e^{kx}, $\sin(kx)$, $\cos(kx)$ and linear combinations of these functions;
- properties of anti-derivatives and definite integrals:
  $$\int (af(x) \pm bg(x))dx = a\int f(x)dx \pm b\int g(x)dx$$
  $$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$$
  $$\int_a^b f(x)dx = -\int_b^a f(x)dx$$
  $$\int_a^a f(x)dx = 0;$$
- integration by recognition that $\dfrac{d}{dx}[f(x)] = f'(x) = g(x)$ implies that $\int g(x)dx = f(x) + c$;
- application of integration to calculating the area of a region under a curve and simple cases of areas between curves.

4. Probability

This area of study will include the study of discrete and continuous random variables, their representation using tables, probability functions or probability density functions (specified by rule and defining parameters as appropriate); and the calculation and interpretation of central measures and measures of spread. The focus is on understanding the notion of a random variable, related parameters, properties and application and interpretation in context for a given probability distribution.

This area of study will include:

- random variables, including:
  - the concept of discrete and continuous random variables;
  - calculation and interpretation of the expected value, variance and standard deviation of a random variable (for discrete and continuous random variables, including consideration of the connection between these);
  - calculation and interpretation of central measures (mode, median, mean);
  - property that, for many random variables, approximately 95 per cent of their probability distribution is within two standard deviations of the mean;
  - bernoulli trials and two-state markov chains;
- discrete random variables:
  - specification of probability distributions for discrete random variables using graphs, tables and probability functions;
  - interpretation of mean (μ) median, mode, variance (σ^2) and standard deviation of a discrete random variable;
  - the binomial distribution, Bi(n, p), as an example of a probability distribution for a discrete random variable (students are expected to be familiar with the binomial theorem and related binomial expansions);
  - the effect of variation in the value(s) of defining parameters on the graph of a given probability function for a discrete random variable;
  - probabilities for specific values of a random variable and intervals defined in terms of a random variable, including conditional probability;
- continuous random variables:
  - construction of probability density functions from non-negative continuous functions of a real variable;
  - specification of probability distributions for continuous random variables using probability density functions;
  - calculation using technology and interpretation of mean (μ) median, mode, variance (σ^2) and standard deviation of a continuous random variable, and their use;
  - standard normal distribution, N(0, 1), and transformed normal distributions N(μ, σ^2), as examples of a probability distribution for a continuous random variable (use as an approximation to the binomial distribution is **not** required);
  - the effect of variation in the value(s) of defining parameters on the graph of a given probability density function for a continuous random variable;
  - probabilities for intervals defined in terms of a random variable, including conditional probability (students should be familiar with the use of definite integrals, evaluated by hand, or numerically using technology, for a probability density function to calculate probabilities; while **not** required, teachers may also choose to relate this to the notion of a cumulative distribution function).

OUTCOMES

For each unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass all of the selected areas of study for each unit. For each of Unit 3 and Unit 4 the outcomes apply to the content from the areas of study selected for that unit.

Outcome 1

On completion of each unit the student should be able to define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.

To achieve this outcome the student will draw on knowledge and related skills outlined in all the areas of study.

Key knowledge

This knowledge includes

- the key features and properties of a function or relation and its graph, and of families of functions and relations and their graphs;
- the effect of transformations (dilation from the coordinate axes; reflection in the coordinate axes and translation) and simple combinations of these transformations on the graphs of a function or relation;
- the concepts of domain, maximal domain, range and asymptotic behaviour of functions;
- the concept of an inverse function, connection between domain and range of the original function and its inverse and the conditions for existence of an inverse function, including the form of the graph of the inverse function for specified functions;
- the concept of combined functions, including composite functions in cases where it can be readily recognised that conditions for existence of a composite are satisfied without requiring domain restriction;
- features which enable the recognition of general forms of possible models for data presented in graphical or tabular form;
- the index (exponent) laws and the logarithm laws;
- analytical, graphical and numerical approaches to solving equations and the nature of corresponding solutions (real, exact or approximate) and the effect of domain restrictions;
- features which link the graph of a function to the graph of the corresponding gradient function or its numerical values, the tangent and normal to a curve at a given point and how the sign and magnitude of the derivative of a function can be used to describe key features of the function and its derivative function;
- chain, product and quotient rules for differentiation;
- related rates of change;
- properties of anti-derivative and definite integrals;
- the concept of approximation to the area under a curve using rectangles, the ideas underlying the fundamental theorem of calculus and the relationship between the definite integral and area;
- the concepts of random variable (discrete and continuous), bernoulli trials and markov chains, probability distribution, parameters used to define a distribution, properties of distributions and their graphs;
- the conditions under which a bernoulli trial or markov chain, or a probability distribution, may be selected to suitably model various situations.

Key skills

These skills include the ability to

- identify key features and properties of the graph of a function or relation and draw the graphs of specified functions and relations, clearly identifying their key features and properties;
- describe the effect of transformations (dilation from the coordinate axes; reflection in the coordinate axes and translation) and simple combinations of these transformations on the graphs of a function or relation;
- find the rule of an inverse function and give its domain and range;
- find the rule of a composite function and give its domain and range;
- sketch by hand graphs of polynomial functions up to degree 4; $y = x^n$ where $n \in N$, $y = a^x$ (using key points $(-1, \frac{1}{a})$, $(0, 1)$ and $(1, a)$), $\log_e(x)$; $\log_{10}(x)$, $|\,x\,|$ and simple transformations of these);
- apply a range of analytical, graphical and numerical processes to obtain solutions (exact or approximate) to equations over a given domain and be able to verify solutions to equations over a given domain;
- solve by hand equations of the form $\sin(ax) = b$, $\cos(ax) = b$, and $\tan(ax) = b$ with exact solutions over a given interval;
- apply algebraic, logarithmic and trigonometric properties to the simplification of expressions and the solution of equations;
- evaluate derivatives of basic and combined functions and apply differentiation to curve sketching and related optimisation problems;
- find derivatives and anti-derivatives of polynomial and power functions, functions of the form $f(ax + b)$ where f is x^n and $n \in Q$, sine, cosine, tangent (derivatives only), e^x or $\log_e(x)$ (derivatives only) and simple linear combinations of these, using pattern recognition or by hand;
- apply the product, chain and quotient rules for differentiation to simple combinations of functions by hand;
- find derivatives of basic and more complicated functions and apply differentiation to curve sketching, related rates and optimisation problems;
- evaluate rectangular area approximations to the area under a curve, find and verify anti-derivatives of specified functions and evaluate definite integrals;
- apply definite integrals to the evaluation of area under a curve and between curves over a specified interval;
- analyse a probability function or probability density function and the shape of its graph in terms of the defining parameters for the probability distribution and the mean and variance of the probability distribution;
- calculate and interpret the probabilities of various events associated with a given probability distribution, by hand in cases where simple arithmetic computations can be carried out;
- apply probability distributions to modelling and solving related problems.

Outcome 2

On completion of each unit the student should be able to apply mathematical processes in non-routine contexts, and analyse and discuss these applications of mathematics.

To achieve this outcome the student will draw on knowledge and related skills outlined in one or more areas of study.

Key knowledge

This knowledge includes

- mathematical content from one or more areas of study relating to a given context for investigation;
- specific and general formulations of concepts used to derive results for analysis within a given context for investigation;
- the role of examples, counter-examples and general cases in developing mathematical analysis;
- inferences from analysis and their use to draw valid conclusions related to a given context for investigation.

Key skills

These skills include the ability to

- specify the relevance of key mathematical content from one or more areas of study to the investigation of various questions related to a given context;
- give mathematical formulations of specific and general cases used to derive results for analysis within a given context for investigation;
- develop functions as models for data presented in graphical or tabular form and apply a variety of techniques to decide which function provides an appropriate model for a given set of data;
- use a variety of techniques to verify results;
- make inferences from analysis and use these to draw valid conclusions related to a given context for investigation;
- communicate conclusions using both mathematical expression and everyday language, in particular, in relation to the context for investigation.

Outcome 3

On completion of each unit the student should be able to select and appropriately use technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.

To achieve this outcome the student will draw on knowledge and related skills outlined in all the areas of study.

Key knowledge

This knowledge includes

- exact and approximate specification of mathematical information such as numerical data, graphical forms and the solutions of equations produced by the use of technology;
- domain and range requirements for the technology-based specification of graphs of functions and relations;
- the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions or equations;
- the similarities and differences between formal mathematical expressions and their representation in various technology applications;
- the appropriate selection of a technology application in a variety of mathematical contexts.

Key skills

These skills include the ability to

- distinguish between exact and approximate presentations of mathematical results, and interpret these results to a specified degree of accuracy;
- produce results using technology which identify examples or counter-examples for propositions;
- produce tables of values, families of graphs or collections of other results using technology which support general analysis in problem-solving, investigative or modelling contexts;
- use appropriate domain and range specifications which illustrate key features of graphs of functions and relations;
- identify the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions or equations;
- specify the similarities and differences between formal mathematical expressions and their representation in various technology applications;
- make appropriate selections for technology applications in a variety of mathematical contexts, and provide a rationale for these selections;
- relate the results from a particular application to the nature of a particular mathematical task (investigative, problem solving or modelling).

ASSESSMENT

The award of satisfactory completion for a unit is based on a decision that the student has demonstrated achievement of the set of outcomes specified for the unit. This decision will be based on the teacher's assessment of the student's overall performance on assessment tasks designated for the unit. The Victorian Curriculum and Assessment Authority publishes an assessment handbook that includes advice on the assessment tasks and performance descriptors for assessment.

The key knowledge and skills listed for each outcome should be used as a guide to course design and the development of learning activities. The key knowledge and skills do not constitute a checklist and such an approach is not necessary or desirable for determining the achievement of outcomes. The elements of key knowledge and skills should not be assessed separately.

For this unit students are required to demonstrate achievement of three outcomes. As a set these outcomes encompass all areas of study.

Assessment of levels of achievement

The student's level of achievement for Units 3 and 4 will be determined by school-assessed coursework and two end-of-year examinations.

Contribution to final assessment

School-assessed coursework for Unit 3 will contribute 20 per cent and for Unit 4 will contribute 14 per cent to the study score.

Units 3 and 4 will also be assessed by two end-of-year examinations, which together will contribute 66 per cent to the study score.

School-assessed coursework

Teachers will provide to the Victorian Curriculum and Assessment Authority a score representing an assessment of the student's level of achievement.

The score must be based on the teacher's rating of performance of each student on the tasks set out in the following table and in accordance with an assessment handbook published by the Victorian Curriculum and Assessment Authority. The assessment handbook also includes advice on the assessment tasks and performance descriptors for assessment.

Assessment tasks must be a part of the regular teaching and learning program and must not unduly add to the workload associated with that program. They must be completed mainly in class and within a limited timeframe. Where optional assessment tasks are used, teachers must ensure that they are comparable in scope and demand. Teachers should select a variety of assessment tasks for their program to reflect the key knowledge and skills being assessed and to provide for different learning styles.

Outcomes	Marks allocated*	Assessment tasks
Unit 3		A function and calculus *application task* based on content from the areas of study, with three components of increasing complexity: • introduction of a context through specific cases or examples • consideration of general features of this context • variation, or further specification, of assumptions or conditions involved in the context to focus on a particular feature related to the context. **and** Two equally weighted tests that consist of an appropriate combination of multiple-choice, short-answer and extended response items. The outcomes are to be assessed across the *application task* and the two *tests*.
Outcome 1 Define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.	30	15 *The application task* 15 *Tests*
Outcome 2 Apply mathematical processes in non-routine contexts, and analyse and discuss these applications of mathematics.	20	20 *The application task*
Outcome 3 Select and appropriately use technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.	10	5 *The application task* 5 *Tests*
Total marks	**60**	

*School-assessed coursework for Unit 3 contributes 20 per cent to the study score.

Outcomes	Marks allocated*	Assessment tasks
Unit 4		Two *analysis tasks*, with the three outcomes assessed across the tasks. Each *analysis task* is a short item of 2–4 hours duration over 1–2 days selected from: • an assignment where students have the opportunity to work on a broader range of problems in a given context; **or** • a short and focused investigation, challenging problem or modelling task; **or** • a set of application questions requiring extended response analysis in relation to a particular topic or topics; **or** • item response analysis for a collection of multiple-choice questions, including analysis of item distractors and their relationship to conceptual, process or reasoning error. The two *analysis* tasks are to be of a different type. One analysis task is to be related to the 'Probability' area of study.
Outcome 1 Define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.	15	7 *Analysis task 1* 8 *Analysis task 2*
Outcome 2 Apply mathematical processes in non-routine contexts, and analyse and discuss these applications of mathematics.	15	8 *Analysis task 1* 7 *Analysis task 2*
Outcome 3 Select and appropriately use technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.	10	5 *Analysis task 1* 5 *Analysis task 2*
Total marks	**40**	

*School-assessed coursework for Unit 4 contributes 14 per cent to the study score.

End-of-year examinations

The student's level of achievement for Units 3 and 4 will also be assessed by two examinations based on tasks related to Outcomes 1 to 3.

Examination 1

Description

Students are required to respond to a collection of short-answer and some extended-answer questions covering all areas of the study in relation to Outcome 1. The task is designed to assess students' knowledge of mathematical concepts, their skills in carrying out mathematical algorithms and their ability to apply concepts and skills in standard ways without the use of technology. This examination is common to Mathematical Methods Units 3 and 4 and Mathematical Methods (CAS) Units 3 and 4.

All questions are compulsory.

Students will complete the examination using a structured answer booklet.

The examination will be set by a panel appointed by the Victorian Curriculum and Assessment Authority.

Conditions

The task will be completed under the following conditions:

- Duration: one hour.
- Date: end-of-year, on a date to be published annually by the Victorian Curriculum and Assessment Authority.
- Victorian Curriculum and Assessment Authority examination rules will apply. Details of these rules are published annually in the *VCE and VCAL Administrative Handbook*.
- No calculators or notes of any kind are permitted. A sheet of formulas will be provided with the examination.
- The task will be marked by a panel appointed by the Victorian Curriculum and Assessment Authority.

Contribution to final assessment

The examination contributes 22 per cent to the study score.

Examination 2

Description

Students are required to respond to a collection of multiple-choice questions and extended-answer questions covering all areas of study in relation to all three outcomes, with an emphasis on Outcome 2. Student access to an approved graphics calculator will be assumed by the setting panel.

The task is designed to assess students' ability to understand and communicate mathematical ideas, and to interpret, analyse and solve both routine and non-routine problems. Students should attempt all the multiple-choice questions in Part I of the examination and all of the extended-answer questions, involving multi-stage solutions of increasing complexity in Part II of the examination.

All questions are compulsory.

Students will complete the examination using a structured answer booklet.

The examination will be set by a panel appointed by the Victorian Curriculum and Assessment Authority.

Conditions

The examination will be completed under the following conditions:

- Duration: two hours.
- Date: end-of-year, on a date to be published annually by the Victorian Curriculum and Assessment Authority.
- Victorian Curriculum and Assessment Authority examination rules will apply. Details of these rules are published annually in the *VCE and VCAL Administrative Handbook*.
- An approved graphics calculator and **one** bound reference, text (which may be annotated) or lecture pad, may be brought into the examination. A sheet of formulas will be provided with the examination.
- The examination will be marked by a panel appointed by the Victorian Curriculum and Assessment Authority.

Contribution to final assessment

The examination will contribute 44 per cent to the study score.

Advice for teachers (Units 3 and 4: Mathematical Methods)

DEVELOPING A COURSE

A course outlines the nature and sequence of teaching and learning necessary for students to demonstrate achievement of the set of outcomes for a unit. The areas of study describe the knowledge required for the demonstration of each outcome. Outcomes are introduced by summary statements and are followed by the key knowledge and skills which relate to the outcomes.

Teachers must develop courses that include appropriate learning activities to enable students to develop the knowledge and skills identified in the outcome statements in each unit.

In Units 3 and 4, assessment is structured. For school-assessed coursework, assessment tasks are prescribed. The contribution that each task makes to the total school-assessed coursework is also stipulated.

Approaches to course development

This study design provides a structure for the development of detailed courses by schools. While Mathematical Methods Units 3 and 4 is a fully prescribed course, there is considerable flexibility at the local level in the design of a course. Planning decisions need to be made about the order of topics, the time devoted to each topic, and connections between the material developed in particular topics and the outcomes for each unit. These decisions also need to take into account the timing of school-assessed coursework.

Some students will undertake Specialist Mathematics Units 3 and 4 concurrently with Mathematical Methods Units 3 and 4, which contains assumed knowledge and skills for Specialist Mathematics Units 3 and 4. Care needs to be taken in giving consideration to how the timing and development of material in Mathematical Methods Units 3 and 4 is related to supporting and facilitating the development of material in Specialist Mathematics Units 3 and 4. The sample topic sequence outlined below illustrates a possible implementation across 27 weeks; schools and teachers are encouraged to develop their own sequences or variations on this sample sequence.

Sample teaching sequence

Week	Unit 3 content
	Functions and graphs
1	• many to one and one to one functions
	• sums/difference, product and composition of functions
2	• existence of inverse functions
	• introduction to finding inverse relations and to graphs of inverse relations and functions
	Transformations
3	• use of transformations to derive graphs from known or given graphs
	• transformations of linear, quadratic and some cubic/quartic polynomial functions
4	• graphs of $y = x^{-1}$, $y = x^{-2}$ $y = \sqrt{x}$, $y = \|x\|$ and transformations
	• combinations of transformed functions
	• modelling with these functions
	Polynomial functions
5	• binomial theorem
	• degree of polynomial, substitution
	• polynomial re-expression/division
6	• remainder and factor theorems
	• factorisation of polynomials and graphs of quadratic, cubic and quartic polynomial functions
	• modelling with polynomial functions
	Exponential and logarithmic functions
7	• definition of $\log_a(x)$, logarithm laws
	• graphs of a^x, $y = \log_a(x)$
	• introduction of e: graphs of $y = e^x$, $y = \log_e(x)$
8	• graphs of exponential and logarithmic functions – simple reflections, translations or dilations of the above
	• solution of exponential and logarithmic equations
	• modelling with exponential and logarithmic functions
	Test 1 – functions, in week 9
	Circular functions
9	• radians and unit circle
	• exact values
10	• revision of graphs
	• solving equations
11	• graphs involving transformations
	• graphs involving combined functions
	• modelling with circular functions
	Differentiation of functions
12	• limits, continuity, differentiability
	• derivatives of key functions x^n, e^x, $\log_e(x)$, $\sin(x)$, $\cos(x)$
13	• graph of the gradient function
	• product, quotient and chain rules for differentiation and application to combinations of functions
	Application task
14	(this could be held earlier or later depending on coverage of relevant concepts, skills and processes with
15	respect to content and context for the application task)

Applications of differentiation

16 • tangents and normals
• sketch graphs of combined functions, including stationary points
17 • application of calculus to maximum/minumum problems
• rates of change

Integration and anti-differentiation

18 • integration – approximation methods
• informal treatment of the fundamental theorem of calculus
19 • link between graph of anti-derivative and graph of original function
• properties of integrals
• evaluation of definite integrals

Test 2 – Differentiation, week 17 or 18

Unit 4 content

20 • anti-differentiation (integration) by rule of x^n, $(ax + b)^n$, e^{kx}, $\sin(kx)$, $\cos(kx)$ and linear combinations of these functions
• anti-differentiation by recognition that if $\frac{d}{dx}[f(x)] = f'(x) = g(x)$ then $\int g(x)dx = f(x) + c$
• area between curves

Analysis task 1

21 *a short and focused investigation, challenging problem or modelling task*, related to an application of differentiation/anti-differentiation and integration (alternatively, this task could be conducted preceding the analysis task 2 on probability at the end of Term 3).

Discrete random variables and distributions

22 • revision of basic probability principles
• concept of a random variable; construction of discrete probability distributions
• bernoulli trials and markov sequences
23 • specification through tables, graphs or function definitions
• calculation and interpretation of expected value (μ) and variance (σ)
• effect of variation of defining parameters on the graph
24 • investigation of property that, for many random variables, ≈ 95% of the distribution is within 2σ of the mean
• $E(kX + b)$ and $Var(kX + b)$
• binomial distribution as a particular discrete distribution

Continuous random variables and distributions

25 • probability density functions from non-negative continuous functions
• probability by area under curve
26 • mean, median, mode, variance and standard deviation for any continuous distribution
• effect of variation of defining parameters on the graph
• applications
• normal distribution as example of continuous distribution
• standard normal curve $Z \sim N(0,1)$ to $X \sim N(\mu, \sigma^2)$
• effect of values of μ and σ on shape
• applications

	Analysis task 2 or Analysis tasks 1 and 2
27 or 26 and 27	If the first analysis task has not been conducted earlier it will need to be conducted at this stage. The first task could be a short and focused investigation, challenging problem or modelling task, related to an application of differentiation/anti-differentiation and integration. The second analysis task is related to the probability area of study, and could be an *assignment where students have the opportunity to work on a broader range of problems in a given context* **or** *a set of application questions requiring extended analysis in relation to a particular topic or topics,* **or** *an item response analysis for a collection of multiple-choice questions, including analysis of item distractors and their relationship to conceptual, process or reasoning error.*

This sample sequence covers material related to functions, graphs and algebra in detail before proceeding to work on differentiation in Unit 3 followed by coverage of material related to anti-differentiation and integration, and probability in Unit 4. Variations on this sequence might, for example:

- cover material on polynomial and power functions and incorporate some related material on differentiation, anti-differentiation and integration of these functions in the first part of Unit 3; followed by applications of differentiation and integration across the range of functions covered in the course, and probability in the latter part of Unit 3 and in Unit 4;
- cover material on each of the functions required in the course and related differentiation and applications in Unit 3, followed by anti-differentiation and integration across all functions, related applications and probability in Unit 4;
- cover material on discrete random variables and probability distributions at the end of Unit 3; and cover material on continuous random variables at the end of Unit 4 following on directly from work on anti-differentiation and integration earlier in Unit 4.

