2005 Further Mathematics GA 3: Written examination 2

GENERAL COMMENTS

The number of students who presented for the Further Mathematics examination 2 in 2005 was 21815. The selection of modules chosen by the students in 2004 and 2005 is shown in the table below.

MODULE	% 2004	% 2005
1 – Number patterns and applications	47.48	49.06
2 – Geometry and trigonometry	89.84	91.08
3 – Graphs and relations	51.94	52.51
4 – Business-related mathematics	68.06	64.36
5 – Networks and decision mathematics	40.64	41.75

Within each module, questions were designed to become increasingly difficult. In some instances, students had to rely on a previously calculated result for a subsequent answer. The marking scheme was structured to accommodate this.

In general, rounding of numerical answers too early in a question remains problematic. Where a calculated number is used to perform a subsequent calculation, a rounded version of that number should **not** be used unless it is the answer to a previous question asked in the module. Only the final answer should be rounded if so directed by a question. Calculations within a question should be stored on the calculator and not rounded.

It was evident that some students either did not read questions carefully enough or had only a limited understanding of some of the concepts in the course. Some very inappropriate solutions were evident, such as using Pythagoras' theorem to calculate a straight line equation, or $A = PR^n$ calculated as $A = P \times R \times N$.

Many students provided answers without suitable working, usually in the Geometry and Business mathematics modules. Where the answer was correct full marks were awarded but, without working, incorrect answers or consequential answers did not receive any marks.

Rounding errors were penalised only once for the paper. Answers written to fewer decimal places than required were not considered rounding errors and scored zero each time. Where students engaged with a question beyond the required answer, a penalty was applied if the further working was in error (for example, an incorrect simplification).

Several instances were observed where the student had crossed out a solution without providing a replacement. Deleted work is not assessed. Students should avoid deleting work without providing an alternative solution.

Areas of strengths

Core

- labelling the dependent variable axis
- recognising that a random residual plot indicates that an attempt to linearise data has been successful
- describing the direction, form and strength of a scatterplot

Number patterns and applications

- finding t_n for an arithmetic sequence
- calculating a distance given a ratio

Geometry and trigonometry

- reading contour lines
- applying the cosine rule to find a length (although some students forgot the square root at the end)
- angles in an isosceles triangle

Graphs and relations

- substituting into linear formulas
- writing simple linear inequalities

- writing a cost equation from given information

Business-related mathematics
- using a simple interest formula
- calculating an effective interest rate

Networks and decision mathematics
- degree of a vertex and its application to an Euler circuit
- drawing a Hamiltonian path
- finding a critical path
- recognising activities that are not on the critical path

Areas of weakness
It is recommended that teachers address the following issues with students.
- Many students relied heavily on using formulas they did not understand. This applied especially in Geometry and Business mathematics.
- Many students did not seem to read the questions correctly. Students should be instructed on the effective use of reading time.
- Instructions to round answers are often ignored. Unless directed otherwise, amounts of money should be written correct to two decimal places, to represent cents.
- If an answer to a previous question had to be rounded and used in a subsequent question, then the rounded answer should be used.
- Numbers (for example, the length of an object) found during working out **within** a question must **not** be rounded when used to determine the answer to that question.
- Students often did not correctly use brackets when using their calculators.
- If working out is absent or difficult to follow, then marks cannot be awarded for applying the correct method if the answer is wrong.
- When using a TVM function, a table to show the inputs should be written as ‘working out’, including relevant negative signs and decimal places.
- Labelling and/or drawing diagrams may help students to gain marks for applying the correct method.
- Often students did not adequately answer questions that required them to ‘Show that …’

Core
- converting from a value for r^2 to a negative value of r
- explaining why a residual plot might suggest a non-linear relationship may be more appropriate
- obtaining log(3) on their calculator and applying it to a formula
- explaining the way that a log (y) or $\frac{1}{y}$ transformation may produce a more linear pattern

Number patterns and applications
- finding a sum of terms between t_3 and t_{12} inclusive
- recognising when a question requires the sum of a geometric sequence rather than a term of that sequence
- understanding that the specification of a difference equation of t_{n+1} in terms of t_n requires a stated value for t_1

Geometry and trigonometry
- scales of maps
- the slope of a line
- determining bearings
- calculating the volume of a triangular prism

Graphs and relations
- finding a feasible region changed by an additional constraint
- understanding that there may be many solutions for a minimum cost in a linear programming question

Business-related mathematics
- understanding what an effective interest rate represents
- finding the annual reducing balance depreciation rate

- understanding the role of the variables A, P, R and n in the equation $A = PR^n$
- understanding the annuities formula, as may be applied to regular payments in an investment
- understanding that regular additions to an investment also earn interest

Networks and decision mathematics
- altering a network diagram to account for changed conditions

SPECIFIC INFORMATION

Question 1

1a.

Marks	0	1	Average
%	24	76	**0.8**

value

1b–d.

Marks	0	1	2	3	4	5	6	Average
%	18	13	12	12	14	17	15	**3.0**

1bi.
–0.952

$\sqrt{0.9058} = 0.9517$ was a frequent incorrect answer; however, the negative sign was necessary.

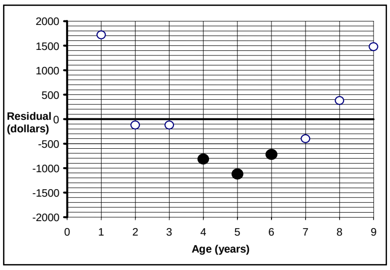
1bii.
91%, 90.6% and 90.58% were all accepted

Common incorrect answers quoted the value of r as 95.2%.

1c.
17 500, –1200

Many students reversed these numbers. These responses scored a maximum of one mark so long as the negative sign was present in –1200.

1d.



The first mark was given for three points that were on the lines for *Age* = 4, 5 and 6 and were all below the horizontal axis. The second mark was given if the outer two points were within *Residual* = –500 and –1000 and the middle point was within *Residual* = –1000 and –1500.

1e–f.

Marks	0	1	2	Average
%	36	37	26	**0.9**

1e.
Yes: a non-linear model is suggested by the clear pattern of the points.

The explanation had to mention that a pattern was evident.

1f.
0.95

Common incorrect answers were 0.91 and 0.92, which were apparently achieved from looking at some sequence obtained from differences along the log(*age*) row of figures.

1g–h.

Marks	0	1	2	3	4	Average
%	11	8	14	27	39	2.8

1g.
negative, linear and strong

1h.
$13 100

A common error was for students to disregard the *log* part of the equation and calculate
$18\,300 - 10\,800 \times 3 = -14\,100$.

1i–j.

Marks	0	1	2	Average
%	60	32	8	0.5

1i.
the random placement of dots

Several incorrect answers referred only to the points being 'scattered'. Others mentioned that 'dots were on both sides of the line' or 'the horizontal line is in the middle of the plot'.

1j.
Either $\frac{1}{y}$ or log (y) (only one was needed). The points closer to the vertical (value) axis need to be compressed more toward the horizontal axis to linearise the data.

Many students answered this poorly. Transformation involving x was common. Vague references to 'straightening the points' gave no indication that the students understood which points were affected the most and in which direction. An explanation such as 'you always do this transformation to a graph shaped this way' was insufficient.

Module 1 – Number patterns and applications

Question 1
1a–d.

Marks	0	1	2	3	4	5	Average
%	3	7	23	15	30	22	3.3

1a.
5.4 km

1b.
Day 11

1c.
61.5 km

Some students listed, then added, all the terms from t_3 to t_{12}, and received full marks. Others stopped at t_{11} and did not gain full marks.

A number of students understood that $S_{12} - S_2$ was required, but many incorrectly said this was equal to S_9 or S_{10}.

1d.
3.9

A common incorrect answer was 4.2.

Question 2
2a–c.

Marks	**0**	**1**	**2**	**3**	**4**	**Average**
%	34	10	22	8	26	**1.8**

2a.
$3.5 - 3 \neq 4.05 - 3.5$ therefore the sequence is not arithmetic; and $\frac{3.5}{3} \neq \frac{4.05}{3.5}$, therefore the sequence is not geometric.