The relationship between the timing of coursework assessment and content coverage would need to be adjusted accordingly.

USE OF INFORMATION AND COMMUNICATIONS TECHNOLOGY

In designing courses and developing learning activities for Mathematical Methods, teachers should make use of applications of information and communications technology and new learning technologies, such as computer-based learning, multimedia and the World Wide Web, where appropriate and applicable to teaching and learning activities.

KEY COMPETENCIES AND EMPLOYABILITY SKILLS

Students undertaking the following types of assessment, in addition to demonstrating their understanding and mastery of the content of the study, typically demonstrate the following key compentencies and employability skills.

Assessment task	Key competencies and employability skills
Analysis task	Planning and organising, solving problems, using mathematical ideas and techniques, (written) communication, use of information and communications technology
Tests	Self management, solving problems
Application task	Planning and organising, solving problems, using mathematical ideas and techniques, (written) communication, use of information and communications technology

LEARNING ACTIVITIES

Examples of learning activities for each unit are provided in the following sections. Extended examples are highlighted by a shaded box. The examples that make use of information and communications technology are identified by this icon ICT.

Units 3 and 4: Mathematical Methods

Outcome 1

Define and explain key concepts, as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.

Examples of learning activities

skills practice on standard mathematical routines through appropriate exercises; this should include development of the ability to identify problems as either being in a standard form, or readily transformable to a standard form, the efficient and accurate application of the relevant mathematical routine (e.g. the application of differentiation rules to combinations of functions), and systematic checking of both the reasonableness and accuracy of results (does the graph of the rule which has been developed for the derived function correspond to a sketch of the derivative function obtained from inspection of the graph of the original function?); the appropriate use of technology, such as checking the numerical derivative value from a graphics calculator with the form of the derivative graph sketch would link this work to Outcome 3

error identification and analysis exercises, where students work through a range of 'worked solutions' and identify and rectify missing steps in working or errors in working

construction of summary or review notes related to a topic or area of study (e.g. the conditions under which different probability distributions can be appropriately applied to solve problems)

presentation of a range of typical problems associated with an area of study and worked solutions (e.g. use of tree diagrams to calculate probabilities associated with bernoulli trials and markov chains, applications of integration to area and areas between curves)

assignments structured around the development of sample cases of standard applications of mathematical skills and procedures in readily recognisable situations (e.g. the development of different sketch graphs in terms of defining parameters for a type of relation – this can be checked by the use of graphics calculators, spreadsheets, function graphers or computer algebra systems and hence linked to Outcome 3)

Outcome 2

Apply mathematical processes in non-routine contexts, and analyse and discuss these applications of mathematics.

Examples of learning activities

investigative projects such as the use of exponential functions to describe growth and/or decay

problem-solving tasks, e.g. optimising volume/cost constraints in container design

modelling tasks (e.g. modelling periodic behaviour)

a set of applications questions requiring analysis and extended response related to a particular context

a report on item response analysis for a collection of multiple-choice questions (e.g. distinguishing between 'competing functions' as models for various data represented graphically and/or in table form)

presentation on research into a particular application of mathematics (e.g. the role of different probability distribution in sampling contexts)

Outcome 3

Select and appropriately use technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.

Examples of learning activities

ICT technology should be used where appropriate and applicable to enhance the teaching and learning of mathematics throughout the course; it is expected that all students will have access to graphics calculators in Mathematical Methods Units 3 and 4; teachers should develop courses that encourage the appropriate use of this technology; supporting technology should be used extensively in the classroom, with appropriate contexts for each of the outcomes and regard for a balance between analytical and numerical approaches in terms of efficiency, effectiveness and elegance of use; graphics calculators offer many opportunities to explore mathematical ideas in ways which will enhance students' understanding of their underlying concepts and subsequently formalised; an example is the exploration of the ideas surrounding the dot point 'the concept of approximation to the area under a curve using rectangles, the ideas underlying the fundamental theorem of calculus and the relationship between the definite integral and area' in Outcome 1

Detailed example

CUBIC POLYNOMIAL FUNCTIONS, TANGENTS AND HORIZONTAL AXIS INTERCEPTS

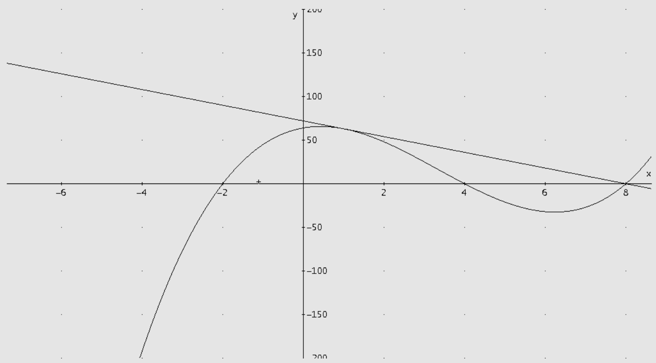
There is a relationship between the horizontal axis intercepts of the graph of a cubic polynomial function (where there is more than one such intercept) and the tangent to the graph of the function at the point with x coordinate halfway between the *x* coordinates of any two of the intercepts.

This relationship can be used as the basis of an activity related to all three outcomes, in particular Outcomes 1 and 3 for the case of specific functions and their graphs, and Outcomes 2 and 3 for systematic analysis leading to consideration of the general case. This activity covers graphs, equations and calculus related to cubic polynomial functions.

Part 1 (Outcomes 1 and 3)

Let $f: R \to R$, $f(x) = (x + 2)(x - 4)(x - 8)$. Show that the equation $f(x) = 0$ has roots –2, 4 and 8 and the graph of f has x axis intercepts at (–2, 0), (4, 0) and (8, 0) respectively.

Consider the two x axis intercepts (–2, 0) and (4, 0) and the point on the graph of f whose x value is half-way between –2 and 4. Use calculus to show that the tangent to the graph of f at this point passes through the third intercept (8, 0):



Repeat this for the pairs of intercepts (–2, 0) and (8, 0) and (4, 0) and (8, 0). Draw the corresponding graphs in each case.

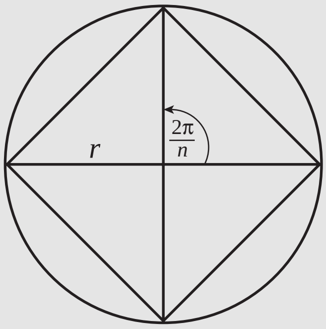
Part 2 (Outcomes 2 and 3)

Investigate the general case for $f: R \to R$ where $f(x) = (x - a)(x - b)(x - c)$ where a, b and c are (not necessarily distinct) real constants.

Detailed example

APPROXIMATION TO THE AREA UNDER A CURVE

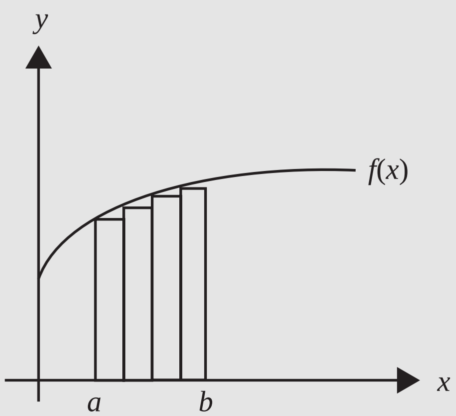
An approach which is sometimes used to introduce the notion of a limiting process being used to obtain an area is to consider finding the area of a circle using n congruent triangles.



$n = 4$ in this diagram

For a circle as shown in the diagram an approximate area is given by: Area = $n\frac{r^2}{2}\sin\frac{2\pi}{n}$ where n represents the number of triangles.

When r has the value 1, $r^2 = 1$, and we note that the value of the area should be approximately π for sufficently large values of n. We can use a graphics calculator to consider the notion that increasing the number of triangles will increase the accuracy of the approximation. This can be done using either a graph of : $y = \frac{x}{2}\sin\frac{2\pi}{x}$ with different windows, or using a sequence. For the latter approach an appropriate sequence command could be: ‘seq(n/2sin(2π/n), n, 10, 100, 10)’, or similar. This would give the value for the function in terms of n for values from 10 to 100 in increments of 10.



The sequence command can be then used to find the area under a curve using rectangles. The area under this curve could be approximated by a sequence such as: seq (hy1, x, a, b-h, h) where h is the width of the interval and the function rule $f(x)$ is stored as y1.

Further exploration of the area in this manner could also take into account key knowledge and skills related to Outcomes 2 and 3.

Detailed example

MEAN AND VARIANCE FOR CONTINUOUS PROBABILITY DISTRIBUTIONS

An activity can be designed to explicitly explore the link between corresponding representations of concepts such as mean and variance with respect to discrete and continuous random variables.

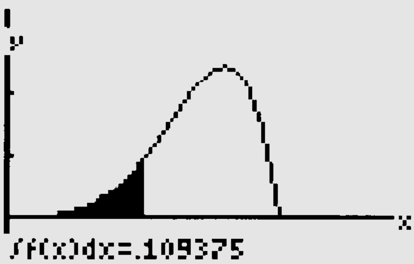
Students would already be familiar with the use of a sum to determine the mean and variance of a discrete distribution and the application of definite integral to the probability density function f of a continuous random variable X to calculate probabilities associated with intervals as $\Pr(a < X < b) = \int_a^b f(x)dx$.

They could be asked to identify the connection between $\Pr(X = x)$ for a discrete distribution, the probability density function f for a continuous distribution and computations involving Σ and $\int$ respectively.

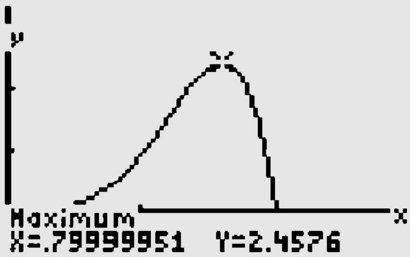
This would include consideration of the connection between $E(X) = \Sigma x \Pr(x)$ for the probability distribution of a discrete random variable and $E(X) = \int_a^b x\, f(x)dx$ for the probability distribution of a discrete random variable; and similarly, for the respective variances $\mathrm{Var}(X) = \Sigma x^2 \Pr(x) - E(X)^2$ and $\mathrm{Var}(X) = \int_a^b x^2\, f(x)dx - E(X)^2$.

Thus, for the probability density function f of a continuous random variable X, where:
$f(x) = 30x^4(1 - x)$ when $0 < x < 1$ and 0 otherwise students can be asked to carry out computation such as:

- $\Pr(X < 0.5) = \int_0^{0.5} 30x^4(1 - x)dx = 0.109375$



- $E(X) = \int_0^1 x \times 30x^4(1 - x)dx = 0.7143$ correct to 4 decimal places
- $\mathrm{Var}(X) = \int_0^1 x^2 \times 30x^4(1 - x)dx - E(X)^2 = 0.5357 - 0.5102 = 0.0255$
- Identify that the mode occurs at $x = 0.8000$, where the maximum value of the probability density function occurs:



As an extension of this work, student could consider informally the link between the uniform (discrete) and rectangular (continuous) distribution, the geometric (discrete) and exponential (continuous) distributions or the binomial (discrete) and normal (continuous) distributions.

APPROACHES TO ASSESSMENT

In Units 3 and 4 assessment tasks must be selected from those designated in the outcome table.

The following are examples of assessment tasks that could be used to demonstrate achievement of the outcomes:

- identification and analysis of key features of functions in graphical, tabular and numerical form (including associated problems involving analytic and/or numeric differentiation and integration and applications)
- production of collections of results in response to variation of parameters used to define a type of function (for example, the role of the coefficient *b* in determining features of the family of graphs of cubic polynomial functions with rules of the form $f(x) = ax^3 + bx^2 + cx + d$)
- developing functions which can be used to model trends in sets of data, comparing empirical and theoretical models for data
- solution of equations (or systems of equations) using graphical, numerical or algebraic approaches (for example, comparison of the behaviour of functions with rules of the form $g(x) = x^n$ and $h(x) = n^x$)
- use of symbolic manipulation to explore and check patterns and conjectures associated with generalisable properties of functions.

In each unit Outcome 3 should be incorporated in the assessment of Outcomes 1 and 2.

Detailed example

SAMPLE NON-CALCULATOR CIRCULAR FUNCTIONS AND CALCULUS TEST

Access to an 'exact values' table would be necessary as minimum assistance for students.

Multiple choice

1. A solution of the equation $\sin(2x) - a\cos(2x) = 0$ is $\frac{\pi}{6}$.

   The value of a is

   A $\sqrt{3}$

   B $\frac{1}{\sqrt{3}}$

   C $\frac{\sqrt{3}}{2}$

   D $\frac{2}{\sqrt{3}}$

   E $\sqrt{3}$

2. For the equation $2\sin(x) - \sqrt{3} = 0$ the sum of the solutions on the interval $[0, 2\pi]$ is

   A π

   B $\frac{5\pi}{3}$

   C 2π

   D $\frac{7\pi}{3}$

   E 3π

3. The function $f: R \to R$, $f(x) = a\cos(bx) + c$, where a, b and c are positive constants, has period

   A a

   B b

   C $\frac{2\pi}{a}$

   D $\frac{2\pi}{b}$

   E $\frac{b}{2\pi}$

4. If $f(x) = a\sin(2x)$, where a is a constant, and $f'(\pi) = 2$, then a is equal to

   A -1

   B $-½$

   C 0

   D $½$

   E 1

Short answer

5. The temperature on a particular day can be modelled by the function

   $C = -4\cos\left(\frac{\pi}{12}t\right) + 16$

   where t is the time elapsed, in hours, after 4:00 am and C is the temperature in degrees Celsius.

   a Calculate the temperature at 8:00 am.

   b At what time is the temperature first 20°C?

6. The graph of $y = \cos(x)$ is transformed into the graph of $y = 2\cos(x)$ by a dilation in the y-direction by a scale factor of 2.

   In each case below state the type of transformation together with any relevant scale factors, distances and directions required to transform the graph of the first equation into the graph of the second equation.

   i $y = \cos(x)$ to $y = 2\cos(0.5x)$

   ii $y = 2\cos(0.5x)$ to $y = 2\cos\left(0.5\left(x - \frac{\pi}{4}\right)\right)$

   iii $y = 2\cos\left(0.5\left(x - \frac{\pi}{4}\right)\right)$ to

   $y = 2\cos\left(0.5\left(x - \frac{\pi}{4}\right)\right) + 2$

continued

Detailed example (continued)

7. The height of the sea level (above a fixed point) on a given day varies over time according to the effect of the tides.

   The height, y cm, is given by the equation
   $y = \frac{2}{5}\sin\left(\frac{\pi t}{6}\right)+1$

   a. Sketch a graph representing the height of the sea level on 21 September 94 from midnight to midday.

   b. State

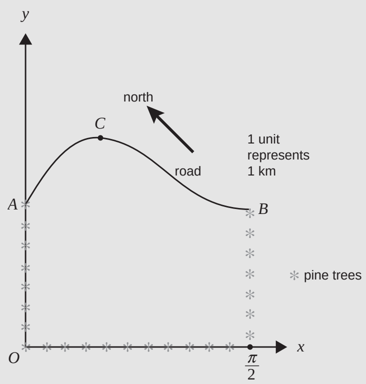
      i The period of the function

      ii The amplitude of the function.

Extended response

8. The diagram below shows a small farm. Rows of pine trees form the boundaries on three sides of the farm and, on the fourth, there is a road. With respect to the axes shown, the road follows the curve with equation

   $y = \sin(x) + 0.5\cos(2x),\ 0 \le x \le \frac{\pi}{2}$



   a. Write down the coordinates of the points A and B.

   b. Find $\frac{dy}{dx}$ and hence show that a stationary point exists when $x = \frac{\pi}{6}$

   c. C is the point $\left(\frac{\pi}{6}, c\right)$. Find the exact value of c.

   A creek which follows the curve with equation $y = \sin(x),\ 0 \le x \le \frac{\pi}{2}$ passes through the farm dividing the land into two sections – a north paddock and a south paddock.

   d. On the diagram given in the question, sketch the curve representing the creek and shade the south paddock.

   e. By solving an appropriate equation, find the exact coordinates of the point where the road crosses the creek.

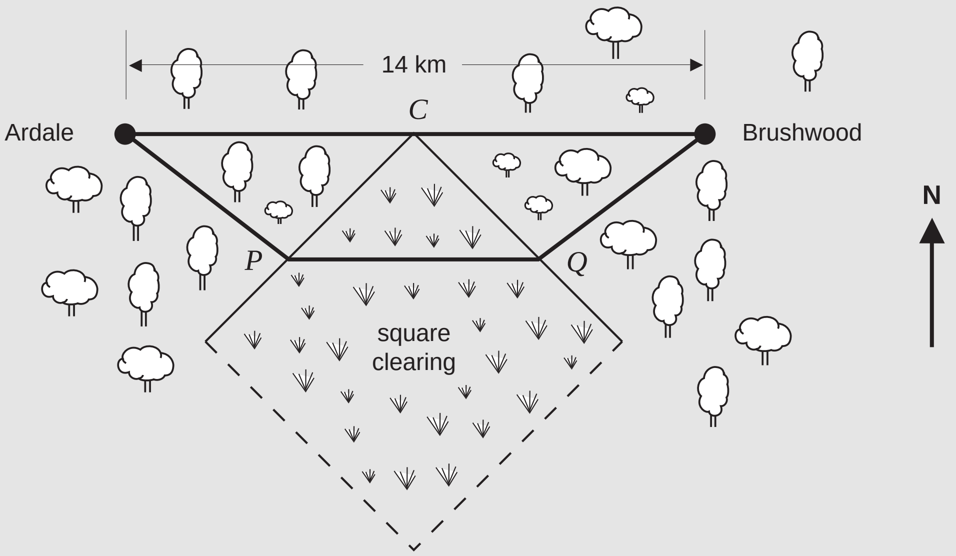
Detailed example

ANALYSIS TASK – BUSHWALKING WITH KIM

Bushwalkers travel through different types of country. The denseness of the bush and the ruggedness of the terrain influence the speed of travel. By planning a route to take such factors into consideration, the total time taken to travel from one point to another can be reduced. In calculating estimates of the time for a particular route, a walker uses his/her average speed for each different type of country.

For a walk through a particular type of country the travel time taken is given by $t = \frac{d}{v}$ where d is the distance in kilometres (km) and v is the average speed in kilometres per hour (km/h) for that particular type of country.

A typical two-day walk for Kim covers a distance of up to 30 km with walking speeds of up to 5 km/h in cleared country.



a. For a typical two-day walk, choose several representative values for Kim's average speed and sketch a graph of the relationship between t and d for each of these values. Similarly, choose several representative values for the distance to be travelled and draw a graph of the relationship between t and v for each of these values. Discuss the key features of each of the two families of graphs, and the differences between the families.

   Next weekend Kim is planning to walk from Ardale to Brushwood. The direct route, a distance of 14 km, will take her entirely through rugged bush country. However, there is a large square clearing situated as shown in the diagram below. This clearing has one diagonal along the perpendicular bisector of the direct route and one corner, *C*, at the midpoint of the direct route.

   Kim believes that time will be saved if she travels from Ardale to Brushwood on a route similar to the one shown above passing through *P* and *Q*, where the section *PQ* is parallel to the direct route. Throughout this investigation assume that Kim's path across the square clearing is parallel to the direct route. The side-length of the square clearing is 7 km.

b. Select an appropriate length variable and hence determine a mathematical relationship which will allow Kim to calculate the total time she would take for any route of this type. Sketch the graph of this relationship and discuss key features. Find and describe the route for which her travelling time will be least and compare it with the direct one.

Detailed example

APPLICATION TASK: SMOOTH SAILING

A context such as the following could be used to investigate the relationship between the average value of a function and its derivative over a given interval. The task is developed in three components, and addresses all three outcomes, with a focus on Outcome 2.

Consider two locations A and B at opposite sides of a bay, such as Port Philip Bay or Sydney Harbour. Conjecture if it is possible to sail a boat from A to B without ever sailing in the exact (straight line) direction from A to B. That is, is it possible to sail from A to B without the instantaneous line of motion ever being parallel to the straight line segment line connecting A and B?

Similarly, if it is also known that the locations A and B are 100 km apart by road around the bay, and are connected by a highway with a speed limit of 100 km/hr, conjecture if it is possible for a car to leave A at 1pm and arrive at B at 1.50 pm without exceeding the speed limit.

Alternatively, in a beach sport event, when a competitor runs 100 metres in 12 seconds, will he/she achieve a speed of 8.33 m/sec at any time during the race? If two competitors run in the same race, both starting and finishing at the same time, is their speed the same at any point during the race?

Each of these situations can be considered as applications of the *mean value theorem for differentiation*. These can be used to develop application tasks in three components where students are asked to consider simple functions used to model such situations and investigate the application of the mean value theorem for differentiation in these situations.