Calculations were expected and students were required to explain what they had found. After their calculations, many did not state that 'the differences are not the same and therefore this is not an arithmetic sequence'. A similar analysis was needed to explain why it was not a geometric sequence. Without such explanations, it was not clear whether the student was able to identify the correct reasoning for their conclusions.

2b.
0.2

2c.
5.32 km

Question 3
3a–c.

Marks	**0**	**1**	**2**	**3**	**4**	**Average**
%	18	17	20	25	20	**2.1**

3a.
10.5

The most common incorrect answer was 10.5125, which is the mean of t_1 and t_3.

3b.
196 km

The most common incorrect answer given was the value for t_{14} rather than S_{14}, with $r = 1.05$.

3ci.
4 km

Dividing 10 km into a ratio of 2:3 appeared to present problems for a number of students.

3cii.
1:3

This part of the question was very poorly done. Many students tried to do a calculation involving the 10 km. Of those who simply worked with the 25%, many came to the incorrect answer of 1:4.

A fractional answer of $\frac{1}{3}$ is not the same as a ratio of 1:3.

3d.

Marks	0	1	2	Average
%	73	22	5	**0.3**

3di.

$t_n = 0.9\, t_{n-1} + 1.2,\ t_1 = 5$

The value for $t_1 = 5$ is an essential requirement for a difference equation written in terms of t_{n-1}. Most students omitted this and could not score any marks.

An acceptable alternative was $t_n = 5\times 0.9^{n-1} + 1.2\times\frac{\left(1-0.9^{n-1}\right)}{1-0.9}$. This form does not require $t_1 = 5$ as the first term is implied by the formula.

3dii.

As the sequence progresses, the value of t_n gradually increases. This increase gets smaller as n gets larger, which shows that the sequence is converging or, getting closer to, a maximum value. A graph suggests that this maximum is 12.

If we let $t_n = 12$, then $t_{n+1} = 0.9 \times t_n + 1.2$
$= 10.8 + 1.2$
$= 12$

The initial loss of 10% of the previous day's travel is exactly made up for by the addition of 1.2.

Students had to provide an explanation along these lines but many simply said that 'going 90% of the previous day's travel must mean the distance is getting less'.

A good mathematical answer seen was:

$$t_n = 5\times 0.9^{n-1} + 1.2\times\frac{\left(1-0.9^{n-1}\right)}{1-0.9}$$
$$= 5\times 0.9^{n-1} + 12\times\left(1-0.9^{n-1}\right)$$
$$=12 - 7\times 0.9^{n-1}$$

but, $7 \times 0.9^{n-1} > 0$

therefore, $t_n = 12 - 7 \times 0.9^{n-1} < 12$ always.

Module 2 – Geometry and trigonometry

Question 1

1a–d.

Marks	0	1	2	3	4	Average
%	15	34	21	16	13	**1.8**

1a.

50 m

A common incorrect answer was 100 m.

1b.

510 m

Many students did not draw a diagram. Others misread the question and used the wrong points.

1c.

1250 m

Many students did not understand that the slope of 0.12 was the gradient. Many used it as the length of a hypotenuse, not realising that a hypotenuse must be the largest side in a right triangle. Others incorrectly used 0.12 as the angle.

1d.
5 cm

Surprisingly, this question proved difficult for most students. Common incorrect answers were 0.5 cm or 0.05 cm, while others did not see any problem with unreasonable answers of up to 8 000 000 000 cm on a map.

Question 2
2a–c.

Marks	0	1	2	3	4	5	Average
%	34	16	13	13	8	16	**1.9**

2ai.
1.6 km

A common incorrect answer was 2.1 km (the length of *GX*).

2aii.
1.2 km

The rounded answer to Question 2ai. needed to be used in the calculation here. If that answer was wrong, marks were still available here if the correct calculation was applied.

The question involved two steps. Many students rounded off their answer to the first step and used this in the second step. Exact numbers should be retained in the calculator and used in the second step **unless** the first step was the answer to a specific question.

Several students applied Pythagoras' theorem to the non-right angled triangle *GXY* to obtain 2.7 km. Others assumed that *X* was twice as far south of *G* as *Y*.