Component 1

Selection of a suitable function, such as a quadratic function, which is continuous on a closed interval $[a, b]$, differentiable on the open interval (a, b), and can be used as a model for one of the above situations, or a similar scenario. Consideration of the graph of the model function, the line segment connecting the interval endpoints $(a, f(a))$ and $(b, f(b))$, and the average rate of change, f_{ave}, over the interval.

Determination of the derivate function f' and the value(s) for $x = c$ in the open interval (a, b) for which $f'(c) = f_{ave}$. Geometric interpretation of these considerations with respect to a conjecture related to the situation being model.

Component 2

Investigation of other models for these situations, including consideration of whether it is always possible to find a value $x = c$ in the open interval (a, b) for which $f'(c) = f_{ave}$, for example, whether there is a possible model which is not continuous or differentiable. Generalisation of these results.

Alternatively, investigation of the conditions under which the above result is generally true, that is, where there exists a point c in (a, b) for which $f'(c) = f_{ave}$. Consideration of the assumption of differentiability and the assumption of continuity.

Component 3

Investigation of one of the other scenarios, where students create their own model.

Alternatively, investigation of the sailing boat context, with A lying due east of B, to extend the result of the mean value theorem for differentiation, by showing that if also $f(a) = f(b)$, then there is a point c in (a, b) for which $f'(c) = 0$.

Important aspects of mathematics to be considered in assessment of student work are:

- sketch graphs of functions that clearly indicate key features;
- the use of calculus to determine the derivative of a given function;.
- solution of equations;
- formulation of general mathematical results/ conjectures;
- consideration of domains.

Units 3 and 4: Mathematical Methods (CAS)

Units 3 and 4: Mathematical Methods (CAS)

Mathematical Methods (CAS) Units 3 and 4 consists of the following areas of study: ‘Functions and graphs’, ‘Calculus’, ‘Algebra’ and ‘Probability’, which must be covered in progression from Unit 3 to Unit 4, with an appropriate selection of content for each of Unit 3 and Unit 4. Assumed knowledge and skills for Mathematical Methods (CAS) Units 3 and 4 are contained in Mathematical Methods Units (CAS) Units 1 and 2, and will be drawn on, as applicable in the development of related content from the areas of study, and key knowledge and skills for the outcomes of Mathematical Methods (CAS) Units 3 and 4.

In Unit 3, a study of Mathematical Methods (CAS) would typically include a selection of content from the areas of study ‘Functions and graphs’, ‘Algebra’ and applications of derivatives and differentiation, and identifying and analysing key features of the functions and their graphs from the ‘Calculus’ area of study. In Unit 4, this selection would typically consist of remaining content from the areas of study: ‘Functions and graphs’, ‘Calculus’, ‘Algebra’ and the study of random variables and discrete and continuous probability distributions and their applications. For Unit 4, the content from the ‘Calculus’ area of study would be likely to include the treatment of anti-differentiation, integration, the relation between integration and the area of regions specified by lines or curves described by the rules of functions, and simple applications of this content.

The selection of content from the areas of study should be constructed so that there is a development in the complexity and sophistication of problem types and mathematical processes used (modelling, transformations, graph sketching and equation solving) in application to contexts related to these areas of study. There should be a clear progression of skills and knowledge from Unit 3 to Unit 4 in each area of study.

Students are expected to be able to apply techniques, routines and processes involving rational and real arithmetic, algebraic manipulation, equation solving, graph sketching, differentiation and integration with and without the use of technology, as applicable. Students should be familiar with relevant mental and by hand approaches in simple cases.

The appropriate use of computer algebra system technology (CAS) to support and develop the teaching and learning of mathematics, and in related assessments, is to be incorporated throughout the course. This will include the use of computer algebra technology to assist in the development of mathematical ideas and concepts, the application of specific techniques and processes to produce required results and its use as a tool for systematic analysis in investigative, problem-solving and modelling work. Other technologies such as spreadsheets, dynamic geometry systems or statistical analysis systems may also be used as appropriate for various topics from within the areas of study.

AREAS OF STUDY

1. Functions and graphs

This area of study covers the behaviour of functions of a single real variable, including key features of their graphs such as axis intercepts, stationary points and points of inflection, domain (including maximal domain) and range, asymptotic behaviour and symmetry. The behaviour of these functions is to be linked to applications in practical situations.

This area of study will include:

- graphs and identification of key features of graphs of the following functions:
  - power functions, $y = x^n$;
  - exponential functions, $y = a^x$;
  - logarithmic functions, $y = \log_e(x)$ and $y = \log_{10}(x)$, the relationship $a = e^k$ where $k = \log_e(a)$;
  - circular functions, $y = \sin(x)$, $y = \cos(x)$ and $y = \tan(x)$;
  - modulus function, $y = |x|$ where $|x| = x$ when $x \geq 0$ and $|x| = -x$ when $x < 0$;
- transformation from $y = f(x)$ to $y = Af(n(x + b)) + c$, where, A, n, b and $c \in R$, and f is one of the functions specified above and the relation between the graph of the original function and the graph of the transformed function (including families of transformed functions for a single transformation parameter) such as:

  $y = \dfrac{0.4}{(x+3)^2}$, $y = -10\sin(\pi(x - c)) - 5,\ c \in R$, $y = A(x + 7)^5 - 2,\ A \in R$;
- graphs of polynomial functions;
- graphs of sum, difference, product and composite functions of f and g where f and g are functions of the types specified above such as:

  $y = \sin(x) + 2x$ $y = |\cos(2x)|$ $y = x^2e^{kx},\ k \in R$

  $y = \dfrac{a}{x^2+1},\ a \in R$ $y = (x^2 - 2)^n,\ n \in N$;
- graphical and numerical solution of equations;
- graphs of inverse functions;
- recognition of the general form of possible models for data presented in graphical or tabular form, using polynomial, power, circular, exponential and logarithmic functions;
- applications of simple combinations of the above functions (including simple hybrid functions), and interpretation of features of the graphs of these functions in modelling practical situations; for example, $y = ax + b + m\sin(nx)$ as a possible pattern for economic growth cycles or $y = ax^ne^{-kx} + b$ as a model for the amount of medication remaining in the blood stream after a dose of the medication.

2. Algebra

This area of study covers the algebra of functions, including composition of functions, simple functional equations, inverse functions and the solution of equations. This area of study includes the identification of appropriate solution processes for solving equations, and systems of simultaneous equations, presented in various forms. It covers recognition of equations and systems of equations that are solvable using inverse operations or factorisation, and the use of graphical and numerical approaches for problems involving equations where exact value solutions are not required or which are not solvable by other methods. This should support work in the other areas of study.

This area of study will include:

- review of algebra of polynomials, equating coefficients and solution of polynomial equations with real coefficients of degree n having up to and including n real solutions;
- the relationship of $f(x \pm y)$, $f(xy)$ and $f(\frac{x}{y})$ to values of $f(x)$ and $f(y)$ for different functions f;
- logarithm laws and exponent laws, recognition of equivalent forms using compound and double angle formulas for sine, cosine and tangent;
- solution of systems of simultaneous linear equations, including consideration of cases where no solution or an infinite number of possible solutions exist; for example, to find a cubic polynomial function f that satisfies the conditions $f'(3) = 0$, $f(3) = 4$ and $f(10) = -1$ (familiarity with matrix representation of systems of simultaneous linear equations with up to five equations in five unknowns will be assumed);
- composition of functions, where f composition g is defined by $f(g(x))$, given $r_g \subseteq d_f$, such as $\log_e(x^2 + 1)$, $e^{2x} - 4e^x - 5$, $|\sin(x)|$;
- functions and their inverses, including conditions for the existence of an inverse function, and use of inverse functions to solve equations involving exponential, logarithmic, circular and power functions;
- solution of equations of the form $af(n(x + b)) + c = k$ and recognition of the inverse function for f (over a suitable principal value domain where necessary) where a, b, c, n and $k \in R$;
- graphical and numerical approaches to solving equations where exact methods may not apply or be required, such as equations of the form $af(n(x + b)) + c = g(x)$, where f and g are power, exponential, logarithmic or circular functions; for example, finding approximate values for the coordinates of the points of intersection of the graphs of $y = 3\sin(2x)$ and $y = e^{-x} + 1$, with specification of values to a required accuracy;
- solution of literal equations such as $ax^3 + b = c$ or $e^{mx+n} = k$;
- general solutions of equations such as $\cos(x) + \cos(3x) = \frac{1}{2}, x \in R$ and the specification of exact solutions or numerical solutions, as appropriate, within a restricted domain;
- solution of general equations which arise from finding the points of intersection of graphs of functions, such as a straight line with a given parabola, or $y = \frac{1}{x}$ with $f(x) = ax^2 + bx + c$.

3. Calculus

This area of study covers graphical treatment of limits, continuity and differentiability (including local linearity) of functions of a single real variable and differentiation, anti-differentiation and integration of these functions. This material is to be linked to applications in practical situations.

This area of study will include:

- deducing the graph of the derivative function from the graph of a function and the relation between the graph of an anti-derivative function and the graph of the original function;
- derivatives of x^n, for $n \in Q$, e^x, $\log_e(x)$, $\sin(x)$ and $\cos(x)$ and $\tan(x)$ (formal derivation is not required);
- properties of derivatives, $(af(x) \pm bg(x))' = af'(x) \pm bg'(x)$ where $a, b \in R$;
- derivatives of $f(x) \pm g(x)$, $f(x) \times g(x)$, $\frac{f(x)}{g(x)}$ and $f(g(x))$ where f and g are polynomial functions, exponential, circular, logarithmic or power functions (or combinations of these functions) such as:

  $x^5 + \sqrt{1 - x^2}$ $\quad x\sin(2x)$ $\quad e^{\cos(x)}$ $\quad \frac{\log(x^2 + 4)}{x}$
- application of differentiation to curve sketching and identification of key features of curves, identification of intervals over which a function is constant, stationary, strictly increasing or strictly decreasing, identification of the maximum rate of increase or decrease in a given application context (consideration of the second derivative is not required) and tangents and normals to curves;

- identification of local maximum/minimum values over an interval and application to solving problems, identification of interval endpoint maximum and minimum values;
- average and instantaneous rates of change, including formulation of expressions for rates of change and related rates of change and solution and interpretation of problems involving rates of change and simple cases of related rates of change;
- the relationship $f(x + h) \approx f(x) + hf'(x)$ for a small value of h and its geometric interpretation;
- anti-derivatives of polynomial functions and of $f(ax + b)$ where f is x^n for $n \in Q$, e^x, $\sin(x)$, or $\cos(x)$ and linear combinations of these;
- definition of the definite integral as the limiting value of a sum $\int_a^b f(x)dx = \lim_{\delta x \to 0} \sum_{i=1}^{n} f(x_i^*)\delta x_i$ where the interval $[a, b]$ is partitioned into n subintervals, with the i th subinterval of length δx_i and containing x_i^*, and $\delta x = \max\{\delta x_i : i = 1, 2, \ldots n\}$ and evaluation of numerical approximations based on this definition;
- examples of the definite integral as a limiting value of a sum involving quantities such as area under a curve, distance travelled in a straight line and cumulative effects of growth such as inflation;
- anti-differentiation by recognition that $F'(x) = f(x)$ implies $\int f(x)dx = F(x) + c$;
- informal treatment of the fundamental theorem of calculus, $\int_a^b f(x)dx = F(b) - F(a)$;
- properties of anti-derivatives and definite integrals:

  $\int (af(x) \pm bg(x))dx = a\int f(x)dx \pm b\int g(x)dx$

  $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$

  $\int_a^b f(x)dx = -\int_b^a f(x)dx$

  $\int_a^a f(x)dx = 0;$
- application of integration to problems involving calculation of the area of a region under a curve and simple cases of areas between curves, such as distance travelled in a straight line; average value of a function; other situations modelled by the use of the definite integral as a limiting value of a sum over an interval; and finding a function from a known rate of change.

4. Probability

This area of study includes the study of discrete and continuous random variables, their representation using tables, probability functions or probability density functions (specified by rule and defining parameters as appropriate); and the calculation and interpretation of central measures and measures of spread. The focus is on understanding the notion of a random variable, related parameters, properties and application and interpretation in context for a given probability distribution.

This area of study will include:

- random variables, including:
  - the concept of discrete and continuous random variables;
  - calculation and interpretation of the expected value, variance and standard deviation of a random variable (for discrete and continuous random variables, including consideration of the connection between these);
  - calculation and interpretation of central measures (mode, median, mean);
  - property that, for many random variables, approximately 95 per cent of their probability distribution is within two standard deviations of the mean;
  - bernoulli trials and two state markov chains, including the length of run in a sequence, steady values for a markov chain (familiarity with the use of transition matrices to compute values of a markov chain will be assumed);

- discrete random variables:
  - specification of probability distributions for discrete random variables using graphs, tables and probability functions;
  - interpretation of mean (μ) median, mode, variance (σ^2) and standard deviation of a discrete random variable and their use;
  - the binomial distribution, Bi(n, p), as an example of a probability distribution for a discrete random variable (students are expected to be familiar with the binomial theorem and related binomial expansions);
  - the effect of variation in the value(s) of defining parameters on the graph of a given probability function for a discrete random variable;
  - probabilities for specific values of a random variable and intervals defined in terms of a random variable, including conditional probability;
- continuous random variables:
  - construction of probability density functions from non-negative continuous functions of a real variable;
  - specification of probability distributions for continuous random variables using probability density functions;
  - calculation using technology and interpretation of mean (μ) median, mode, variance (σ^2) and standard deviation of a continuous random variable and their use;
  - standard normal distribution, N(0, 1), and transformed normal distributions, N(μ, σ^2), as examples of a probability distribution for a continuous random variable (use as an approximation to the binomial distribution is **not** required);
  - the effect of variation in the value(s) of defining parameters on the graph of a given probability density function for a continuous random variable;
  - probabilities for intervals defined in terms of a random variable, including conditional probability (students should be familiar with the use of definite integrals, evaluated by hand or using technology, for a probability density function to calculate probabilities; while **not** required, teachers may also choose to relate this to the notion of a cumulative distribution function).

OUTCOMES

For each unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass all of the selected areas of study for each unit. For each of Unit 3 and Unit 4 the outcomes apply to the content from the areas of study selected for that unit.

Outcome 1

On completion of each unit the student should be able to define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.

To achieve this outcome the student will draw on knowledge and related skills outlined in all the areas of study.

Key knowledge

This knowledge includes

- the key features and properties of a function or relation and its graph and of families of functions and relations and their graphs;
- the effect of transformations (dilation from the coordinate axes; reflection in the coordinate axes and translation) and simple combinations of these transformations on the graphs of a function or relation;
- matrix representation of transformations of the plane;
- the concepts of domain, maximal domain, range and asymptotic behaviour of functions;
- the concept of an inverse function, connection between domain and range of the original function and its inverse and the conditions for existence of an inverse function, including the form of the graph of the inverse function for specified functions;
- the concept of combined functions, and the connection between domain and range of the functions involved and the domain and range of the combined functions;
- features which enable the recognition of general forms of possible models for data presented in graphical or tabular form;
- the index (exponent) laws, logarithm laws and compound angle formulas for sine, cosine and tangent;
- analytical, graphical and numerical approaches to solving equations and the nature of corresponding solutions (real, exact or approximate) and the effect of domain restrictions;
- features which link the graph of a function to the graph of the corresponding gradient function or its numerical values, the tangent and normal to a curve at a given point and how the sign and magnitude of the derivative of a function can be used to describe key features of the function and its derivative function;
- chain, product and quotient rules for differentiation;
- related rates of change;
- properties of anti-derivatives and definite integrals;
- the concept of approximation to the area under a curve using rectangles, the ideas underlying the fundamental theorem of calculus and the relationship between the definite integral and area;
- the concepts of a random variable (discrete and continuous), bernoulli trials and markov chains and probability distributions, the parameters used to define a distribution and properties of probability distributions and their graphs;
- the conditions under which a bernoulli trial or markov chain, or a probability distribution, may be selected to suitably model various situation.

Key skills

These skills include the ability to

- identify key features and properties of the graph of a function or relation and draw the graphs of specified functions and relations, clearly identifying their key features and properties;
- describe the effect of transformations (dilation from the coordinate axes; reflection in the coordinate axes and translation) and simple combinations of these transformations on the graphs of a function or relation;
- apply matrices to transformations of functions and their graphs;
- find the rule of an inverse function and give its domain and range;
- find the rule of a composite function and give its domain and range;

- sketch by hand graphs of polynomial functions up to degree 4; power functions $y = x^n$ where $n \in N$, $y = a^x$, (using key points $(-1, \frac{1}{a})$, (0, 1) and (1, a)); $\log_e(x)$; $\log_{10}(x)$; $|x|$ and simple transformations of these;
- apply a range of analytical, graphical and numerical processes, as appropriate, to obtain general and specific solutions (exact or approximate) to equations (including literal equations) over a given domain and be able to verify solutions to a particular equation or equations over a given domain;
- solve by hand equations of the form $\sin(ax) = b$, $\cos(ax) = b$ and $\tan(ax) = b$ with exact value solutions over a given interval;
- apply algebraic, logarithmic and circular fucntion properties to the simplification of expressions and the solution of equations;
- evaluate derivatives of basic and combined functions and apply differentiation to curve sketching and related optimisation problems;
- find derivatives and anti-derivatives of polynomial functions and power functions, functions of the form $f(ax + b)$ where f is x^n and $n \in Q$, sine, cosine, tangent (derivatives only) e^x or $\log_e(x)$ (derivatives only) and simple linear combinations of these, using pattern recognition, or by hand;
- apply the product, chain and quotient rules for differentiation to simple combinations of functions by hand;
- find derivatives of basic and more complicated functions and apply differentiation to curve sketching, related rates and optimisation problems;
- evaluate rectangular area approximations to the area under a curve, find and verify anti-derivatives of specified functions and evaluate definite integrals;
- apply definite integrals to the evaluation of area under a curve and between curves over a specified interval;
- analyse a probability function or probability density function and the shape of its graph in terms of the defining parameters for the probability distribution and the mean and variance of the probability distribution;
- calculate and interpret the probabilities of various events associated with a given probability distribution, by hand in cases where simple arithmetic computations can be carried out;
- apply probability distributions to modelling and solving related problems.

Outcome 2

On completion of each unit the student should be able to apply mathematical processes in non-routine contexts, and analyse and discuss these applications of mathematics.

To achieve this outcome the student will draw on knowledge and related skills outlined in one or more areas of study.

Key knowledge

This knowledge includes

- mathematical content from one or more areas of study relating to a given context for investigation;
- specific and general formulations of concepts used to derive results for analysis within a given context for investigation;
- the role of examples, counter-examples and general cases in developing mathematical analysis;
- inferences from analysis and their use to draw valid conclusions related to a given context for investigation.

Key skills

These skills include the ability to

- specify the relevance of key mathematical content from one or more areas of study to the investigation of various questions related to a given context;
- give mathematical formulations of specific and general cases used to derive results for analysis within a given context for investigation;
- develop functions as models for data presented in graphical or tabular form and apply a variety of techniques to decide which function provides an appropriate model for a given set of data;
- use a variety of techniques to verify results;
- make inferences from analysis and use these to draw valid conclusions related to a given context for investigation;
- communicate conclusions using both mathematical expression and everyday language, in particular, in relation to the context for investigation.

Outcome 3

On completion of each unit the student should be able to select and appropriately use a computer algebra system and other technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.

To achieve this outcome the student will draw on knowledge and related skills outlined all in the areas of study.

Key knowledge

This knowledge includes

- exact and approximate specification of mathematical information such as numerical data, graphical forms and general or specific forms of solutions of equations produced by use of a computer algebra system;
- domain and range requirements for a computer algebra system's specification of graphs of functions and relations;
- the role of parameters in specifying general forms of functions and equations;
- the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions or equations;
- the similarities and differences between formal mathematical expressions and their computer algebra system representation;
- the appropriate selection of a technology application, in particular, computer algebra systems, in a variety of mathematical contexts.

Key skills

These skills include the ability to

- distinguish between exact and approximate presentations of mathematical results produced by a computer algebra system, and interpret these results to a specified degree of accuracy;
- produce results using a computer algebra system which identify examples or counter-examples for propositions;
- produce tables of values, families of graphs or collections of other results using a computer algebra system, which support general analysis in problem-solving, investigative or modelling contexts;
- use appropriate domain and range specifications to illustrate key features of graphs of functions and relations;

- identify the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions or equations;
- specify the similarities and differences between formal mathematical expressions and their computer algebra system representation, in particular, equivalent forms of symbolic expressions;
- make appropriate selections for a computer algebra system and other technology applications in a variety of mathematical contexts, and provide a rationale for these selections;
- relate the results from a particular application to the nature of a particular mathematical task (investigative, problem solving or modelling) and verify these results;
- specify the process used to develop a solution to a problem using a computer algebra system, and communicate the key stages of mathematical reasoning (formulation, solution, interpretation) used in this process.

ASSESSMENT

The award of satisfactory completion for a unit is based on a decision that the student has demonstrated achievement of the set of outcomes specified for the unit. This decision will be based on the teacher's assessment of the student's overall performance on assessment tasks designated for the unit. The Victorian Curriculum and Assessment Authority will publish annually an assessment handbook which will include advice on the scope of the assessment tasks and criteria for assessment.

The key knowledge and skills listed for each outcome should be used as a guide to course design and the development of learning activities. The key knowledge and skills do not constitute a checklist and such an approach is not necessary or desirable for determining the achievement of outcomes. The elements of key knowledge and skills should not be assessed separately.

Assessment of levels of achievement

The student's level of achievement for Units 3 and 4 will be determined by school-assessed coursework and two end-of-year examinations.

Contribution to final assessment

School-assessed coursework for Unit 3 will contribute 20 per cent and for Unit 4 will contribute 14 per cent to the study score.

Units 3 and 4 will also be assessed by two end-of-year examinations, which together will contribute 66 per cent to the study score.

School-assessed coursework

Teachers will provide to the Victorian Curriculum and Assessment Authority a score representing an assessment of the student's level of performance in achieving the set of outcomes.

The score must be based on the teacher's rating of performance of each student on the tasks set out in the following table and in accordance with an assessment guide published annually by the Victorian Curriculum and Assessment Authority. The assessment handbook also includes advice on the scope of the task and the criteria for assessment.

Assessment tasks must be a part of the regular teaching and learning program and must not unduly add to the workload associated with that program. They must be completed mainly in class and within a limited timeframe. Where optional assessment tasks are listed teachers must ensure that the tasks they select are comparable in scope and demand.

Outcomes	Marks allocated*	Assessment tasks
Unit 3		A function and calculus *application task* based on content from the areas of study, with three components of increasing complexity: • introduction of a context through specific cases or examples • consideration of general features of this context • variation, or further specification, of assumptions or conditions involved in the context to focus on a particular feature related to the context. **and** Two equally weighted *tests* that consist of an appropriate combination of multiple-choice, short-answer and extended response items. The outcomes are to be assessed across the *application task* and the two *tests*.
Outcome 1 Define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.	30	15 *The application task* 15 *Tests*
Outcome 2 Apply mathematical processes in non-routine contexts, and analyse and discuss these applications of mathematics.	20	20 *The application task*
Outcome 3 Select and appropriately use a computer algebra system and other technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.	10	5 *The application task* 5 *Tests*
Total marks	**60**	

*School-assessed coursework for Unit 3 contributes 20 per cent to the study score.

Outcomes	Marks allocated*	Assessment tasks	
Unit 4		Two *analysis tasks*, with the three outcomes assessed across the tasks. Each *analysis task* is a short item of 2–4 hours duration over 1–2 days selected from: • an assignment where students have the opportunity to work on a broader range of problems in a given context; **or** • a short and focused investigation, challenging problem or modelling task; **or** • a set of application questions requiring extended response analysis in relation to a particular topic or topics; **or** • item response analysis for a collection of multiple-choice questions, including analysis of item distractors and their relationship to conceptual, process or reasoning error. The two *analysis* tasks are to be of a different type. One analysis task is to be related to the 'Probability' area of study.	
Outcome 1 Define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.	15	7	*Analysis task 1*
		8	*Analysis task 2*
Outcome 2 Apply mathematical processes in non-routine contexts, and analyse and discuss these applications of mathematics.	15	8	*Analysis task 1*
		7	*Analysis task 2*
Outcome 3 Select and appropriately use a computer algebra system and other technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.	10	5	*Analysis task 1*
		5	*Analysis task 2*
Total marks	**40**		

*School-assessed coursework for Unit 4 contributes 14 per cent to the study score

End-of-year examinations

The student's level of achievement for Units 3 and 4 will also be assessed by two examinations based on tasks related to Outcomes 1 to 3.

Examination 1

Description

Students are required to respond to a collection of short-answer and some extended-answer questions covering all areas of study in relation to Outcome 1. The task is designed to assess students' knowledge of mathematical concepts, their skills in carrying out mathematical algorithms and their ability to apply concepts and skills in standard ways without the use of technology. This examination is common to Mathematical Methods (CAS) Units 3 and 4 and Mathematical Methods Units 3 and 4. All questions are compulsory.

Students will complete the examination using a structured answer booklet.

The examination will be set by a panel appointed by the Victorian Curriculum and Assessment Authority.

Conditions

The task will be completed under the following conditions:

- Duration: one hour.
- Date: end-of-year, on a date to be published annually by the Victorian Curriculum and Assessment Authority.
- Victorian Curriculum and Assessment Authority examination rules will apply. Details of these rules are published annually in the *VCE and VCAL Administrative Handbook*.
- No calculators, CAS or notes of any kind are permitted. A sheet of formulas will be provided with the examination.
- The task will be marked by a panel appointed by the Victorian Curriculum and Assessment Authority.

Contribution to final assessment

The examination contributes 22 per cent to the study score.

Examination 2

Description

Students are required to respond to a collection of multiple-choice questions and extended-answer questions covering all areas of study in relation to all three outcomes, with an emphasis on Outcome 2. Student access to an approved CAS will be assumed by the setting panel.

The task is designed to assess students' ability to understand and communicate mathematical ideas, and to interpret, analyse and solve both routine and non-routine problems. Students should attempt all of the multiple-choice questions in Part I of the examination and all of the extended-answer questions, involving multi-stage solutions of increasing complexity in Part II of the examination.

All questions are compulsory.

Students will complete the examination using a structured answer booklet.

The examination will be set by a panel appointed by the Victorian Curriculum and Assessment Authority.

Conditions

The task will be completed under the following conditions:

- Duration: two hours.
- Date: end-of-year, on a date to be published annually by the Victorian Curriculum and Assessment Authority.
- Victorian Curriculum and Assessment Authority examination rules will apply. Details of these rules are published annually in the *VCE and VCAL Administrative Handbook*.
- An approved CAS and **one** bound reference, text (which may be annotated) or lecture pad, may be brought into the examination. A sheet of formulas will be provided with the examination.
- The task will be marked by a panel appointed by the Victorian Curriculum and Assessment Authority.

Contribution to final assessment

The examination contributes 44 per cent to the study score.

Advice for teachers (Units 3 and 4: Mathematical Methods (CAS))

DEVELOPING A COURSE

A course outlines the nature and sequence of teaching and learning necessary for students to demonstrate achievement of the set of outcomes for a unit. The areas of study describe the knowledge required for the demonstration of each outcome. Outcomes are introduced by summary statements and are followed by the key knowledge and skills which relate to the outcomes.

Teachers must develop courses that include appropriate learning activities to enable students to develop the knowledge and skills identified in the outcome statements in each unit.

In Units 3 and 4, assessment is structured. For school-assessed coursework, assessment tasks are prescribed. The contribution that each task makes to the total school-assessed coursework is also stipulated.

Approaches to course development

This study design provides a structure for the development of detailed courses by schools. While Mathematical Methods (CAS) Units 3 and 4 is a fully prescribed course there is considerable flexibility at the local level in the design of a course. Planning decisions need to be made about the order of topics, the time devoted to each topic, and connections between the material developed in particular topics and the outcomes for each unit. These decisions also need to take into account the timing of school-assessed coursework.

Some students will undertake Specialist Mathematics Units 3 and 4 concurrently with Mathematical Methods (CAS) Units 3 and 4, which contains assumed knowledge for Specialist Mathematics Units 3 and 4. Care needs to be taken in giving consideration to how the timing and development of material in Mathematical Methods (CAS) Units 3 and 4 is related to supporting and facilitating the development of material in Specialist Mathematics Units 3 and 4. The sample course sequence outlined below illustrates one possible implementation across 27 weeks; schools and teachers are encouraged to develop their own sequences or variations to this sample sequence.

Sample course sequence

Topic	Approximate time
Polynomials and power functions including differential calculus	**5 weeks**
Polynomial functions and their graphs, average and instantaneous rates of change, use of f' to define increasing and decreasing, use of linear approximation relationship.	2 weeks
Power functions and generalisation to rational powers on positive real domain. Review of transformations in the context of power functions. Introduction of modulus function, and transformations of this function, as an example of a function that is continuous, but not differentiable at a given point (graphical treatment).	1 week
Sums, differences and products of polynomial and power functions and their graphs (including some simple cases involving the modulus function), consideration of this type of function, differentiation of these functions, related modelling and max/min problems and other applications.	2 weeks
Exponential and logarithmic functions including differential calculus	**3 weeks**
Review of exponential and logarithmic functions and transformations of these functions and their graphs, the inverse relation and equation solving.	1 week
Sums, differences and products of exponential, logarithmic, polynomial and power functions, differentiation involving these functions, and their graphs.	1 week
Consideration of these combinations of functions in related modelling and max/min problems and other applications.	1 week
Test 1: functions and graphs	
Circular functions including differential calculus	**3 weeks**
Review of circular functions and transformations of these functions, and their graphs, angle formulas, inverse functions and equation solving.	2 weeks
Sums, differences and products of circular exponential, logarithmic, polynomial and power functions, their derivatives and their graphs.	1 week
Application task (this could be held earlier or later depending on coverage of relevant concepts, skills and processes with respect to content and context for the application task)	2 weeks
Calculus applications	**4 weeks**
Consideration of these combinations of functions and derivatives in related modelling and max/min problems and other applications.	1 week
Composition of functions, including functions of the form $\lvert f(x) \rvert$ and graphs of these functions.	1 week
Chain rule for composite functions and related rates of change.	2 weeks
Test 2: differentiation	
Anti-differentiation and integral calculus	**3 weeks**
Anti-differentiation, integration and applications.	3 weeks
Analysis task 1: anti-differentiation and integration	**1 week**

Probability	**5 weeks**
Random variables, bernoulli trials and discrete distributions, including the binomial distribution.	2 weeks
Transition matrices and two-state markov chains	1 week
Continuous probability distributions, including the normal distribution	2 weeks
Analysis task 2: probability	1 week

USE OF INFORMATION AND COMMUNICATIONS TECHNOLOGY

In designing courses and developing learning activities for Mathematical Methods, teachers should make use of applications of information and communications technology and new learning technologies, such as computer-based learning, multimedia and the World Wide Web, where appropriate and applicable to teaching and learning activities.

KEY COMPETENCIES AND EMPLOYABILITY SKILLS

Students undertaking the following types of assessment, in addition to demonstrating their understanding and mastery of the content of the study, typically demonstrate the following key compentencies and employability skills.

Assessment task	Key competencies and employability skills
Analysis task	Planning and organising, solving problems, using mathematical ideas and techniques, (written) communication, use of information and communications technology
Tests	Self management, use of information and communications technology, solving problems
Application task	Planning and organising, solving problems, using mathematical ideas and techniques, (written) communication, use of information and communications technology

LEARNING ACTIVITIES

Examples of learning activities for each unit are provided in the following sections. Extended examples are highlighted by a shaded box. The examples that make use of information and communications technology are identified by this icon ICT.

Outcome 1

Define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.

Examples of learning activities

skills practice on standard mathematical routines through appropriate exercises; this should include development of the ability to identify problems as either being in a standard form, or readily transformable to a standard form, the efficient and accurate application of the relevant mathematical routine (e.g. the application of differentiation rules to combinations of functions), and systematic checking of both the reasonableness and accuracy of results (does the graph of the rule which has been developed for the derived function correspond to a sketch of the derivative function obtained from inspection of the graph of the original function?); the appropriate use of technology, such as checking the numerical derivative value from a CAS with the form of the derivative graph sketch would link this work to Outcome 3

error identification and analysis exercises, where students work through a range of 'worked solutions' and identify and rectify missing steps in working or errors in working

construction of summary or review notes related to a topic or area of study (e.g. the conditions under which different probability distributions of discrete and continuous random variables can be appropriately applied to solve problems)

presentation of a range of typical problems associated with an area of study and worked solutions (e.g. applications of integration to area and areas between curves)

assignments structured around the development of sample cases of standard applications of mathematical skills and procedures in readily recognisable situations (e.g. the development of different sketch graphs in terms of defining parameters for a type of relation – this can be checked by the use of CAS or spreadsheets and hence linked to Outcome 3)

Outcome 2

Apply mathematical processes in non-routine contexts, and analyse and discuss these applications of mathematics.

Examples of learning activities

investigation of reflection symmetry of the graphs of quadratic functions about the vertical line through their turning point(vertex) and the half-turn rotational symmetry of the graph of a cubic function about its point of inflection

problem-solving tasks, e.g. optimising volume/cost constraints in container design

modelling tasks, e.g. modelling periodic behaviour with circular functions or modelling tax scales with modulus functions

ICT a set of applications questions requiring analysis and extended response related to a particular context, e.g. functional equations

a report on item response analysis for a collection of multiple-choice questions (e.g. distinguishing between ‘competing functions’ as models for various data represented graphically and/or in table form)

presentation on research into a particular application of mathematics (e.g. the use of transition matrices to model conditional probabilities in two-state markov chains)

Outcome 3

Select and appropriately use a computer algebra system and other technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.

Examples of learning activities

ICT exploration of the ideas surrounding the concept of approximation to the area under a curve using rectangles, the ideas underlying the fundamental theorem of calculus and the relationship between the definite integral and area

ICT evaluation of definite integrals from first principles and their numerical approximation

ICT investigation of the effect of variation of parameters on the graphs of probability distributions for discrete and continuous random variables

ICT solution of equation involving parameters either in the formulation of the equation, and/or the specification of solutions

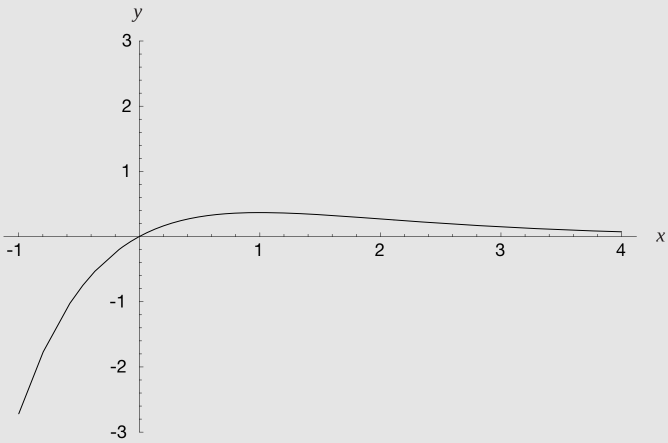
Detailed example

A FAMILY OF FUNCTIONS

This activity relates to all three outcomes and involves functions, graphs and calculus. Part 1 of the activity should be completed without the assistance of technology, and relates to Outcome 1, while technology would be used to assist in the analysis for Part 2, which relates to Outcomes 2 and 3.

Part 1

Let $f: R \to R$ and $f(x) = xe^{-x}$. Part of the graph of f is shown below:



a. Sketch the graphs of $y = x$ and $y = e^{-x}$ on the same axes and hence explain how the general shape of the graph of f can be deduced from the graphs of $y = x$ and $y = e^{-x}$.

b. Find the derivative of f and hence state the exact coordinates of the local maximum.

c. Show that $y = x$ is tangent to the graph of f at the origin.

Part 2

a. Let $f_n: R \to R$ and $f_n(\boldsymbol{x}) = x^n e^{-x}$ for $n \in N$. Draw the graph of f_n for several different values of n.

b. Find a general formula for the location of the stationary points of f_n and summarise the nature and location of these in terms of n.

c. Let g be a differentiable function with domain R. Determine when $h(x) = g(x)e^{-x}$ is increasing, stationary or decreasing in terms of g and its derivative.

Detailed example

FUNCTIONAL EQUATIONS

In this activity students consider properties of the graphs of functions, such as symmetry and periodicity, as well as algebraic equivalence to identify continuous functions which satisfy the following functional equations over their maximal domain. This relates to Outcomes 1 and 2 with respect to the identification of a particular function, and Outcomes 2 and 3 where a family or class of related functions is considered.

Functional equation	**Solution function**
$f(x) = f(-x)$	
$f(-x) = -f(x)$	
$f(kx) = kf(x)$	
$f(x + k) = f(x)$	
$f(f(x)) = x$	
$f(x + y) = f(x) + f(y)$	
$f\left(\frac{x+y}{2}\right) = \frac{f(x)+f(y)}{2}$	
$f(x \times y) = f(x) \times f(y)$	
$f(x \times y) = f(x) + f(y)$	
$f(x + y) = f(x) \times f(y)$	
$f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}$	
$f\left(\frac{x}{y}\right) = f(x) - f(y)$	

Detailed example

PRODUCT FUNCTIONS AND THE CUCKOO CLOCK

The following activity is an investigation of the behaviour of product functions formed from a combination of transformed circular exponential functions, and their application to modelling the swing of a pendulum in a cuckoo clock. The activity relates to all three outcomes, and could be used as the basis for developing an application task.

Part A – graphs of exponential and circular functions

a. Sketch the graphs of $y = f(t) = 5e^{-kt}$ for several values of $0 < k < 1$ on the same set of axes.

b. Draw the corresponding graphs of $f(-t)$ and $-f(t)$ on the same set of axes, and comment on the similarities and differences between these graphs.

c. Sketch the graphs of $y = g(t) = \sin(at)$, $t \in [0, 2\pi]$ for $a = 1, 2, 3$ together on the same set of axes. State the period for each function, and comment briefly on the similarities and differences between these graphs.

Part B – product functions

a. For $k = 0.2$; $a = 1$, sketch the graphs of $y = f_1(t) = 5e^{-kt}$, $y = f_2(t) = -5e^{-kt}$ and $y = s(t) = 5e^{-kt}\sin(at)$, where $t \in [0, 4\pi]$.

b. Find the derivative of $s(t)$, in terms of t, k and a, and hence for $k = 0.2$; $a = 1$, find the coordinates of the first two maximum/minimum points for $s(t)$ with x values closest to the y-axis.

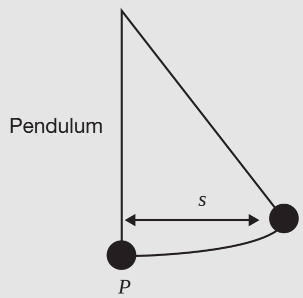
c. Find the coordinates of any points of contact between the graph $y = s(t)$ and $y = f_1(t)$ and between $y = s(t)$ and $y = f_2(t)$. Briefly comment on the relationship between these points of contact and the graph of $y = \sin(t)$. Hence, state the exact x value for the point of intersection closest to the y-axis.

d. State the coordinates of intersection between $y = s(t)$ and $y = \sin(t)$, over the given domain. Comment on these findings. Hence, give the exact coordinates of these intersection points.

Part C – the cuckoo clock

Briana has recently purchased a cuckoo clock. The rate at which the hands of the clock move is controlled by a pendulum which is kept in regular motion by slowly-descending weighted chains.

When the weights reach their lower point and stop moving, the pendulum swing begins to change, causing the hands of the clock to slow down and gradually stop. From the time when the swing begins to change, the horizontal displacement, s cm, of the point, P, at the end of the pendulum, from the vertical, as shown in the following diagram, can be modelled by functions with the rule $s(t) = 5e^{-kt} \times \sin(at)$, $t \geq 0$ where t is the time in seconds after the pendulum swing begins to change and k and a are real constants. For Briana's clock, $k = 0.2$ and $a = 1$.



a. Find the horizontal displacement of the pendulum for several seconds after the weights stop descending, and draw a series of diagrams corresponding to the position of the pendulum at these times.

b. Draw a series of diagrams of the position of the pendulum the first several maxima.

c. If the pendulum is deemed to have come to rest when the swing is less that 0.01 cm, find how long the pendulum takes to come to rest.

d. Briana has a friend, John, who also bought a similar cuckoo clock. John's clock is modelled by the same rule for the horizontal displacement, when the weights stop descending, where $k = 0.4$ and $a = 1$. Draw the graph of the two pendulums' horizontal displacement for $t \geq 22$. Compare the behaviour of the two pendulums and discuss how the different values for k affect the motion of the point P after the swing of the pendulum begins to change.

Detailed example

MATRIX TRANSFORMATIONS AND TRANSITION MATRICES

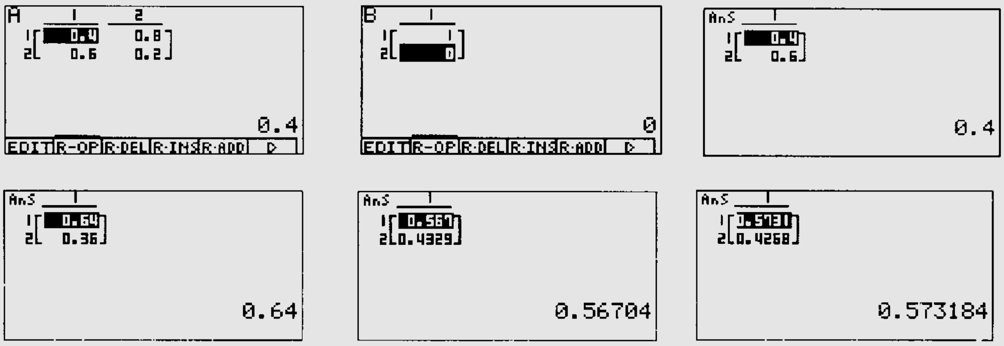
In the first part of the activity, students work with the application of a transformation matrix to sets of points in the plane that form simple geometric shapes, such as triangles, rectangles, parallelograms and trapezia. For example, the matrix transformation $A = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$ or similar could be applied to the trapezium formed by the line segments connecting the corner points (0, 0), (5, 0), (3, 4) and (1,4) and other shapes to note that this sort of transformation maps pairs of parallel lines onto image pairs parallel lines. The particular transformations corresponding to dilations from the origin, reflections in the lines $x = 0$, $y = 0$ and $y = x$, and translations are introduced and their effects observed on simple geometric shapes and sets of points from well-known functions. This work would relate to Outcomes 1 and 3.