2b.
115°

Common errors included answers of 75°, 150° and 215°.

2c.
3.2 km

The rounded answer to Question 2b. was required here. If that answer was wrong, a consequential mark was still available here for correct use of the cosine rule.

Many students who used the cosine rule forgot to take the square root at the end. Pythagoras' theorem was also incorrectly used here by some students.

Questions 2d–3a.

Marks	0	1	2	3	4	Average
%	40	29	15	8	8	**1.1**

2d.
111° T

This question was quite poorly done, and, as working out was often not clear, students were sometimes unable to even earn one mark for method. Bearings continued to be poorly understood by many students.

3a.
3.2 m

A method mark was awarded for showing the correct location of either 25° or 65° on a diagram.

Common incorrect answers included those from students who had found no angle or who got lost in the two steps required here. Unreasonable answers included values of up to 219 metres and the use of the formula for the area of a triangle.

3b.

Marks	0	1	2	Average
%	96	2	1	**0.1**

$\frac{4}{25}$

This question was poorly answered by most students. The common misconception was that the volume fraction was the cube of a length fraction and many students obtained the incorrect answer of $\left(\frac{2}{5}\right)^3 = \frac{8}{125}$. This would apply if the lengths in all three dimensions had been enlarged; however, only the height and width of the two end triangles were scaled up.

The lengths of the triangular prisms were equal and the fraction for volume is the same as the fraction for end area: $\left(\frac{2}{5}\right)^2 = \frac{4}{25}$.

Module 3 – Graphs and relations

Question 1

1a–b.

Marks	0	1	2	Average
%	7	24	69	**1.6**

1a.
Line A

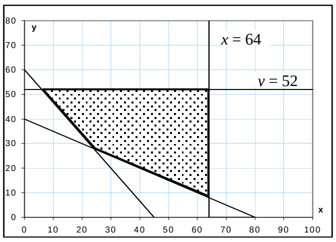
1b.
(24, 28)

1c–d.

Marks	0	1	2	3	4	Average
%	32	11	4	11	41	**2.1**

1c.
$x \le 64$ and $y \le 52$

1d.



The question required the feasible region to be shaded. Some texts leave the feasible region unshaded. If students prefer to use this convention, they are expected to provide a legend to specify the feasible region.

1e–g.

Marks	0	1	2	3	4	Average
%	18	16	22	19	24	**2.1**

1e.
$C = 100x + 70y$

1f.
$4240

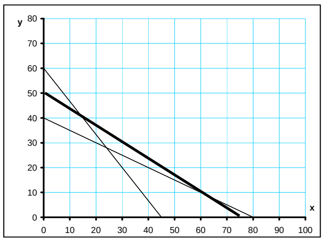
The answer comes from the point (6, 52). A mark was available for the correct substitution of one appropriate point into the formula from Question 1e. Marks were available for the correct use of the feasible region from Question 1d. and the equation from Question 1e., even if these were incorrect.

1g.
$2x + 3y \geq 150$

1h–i.

Marks	0	1	2	Average
%	32	28	39	**1.1**

1h.



1i.
No: he will sell 2 × 25 + 3 × 32 = 146 books, which is fewer than the required 150 books.

A number of students answered this question with an incorrect use of the inequality sign. Their calculation showed that 146 books would be sold and they then wrote that 'since 146 > 150, he will not sell enough books'.

1j.

Marks	0	1	2	3	Average
%	73	18	9	1	**0.4**

1ji.
$5400

The point (24, 28) does not belong to the feasible region and could not be part of an answer here. It gave the common incorrect answer of $5250.

1jii.
All points on the line $20x + 15y = 900$ between (0, 60) and (15, 40) inclusive.

The minimum cost could be found from the points (0, 60), (6, 52) or from (15, 40). An answer that included any two of these points earned one mark. The second mark came from explaining that all the points along the line segment between (0, 60) and (15, 40) were solutions. Very few students realised that all these points were solutions.

Module 4 – Business-related mathematics

In general, this module was not well done by students. Many students seemed to have a limited understanding of important financial concepts at this level and often provided unreasonable answers or used inappropriate formulas. Many seemed to rely on substitution into formulas they had been given in class but often did not know what the elements of the formula really represented.