Students systematically investigate the effect of combined transformations of the form:

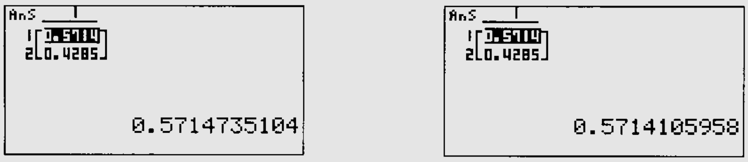
$$\begin{pmatrix} x' \\ y' \end{pmatrix} = A\begin{pmatrix} x \\ y \end{pmatrix} + B \quad \text{or} \quad \begin{pmatrix} x' \\ y' \end{pmatrix} = A\left(\begin{pmatrix} x \\ y \end{pmatrix} + B\right)$$

where A is a dilation and/or reflection and B is a translation on power functions $y = x^n$ for $n = -2, -1, \frac{1}{2}$, 1, 2 and 3. This is extended to the development for the general form of the rule of such functions under these combinations of transformations, and for other functions such as $y = \sin(x)$ or $y = e^x$ and would relate to Outcomes 2 and 3.

In the second part of the activity, students consider the application of a transition matrix, or example $T = \begin{pmatrix} 0.4 & 0.8 \\ 0.6 & 0.2 \end{pmatrix}$ to an initial state such as $S_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and evaluate the sequence (chain) for several transitions (the results of the first four transitions are shown below):



In particular, students identify informally likely convergence to a steady state by recognition for sufficiently large values of n that $S_{n+1} = T \times S_n \approx S_n$, for example, the following results for the 10th and 11th transitions show little change:

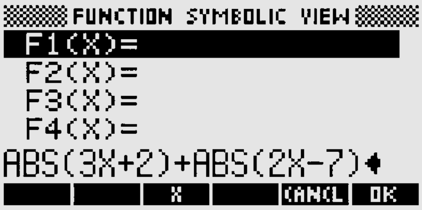


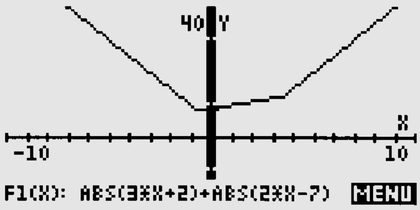
They determine the exact steady state values by solving the linear equation obtained from the transition matrix equation $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ simultaneously with $x + y = 1$.

Detailed example

MODULUS FUNCTIONS AND COMPOSITION OF FUNCTIONS

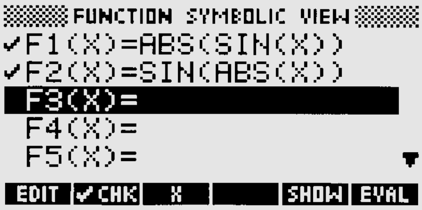
In the first part of this activity students investigate the graphical behaviour of functions formed by sums of functions with rule $y = |ax + b|$. For example, the graph of the function with rule $f(x) = |3x + 2| + |2x - 7|$ is shown below:

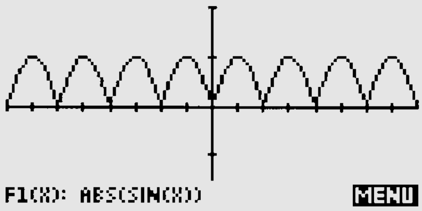


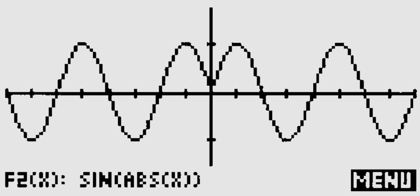


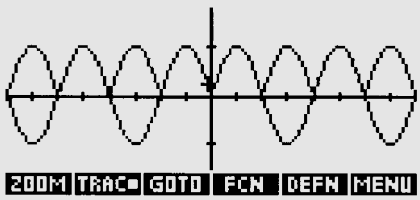
Students could be asked to determine the general location of axis intercepts and points of non-differentiability of functions of the form: $f: R \to R$ and $f(x) = |ax + b| + |cx + d|$ and subsequently use sums of modulus of linear functions to model various tax scales.

In the second part of the activity, students work with the composite functions $|f(x)|$ and $f(|x|)$ for various functions f. They graph these functions, and identify points in their maximal domains where they are not differentiable. For example, in the case where $f(x) = \sin(x)$ the graphs of the composite functions $y = |\sin(x)|$ and $y = \sin(|x|)$ are shown below:









The location of the points of non-differentiability of these functions can be specified in parametric form, for example, for $y = |\sin(x)|$ these occur at $x = n\pi$ where $n \in Z$. Similar analysis can also be carried out for other $f(x)$ such as e^x, $\frac{1}{x}$, $x^2 - 4$, and $\log_e(x)$.

APPROACHES TO ASSESSMENT

In Units 3 and 4 assessment tasks must be selected from those designated in the outcomes table.

The following are examples of assessment tasks that could be used to demonstrate achievement of the outcomes:

- identification and analysis of key features of functions in symbolic, graphical, tabular and numerical form (including associated problems involving analytic and/or numeric differentiation and integration and applications)
- production of collections of results in response to variation of parameters used to define a type of function (for example, the role of the coefficient b in determining features of the family of graphs of cubic polynomial functions with rules of the form $f(x) = ax^3 + bx^2 + cx + d$)
- developing functions which can be used to model trends in sets of data, comparing empirical and theoretical models for data
- solution of equations (or systems of equations) using graphical, numerical or algebraic approaches (for example, comparison of the behaviour of functions with rules of the form $g(x) = x^n$ and $h(x) = n^x$)
- use of symbolic manipulation to explore and check patterns and conjectures associated with generalisable properties of functions.

In each unit Outcome 3 should be incorporated in the assessment of Outcomes 1 and 2.

Detailed example

SAMPLE TEST QUESTIONS

The following provides some examples of multiple-choice and short-answer questions that might be incorporated in a component of a test where students do not use technology. Relevant multiple-choice questions could be conceptual questions, questions concerning parameters and aspects of functions and graphs, algebra and calculus, or forms of mathematical relations and their interpretation, for example:

1. If $f(x) = e^{2x}$ for all $x \in R$, then $[f(x)]^2 = f(y)$, where y is equal to

   A. $2x$ B. $4x$ C. x^2 D. $2x^2$ E. $4x^2$

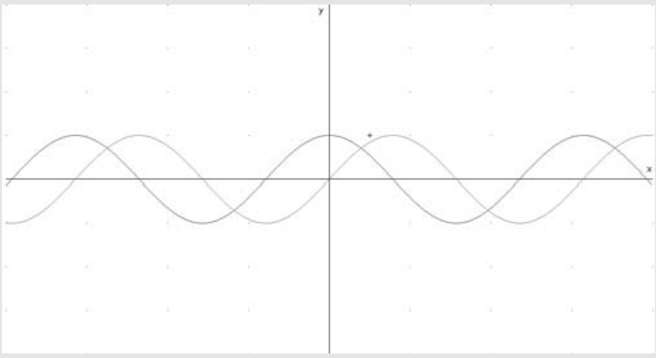
2. If $\int_a^b f(x)dx = 3$, then $\int_a^b (4 - 2f(x))\, dx$ is equal to:

   A. $4(b - a) - 6$ B. $4(b - a) - 6$ C. $4(b - a) + 6$ D. -2 E. 10

Suitable short-answer questions might be:

3. Let $f: R \rightarrow R$, $f(x) = x^2$ and $g: R \rightarrow R$, $g(x) = x^4$.
   - Sketch the graphs of f and g on the same set of axes, labelling axis intercepts and any points of intersection with their coordinates.
   - State all real values for which $|f'(x)| > |g'(x)|$
   - Sketch the graph of $g(x) - f(x)$ and label the stationary points with their exact coordinates.

4. Let $h: R \rightarrow R$, $h(x) = \sin(x) + \cos(x)$. Find the equation of the tangent to the graph of h at $x = \frac{\pi}{2}$. The graphs of $\sin(x)$ and $\cos(x)$ are shown below, sketch the graph of h on the same set of axes, and draw in the tangent.



Detailed example

ANALYSIS TASK – CONTINUOUS RANDOM VARIABLES

The following illustrates a range of questions that might be used to develop an analysis task.

Students could be asked to differentiate a function such as $f(x) = x \times e^{-kx}$ and hence find the anti-derivative $\int x \times e^{-kx} dx$. They could then be asked to evaluate $\int_0^a x \times e^{-kx} dx$ in terms of the parameters a and k and hence, show that $\int_0^a x \times e^{-kx} dx \to \frac{1}{k^2}$ as $a \to \infty$.

Now consider the probability density function for the continuous random variable, X:

$$f(x) = \begin{cases} k \times e^{-kx} & x \geq 0, \\ 0 & \text{elsewhere} \end{cases} \quad \text{where } k \in R^+$$

Students could then be asked various related question, for example, find a value for k, correct to two decimal places, such that $\Pr(0 \leq X \leq 5) = 0.9$, evaluate various probabilities such as $\Pr(0.1 < X \leq 2)$; and $\Pr(0.1 < X \leq 2 | X \leq 3)$; and determine the mean, variance, mode and median of the random variable.

They could then be asked various questions related to this probability density function in practical contexts, such as the time in minutes for discharge of a capacitor. If this is a random variable, T and its probability density function $f(t)$ is given by

$$f(t) = \begin{cases} Ae^{-kt}, & 0 \leq t < \infty, \\ 0 & \text{elsewhere} \end{cases} \quad \text{and } k > 0.$$

Students could be asked to:

- obtain A in terms of k
- estimate the value of k if it is observed by experimentation that, out of 1000 such capacitors, 425 lose 50% of their charge within two minutes of being turned off
- calculate the mean and variance of T
- find the probability that if If two such capacitors are turned off together, one of them loses 50% of its charge within the first minute and the other takes more than three minutes to lose 50% of its charge.

Detailed example

ANALYSIS TASK – POLYNOMIAL FUNCTIONS, COEFFICIENTS AND EQUATIONS

The following outline for an analysis task is presented in two parts, the first part is a practical application of simple polynomial functions to coding, the second part is a theoretical investigation into constructing polynomial functions f with rational roots to $f(x) = 0$ and $f'(x) = 0$.

Part 1

This part of the task is based on a much simplified version of the Reed-Solomon error correction coding scheme. Reed-Solomon error correction is used to correct *bursts* of errors associated with electronic media defects, and has applications in disc drives, CDs (it underpins the fact that error bursts resulting from scratches on the surface of up to 2.5 cm in length can be corrected), telecommunications (in particular, satellite messages) and digital broadcasts.

A simple coded message consists of a sequence of natural numbers, which can be represented uniquely as the coefficients of a polynomial function of suitable degree. For example, the message $<a, b>$, where a and b are real numbers, would be coded by the linear function $f(x) = a + bx$. To send the message a sample of coded values $\{f(0), f(1), f(2), f(3) \ldots\}$ is sent – it can be assumed that the data is sent in this sequence – and while most of the data received will be correct, it may also contain errors. For example, the received sample of coded values might be {5, 7, 11, 11}, which contains a single error.

For a linear function, only two correct values are needed to find the rule of the linear polynomial and thus decode the message – but this assumes there is no error. To deal with this in practice, an over-sampled set of data is sent, this contains correct data and (possibly) some errors. To identify the correct message and eliminate errors, linear function rules are constructed for each pair of values in the sample, and all of the resultant rules compared. The corresponding analysis can be carried out graphically and analytically, the majority of rules which are the same correspond to the correct rule.

Following some introductory work on fitting polynomial functions to data using systems of simultaneous linear equations, students could, for example, be asked to decode and correct:

1. a message $<a, b>$ where there is one error in the sample {5, 7, 11, 11};
2. a message $<a, b>$ where there are two errors in the sample {7, 8, 11, 13, 17, 20};
3. a message $<a, b, c>$ where there is one error in the sample {3, 6,13 , 26, 39};
4. investigate the relationship between the size of the sample and the number of errors for a given length of the message; and
5. extend the investigation to the case of a cubic polynomial function.

Part 2

If f is a cubic polynomial function, then its derivative f' is a quadratic polynomial function. The horizontal axis intercepts of f are determined by the roots of the equation $f(x) = 0$, and the stationary points of f are determined by the roots of the equation $f'(x) = 0$. These roots may be either rational or irrational.

Find a general process for determining the coefficients of f so that its graph does not pass through the origin and the x values of the coordinates of both the horizontal axis intercepts and the stationary points are rational.

Detailed example

APPLICATION TASK: BEZIER CURVES

A context such as the following could be used to investigate a contemporary application of a special type of cubic polynomial function, whose graphs are called *bezier* curves in design (after the French automobile engineer *Pierre Bezier* who developed their application to a new computer aided design tool for the Rénault car manufacturing corporation in the 1960s).

The task is developed in three components, and addresses all three outcomes, with a focus on Outcomes 2 and 3.

Curves drawn by drawing packages and the outlines of letters produced by printers are typically based on a set of routines that utilise curves known as bezier curves. Bezier curves use polynomial functions of low degree, such as cubic polynomials over a restricted domain, to specify the coordinates of the points which make up these curves. These functions provide local control of shape, based on a small set of points called *control points*, and have graphs that are smooth and continuous curves, for which the derivative can be found at any point on the curve.

A cubic bezier curve drawn over the interval $0 \le t \le 1$, is produced by a relation which has its x and y coordinates, respectively, specified by the cubic polynomial functions:

$x = a(1-t)^3 + 3ct(1-t)^2 + 3et^2(1-t) + gt^3$ and
$y = b(1-t)^3 + 3dt(1-t)^2 + 3ft^2(1-t) + ht^3$

where the coefficients a, b, c, d, e, f, g and h are obtained from the coordinates of the four control points (a, b), (c, d), (e, f) and (g, h). The gradient of the curve at a particular value of t, can be determined using the chain rule for differentiation, by the relationship:

$$\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{y'(t)}{x'(t)} \text{ provided } \frac{dx}{dt} \neq 0$$

The shape of the bezier curve produced depends on the selection of coordinate values for the control points. Where more than one bezier curve is used to produce a required shape, these curves will need to be joined smoothly to produce a good image.

Component 1

Selection of four distinct points as control points and determination of the equations for the x and y coordinates as functions of t to specify bezier curve. Consideration of the corresponding bezier curve using a table and its graph. Consideration of the slope of the tangent to the curve at various points in the curve, including the relation between the tangents to the first and last control points and the location of the second and third control points.

Component 2

Variation of control points and consideration of the bezier curves produced, including cases which lead to, for example, straight lines and loops. Selection of control points to produce a reasonable representation of a particular shape, such as the letter **C** or the letter **V**. Consideration of how well the cubic bezier curve matches the required shape and any limitations on possible shapes that can be represented using these curves.

Component 3

Representation of more complicated shapes formed by piecing together several different bezier curves. Identification of the location of control points for a pair of cubic bezier curves that are smoothly joined to represent a letter such as **S**, or for several bezier curves that are smoothly joined to represent a letter or symbol such as **&** or a shape such as the outline or cross section of a car.

Important aspects of mathematics to be considered in assessment of student work are: identification of important features of a shape with respect to selection of a suitable cubic bezier curve, including endpoints:

- sketch graphs of relations that produce bezier curves and identification of key features;
- qualitative and quantitative assessment of how well a bezier curve represents a particular shape;
- smooth joining of bezier curves.

Units 3 and 4: Specialist Mathematics

Units 3 and 4: Specialist Mathematics

Specialist Mathematics consists of the following areas of study: ‘Functions, relations and graphs’ ‘Algebra’, ‘Calculus’, ‘Vectors’ and ‘Mechanics’. The development of course content should highlight mathematical structure and proof. All of this material must be covered in progression from Unit 3 to Unit 4, with an appropriate selection of content for each of Unit 3 and Unit 4. The selection of materials for Unit 3 and Unit 4 should be constructed so that there is a balanced and progressive development of knowledge and skills with connections among the areas of study being developed as appropriate across Unit 3 and Unit 4. Specialist Mathematics Units 3 and 4 assumes concurrent or previous study of Mathematical Methods Units 3 and 4 or Mathematical Methods (CAS) Units 3 and 4. They contain assumed knowledge and skills for Specialist Mathematics, which will be drawn on as applicable in the development of content from the areas of study and key knowledge and skills for the outcomes.

In Unit 3 a study of Specialist Mathematics would typically include content from ‘Functions, relations and graphs’ and a selection of material from the ‘Algebra’, ‘Calculus’ and ‘Vectors’ areas of study. In Unit 4 this selection would typically consist of the remaining content from the ‘Algebra’, ‘Calculus’, and ‘Vectors’ areas of study and the content from the ‘Mechanics’ area of study.

Students are expected to be able to apply techniques, routines and processes, involving rational, real and complex arithmetic, algebraic manipulation, diagrams and geometric constructions, solving equations, graph sketching, differentiation and integration related to the areas of study, as applicable, both with and without the use of technology. The appropriate use of technology to support and develop the teaching and learning of mathematics is to be incorporated throughout the units. This will include the use of some of the following technologies for various areas of study or topics: graphics calculators, spreadsheets, graphing packages, dynamic geometry systems and computer algebra systems. In particular, students are encouraged to use graphics calculators and other technologies both in the learning of new material and the application of this material in a variety of different contexts.

Familiarity with sequence and series notation and related simple applications, the use of sine and cosine rules in non-right-angled triangles and the following mathematics is assumed:

- the solution of triangles in two-dimensional situations;
- the sum of the interior angles of a triangle is 180°;
- the sum of the exterior angles of a convex polygon is 360°;
- corresponding angles of lines cut by a transversal are equal if, and only if, the lines are parallel;
- alternate angles of lines cut by a transversal are equal if, and only if, the lines are parallel;
- opposite angles of a parallelogram are equal;
- opposite sides of a parallelogram are equal in length;

- the base angles of an isosceles triangle are equal;
- the line joining the vertex to the midpoint of the base of an isosceles triangle is perpendicular to the base;
- the perpendicular bisector of the base of an isosceles triangle passes through the opposite vertex;
- the angle subtended by an arc at the centre of a circle is twice the angle subtended by the same arc at the circumference;
- the angle in a semicircle is a right angle;
- angles in the same segment of a circle are equal;
- the sum of the opposite angles of a cyclic quadrilateral is 180°;
- an exterior angle of a cyclic quadrilateral and the interior opposite angle are equal;
- the two tangents to a circle from an exterior point are equal in length;
- a tangent to a circle is perpendicular to the radius to the point of contact;
- the angle between a tangent to a circle and a chord through the point of contact is equal to the angle in the alternate segment.

AREAS OF STUDY

1. Functions, relations and graphs

This area of study covers sums of simple power functions of integer powers, reciprocal functions of quadratic functions and circular functions, inverse circular functions, relations representing circles, simple ellipses and hyperbolas in cartesian and parametric forms (polar coordinates **not** required), graphical representation of these functions and relations, and the analysis of key features of their graphs.

This area of study will include:

- sketch graphs of functions defined by:
  $f(x) = ax^m + bx^{-n}$ for $m, n = 1, 2$ and $f(x) = \dfrac{1}{ax^2+bx+c}$
  their asymptotic behaviour and nature and location of stationary points;
- sketch graphs of ellipses from the general cartesian relation
  $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$
- sketch graphs of hyperbolas (including asymptotic behaviour) from the general cartesian relation
  $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$
  these do **not** involve consideration of focus-directrix properties;
- definition and graphs of the reciprocal circular functions cosecant, secant and cotangent, and simple transformations of these;
- the identities: $\sec^2(x) = 1 + \tan^2(x)$ and $\text{cosec}^2(x) = 1 + \cot^2(x)$;
- compound and double angle formulas for sine, cosine and tangent;
- the restricted circular functions of sine, cosine and tangent over principal domains and their respective inverse functions $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$ (students should be familiar with alternative notations); graphs of these inverse functions, including simple transformations.

2. Algebra

This area of study covers the expression of simple rational functions as a sum of partial fractions; the arithmetic and algebra of complex numbers, including polar form; regions and paths in the complex plane; introduction to factorisation of polynomial functions over the complex field and an informal treatment of the fundamental theorem of algebra.

Rational functions of a real variable, including:

- expression in partial fractions of an algebraic fraction with factorisable quadratic denominator.

Complex numbers, including:

- C, the set of numbers z of the form $z = x + yi$, where x, y are real numbers and $i^2 = -1$;
- real and imaginary parts; complex conjugates;
- equality, addition, subtraction, multiplication and division of complex numbers;
- use of an argand diagram to represent complex numbers;
- polar form (modulus and argument); multiplication and division in polar form, including their geometric representation and interpretation;
- de moivre's theorem and its use to find powers and roots of complex numbers in polar form, including their geometric representation and interpretation;
- factors over C of polynomials with integer coefficients; and informal introduction to the fundamental theorem of algebra, including its application to factorisation of polynomial functions of a single variable over C, for example, $z^2 - i$, $z^3 - (2 - i)\, z^2 + z - 2 + i$ or $z^8 + 1$;
- solution over C of corresponding polynomial equations by completing the square, use of the quadratic formula and using factorised form; occurrence of non-real roots in conjugate pairs (conjugate root theorem).