Question 1
1a–d.

Marks	0	1	2	3	4	Average
%	33	27	17	15	8	**1.4**

1a.
$560

This question was poorly answered. Students were required to analyse the information given in this relatively straight-forward scenario. Many students did not do this. 4560 – (4000 + 500) = $60 was a very common incorrect answer.

1b.

$$R = \frac{100 \times 560}{3500 \times 2} = 8$$

1c.

15.4%

1d.

An effective interest rate is different to a flat (simple) interest rate as a flat rate does not take into account reductions in the balance owing after repayments are made.

Most students were unable to adequately explain the difference. Examples of unacceptable answers included 'Flat rate can't change. Effective interest can change its value,' 'Effective interest rate is what is really being paid,' and 'Effective rate is always double whereas the flat rate isn't.'

Question 2

2a–b.

Marks	0	1	2	3	4	Average
%	45	13	23	8	12	**1.3**

2a.

15%

The annual percentage flat rate of depreciation was expected here. However, an answer of $600 per annum is also a rate of depreciation and was also accepted.

A method mark was awarded for $\frac{4000 - 1000}{5} = 600$.

2b.

24.2%

A method mark was awarded for $4000 \times \left(1 - \frac{y}{100}\right)^5 = 1000$, but many students then incorrectly solved this equation.

Question 3

3a–b.

Marks	0	1	2	3	4	Average
%	31	27	18	16	8	**1.4**

3ai.

P = 10 000, R = 1.004, n = 60

Common incorrect values for R were 1.048, 1.48 and 4.8. Many students confused r with $R = 1 + \frac{r}{100}$. The incorrect value of n = 5 was very common.

3aii.

$12 706.41 or $12 706.40

The formula $A = PR^n$ was given, but some students used the simple interest rate formula instead. A mark was available here for a reasonable, correct answer using the student's incorrect answer to Question 3ai.

Several students gave an answer of $25 480 396.50 without commenting on its validity.

Money answers should, unless otherwise stated, be written correct to the nearest cent. It was expected that $12 706.40 would not be written to only one decimal place.

3b.
$11 848.58

This question presented a challenge to many students. They apparently did not have a full understanding of the components of the annuities formula or the TVM function on their calculator, which permits a loan or an investment to be calculated in similar ways.

An investment and a loan are two sides of the same coin. For instance, a loan of money from a bank is an investment of the money by the bank. In this question, the investment is equivalent to a loan of $4000 and a regular further borrowing (instead of repayment) of $100 each month.

The question can be readily calculated with the TVM function, which is the most appropriate way to approach this one-mark question using the following data. The answer is shown at FV.
$N = 60$
$I = 4.8$
$PV = 4000$
$PMT = 100$
$FV = -11848.58$
$P/Y = 12$

Alternatively, the annuities formula could be applied. The usual way of writing this formula is $A = PR^n - \frac{Q(R^n - 1)}{R - 1}$.

For a loan, this formula shows the principal amount through interest in the first term (PR^n), and the subtracted fraction $\frac{Q(R^n - 1)}{R - 1}$ indicates the repayments. The minus sign indicates that this portion is reducing the amount still owing.

For an investment, the first term (PR^n) represents the growth in the principal invested. If there were regular withdrawals from the investment, then the second term would again be subtracted. However, in this question, there are additional payments (Q) being made, and so the second term $\frac{Q(R^n - 1)}{R - 1}$ must be added.

The appropriate formula to use in this question would be $A = PR^n - \frac{(-Q)(R^n - 1)}{R - 1}$, where A = the value after n additional and regular deposits of $$Q$ per deposit at an interest rate of r% per annum and $R = 1 + \frac{r}{1200}$ for this case of monthly deposits.

Many students tried to use the formula $A = PR^n$, using $P = 4000$, $R = 1.004$ and $n = 60$ and then simply adding 60×100 to account for the monthly deposits. This does not take into account that each additional deposit also earns interest throughout the term of the investment.

3c.