Representation of relations and regions in the complex plane, including:

- lines and rays;
- circles, ellipses and other familiar simple curves;
- combinations of the above;
- regions defined through the above curves.

3. Calculus

This area of study covers advanced calculus techniques for analytic and numeric differentiation and integration of a broad range of functions, and combinations of functions; and their application in a variety of theoretical and practical situations, including curve sketching, evaluation of area and volumes, the solution of differential equations and kinematics.

Differential and integral calculus, including:

- derivatives of inverse circular functions;
- second derivatives, use of notations $f''(x)$ and $\frac{d^2y}{dx^2}$, and their application to the analysis of graphs of functions, including points of inflection (treatment of concavity is not required);
- applications of chain rule to related rates of change and implicit differentiation; for example, implicit differentiation of the relations $x^2 + y^2 = 9$ and $3xy^2 = x + y$;
- anti-differentiation of $\frac{1}{\sqrt{a^2 - x^2}}$ and $\frac{a}{a^2 + x^2}$ for $a > 0$ by recognition that they are derivatives of corresponding inverse circular functions;
- anti-differentiation of $\frac{1}{x}$ to obtain $\log_e(|x|) + c$;
- use of the substitution $u = g(x)$ to anti-differentiate expressions of the form $f(g(x))g'(x)$; for example, anti-differentiation of $2x\sqrt{1 - x^2}$, $\cos^2(x)\sin^3(x)$ using the equivalent form $\cos^2(x)\,(1 - \cos^2(x))\sin(x)$, and using linear substitution to anti-differentiate expressions such as $x^2\sqrt{1 - x}$;

- use of the trigonometric identities $\sin^2(ax) = ½\,(1 - \cos(2ax))$, $\cos^2(ax) = ½\,(1 + \cos(2ax))$ in anti-differentiation techniques;
- anti-differentiation using partial fractions of rational functions with quadratic denominators;
- the relationship between the graph of a function and the graphs of its anti-derivative functions;
- evaluation of definite integrals involving anti-differentiation techniques listed above;
- evaluation of definite integrals numerically using technology;
- application of integration to finding areas of regions bounded by curves and to volumes of solids of revolution of a region about either coordinate axis.

Differential equations, including:

- setting up differential equations in a variety of contexts drawn from the physical, biological and social sciences, but not requiring specialised knowledge of any of these (including some differential equations whose analytic solutions are not required, but can be solved numerically using technology);
- direction (slope) field of a differential equation;
- verification of solutions to differential equations;
- solution techniques, including the fitting of initial conditions, for the types $\frac{dy}{dx} = f(x)$, $\frac{d^2y}{dx^2} = f(x)$, $\frac{dy}{dx} = g(y)$ where $g(y)$ is a linear or quadratic function of y, or a reciprocal of one of these, or $g(y) = \sqrt{a^2 - y^2}$;
- numerical solution of $\frac{dy}{dx} = f(x)$ with $y = b$ when $x = a$, using technology, by evaluation of $y = \int_a^x f(t)dt + b$, for example, $\frac{dy}{dx} = \sin(x^2)$ where $y = 2$ when $x = 0$.
- numerical solution by euler's method (first-order approximation).

Kinematics: rectilinear motion, including:

- the application of differentiation, antidifferentiation and solution of differential equations to rectilinear motion of a single particle, including the different derivative forms for acceleration $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}(½v^2)$;
- velocity–time graphs and their use.

4. Vectors

This area of study covers the arithmetic and algebra of vectors, linear dependence and independence of a set of vectors, proof of geometric results using vectors, vector representation of curves in the plane and vector kinematics in one, two and three dimensions.

Vectors, including:

- addition and subtraction of vectors and their multiplication by a scalar, position vectors;
- linear dependence and independence of a set of vectors and geometric interpretation;
- magnitude of a vector, unit vector, the orthogonal unit vectors $\underset{\sim}{\mathbf{i}}\ \underset{\sim}{\mathbf{j}}\ \underset{\sim}{\mathbf{k}}$;
- resolution of a vector into rectangular components;
- scalar (dot) product of two vectors: definition $\underset{\sim}{\mathbf{a}} . \underset{\sim}{\mathbf{b}} = ab\cos(\theta)$, deduction of dot product for $\underset{\sim}{\mathbf{i}}\ \underset{\sim}{\mathbf{j}}\ \underset{\sim}{\mathbf{k}}$; system; its use to find scalar and vector resolutes;
- parallel and perpendicular vectors;
- sketch graphs of plane curves specified by a vector equation, and the determination of the corresponding cartesian equations (examples of plane curves could include linear, conic sections and combinations of these);

- vector proofs of simple geometric results, for example:
  - the diagonals of a rhombus are perpendicular;
  - the medians of a triangle are concurrent;
  - the angle subtended by a diameter in a circle is a right angle.

Vector calculus, including:

- position vector as a function of time $\underset{\sim}{\mathbf{r}}(t)$; deriving the cartesian equation of a path given $\underset{\sim}{\mathbf{r}}(t)$ and sketching the path;
- differentiation and anti-differentiation of a vector function with respect to time;
- application of vector calculus to curvilinear motion, in simple cases where the vectors $\underset{\sim}{\mathbf{r}}$ (position), $\dot{\underset{\sim}{\mathbf{r}}}$ (velocity) and $\ddot{\underset{\sim}{\mathbf{r}}}$ (acceleration) are given as functions of time;
- motion along a path in two or three dimensions; examples could include:
  - i A particle moves so that its position vector $\underset{\sim}{\mathbf{r}}$ at time t, $t \geq 0$, is given in a vector parametric form such as: $\underset{\sim}{\mathbf{r}} = 3\cos(2t)\,\underset{\sim}{\mathbf{i}} + 4\sin(2t)\,\underset{\sim}{\mathbf{j}}$ or $\underset{\sim}{\mathbf{r}} = \tan(t)\,\underset{\sim}{\mathbf{i}} + 4\sec^2(t)\,\underset{\sim}{\mathbf{j}}$. Answer questions about the equation of the path, and the magnitude and direction of the velocity and acceleration.
  - ii $\underset{\sim}{\mathbf{r}}$ is the position vector at time t of a particle moving in a plane in such a way that $\ddot{\underset{\sim}{\mathbf{r}}} = 5\underset{\sim}{\mathbf{i}}$ always, and when $t = 0$, $\underset{\sim}{\mathbf{r}} = \underset{\sim}{0}$ and $\dot{\underset{\sim}{\mathbf{r}}} = 7\underset{\sim}{\mathbf{i}} = 10\underset{\sim}{\mathbf{j}}$. Find the position vector at time t and answer questions about the motion and its path.

5. Mechanics

This area of study covers statics and an introduction to newtonian mechanics, for both constant and variable acceleration.

This area of study will include:

- inertial mass, momentum, including change of momentum (conservation of momentum and impulse are **not** required), force, resultant force, weight, action and reaction;
- equations of motion using absolute units (Equations of motion should be described from a diagram, showing all the forces acting on the body, and then writing down the equation of motion. Extensions could include cases involving a system of two or more connected particles. Examples are to be restricted to rectilinear motion, including motion on an inclined plane.);
- motion of a body, regarded as a particle under the action of concurrent coplanar forces, including frictional forces; sliding friction and the coefficient of friction.

The case of equilibrium should be regarded as an application, where acceleration is zero. This includes consideration of limiting equilibrium when the body is at rest.

Many cases, but not all, will involve motion with constant acceleration. Use of the standard constant acceleration formulas where appropriate is encouraged.

OUTCOMES

For each unit the student is required to demonstrate achievement of three outcomes. As a set these outcomes encompass all of the areas of study for each unit. For each of Unit 3 and Unit 4 the outcomes apply to the content from the areas of study selected for that unit.

Outcome 1

On the completion of each unit the student should be able to define and explain key terms and concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures. It is expected that students will be able to use technology as applicable in the solution of problems, as well as apply routines and procedures by hand.

To achieve this outcome the student will draw on knowledge and related skills outlined in all the areas of study.

Key knowledge

This knowledge includes

- functions and relations, the form of their sketch graphs and their key features, including asymptotic behaviour;
- complex numbers, cartesian and polar forms, operations and properties and representation in the complex plane;
- the geometric interpretation of vectors in the plane and of complex numbers in the complex plane;
- specification of regions in the complex plane using complex relations;
- techniques for finding derivatives of explicit and implicit functions, and the meaning of first and second derivatives of a function;
- techniques for finding anti-derivatives of functions, the relationship between the graph of a function and the graph of its anti-derivative functions, and graphical interpretation of definite integrals;
- analytical, graphical and numerical techniques for setting up and solving equations involving functions and relations;
- simple modelling contexts for setting up differential equations and associated solution techniques, including numerical approaches;
- the definition and properties of vectors, vector operations, the geometric representation of vectors and the geometric interpretation of linear dependence and independence;
- standard contexts for the application of vectors to the motion of a particle and to geometric problems;
- techniques for solving kinematics problems in one, two and three dimensions;
- newton's laws of motion and related concepts.

Key skills

These skills include the ability to

- sketch graphs and describe behaviour of specified functions and relations with and without the assistance of technology, clearly identifying their key features and using the concepts of first and second derivatives;
- perform operations on complex numbers expressed in cartesian form or polar form and interpret them geometrically;
- represent regions of an argand diagram using complex relations;
- apply implicit differentiation, by hand in simple cases;
- use analytic techniques to find derivatives and anti-derivatives by pattern recognition, and apply anti-derivatives to evaluate definite integrals;
- set up and evaluate definite integrals to calculate areas and volumes;
- set up and solve differential equations of specified forms;
- perform operations on vectors and interpret them geometrically;
- apply vectors to motion of a particle and to geometric problems;
- solve kinematics problems using a variety of techniques;
- set up and solve problems involving newton's laws of motion;
- apply a range of analytical, graphical and numerical processes to obtain solutions (exact or approximate) to equations.

Outcome 2

On the completion of each unit the student should be able to apply mathematical processes, with an emphasis on general cases, in non-routine contexts, and analyse and discuss these applications of mathematics.

To achieve this outcome the student will draw on knowledge and related skills outlined in one or more areas of study.

Key knowledge

This knowledge includes

- key mathematical content from one or more areas of study relating to a given application context;
- specific and general formulations of concepts used to derive results for analysis within a given application context;
- the role of examples, counter-examples and general cases in developing mathematical analysis;
- the role of proof in establishing a general result;
- the use of inferences from analysis to draw valid conclusions related to a given application context.

Key skills

These skills include the ability to

- specify the relevance of key mathematical content from one or more areas of study to the investigation of various questions related to a given context;
- give mathematical formulations of specific and general cases used to derive results for analysis within a given application context;
- develop functions as possible models for data presented in graphical form and apply a variety of techniques to decide which function provides an appropriate model;
- use a variety of techniques to verify results;
- establish proofs for general case results;
- make inferences from analysis and use these to draw valid conclusions related to a given application context;
- communicate conclusions using both mathematical expression and everyday language, in particular in relation to a given application context.

Outcome 3

On completion of each unit the student should be able to select and appropriately use technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.

To achieve this outcome the student will draw on knowledge and related skills outlined in all the areas of study.

Key knowledge

This knowledge includes

- exact and approximate specification of mathematical information such as numerical data, graphical forms and the solutions of equations produced by the use of technology;
- domain and range requirements for the technology-based specification of graphs of functions and relations;

- the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions or equations;
- the similarities and differences between formal mathematical expressions and their representation in various technology applications;
- the appropriate selection of a technology application in a variety of mathematical contexts.

Key skills

These skills include the ability to

- distinguish between exact and approximate presentations of mathematical results produced by the use of technology, and interpret these results to a specified degree of accuracy;
- produce results using technology which identify examples or counter-examples for propositions;
- produce tables of values, families of graphs or collections of other results using technology which support general analysis in problem-solving, investigative or modelling contexts;
- use appropriate domain and range specifications which illustrate key features of graphs of functions and relations;
- identify the relation between numerical, graphical and symbolic forms of information about functions and equations and the corresponding features of those functions or equations;
- specify the similarities and differences between formal mathematical expressions and their representation in various technology applications;
- make appropriate selections for technology applications in a variety of mathematical contexts, and provide a rationale for these selections;
- relate the results from a particular application to the nature of a particular mathematical task (investigative, problem solving or modelling).

ASSESSMENT

The award of satisfactory completion for a unit is based on a decision that the student has demonstrated achievement of the set of outcomes specified for the unit. This decision will be based on the teacher's assessment of the student's overall performance on assessment tasks designated for the unit. The Victorian Curriculum and Assessment Authority will publish annually an assessment handbook which will include advice on the scope of the assessment tasks and criteria for assessment.

The key knowledge and skills listed for each outcome should be used as a guide to course design and the development of learning activities. The key knowledge and skills do not constitute a checklist and such an approach is not necessary or desirable for determining the achievement of outcomes. The elements of key knowledge and skills should not be assessed separately.

Assessment of levels of achievement

The student's level of achievement for Units 3 and 4 will be determined by school-assessed coursework and two end-of-year examinations.

Contribution to final assessment

School-assessed coursework for Unit 3 will contribute 14 per cent and for Unit 4 will contribute 20 per cent to the study score.

Units 3 and 4 will also be assessed by two end-of-year examinations, which together will contribute 66 per cent to the study score.

School-assessed coursework

Teachers will provide to the Victorian Curriculum and Assessment Authority a score representing an assessment of the student's level of performance in achieving the set of outcomes.

The score must be based on the teacher's rating of performance of each student on the tasks set out in the following table and in accordance with an assessment guide published annually by the Victorian Curriculum and Assessment Authority. The assessment handbook also includes advice on the scope of the task and the criteria for assessment.

Assessment tasks must be a part of the regular teaching and learning program and must not unduly add to the workload associated with that program. They must be completed mainly in class and within a limited timeframe. Where optional assessment tasks are listed teachers must ensure that the tasks they select are comparable in scope and demand.

Outcomes	Marks allocated*	Assessment tasks
Unit 3		Two *analysis tasks*, with the three outcomes assessed across the tasks. Each analysis task is a short item of 2–4 hours duration over 1–2 days, selected from: • an assignment where students have the opportunity to work on a broader range of problems in a given context; **or** • a short and focused investigation, challenging problem or modelling task; **or** • a set of application questions requiring extended response analysis in relation to a particular topic or topics; **or** • item response analysis for a collection of multiple-choice questions, including analysis of item distractors and their relationship to conceptual, process or reasoning error. The two *analysis tasks* are to be of a different type.
Outcome 1 Define and explain key terms and concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures. It is expected that students will be able to use technology as applicable in the solution of problems, as well as apply routines and procedures by hand.	15	7 *Analysis task 1* 8 *Analysis task 2*
Outcome 2 Apply mathematical processes, with an emphasis on general cases, in non-routine contexts, and analyse and discuss these applications of mathematics.	15	8 *Analysis task 1* 7 *Analysis task 2*
Outcome 3 Select and appropriately use technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.	10	5 *Analysis task 1* 5 *Analysis task 2*
Total marks	**40**	

*School-assessed coursework for Unit 3 contributes 14 per cent to the study score.

Outcomes	Marks allocated*	Assessment tasks
Unit 4		A single problem-solving or modelling *application task* based on content from the areas of study, with three components of increasing complexity: • introduction of a context through specific cases or examples; • consideration of general features of this context; • variation, or further specification, of assumptions or conditions involved in the context to focus on a particular feature related to the context. **and** Two equally weighted tests that consist of an appropriate combination of multiple-choice, short-answer and extended-response items. The outcomes are to be assessed across the *application task* and the two *tests*.
Outcome 1 Define and explain key terms and concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures. It is expected that students will be able to use technology as applicable in the solution of problems, as well as apply routines and procedures by hand.	30	15 *The application task* 15 *Tests*
Outcome 2 Apply mathematical processes, with an emphasis on general cases, in non-routine contexts, and analyse and discuss these applications of mathematics.	20	20 *The application task*
Outcome 3 Select and appropriately use technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.	10	5 *The application task* 5 *Tests*
Total marks	**60**	

*School-assessed coursework for Unit 3 contributes 20 per cent to the study score.

End-of-year examinations

The student's level of achievement for Units 3 and 4 will also be assessed by two examinations based on tasks related to Outcomes 1 to 3.

Examination 1

Description

Students are required to respond to a collection of short-answer and some extended-answer questions covering all areas of study in relation to Outcome 1. The task is designed to assess students' knowledge of mathematical concepts, their skills in carrying out mathematical algorithms and their ability to apply concepts and skills in standard ways without the use of technology.

All questions are compulsory.

Students will complete the examination using a structured answer booklet.

The examination will be set by a panel appointed by the Victorian Curriculum and Assessment Authority.

Conditions

The task will be completed under the following conditions:

- Duration: one hour.
- Date: end-of-year, on a date to be published annually by the Victorian Curriculum and Assessment Authority.
- Victorian Curriculum and Assessment Authority examination rules will apply. Details of these rules are published annually in the *VCE and VCAL Administrative Handbook*.
- No calculators, CAS or notes of any kind are permitted. A sheet of formulas will be provided with the examination.
- The task will be marked by a panel appointed by the Victorian Curriculum and Assessment Authority.

Contribution to final assessment

The examination contributes 22 per cent to the study score.

Examination 2

Description

Students are required to respond to a collection of multiple-choice questions and extended-answer questions covering all areas of the study in relation to all three outcomes, with an emphasis on Outcome 2. Student access to an approved graphics calculator or CAS will be assumed by the setting panel.

The task is designed to assess students' ability to understand and communicate mathematical ideas, and to interpret, analyse and solve both routine and non-routine problems. Students should attempt all of the multiple-choice questions in Part I of the examination and all of the extended-answer questions, involving multi-stage solutions of increasing complexity in Part II of the examination.

All questions are compulsory.

Students will complete the examination using a structured answer booklet.

The examination will be set by a panel appointed by the Victorian Curriculum and Assessment Authority.

Conditions

The task will be completed under the following conditions:

- Duration: two hours.
- Date: end-of-year, on a date to be published annually by the Victorian Curriculum and Assessment Authority.
- Victorian Curriculum and Assessment Authority examination rules will apply. Details of these rules are published annually in the *VCE and VCAL Administrative Handbook*.
- An approved graphics calculator or CAS and **one** bound reference, text (which may be annotated) or lecture pad, may be brought into the examination. A sheet of formulas will be provided with the examination.
- The task will be marked by a panel appointed by the Victorian Curriculum and Assessment Authority.

Contribution to final assessment

The examination contributes 44 per cent to the study score.

Advice for teachers (Units 3 and 4: Specialist Mathematics)

DEVELOPING A COURSE

A course outlines the nature and sequence of teaching and learning necessary for students to demonstrate achievement of the set of outcomes for a unit. The areas of study describe the learning context and the knowledge required for the demonstration of each outcome. Outcomes are introduced by summary statements and are followed by the key knowledge and skills which relate to the outcomes.

Teachers must develop courses that include appropriate learning activities to enable students to develop the knowledge and skills identified in the outcome statements in each unit. In particular, teachers will need to consider the progression of material in complementary implementation of Mathematical Methods Units 3 and 4 or Mathematical Methods (CAS) Units 3 and 4 as applicable.

In Units 3 and 4, assessment is more structured. For some outcomes, or aspects of an outcome, the assessment tasks are prescribed. The contribution that each outcome makes to the total score for school-assessed coursework is also stipulated.

The following material illustrates sequences for implementation of Specialist Mathematics Units 3 and 4. Sample sequence 1 covers non-calculus based material extensively at the beginning of Unit 3, while calculus and corresponding techniques that underpin related content for Specialist Mathematics, are progressively developed in detail in Mathematical Methods Unit 3 or Mathematical Methods (CAS) Unit 3.

Sample sequence 2 covers material on vectors and relations involving ellipses and hyperbolas in conjunction with work on vector calculus and mechanics in the latter part of Unit 4. In this sequence, calculus and related techniques required in Unit 3 are introduced and used on an operational basis. The following outlines a possible implementation for each of these types of sequence over 27 teaching weeks. Schools and teachers are encouraged to develop their own sequences, or variations to those outlined.

Sample sequence 1

Topic	Approximate time
Complex numbers	4 weeks
• algebra of complex numbers and relations and regions in the complex plane	
Functions, relations and graphs	2 weeks
Sketch graphs of functions of the type	
• $f(x) = ax^m + bx^{-n}$ for m, n = 1 or 2	
• $f(x) = \dfrac{1}{ax^2+bx+c}$	
Sketch graphs of relations (ellipses and hyperbolas) of the type	
• $\dfrac{(x-h)^2}{a^2} \pm \dfrac{(y-k)^2}{b^2} = 1$	
Circular functions and sketch graphs	2 weeks
• reciprocal circular functions	
• trigonometric identities	
• inverse circular functions	
Vectors	4 weeks
• vector algebra and application to geometric proofs	
• sketch graphs of plane curves	
Calculus	
Differential calculus and applications	2 weeks
Integral calculus (including use of partial fractions) and applications	4 weeks
Differential equations	3 weeks
Kinematics	1 week
• rectilinear motion	
Vectors	2 weeks
• vector calculus	
Mechanics	3 weeks

Sample sequence 2

Topic	Approximate time
Circular functions	2 weeks
• trigonometric identities, reciprocal and inverse circular functions	
Differential calculus	3 weeks
• derivatives of elementary transcendental functions, including exponential, logarithmic, reciprocal circular and inverse circular functions	
• derivatives of combined functions and applications, including derivatives of inverse functions and implicit differentiation	
• sketch graphs of functions of the type $f(x) = ax^m + bx^{-n}$ and $f(x) = \dfrac{1}{ax^2+bx+c}$	
• parametrically defined curves	
Integral calculus	4 weeks
• fundamental theorem of calculus and sketch graphs of f from the graph of f'	
• integration of elementary transcendental functions, including exponential, logarithmic, and circular, inverse circular	
• use substitution and partial fractions to find anti-derivatives	
• applications of integration to area and volume	

Differential equations — 3 weeks

- use of direction (slope) fields to identify solutions of differential equations and approximate solutions to differential equations using euler's method and the fundamental theorem of calculus for the case $y' = f(x)$
- formulation and solution of problems involving related rates, implicit differentiation and differential equations

Kinematics: rectilinear motion — 1 week

Vectors — 4 weeks

- vector algebra and application to geometric proofs

Function, relations and graphs

- sketch graphs of ellipses and hyperbolas — 1 week

Complex numbers — 1 week

- algebra of complex numbers and relations and regions in the complex plane

Vector calculus — 2 weeks

Mechanics — 3 weeks

USE OF INFORMATION AND COMMUNICATIONS TECHNOLOGY

In designing courses and developing learning activities for Specialist Mathematics, teachers should make use of applications of information and communications technology and new learning technologies, such as computer-based learning, multimedia and the World Wide Web, where appropriate and applicable to teaching and learning activities.

KEY COMPETENCIES AND EMPLOYABILITY SKILLS

Students undertaking the following types of assessment, in addition to demonstrating their understanding and mastery of the content of the study, typically demonstrate the following key compentencies and employability skills.

Assessment task	Key competencies and employability skills
Analysis task	Planning and organising, solving problems, using mathematical ideas and techniques, (written) communication, use of information and communications technology
Tests	Self management, solving problems, technology
Application task	Planning and organising, solving problems, (written) communication, using mathematical ideas and techniques, use of information and communications technology

LEARNING ACTIVITIES

Examples of learning activities for each unit are provided in the following sections. Extended examples are highlighted by a shaded box. The examples that make use of information and communications technology are identified by this icon ICT.

Units 3 and 4: Specialist Mathematics

Outcome 1

Define and explain key terms and concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.
It is expected that students will be able to use technology as applicable in the solution of problems, as well as apply routines and procedures by hand.

Examples of learning activities

skills practice on standard mathematical routines through appropriate exercises; development of the ability to identify problems as either being in a standard form, or readily transformable to a standard form, the efficient and accurate application of the relevant mathematical routine (e.g. the application of anti-differentiation methods to combinations of functions, such as different types of rational and other quotient functions), and systematic checking of both the reasonableness and accuracy of results (does the anti-derivative function fit with the characteristic types of functions which result from identifiable cases of this type of problem, and does differentiation yield the original function?); the appropriate use of technology, such as the use of a graphics calculator, spreasheet or computer algebra system to check working and explore variations on the rule of the original function would link this work to Outcome 3

error identification and analysis exercises, where students work through a range of ‘worked solutions’ and identify and rectify missing steps or errors in working

construction of concept maps, summary or review notes related to a topic or area of study (e.g. the conditions under which different combinations of coefficients for a polynomial function can be related to the existence and number of real roots)

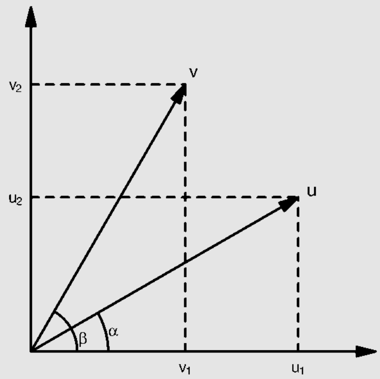
presentation of a range of typical problems associated with an area of study and worked solutions (e.g. applications of integration to volumes of revolution about either axis, and the generation of ‘interesting shapes’, by forming such solids from the area between curves to be rotated about the relevant axis)

assignments structured around the development of sample cases of standard applications of mathematical skills and procedures in readily recognisable situations (e.g. the application of parametric forms, including vector parametric form to specify curves in two and three dimensions – this can be checked by the use of graphics calculators, function graphers or computer algebra systems and hence linked to Outcome 3)

Detailed example 1

A COLLECTION OF VECTOR QUESTIONS

1. Consider the diagram:



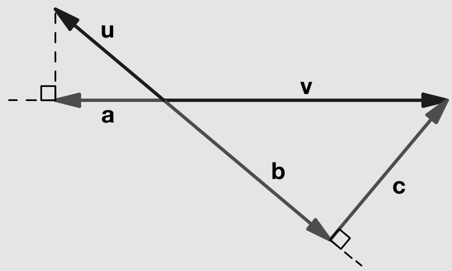
Express u_1 and u_2 in terms of |**u**| and **α** and v_1 and v_2 in terms of |**v**| and **ß**; also express **u·v** in terms |**u**|, |**v**| and the angle included between **u** and **v**. Show that **u·v** =(|**u**| cos α)(|**v**| cos **ß**) + (|**u**| sin **α**)(|**v**| sin **ß**) and hence $\mathbf{u}\cdot\mathbf{v} = \mathbf{u}_1\mathbf{v}_1 + \mathbf{u}_2\mathbf{v}_2$.

2. Let **u** and **v** be non-zero vectors and **w** = |**v**|**u** + |**u**|**v**. Show that $\hat{\mathbf{u}}.\mathbf{v} = \mathbf{u}.\hat{\mathbf{v}}$ and hence show that **w** bisects the angle between **u** and **v**.

3. Three points *P*, *Q* and *R* have the respective position vectors **p**, **q** and *k* (**p** + **q**), where *k* is a positive real scalar. Find respectively the values of *k* for which: $\overrightarrow{QR}$ is parallel to **p**; $\overrightarrow{PR}$ is parallel to **q**; and *P*, *Q* and *R* are collinear.

4. Two distinct vectors **p** and **q** have the same magnitude. Draw a diagram representing **p** + **q** and **p** – **q** and use geometry to show that the vectors **p** + **q** and **p** – **q** are perpendicular. Use scalar product to verify this result.

5. Consider the diagram

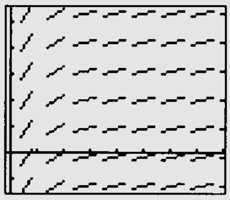


Use the projection of **u** along **v** to express **a**, **b** and **c** respectively in terms of **u** and **v**.

Detailed example 2

A COLLECTION OF IMPLICIT DIFFERENTIATION, RELATED RATES AND DIRECTION (SLOPE) FIELD QUESTIONS

1. Use implicit differentiation to find $\frac{dy}{dx}$ where $x^2y + xy^2 = 6$
2. Use calculus to find the points (ordered pairs) where the graph of the given equation $4x^2 + y^2 – 8x + 4y + 4 = 0$ has a vertical or horizontal tangent line.
3. Grain spills from a truck at a rate of 10 m^3/min onto the top of a conical pile. The height of the pile is always three-eighths of the base diameter. How fast are the height and radius changing when the pile is 4 m high?
4. A particle moves along the parabola $y = x^2$ in the first quadrant in such a way that its x– coordinate increases at a steady rate of 10 m/s. How fast is the angle of inclination θ of the line joining the particle to the origin changing when $x = –4$?
5. The slope field from a certain differential equation is shown below:



6. Which one of the following could be a specific solution to that differential equation?

(A) $y = x^2$ (D) $y = \cos(x)$

(B) $y = e^x$ (E) $y = \log_e(x)$

(C) $y = e^{–x}$

Outcome 2

Apply mathematical processes, with an emphasis on general cases, in non-routine contexts, and analyse and discuss these applications of mathematics.

Examples of learning activities

investigative projects (e.g. functions mapping points in the complex plane; direction (slope) fields for differential equations, including those suitably tackled by numerical techniques such as euler’s method)

problem-solving tasks (e.g. finding volume of different shaped containers)

modelling tasks (e.g. heating a greenhouse)

a set of applications questions requiring analysis and extended response related to a particular context (e.g. vector proofs of geometric results)

a report on item response analysis for a collection of multiple-choice questions (e.g. integrals of rational or quotient function)

presentation on research into a particular application of mathematics (e.g. the application of laws of motion to describing the behaviour of physical systems)

group work in the development of conjectures for general case propositions and the development and presentation (possibly as an oral presentation) of results involving general case arguments and proof

Detailed example 1

A MODELLING TASK

Mass balance problems refer to a large collection of contexts where mass of a component of interest is flowing into and out of a confined space (the control volume). In addition to these mass flows, mass of the component may be generated within the control volume by conversion from another component, or may be reduced by decay to another component.

The law of conservation of mass which states that:

> *accumulation rate = inflow rate – outflow rate + generation rate – decay rate*

can be used to provide a starting point for a range of modelling contexts.

Mass balance problems give the opportunity to apply modelling skills in new and interesting contexts of interest. While there are many mass balance contexts around engineering, they are mainly used to model environmental systems such as pollution in a lake and/or salinity. Assuming the control volume is well mixed, the corresponding differential equation is:

$$\frac{dM}{dt} = V\frac{dC}{dt} = C_iQ_i - CQ_o + g(C,t) - d(C,t)$$

with the model parameters:

C = spatially uniform mass concentration of component within control volume

C_i = mass concentration of component in inflow stream

Q_i = the volumetric inflow rate

Q_o = the volumetric outflow rate

V = the volume within control volume (e.g. lake)

$M = CV$ = the mass of component (e.g. salt or a contaminant)

g = generation rate (context specific)

d = decay rate (context specific)

An approach to generating tasks for such contexts to address Outcome 1 and Outcome 2 could involve:

1. investigation of model assumptions (e.g. well mixed assumption)
2. the prescription of model parameters (e.g. $Q_i = Q_o$, $g(C,t) = 0$ and $d(C,t) = kCV$)
3. the verification and interpretation of a proposed analytical solution
4. analyse dependence of analytical solution on initial conditions
5. interpret effect of the relaxation of a model assumption (e.g. $Q_i \neq Q_o$ and/or $C_i = f(t)$)

To also address Outcome 3, a task could be developed to incorporate:

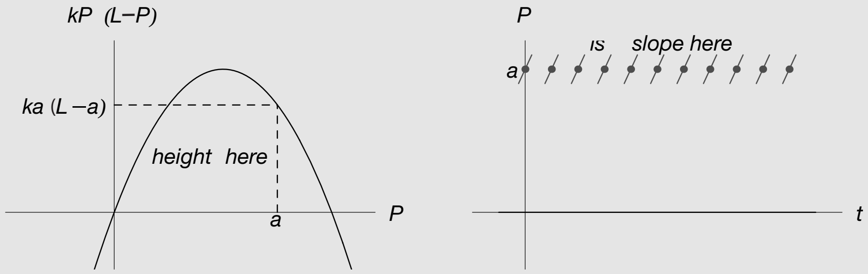
6. the numerical solution of the relaxed model
7. an investigation of the general features of solution via the direction (slope) field of the governing equation (e.g. does the system approach steady state for a given $C_i = f(t)$)

Detailed example 2

LOGISTIC EQUATION INVESTIGATION

The *logistic equation* is a model governing population growth in an environment with limited available resources. If $P(t)$ is the population at time t, then the logistic equation is given by : $\frac{dP}{dt} = kP(L - P)$

Where the constants k and L are related to the particular population and depend on the characteristics of its environment. The slope of line segment at any point in the corresponding direction (slope) field, is given by the height of the parabola for $kP(L - P)$ at each value of P as shown in the following diagram:



Students could be asked to depict the slope field of the *logistic equation* over an appropriate region of the (t, P) plane and to explain their choice of region.

At each point (t_m, P_n) on a well defined grid in the (t, P) plane, a short line segment with a slope $kP_n(L - P_n)$ can be marked for that point.

Students could then be asked to sketch the set of particular solutions curves satisfying initial conditions such as $P(0) = L/2$, $P(0) = L$ and $P(0) = 3L/2$ over the direction (slope) field; and to comment on the form of the slope field in the land surrounding. This could include consideration of which population has the greatest growth rate.

An *isocline* is the locus of points within the slope field along which the slope is constant. Students could be asked to describe the isoclines of the *logistic equation*. The *logistic equatio*n is autonomous, that is, the independent variable is not present in the right-hand side of the equation. Students could consider how the autonomy of the governing equation affect the corresponding direction (slope) field.

The population of trout within a lake could be modelled using the *logistic equation*. It has been proposed by the managers of the lake to allocate a fixed quota of these trout to fishing. Students could be asked to explain why the equation governing the trout population under the fixed quota policy becomes $\frac{dP}{dt} = kP(L - P) - q$

and to investigate the effect of q on the *logistic equation* parabola and how this affects the direction (slope) field. This investigation could include justification for the selection of a maximum q and analysis of how the maximum of q is related to the maximum of $kP(L - P)$ and consideration of the minimum sustainable fish population for a given choice of q.

Outcome 3

Select and appropriately use technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.

Examples of learning activities

technology should be used to enhance the learning of mathematics throughout the course; students will have the opportunity to use a variety of technologies as part of their mathematics learning and learning activities should reflect this; it is assumed that all students will have access to an approved graphics calculators or CAS and teachers should develop courses that encourage the appropriate use of this technology; students should be able to recognise limitations of technology and decide when the use of technology is appropriate or not in a given context

identification and analysis of key features of functions (e.g. reciprocal functions) in graphical, tabular and numerical form (including associated problems involving analytical and/or numerical differentiation and integration and applications)

production of collections of results in response to variation of parameters used to define a type of function (e.g. the role of boundary conditions, in particular, initial conditions in determining features of the family of graphs that represent functions which are solutions to a differential equation), including the use of graphics calculator, spreadsheet or CAS technology to generate direction (slope) fields

use of dynamic geometry software to explore conic sections in cartesian and parametric forms, transformations of the plane, vector algebra and constructions associated with geometric proofs

drawing families of parametrically described functions in two dimensions and three dimensions and drawing regions in the complex plane using graphics calculators or CAS

solving equations which need numerical approaches, such as a polynomial equation with a circular function, and the numerical solution of differential equations using a graphics calculator, spreadsheet or CAS

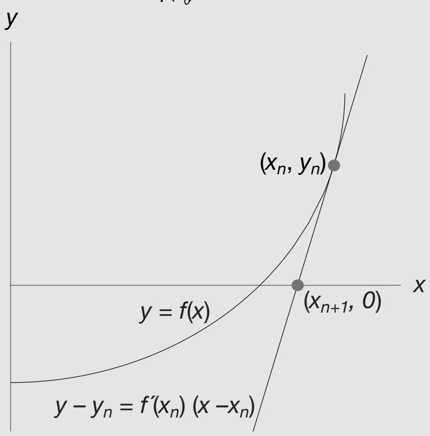
use of symbolic manipulation to explore and check patterns and conjectures associated with generalisable algebraic arguments

Supporting technology should be used extensively in the classroom, with appropriate contexts for each of the outcomes. Attention needs to be given to assisting students in developing recording skills. Thus, if students are using graphics calculators to deduce the relationship between a graph of a function and the graphs of their anti-derivatives, they should be encouraged to sketch and annotate these graphs as they are produced. Students should also be encouraged to take advantage of the capacity of a graphic calculator to be programmed. The appropriate use of programs for graphics calculators, spreadsheets or CAS, where these are clearly related to the task at hand, provides students with an opportunity to develop an understanding of underlying processes related to the development of algorithms to implement mathematical processes and techniques.

Detailed example

CONSTRUCTING IMAGES IN THE COMPLEX PLANE

An iterative procedure using tangents to a curve can be employed to solve equations in a single real variable x of the form $f(x) = 0$ provided that $f(x)$ is sufficiently smooth, and its derivative $f'(x)$ is available. Let $y_n = f(x_n)$ where x_n is near a root and construct the tangent line approximation $(y - y_n) = f'(x_n)(x - x_n)$ as shown in the following diagram. Solving for the x-intercept of the tangent line gives $x = x_n - \frac{f(x_n)}{f'(x_n)}$ where y_n has been replaced with $f'(x_n)$. This process can be repeated using $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$. An initial value $x = x_0$ is chosen such that $f(x_0)$ is close to zero.

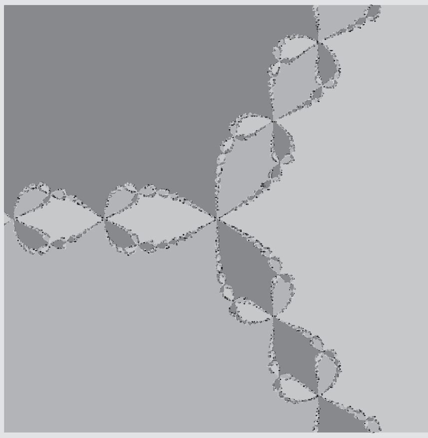


This process can be continued until the *fixed point* is approximated to sufficient accuracy, i.e. $|x_{n+1} - x_n| \le \varepsilon$ for some prescribed tolerance ε, where ε is a small positive real constant.

This process can be adapted to examine roots of polynomial equations in the *complex* plane, that is, where a complex variable z is used instead of a real variable x.

Consider the complex polynomial equation $z^3 - 1 = 0$. The set of points in the complex plane such that initial conditions chosen in this set dynamically evolve to a particular solution (root) of the equation $z^3 - 1 = 0$ is called the *basin of attraction* of the root. These basins depict a fractal image, and starting points on the boundary between different basins exhibit chaotic behaviour (that is, the fixed point for convergence is extremely sensitive to the initial condition).

A graphics calculator, spreadsheet or CAS can be used to carry out the iterative processes, using a program or algorithm such as the following:



Students could investigate basins of attraction related to the equation $z^n - 1 = 0$ for different values of n.

APPROACHES TO ASSESSMENT

Assessment tasks must be selected from those designated for the study, the following detailed examples illustrate possible application tasks. Each of the preceding activities could be suitably developed as an assessment task. The specific or general nature of various components will depend on whether the intended tasks is a test, analysis task or application task. Teachers also need to consider which tasks and/or components of tasks are suited to mental, by hand or technology assisted/expected approaches. In each case, the intended tasks should provide students with the opportunity to demonstrate key knowledge and skills for the outcomes, drawing on content from the areas of study for the course.

Detailed example

TEST OR ANALYSIS TASK ON RATIONAL FUNCTIONS

The following context could be used to develop either a test or an analysis task, depending on the use of specific or general formulation of aspects of the task. This could include a suitable selection and/or combination of multiple-choice, short-answer or extended-response items.

Let $q: R \to R$ where $q(x) = ax^2 + bx + c$ and define $f: X \to R$ where $f(x) = \frac{k}{q(x)}$ and k is a non-zero real constant.

The first component of the assessment could focus on finding the maximal domain and range, and determining the existence and equations/coordinates of asymptotes, intercepts, stationary points and points of inflection for specific combinations of values of the parameters (test) or generally in terms of the parameters, a, b, c and k (analysis task), and the nature of $q(x)$.

The second component of the assessment could focus on techniques of anti-differentiation, for specific cases (test) or in general (analysis task) in terms of the factorised or perfect square form of $q(x)$.

The third component of the assessment could focus on consideration of related definite integrals and areas, and the conditions under which the definite integral of $f(x)$ over $[a, b]$ is not defined for different values of the parameters a, b, c and k.

Detailed example

ANALYSIS TASK ON VECTORS AND GEOMETRIC PROOFS

Quadrilaterals can be classified in terms of properties of their diagonals. Students could be asked to use vector proof methods to establish these results for a selection of quadrilaterals listed in the following table:

	Properties of diagonals		
Shape	**Equal in length?**	**Perpendicular?**	**Bisect each other?**
trapezium	no	no	no
kite	no	yes	no
parallelogram	no	no	yes
rectangle	yes	no	yes
rhombus	no	yes	yes
square	yes	yes	yes

A particular selection of these could be combined with questions of similar design to extended analysis vector questions from previous examination 2 papers, and consideration of linear independence of a set of vectors.

Detailed example

APPLICATION TASK: MODELLING THE 2 BODY PROBLEM IN THE COMPLEX PLANE

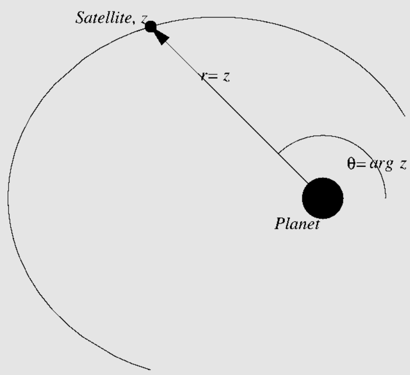
The motion of a celestial body such as satellite, about a planet can be approximately considered as motion in a plane, and this motion can be described using complex numbers. Assume that the location of a planet of mass *M* is given at the origin of the argand diagram shown below. The position of the satellite, with (very much smaller) mass *m*, relative to the planet, is given by the complex number $z = x + iy = r\,\text{cis}\theta$ which is itself a function of time, *t*, according to newton's second law:

$$m\frac{d^2z}{dt^2} = -\frac{GMm}{r^2}\text{cis}\theta$$

Unfortunately, a straightforward analytical solution of this equation is not available, so numerical methods similar to euler's method are used instead. By changing the independent variable from t to $\theta = \arg(z)$, the dependent variable to $\frac{1}{r}$ (the reciprocal of $|z|$), and several applications of the chain rule for differentiation, newtons second law can be transformed to the simpler differential equation:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = \frac{1}{a(1-e^2)} \qquad (2)$$

which governs the angular (θ) dependence of the radial distance (r) of a satellite orbiting around a planet and involves positive real constants a and e, which need to be determined.