Marks	**0**	**1**	**2**	**3**	**Average**
%	94	3	1	2	**0.1**

3ci.
$129.80

Regardless of prior answers, full, consequential marks were available to all students for Questions 3ci. and 3cii., but many students did not seem to understand the requirements of these questions.

The value of the investment in Question 3b. after two years had to be found first. The correct value for this was \$6915.90. The TVM function could then be used with the following data to calculate the new monthly deposits to achieve a \$13 000 value after an additional three years.
$N = 36$
$I = 4.8$
$PV = 6915.90$
$PMT = 129.80$
$FV = –13\ 000$
$P/Y = 12$

3cii.
\$1927.20

A mark was available here for students who had calculated the total deposits made in the investment for the three year period using an incorrect answer from Question 3ci.

Total deposits = $4000 + 24 \times 100 + 36 \times 129.80$ = \$11 072.80

A mark was then available for finding the final answer by subtracting the total deposits from the \$13 000 value at the end of five years.

Module 5 – Networks

Question 1
1a–b.

Marks	0	1	2	Average
%	8	40	53	1.5

1a.
5

1b.
30 km

The correct path was *B–C–D–C–W*. There was no restriction about passing through *C* twice. Common incorrect answers included 38, for the path *B–D–C–W*, and 41 for *B–D–W*.

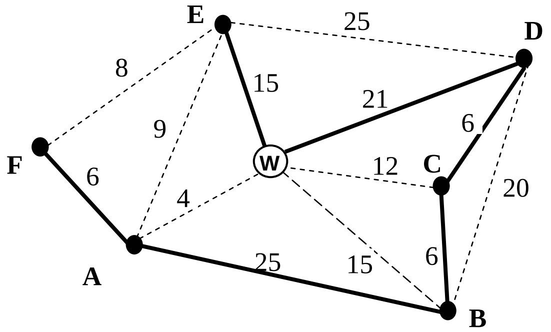
1c–d.

Marks	0	1	2	3	4	Average
%	9	16	23	27	25	2.4

1ci.
Hamiltonian path

Incorrect answers included Hamiltonian circuit and Euler path.

1cii.



1di.
Not all vertices are of even degree.

1dii.
Vertex *C*

Question 2
2a–b.

Marks	0	1	2	3	Average
%	17	22	18	43	**1.9**

2a.
D, *F*, *G* and 50

Common errors included not giving all three predecessors for *H*, and giving *EST* = 25.

2b.
125 minutes

2c–d.

Marks	0	1	2	3	4	5	6	Average
%	28	18	15	15	13	8	3	**2.0**

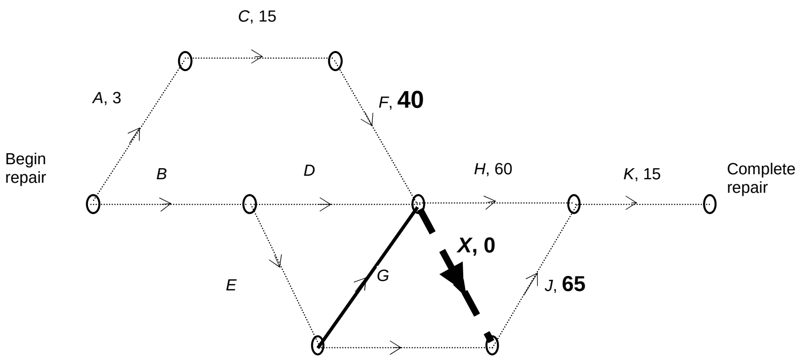
2ci.
Activity *F* is not on the critical path.

2cii.
32 minutes

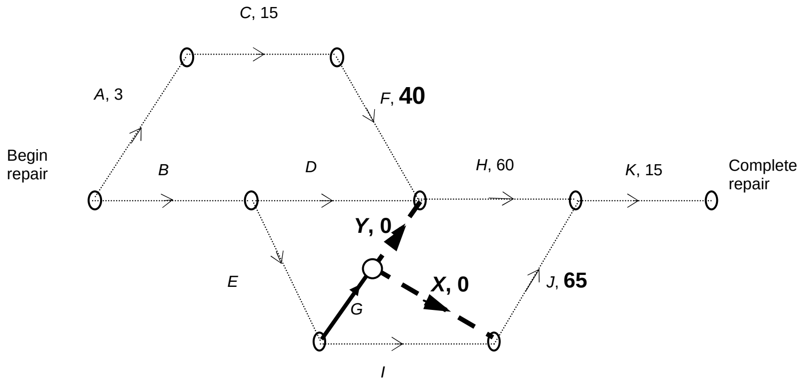
The mark was awarded if the student wrote 'the duration for *F* can be increased by 12 minutes'. A common incorrect answer was 12 minutes, without reference to float or slack time or a relevant description of this.

2di.
Two alternatives were accepted which showed an activity of zero duration. Such a line must have had an arrow to indicate direction and a duration of zero stated. A label of 'dummy' or a dotted line was assumed to indicate zero duration, but an arrow was still necessary. Many students did not draw any line on the network diagram.

The intent of the question was for the diagram to be altered as shown below, and most students drew a dummy activity in this location. This diagram was accepted so long as the line had an arrow and zero duration indicated. However, this diagram implies that *F* and *D* have become predecessors for *J*, which was not a requirement actually stated in the question.



Another appropriate diagram for the stated conditions is shown below. This diagram shows that *J* must wait for both *G* and *I* to finish before starting. Importantly, it indicates that *G* is still also a predecessor of *H* and that *D* and *F* are not predecessors of *J*. However, not many students answered this way.

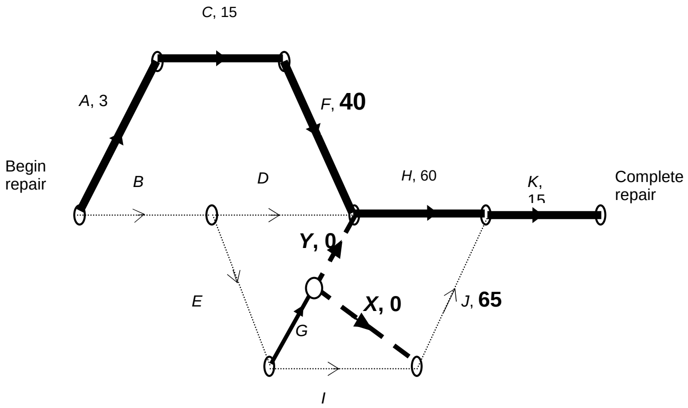


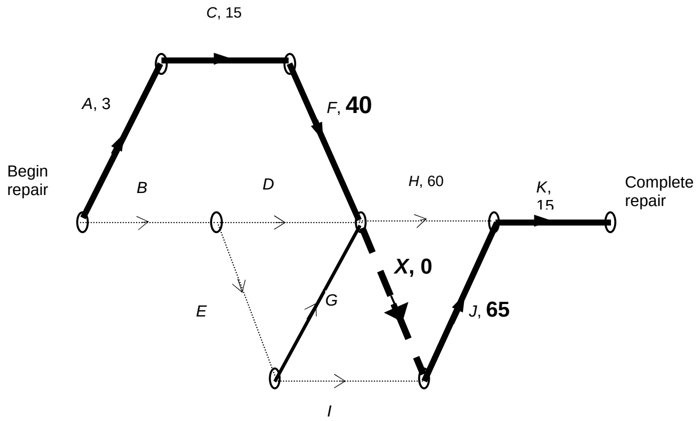
2dii.
58 minutes (63 minutes was also accepted).

A common incorrect answer was 65 minutes.

The correct critical path for this question is shown in bold in the first diagram below. This path gives the LST for *H* = 58 minutes.

The alternative answer of LST for *H* = 63 minutes comes from the second diagram below and was also accepted.





2diii.
133 minutes (138 minutes could also have been accepted).

An answer of 133 minutes (from the first diagram) was accepted here only if the answer to Question 2dii. did not come from the second diagram. Alternatively, an answer of 138 minutes (from the second diagram) was accepted here only if answer Question 2dii. did not come from the first diagram.

2div.
Activity *D*

This answer was unaffected by the choice of either diagram.