Component 1

In the first part of the application task, students could be asked to tackle some specific problems which familiarise them with the context. For example, they could be asked to:

Verify that

$$\frac{1}{r} = A\cos\theta + B\sin\theta + \frac{1}{a(1-e^2)},\ 0 \le e < 1$$

is the general solution of the transformed governing equation.

They could then be asked to choose constants *A* and *B* that satisfy the initial conditions

$$\left.\frac{d}{d\theta}\frac{1}{r}\right|_{\theta=0} = 0 \text{ and } \left.\frac{d^2 1}{d\theta^2 r}\right|_{\theta=0} = \frac{-e}{a(1-e^2)}$$

and comment on the local behaviour of the solution $\frac{1}{r}$ in a small interval of $\theta = 0$.

Students could also be asked to:

- Show that the position of the satellite at angle θ is given by the complex number

  $$z = \frac{a(1-e^2)}{1+e\cos\theta}\text{cis}\theta$$

- Find the minimum (periapsis) and maximum (apoapsis) values of $|z|$ on the orbit of the satellite, along with the corresponding values of θ at these points.

- Plot the complex region $\left\{z : z = \frac{a(1-e^2)}{1+e\cos\theta}\text{cis}\theta\right\}$ with $\theta \in [-\pi, \pi]$, $e = 0.5$ and $a = 1$ and comment on the form of this region.

Component 2

Celestial bodies such as comets can follow a more general path where:

$$\left\{z : z = \frac{\alpha}{1+e\cos\theta}\text{cis}\theta\right\} \text{where } e \ge 0,\ \alpha \ge 0$$

Students could be asked to investigate a family of these paths for appropriate sets of e and α.

Component 3

Student could be asked to show that the cartesian form of the paths

$$\left\{z = x + iy : |z| = \frac{\alpha}{1+e\cos\theta} = \frac{\alpha}{1+e\text{Re}(z)/|z|}\right\}$$

in Component 2 can be re-written as $(1 - e^2)x^2 + y^2 + 2\alpha ex - \alpha^2 = 0$ and to generate all possible types of path by restricting e and α to membership of explicitly stated sets.

SUITABLE RESOURCES

Some of the print resources listed in this section may be out of print. They have been included because they may still be available from libraries, bookshops and private collections.

At the time of publication the URLs (website addresses) cited were checked for accuracy and appropriateness of content. However, due to the transient nature of material placed on the web, their continuing accuracy cannot be verified. Teachers are strongly advised to prepare their own indexes of sites that are suitable and applicable to the courses they teach, and to check these addresses prior to allowing student access.

BOOKS AND CD-ROMs

Foundation Mathematics Units 1 and 2

Flitman, Evelyn & Sharrock, Helen 2000, *Foundation Mathematics for the Real World*, Longman, Melbourne.

Hodgson, B, Richards, M, Corkhill, P, Gary, A, Umbers, B & Vale, C 1991, *Investigations – Guided Projects in Space and Number*, Jacaranda, Milton, Queensland.

Johnstone, J, Kissane, B, Lowe, I & Willis, S 1993, *Access to Algebra Books 1–4*, Curriculum Corporation, Carlton, Victoria.

Lawrence, Gail & Moule, James 1989–1990, *Mathematics in practice series: (Accommodation, Design, Health, Social Issues, The consumer, Transport, travel;)*, Harcourt Brace Jovanovich, Marrickville.

Lowe, I 1991, *Mathematics at Work modelling your world*, Volumes I and II, Australian Academy of Science, Canberra.

O'Connor, M & Gaton, B 2000, *Foundation Mathematics*, Oxford, Melbourne.

Thomson, Sue & Forster, Ian 1998, *Trade and business maths 1*, Addison Wesley Longman Australia, South Melbourne.

Thomson, Sue & Forster, Ian 1999, *Trade and business maths 2*, Pearson Education, South Melbourne.

General Mathematics Units 1 and 2

Berggreb, L, Borwein, J & Borwein, P 2004, *Pi : A Source Book*, 3rd edn Springer, New York.

Berk, K & Carey, P 1995, *Data Analysis with Microsoft Excel 5.0 for Windows*, Nelson, Melbourne. (This resource comes with a disk which includes a treatment of box-plots and correlation matrices.)

Gremillion, D & Keyton, M 1994, *Cabri Geometry II*, Texas Instruments, USA.

Hammel-Garland, T 1987, *Fascinating Fibonaccis*, Dale Seymour, Palo Alto, California.

Hodgson, B, Richards, M, Corkhill, P, Gary, A, Umbers, B & Vale, C 1991, *Investigations – Guided Projects in Space and Number,* Jacaranda, Milton, Queensland.

Johnstone, J, Kissane, B, Lowe, I & Willis, S 1993, *Access to Algebra Books 1–4*, Curriculum Corporation, Carlton, Victoria.

Lipson, K, Sharpe, K & Watson, R 1990, *Projects in Probability and Statistics*, Mathematical Association of Victoria, Brunswick.

Livio, M 2003, *The Golden ratio: the Story of Phi*, Headline, London.

Lowe, I 1991, *Mathematics at Work Modelling your World*, Volumes I and II, Australian Academy of Science, Canberra.

Ruth-Eagle, M 1995, *Exploring Mathematics Through History*, Cambridge University Press, Cambridge.

Serra, M 1989, *Discovering Geometry: An Inductive Approach*, Key Curriculum Press, Berkeley, California.

Stillwell, J 2003, *Elements of Number Theory*, Springer, New York.

Further Mathematics Units 3 and 4

Berk, K & Carey, P 1995, *Data Analysis with Microsoft Excel 5.0 for Windows*, Nelson, Melbourne. (This resource comes with a disk which includes a treatment of boxplots and correlation matrices.)

Bolt, B 1987, *Even More Mathematical Activities*, pp 68–70, 108–109 Cambridge University Press, Cambridge.

Fiekler, D et al. 1973, *Geometry for Enjoyment*, Granada Publishing, UK.

Gremillion, D & Keyton, M 1994, *Cabri Geometry II*, Texas Instruments, USA.

Hammel-Garland, T. 1987, *Fascinating Fibonaccis*, Dale Seymour, Palo Alto, California.

Kannegiesser, H & Mirsky, M 1969, *Business Mathematics*, Ch 30 MacMillan Co, Melbourne.

Lipson, K, Sharpe, K & Watson, R 1990, *Projects in Probability and Statistics*, Mathematical Association of Victoria, Brunswick.

Mottershead, L 1985, *Investigations in Mathematics*, McGraw-Hill Book Co., Sydney.

Mottershead, L, Goodman, H & Cody, M 1991, *Maths Blackline Masters 10*, pp 46–47, McGraw-Hill Book Co., Sydney.

Newhouse, P, & Bray, C 1993, *Mathematics in Practice*, Napier Publications, Western Australia.

Quadling, D 1987, *Statistics and Probability*, Cambridge University Press, Cambridge.

Serra, M 1989, *Discovering Geometry: An Inductive Approach*, Key Curriculum Press, Berkeley, California.

Shell Centre for Mathematical Education 1985, *The Language of Functions and Graphs*, Joint Matriculation Board, Manchester.

Mathematical Methods Units 1–4

Barnes, M 1991, *Investigating Change Units 1–10*, Curriculum Corporation, Carlton, Victoria.

Courant, R & Robbins, H 1941 (re-printed 1981), *What is mathematics?*, Oxford University Press, New York.

Estep, D 2002, *Practical Analysis in One Variable*, Springer, New York.

Grossman, S 1992, *Calculus* (5th edn), Harcourt-Brace, New York.

Johnstone, J, Kissane, B, Lowe, I & Willis, S 1993, *Access to Algebra Books 1–4*, Curriculum Corporation, Carlton, Victoria.

Kreyszig, E 1999, *Advanced Engineering Mathematics*, 8th edn, Wiley, New York.

Mendenhall, W, Scheaffer, RL & Wackerley, DD 1981, *Mathematical statistics with applications*, Duxburry Press, Boston.

Moore, DS & McCabe, GP 1999, *Introduction to the practise of statistics*, (with CD-ROM), Freeman and Co., New York.

Quadling, D 1987, *Statistics and Probability*, Cambridge University Press, Cambridge.

Ruth Eagle, M 1995, *Exploring Mathematics Through History*, Cambridge University Press, Cambridge.

Stillwell, J 2002, *Mathematics and its History*, 2nd edn, Springer, New York.

Thomas, GB, & Finney, RC 1996, *Calculus and Analytic Geometry* (9th ed), Addison-Wesley, New York.

Mathematical Methods (CAS) Units 1–4

Ball, L, Dowsey, J & Tynan, D 2004, *CAS – Active Mathematics. A Student Guide to the TI-89 in VCE Mathematics*, Macmillan, South Yarra.

Barnes, M 1991, *Investigating Change Units 1–10*, Curriculum Corporation, Carlton, Victoria.

Barton, R, & Diehl, J 1999, *Advanced Placement Calculus with the T1-89: Explorations*, Texas Instruments, Dallas.

Cohen, JS 2002, *Computer Algebra and Symbolic Computation – Elementary Algorithms*, AK Peters, Natick, Massachusetts.

Cohen, JS 2003, *Computer Algebra and Symbolic Computation – Mathematical Methods*, AK Peters, Natick, Massachusetts.

Croft, C 2003, *Mastering the hp 39g+: A guide for teachers, students and other users of 39g+, hp39g and hp40g*, St Hilda's Anglican School, Perth.

De Graeve, R 2000, *HP40G Computer Algebra System*, Moravia Consulting, Czech Republic.

Estep, D 2002, *Practical Analysis in One Variable*, Springer, New York.

Etchells, T, Humber , M, Monoghan, J, Pozzi, S & Rothey, A 1997, *Mathematical Activities with Computer Algebra – a photocopiable resource book*, Chartwell-Bratt, Bromley, Kent.

Evans, M, Greenwood, D, Woolacott, B & Taylor, N 2003, *Essential Applications of Technology in Mathematics*, Cambridge, Port Melbourne.

Finney, RL, Thomas, BT, Demana, F D & Waits, KB 1994, *Calculus, Graphical, Numerical, Algebraic*, Addison-Wesley, Sydney.

Garner, S, McNamara, A & Moya, F 2003, *CAS Analysis Supplement for Maths Methods*, Pearson, South Melbourne.

Grabmeier, J, Kaltofen, E & Weispfenning, V 2003, *Computer Algebra Handbook – Foundations, Applications, Systems*, Springer, Berlin.

Hughes-Hallett, D & McCallum, WG 2000, *Computer Algebra Problems*, Wiley.

Johnstone, J, Kissane, B, Lowe, I & Willis, S 1993, *Access to Algebra Books 1–4*, Curriculum Corporation, Carlton, Victoria.

Kreyszig. E 1999, *Advanced Engineering Mathematics*, 8th edn, Wiley, New York.

Kutzler, B & Kokol-Volj, V 2000, *An Introduction to Derive*, Texas Instruments, Dallas.

Laughbaum, ED et al. 2000, *Hand-held Technology in Mathematics and Science Education: A Collection of Papers*, E. Laughbaum (ed.), Ohio-State University, Columbus.

Morphett, Brian 2003, *Essential CAS Calculator Companion*, Cambridge, East Melbourne.

Oldknow, A & Flower, J (eds) 1996, *Symbolic Manipulation by Computers and Calculators – Information, Ideas and Implications for teaching mathematics 14–21*, The Mathematical Association, Leicester.

Quadling, D 1987, *Statistics and Probability*, Cambridge University Press, Cambridge.

Ruth, M 1995, *Exploring Mathematics Through History*, Cambridge University Press, Cambridge.

Stillwell, J 2002, *Mathematics and its History*, 2nd edn, Springer, New York.

Texas Instruments 2005, *'CAS' RESOURCE MATERIALS – everything you need to know about CAS Version2.0*, CD-ROM, Texas-Instruments Australasia, Melbourne.

Victorian Curriculum and Assessment Authority 2004, *Mathematical Methods (CAS) Teacher Support Resources 2004,* CD-ROM, East Melbourne.

Specialist Mathematics Units 3 and 4

Barnes, M 1991, *Investigating Change Units 1–10*, Curriculum Corporation, Carlton, Victoria.

Courant, R & Robbins, H 1941 (re-printed 1981), *What Is Mathematics?,* Oxford University Press, New York.

Estep, D 2002, *Practical Analysis in One Variable*, Springer, New York.

Evans, Avery, Greenwood, Woolacott & Taylor 1995, *Essential Technology In Mathematics*, Cambridge University Press, Cambridge.

Finney, RL, Weir, MD & Giordano, FR 2001, *Thomas' Calculus*, 10th edn, Addison-Wesley Publishing Co.

Giordano, F & Weir, M 1977, *A First Course in Mathematical Modelling*, Brooks-Cole, Belmont, California.

Grossman, S 1992, *Calculus* (5th edn), Harcourt-Brace, New York.

Hughes-Hallett, D, Gleason, A et al. 2001, *Calculus, Single Variable*, 3rd edn, Wiley.

Johnstone, J, Kissane, B, Lowe, I & Willis, S 1993, *Access to Algebra Books 1–4*, Curriculum Corporation, Carlton, Victoria.

Kreyszig. E 1999, *Advanced Engineering Mathematics*, 8th edn, Wiley, New York.

Larson, R, Hostetler, RP & Edwards, BH 2001, *Calculus*, 7th edn, Houghton-Mifflin.

Larson, R, Hostetler, RP, Edwards, BH & Heyd, DE 2003, *Calculus: Early Transcendental Functions*, 3rd edn, Houghton-Mifflin.

Numerical Methods (16–19 Mathematics Series), Cambridge University Press, Cambridge. (Contains a good discussion of euler's Method.)

Sokolnikoff, IS, & Redheffer, RM 1966, *Mathematics of Physics and Modern Engineering*, International Student Edition, McGraw-Hill, Kogakusha.

Spode Group 1997, *Motivating A level Mathematics*, Oxford University Press, Oxford.

Stewart J 2003, *Calculus Single Variable*, 5th edn, Brooks Cole.

Stillwell, J 2002, *Mathematics and its History*, 2nd edn, Springer, New York.

Student Research Projects in Calculus, The Mathematical Association of America, Spectrum Series.

Thomas, GB & Finney, RC 1996, *Calculus and Analytic Geometry,* 9th edn, Addison-Wesley, New York.

Tomastik, EC 2004, *Calculus Applications and Technology*, 3rd edn, Brooks Cole.

Tucker, TW (ed.) *Priming the Calculus Pump*, Mathematical Association of America.

JOURNALS AND PERIODICALS

Mathematics Teaching, Association of Teachers of Mathematics, Derby, UK.

Teaching Statistics, RSS Centre for Statistical Education, University of Nottingham, England.

The Australian Senior Mathematics Journal, Australian Association of Mathematics Teachers, Adelaide.

The Australian Association of Mathematics Teachers (AAMT) Biennial Conference Handbook provides a rich source of ideas and approaches relating to the teaching and assessment of VCE mathematics, including the use and application of technology.

The Australian Mathematical Society Gazette, AustMS, ACT, contains a range of articles and applications of mathematics relevant to senior secondary mathematics.

The Mathematical Association of Victoria (MAV) Annual Conference Handbook provides a rich source of ideas and approaches relating to the teaching and assessment of VCE mathematics, including the use and application of technology.

The Mathematics Teacher, NCTM (National Council of Teachers of Mathematics), Virginia, USA.

Vinculum, Mathematical Association of Victoria, Brunswick.

WEBSITES

A Dictionary of Units, Frank Tapson
A comprehensive summary of many of the units of measurement in use around the world today, some units of historical interest and conversions into standard SI units. It also contains links to other sites related to units and measurement.
www.ex.ac.uk/cimt/dictunit/dictunit.htm

Cut The Knot, Alex Bogomolny
An extensive mathematical glossary, items of interest, mathematical games and puzzles and is a mathematics forum.
www.cut-the-knot.org/glossary/atop.shtml

Math Archives
An archive of mathematics related websites.
http://archives.math.utk.edu/

Mathworld
A mathematics forum and encyclopaedia. provided by Wolfram Research. It contains an extensive collection of examples supported by Mathematica.
http://mathworld.wolfram.com/

School of Mathematics and Statistics at the University of St Andrew's, Scotland
Topic, context, chronology and biographical historical references, and links to other history of mathematics sites.
www-history.mcs.st-and.ac.uk/history/

The Math Forum (Swarthmore)
This site includes Dr Maths.
www.forum.swarthmore.edu

The Victorian Curriculum and Assessment Authority publishes resources and advice for all mathematics studies including past examination papers and related assessment reports. The approved technology for VCE mathematics examinations is published annually in the *VCAA Bulletin*.
www.vcaa.vic.edu.au/vce/studies/index.html#M

Wik1pedia
An online encyclopaedia with an extensive set of mathematics references.
http://en.wikipedia.org/wiki/Mathematics

Technology applications

Teachers need to check the list of approved technology, which is published annually in the *VCAA Bulletin*.

The Victorian Curriculum and Assessment Authority website section for Mathematical Methods (CAS) Units 1–4 and related materials.
www.vcaa.vic.edu.au/vce/studies/mathematics/caspilot/casindex.html

CAS-CAT project website
This research project was supported by Commonwealth Government Australian Research Council SPIRT grant funding. It was a partnership between the University of Melbourne (DSME) as the research institution and the VCAA, CASIO (SHRIRO), Hewlett-Packard and Texas Instruments as industry partners and supported the first stage of the VCAA Mathematical Methods (CAS) pilot study with three trial schools 2000 – 2002. From 2003 CAS-CAT has continued as a project of the University of Melbourne.
http://extranet.edfac.unimelb.edu.au/DSME/CAS-CAT/

Computer Algebra in Mathematics Education (CAME) website
CAME is an open, international organisation for those interested in the use of computer algebra software in mathematics education.
www.mathstore.ac.uk/came/

Casio websites for hand-held technology, including CAS: Algebra FX 2.0, ClassPad 300
www.casio.co.jp/edu_e
www.casio.edu.shriro.com.au

Hewlett-Packard website (Moravia Consulting) for hand-held technology, including CAS: HP 40G , HP 48, HP 49
www.calcsandmore.com

International Journal of Computer Algebra in Mathematics Education (IJCAME) website
www.tech.plym.ac.uk/maths/CTMHOME/ijcame.htm

Sharp website, educational section, resources for graphic and scientific calculators
http://support.sharp.net.au/drivers/calcsw.htm

Texas Instruments website for the computer based CAS Derive
http://education.ti.com/product/software/derive/features/features.html

Texas Instruments websites for hand-held technology, including CAS: T1-89, T1-92, Voyage 200
www.connecting-t3.com
www.education.ti.com/educationportal

Waterloo Maple
Maplesoft website for the computer based CAS Maple.
www.maplesoft.com/

Wolfram Research website for the computer based CAS Mathematica
www.wolfram.com/

Mathematical associations and other educational organisations

The following list of websites provides a selection of organisations which have sections related to both general issues and specific topics in VCE mathematics and mathematics education. There are other relevant sites, including school-based sites, which can be found through web searches. Teachers will need to check the current website addresses on a regular basis.

DE&T sofweb
Includes resources related to mathematics teaching and learning technologies
www.sofweb.vic.edu.au/

Educational Network of Australia
www.edna.edu.au

Mathematical Association of America
www.maa.org/welcome.html

National Council of Teachers of Mathematics (NCTM, USA)
www.nctm.org

The Australian Association of Mathematics Teachers (AAMT) provides mathematics teachers with a range of supporting text and web-based resources relevant to mathematics curricula in Australian states and territories. AAMT publishes several editions of the Australian Senior Mathematics Journal (ASMJ) annually and proceedings from its biennial conference.
The Australian Association of Mathematics Teachers (AAMT)
GPO Box 1729
Adelaide, SA 5001
Tel: (08) 83630288
Fax: (08) 83629288
Email: office@aamt.edu.au
Website: www.aamt.edu.au

The Mathematical Association of Victoria (MAV) is the Victorian mathematics teacher professional association (affiliated with the AAMT) and provides teachers with professional development, resources materials and advice on course implementation for P–12 mathematics curriculum in Victoria. The MAV publishes the mathematics 7–12 journal *Vinculum* quarterly and proceedings from its annual conference.
The Mathematical Association of Victoria (MAV)
61 Blyth Street
Brunswick, Vic 3057
Tel: (03) 9380 2399
Fax: (03) 9389 0399
Email: office@mav.vic.edu.au
Website: www.mav.vic.edu.au

The Australian Mathematical Sciences Institute is funded by the Department of Education, Science and Training and manages the International Centre of Excellence for Education in Mathematics (ICE-EM). The Centre provides professional development and resource material support for teachers, with an emphasis on mathematical content.
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c/o The University of Melbourne, Vic 3010
Tel: (03) 8344 1789
Fax: (03) 9349 4106
Email: michael@amsi.org.au
Website: www.ice-em.org.au

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